DAA - Analytical Assignment - 1

NAME : ABIRAMY. K. J
REG NO : 192311016
CODE : CSA0669
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1. Solve the following recurrence relations.

a) $x(n) = x(n-1) + 5$ for $n > 1$, $x(1) = 0$

Solution:

$x(n) = x(n-1) + 5 \rightarrow (1)$

$x(n-1) = x(n-1-1) + 5$

$x(n-1) = x(n-2) + 5 \rightarrow (2)$

$x(n-2) = x(n-2-1) + 5$

$x(n-2) = x(n-3) + 5 \rightarrow (3)$

Sub eq (3) in (2);

$x(n-1) = x(n-3) + 5 + 5$

$x(n-1) = x(n-3) + 10 \rightarrow (4)$

Sub eq (4) in (1);

$x(n) = x(n-3) + 10 + 5$

$x(n) = x(n-3) + 15 \rightarrow (5)$

for some k,

$x(n) = x(n-k) + 5k \rightarrow (6)$

$n - k = 1$, $n - 1 = k$

Eq (6) $\rightarrow x(n) = x(1) + 5(n-1) = 0 + 5n + 5$

$x(n) = 5n - 5$

$\therefore$ Time complexity = $O(n)$

3) $x(n) = 3x(n-1)$ for $n > 1$, $x(1) = 4$

Solution:

$x(n) = 3x(n-1) \rightarrow (1)$

$x(n-1) = 3x(n-1-1) = 3x(n-2) \rightarrow (2)$

$x(n-2) = 3x(n-3) \rightarrow (3)$

Sub eq (3) in (2),

$x(n-1) = 3[3x(n-3)]$

$x(n-1) = 9x(n-3) \rightarrow (4)$

Sub eq (4) in (1),

$x(n) = 3[9x(n-3)]$

$x(n) = 27x(n-3)$.

At some k,

$x(n) = 3^k x(n-k) \rightarrow (5)$

$n-k = 1$

$\Rightarrow k = n-1$

Eq (5) $\Rightarrow x(n) = 3^{n-1} x(1)$

$= 3^{n-1} \cdot 4 = 3^n \cdot 3^{-1} \cdot 4$

$= 3^n$

Time complexity $= O(3^n)$

$\hookrightarrow$ $x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$

(solve for $n = 2k$)

Solution:

$x(n) = x(n/2) + c \rightarrow$ ①

$x(n/2) = x(n/4) + c \rightarrow$ ②

$x(n/4) = x(n/8) + c \rightarrow$ ③

sub ② in ①,

$x(n) = x(n/4) + c + c$

$x(n) = x(n/4) + 2c \rightarrow$ ④

$= x(n/2^2) + 2c$

sub ③ in ④

$x(n) = x(n/8) + c + 2c$

$x(n) = x(n/2^3) + 3c$

$\vdots$

$x(n) = x(n/2^k) + kc \rightarrow$

$n/2^k = 1$

$n = 2^k$

$\log n = k \log 2$

$\Rightarrow \boxed{k = \log n}$

$n = 2k$

$\therefore \boxed{k = n/2}$

$\boxed{n = 2k}$; $\boxed{x(1) = 1}$

$x(n) = x(\cancel{n}/\cancel{n}) + kc$

$x(n) = 1 + kc$

$x(n) = 1 + \log n \cdot c$

Time complexity $= O(\log n)$

d) $x(n) = x(n/3) + 1$ for $n > 1$ $x(1) = 1$

(solve for $n = 3^k$).

Solution:

$x(n) = x(n/3) + 1 \rightarrow ①$

$x(n/3) = x(n/9) + 1 \rightarrow ②$

$x(n/9) = x\left(\frac{n}{27}\right) + 1 \rightarrow ③$

Sub ② in ①,

$x(n) = x(n/9) + 2 \rightarrow ④$

$= x(n/3^2) + 2$

Sub ③ in ④,

$x(n) = x(n/27) + 3 \rightarrow ⑤$

$= x(n/3^3) + 3$

$x(n) = x(n/3^k) + k$

$n/3^k = 1$

$\boxed{n = 3^k}$

$\log n = \log 3^k$

$\therefore \boxed{k = \log n}$

$x(n) = x(n/3^k) + k$

$= x(\cancel{n}/\cancel{n}) + k$

$= x(1) + k$

$= 1 + k$

$x(n) = \log n$

$\therefore$ Time complexity $= O(\log n)$

2. Evaluate following recurrence completely.

i) $T(n) = T(n/2) + 1$ where $n = 2^k$ for all $k \geq 0$.

Given,

$$T(n) = T(n/2 + 1), n = 2^k$$

Substitute $n = 2^k$

$$T(2^k) = T\left(\frac{2^k}{2}\right) + 1 = T(2^{k-1}) + 1$$

Now,

$$T(2^{k-1}) = T\left(\frac{2^{k-1}}{2}\right) + 1 = T(2^{k-2}) + 1$$

$$T(2^{k-2}) = T(2^{k-2}/2) + 1 = T(2^{k-3}) + 1$$

$$T(2^1) = T(2^0) + 1$$

$n = 2^k \Rightarrow k = \log_2 n$

$$T(2^k) = T(2^{k-1}) + 1 = T(2^{k-2}) + 1 + 1 = \cdots = T(2^0) + k.$$

Since,

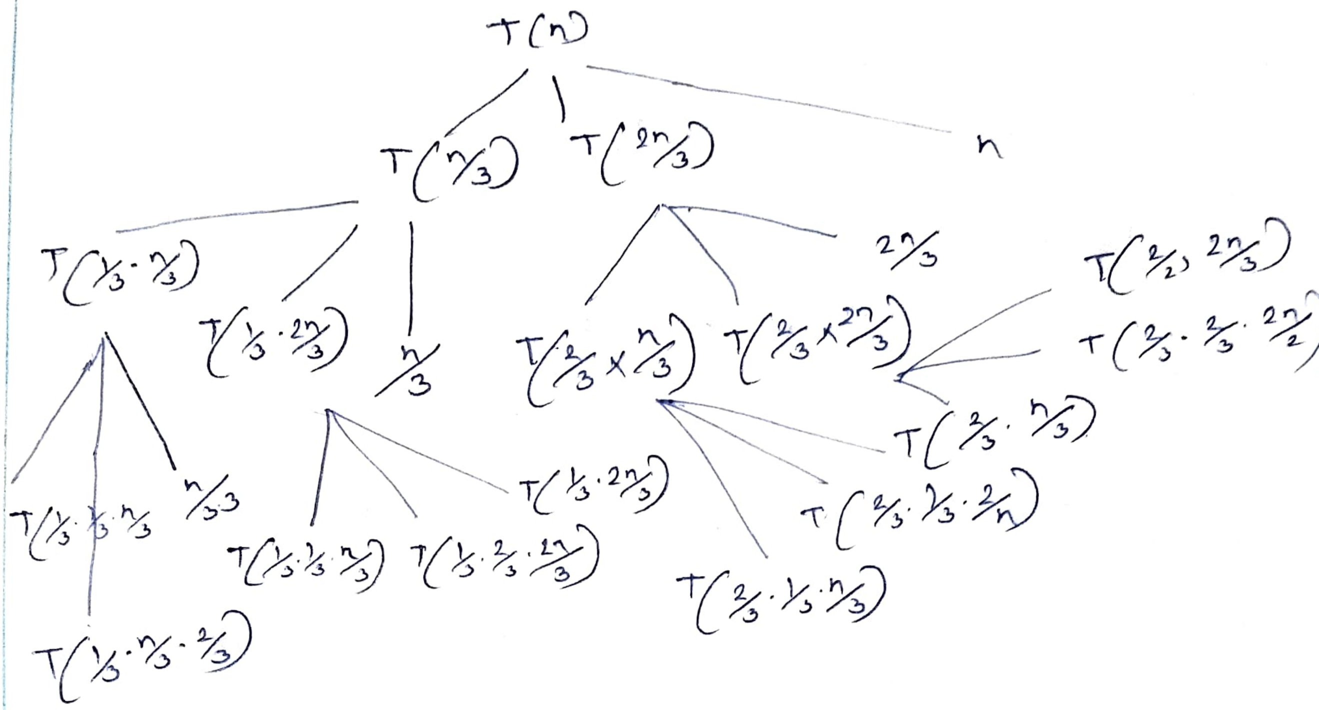
$2^0 = 1$, $T(2^0) = T(1)$

$T(2^k) = 1 + k$

$$\boxed{T(n) = 1 + \log_2 n}$$

Time complexity $\} = O(\log n)$

ii) $T(n) = T(n/3) + T(2n/3) + cn.$



assume, $\frac{2^k}{3^k} n = 1$

$$\Rightarrow \frac{n}{\left(\frac{3}{2}\right)^k} = 1$$

$$n = \left(\frac{3}{2}\right)^k$$

$$\log n = k \log\left(\frac{3}{2}\right)$$

$$k = \frac{\log n}{\log \frac{3}{2}}$$

$$k = \log_{3/2} n$$

$$\boxed{k = \log_{3/2} n}$$

Time complexity $= O(nk)$

$\therefore$ Time complexity $= O(n \log_{3/2} n)$

3. Consider following algorithm.

min 1 (A[0.....n-1])

if n=1 return A[0]

Else temp = Min 1 [A [0....n-2]]

if temp <= A [n-1] return temp

Else

Return A [n-1]

a) what does this algorithm compute?

b) Setup a recurrence relation for the algorithms basic operation count and solve it.

a) This algorithm computes minimum element in an array A of size n.

If i < n, A[i] is smaller than all elements, then A[j], j = i+1 to n-1, then it returns A[i]. It also returns the leftmost minimal element.

b) $T(n) = T(n-1) + 1$, when $n > 1$ (one comparison at every step except, $n=1$)

$T(1) = 0$ (No compare $n = 1$).

$T(n) = T(1) + (n-1) * 1$

$= 0 + (n-1)$

$= n-1$

$\therefore$ Time complexity $= O(n)$.

4. Analyze order of growth.

(i) $F(n) = 2n^2 + 5$ and $g(n) = 7n$.

use the $\Omega(g(n))$ notation.

Given,

$F(n) = 2n^2+5$

$c \cdot g(n) = 7n$

$$\boxed{F(n) \geq c \cdot g(n)}$$

$n=1$

$F(1) = 2(1)^2+5 = 7$

$g(1) = 7$

$n=1, \; 7=7$

$n=2, \; 13=14$

$n=3, \; 23=21$

$n \geq 3, \; F(n) \geq g(n) \cdot c$.

$\boxed{n=2}$

$F(2) = 2(2)^2+5$

$= 8+5 = 13$

$g(2) = 7 \times 2 = 14$

$\boxed{n=3}$

$F(3) = 2(3)^2+5 = 18+5 = 23$

$g(3) = 21$.

$F(n) \geq c \cdot g(n)$ when, n value is $\geq$ to 3.

$\therefore \; F(n) = \Omega(g(n))$.

$\Rightarrow$ $F(n)$ is more than $g(n)$ from asymptotically.