Analytical Assignment - 3

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1. Calculate the No. of ways to achieve a sum of 15 when rolling four six-sided dice. Provide a detailed step-by-step solution.

No. of Solutions:

$x_1 + x_2 + x_3 + x_4 = 15$ where $1 \le x_i \le 6$ into $0 \le y_i \le 5$.

This becomes.

$(y_1+1) + (y_2+1) + (y_3+1) + (y_4+1) = 15$

$y_1 + y_2 + y_3 + y_4 = 11$

using Inclusion - Exclusion Principle "stars & bars"

$$\binom{11+4-1}{4-1} = \binom{14}{3}$$

$$\binom{14}{3} = \frac{14 \times 13 \times 12}{3 \times 2 \times 1} = 364$$

Set $y_1' = y_1 - 6$ then

$y_1' + y_2 + y_3 + y_4 = 5$

$$\binom{5+4-1}{4-1} = \binom{8}{3}$$

$$\binom{8}{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

Since any of four variables $= 4 \times 56 = 224$

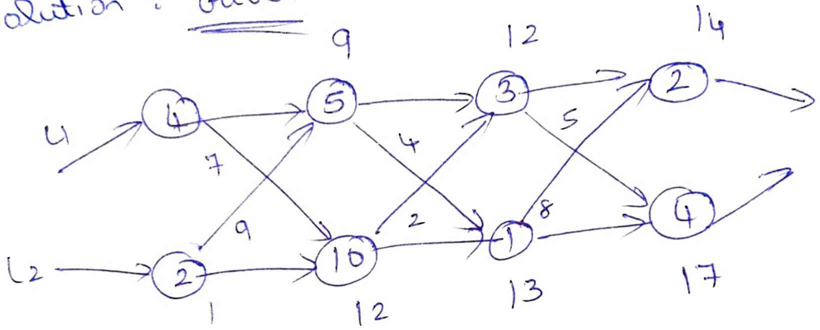
set $y_1' = y_1 - 6$ and $y_2' = y_2 - 6$ then

$y_1' + y_2' + y_3 + y_4 = -1$

$\therefore$ the no. of valid solutions is: $364 - 225 = 140$

2. Two assembly lines have station times as follows. line 1 : [4, 5, 3, 2], line 2 : [2, 10, 1, 4], L1 to L2 : [7 4, 5] & L2 to L1 : [9, 2, 8]. Calculate min time to assemble a product.

Solution : Given



	1	2	3	4
$F_1[j]$	4	9	12	14
$F_2[j]$	2	12	13	17

	1	2	3	4
$L_1[j]$	1	1	1	1
$L_2[j]$	2	2	2	2

$$F_1[j] = \min\{(f_1(j-1) + a_{1j}), (f_2(j-1) + (t_2, j-1) + a_{1j})\} = \min\{9, 12\} = 9$$

$$F_2[j] = \min\{(f_1(j-1) + (t_1, j-1) + a_{2j}), (f_2(j-1) + (a_{2j})\} = \min\{21, 12\} = 12$$

3. Given keys {10, 20, 30, 40}) {10, 20, 30, 40}) {10, 20, 30, 40} with access probabilities {0.1, 0.2, 0.4, 0.3} | {0.1, 0.2, 0.4, 0.3} respectively, construct optimal BST. Calculate Total cost of tree.

Sol:

Given keys with access probabilities:

{10, 20, 30, 40}

{0.1, 0.2, 0.4, 0.3}

$j - i = 1$

$1 - 0 = 1$ (0,1) (1,1)

$2 - 1 = 1$ (1,2) (2,2)

$3 - 2 = 1$ (2,3) (3,3)

$4 - 3 = 1$ (3,4) (4,4)

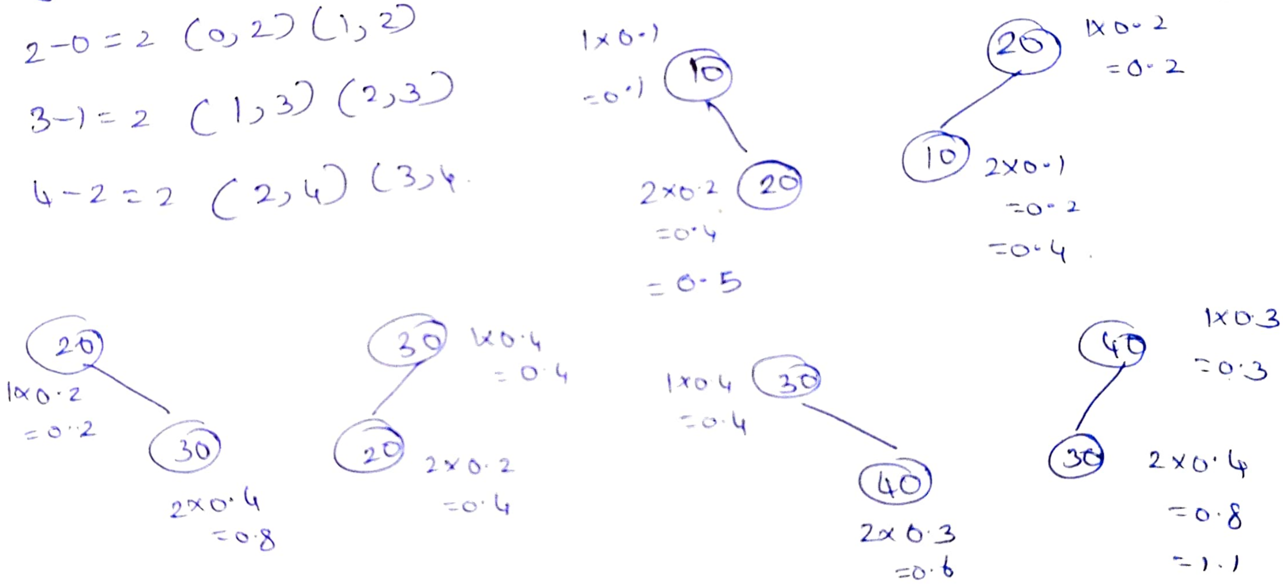
	0	1	2	3	4
0	0	0.1	[9] 0.4	[3] 1.1	[3] 1.7
1		0	0.2	[3] 0.8	[2] 1.4
2			0	0.4	[3] 1.0
3				0	0.3
4					0

$j - i = 2$

$2 - 0 = 2$ (0,2) (1,2)

$3 - 1 = 2$ (1,3) (2,3)

$4 - 2 = 2$ (2,4) (3,4)



$j - i = 3$

$3 - 0 = 3$ (0,3) (1,3)

$4 - 1 = 3$ (1,4) (2,4)

$$\text{cost}(i, j) = \min\{\text{cost}(i, n-1) + \text{cost}(k, j)\} + w_i$$

$$\text{cost}(0,3) = \min_{k=1,2,3} \left\{ \begin{array}{l} \text{cost}(0,0) + \text{cost}(1,3) \\ \text{cost}(0,1) + \text{cost}(2,3) \\ \text{cost}(0,2) + \text{cost}(3,3) \end{array} \right\} + 0.7$$

$$= \min \begin{Bmatrix} 0+0.8 \\ 0.1+0.4 \\ 0.4+0 \end{Bmatrix} + 0.7$$

$$= \min \begin{Bmatrix} 1.5 \\ 1.2 \\ 1.1 \end{Bmatrix} = 1.1$$

$$cost(1,4) = \min_{k=2,3,4} \begin{Bmatrix} cost(1,1) + cost(2,4) \\ cost(1,2) + cost(3,4) \\ cost(1,3) + cost(4,4) \end{Bmatrix} + 0.9$$

$$= \min \begin{Bmatrix} 0+1.0 \\ 0.2+0.3 \\ 0.8+0 \end{Bmatrix} + 0.9$$

$$= \min \begin{Bmatrix} 1.9 \\ 1.4 \\ 1.7 \end{Bmatrix} = 1.4$$

$$cost(0,4) = \min \begin{Bmatrix} cost(0,0) + cost(1,4) \\ cost(0,1) + cost(2,4) \\ cost(0,2) + cost(3,4) \\ cost(0,3) + cost(4,4) \end{Bmatrix} + 1.0$$

$$= \min \begin{Bmatrix} 0+1.4 \\ 0.1+1.0 \\ 0.4+0.3 \\ 1.1+0 \end{Bmatrix} + 1.0 = \min \begin{Bmatrix} 2.4 \\ 2.1 \\ 1.7 \\ 2.1 \end{Bmatrix} = 1.7$$

4. Solve the TSP for the following 5-city distance using dp.

A : [0, 29, 20, 21, 17]

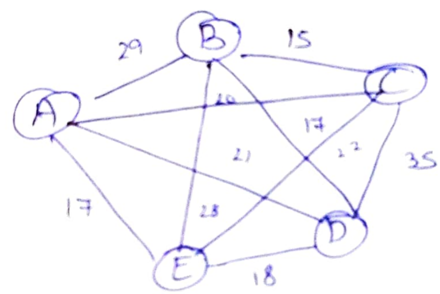
B : [29, 0, 15, 17, 28]

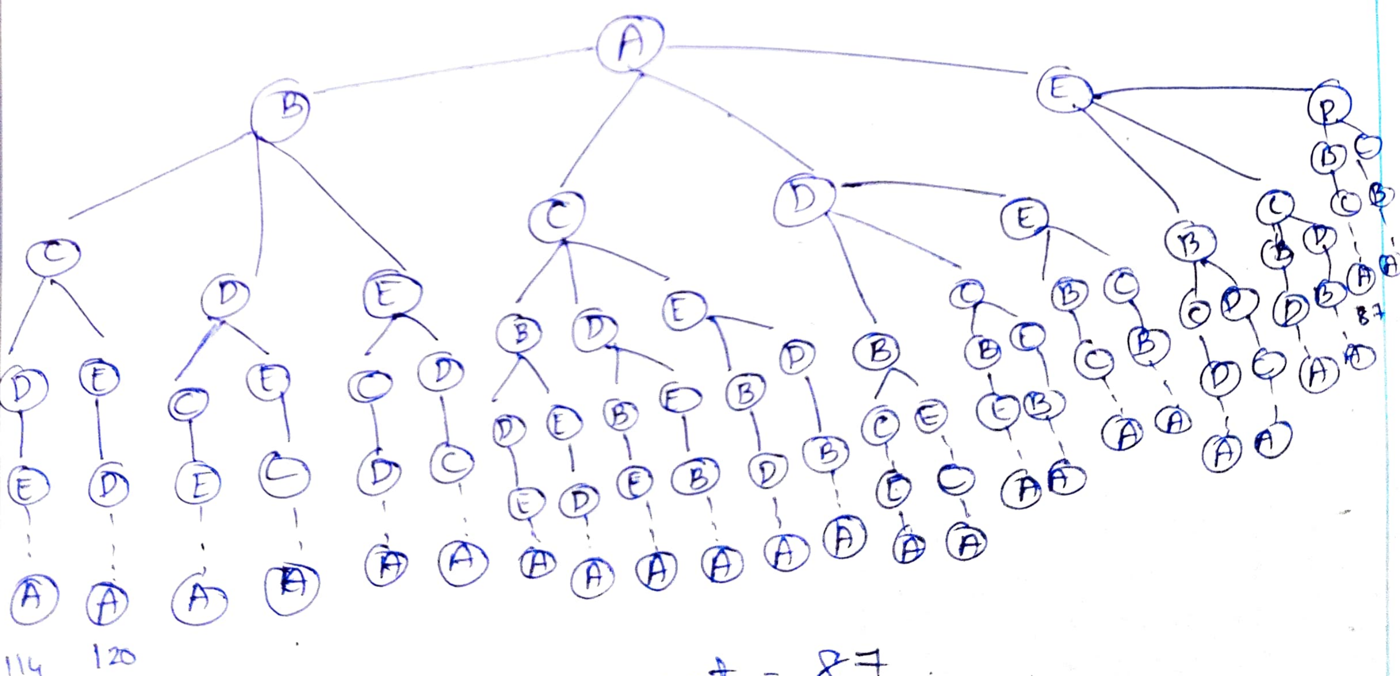
C : [20, 15, 0, 35, 22]

D : [21, 17, 35, 0, 18]

E : [17, 28, 22, 18, 0].

Sol:





∴ The minimum cost = 87.

A — E — D — B — C — A

5. you have a knapsack with a capacity of 50
There are n items the following weights and valu-

Item 1 : weight = 10, value = 60

Item 2 : weight = 20, value = 100

Item 3 : weight = 30, value = 120

Items 4 : weight = 40, value = 200

Determine max value that can be obtained using 0/1 knapsack problem approach :

Sol: Given capacity 50 units

Item	weight	value (P)
1.	10	60
2.	20	100
3.	30	120
4.	40	200

V/W	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	160	180	220
4	0	60	100	160	200	260

Formula:

$$V[i,w] = \max\{V[i-1,w],\ V[i-1, w-w[i]] + value[i]\}$$

$$V[4,50] = \max\{V[3,50],\ V[3, 50-200] + value[4]\}$$

$$= \max\{220, 260\} = 260$$

6. Given the following directed graph with vertices A, B, C, DA, B, C, DA, B, C, D and edges with weights:

A → BA) right arrow BA → B with weight 1

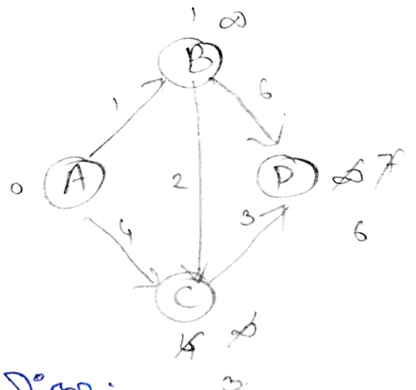
A → CA \ right arrow CA → C with weigh 4

B → CB \ right arrow CB → C with weight 2

B → DB) right arrow DB → D with weight 6

C → DC \ right arrow DC → D with weight 3

use the Bellman - Ford algorithm to find the shortest path from vertex AAA to all other vertices. Show the steps.



A → B	1
A → C	4
B → C	2
B → D	6
C → D	3

Initialize:

V	A	B	C	D
d	0	∞	∞	∞
p	–	–	–	–

1

V	A	B	C	D
d	0	1	4	∞
p	–	A	A	B

(2)

V	A	B	C	D
d	0	1	3	7
k	-	A	B	B

(3)

V	A	B	C	D
d	0	1	3	6
k	-	A	B	C

Path	Shortest distance	Shortest path
A-B	1	A-B
A-C	3	A-B-C
A-D	6	A-B-C-D

O/P → A → B → C → D

(7) Determine the probability of rolling five dice such that the sum is exactly 20. Include a combinato[rial] approach to arrive at the solution.

Sol:

$6^5 = 7776$

$x_1 + x_2 + x_3 + x_4 + x_5 = 20$, where $1 \le x_i \le 6$

$y_i = x_i - 1$ for $i = 1, 2, 3, 4, 5$.

$(y_1+1) + (y_2+1) + (y_3+1) + (y_4+1) + (y_5+1) = 20$

$y_1 + y_2 + y_3 + y_4 + y_5 = 15$

where $0 \le y_i \le 5$.

By "stars & bars"

$$\binom{15+5-1}{5-1} = \binom{19}{4}$$

$$\binom{19}{4} = \frac{19\times18\times17\times16}{4\times3\times2\times1} = 3876$$

If $y_1 \geq 6$, let $y_1' = y_1 - 6$

$y_1' + y_2 + y_3 + y_4 + y_5 = 9$

$$\binom{9+5-1}{5-1} = \binom{13}{4}$$

$$\binom{13}{4} = \frac{13\times12\times11\times10}{4\times3\times2\times1} = 715$$

There are 5 such variables,

$5 \times 715 = 3575$

If two variables $y_i, y_j \geq 6$, let $y_1' = y_1 - 6$ and

$y_2' = y_2 - 6$.

$y_1' + y_2' + y_3 + y_4 + y_5 = 3$

$$\binom{3+5-1}{5-1} = \binom{7}{4}$$

$$\binom{7}{4} = \frac{7\times6\times5\times4}{4\times3\times2\times1} = 35$$

$$\binom{5}{2}\times35 = 10\times35 = 350$$

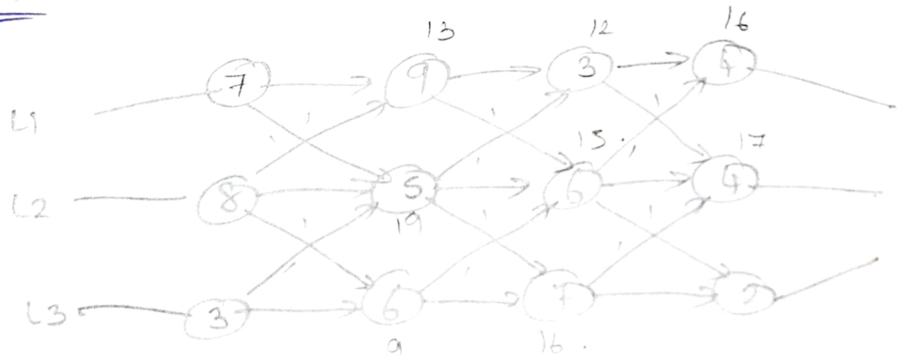
using inclusion - exclusion principle:

$3876 - 3575 + 350 = 651$

$$\frac{651}{7776} = \frac{651}{7776} \simeq 0.0837$$ //

(8.) For 3 assembly lines with station times: Line 1: [7, 9, 3, 4], Line 2: [8, 5, 6, 4], Line 3: [3, 6, 7, 2], & transfer times b/w lines given, determine the optimal scheduling on the total minimum assembly time.

Sol :-



$$F_1[j] = \min\{(f_1(j-1) + a_{1j}), (f_2(j-1) + (t_2, j-1) + a_{1j}), (f_3(j-1) + (t_3, j-1) + a_{1j})\}$$

$$= \min\{(7+9), (8+1+9), (3+1+9)\} = 13.$$

	1	2	3	4
$F_1[j]$	7	13	12	16
$F_2[j]$	8	9	15	17
$F_3[j]$	3	9	16	15

	1	2	3	4
$L_1[j]$	1	3	1	1
$L_1[j]$	2	3	2	1
$L_1[j]$	3	3	3	1

9. consider keys {15, 25, 35, 45, 55}) {15, 23, 35, 45, 55}
{15, 25, 35, 45, 55} with access probabilities
{0.05, 0.15, 0.4, 0.25, 0.15}. Determine the structure
of the optimal binary search tree & compute cost.

Sol :

{15, 25, 35, 45, 55}
{0.05, 0.15, 0.4, 0.25, 0.15}

	0	1	2	3	4	5
0	0	0.05	[2] 0.25	[3] 0.85	[3] 1.35	[3] 1.80
1		0	0.15	[3] 0.70	[3] 1.20	[4] 1.80
2			0	0.4	[3] 0.90	[3,4] 1.35
3				0	0.25	[4] 0.55
4					0	0.15
5						0

$j - i = 1$

1-0 = 1 (0, 1) (1, 1)

2-1 = 1 (1, 2) (2, 2)

3-1 = 1 (2, 3) (3, 3)

4-2 = 1 (3, 4) (4, 4)

5-4 = 1 (4, 5) (5, 5)

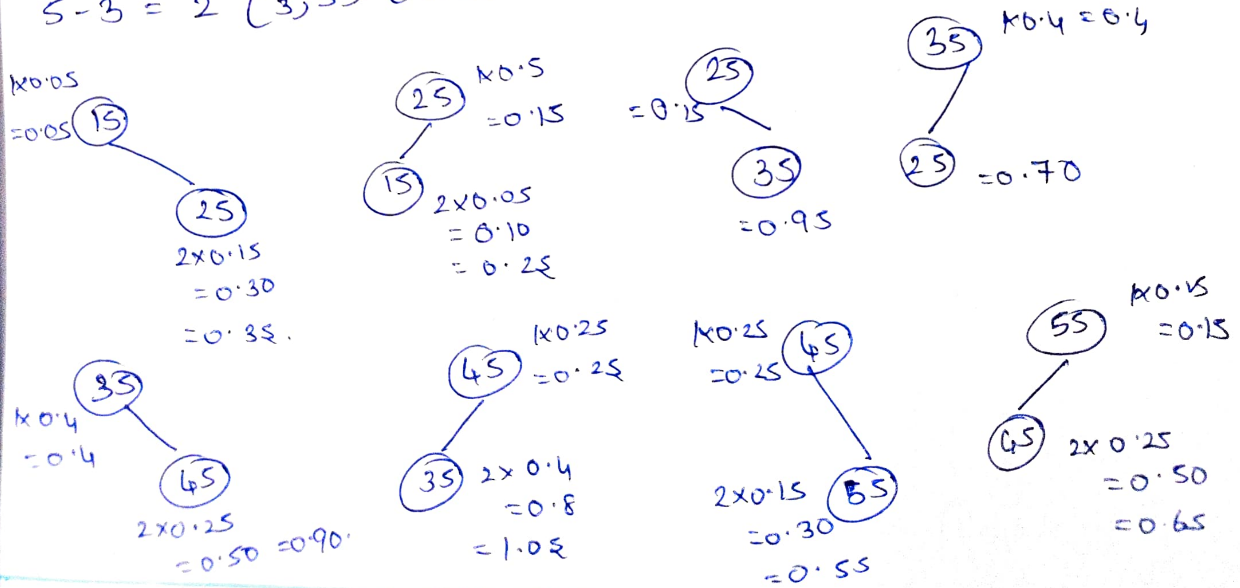
$j - i = 2$

2-0 = 2 (0, 2) (1, 2)

3-1 = 2 (1, 3) (2, 3)

4-2 = 2 (2, 4) (3, 4)

5-3 = 2 (3, 5) (4, 5)



$j - i = 3$

$3 - 0 = 3 \quad (0,3)\ (1,3)$

$4 - 1 = 3 \quad (1,4)\ (2,4)$

$5 - 2 = 3 \quad (2,5)\ (3,5)$

$cost(i,j) = \min \{ cost(i, k-1) + cost(k, j) \} + w_i$

$$cost(0,3) = \min_{k=1,2,3} \begin{Bmatrix} cost(0,0) + cost(1,3) \\ cost(0,1) + cost(2,3) \\ cost(0,2) + cost(3,3) \end{Bmatrix} + 0.6$$

$$= \min \begin{Bmatrix} 0 + 0.70 \\ 0.05 + 0.4 \\ 0.25 + 0 \end{Bmatrix} + 0.6$$

$$= \min \begin{Bmatrix} 1.30 \\ 1.05 \\ 0.85 \end{Bmatrix} = 0.85$$

$$Cost(1,4) = \min_{k=2,3,4} \begin{Bmatrix} 0 + 0.90 \\ 0.15 + 0.25 \\ 0.70 + 0 \end{Bmatrix} + 0.8 = 1.20$$

$$cost(2,5) = \min_{k=3,4,5} \begin{Bmatrix} cost(2,2) + cost(3,5) \\ cost(2,3) + cost(4,5) \\ cost(2,4) + cost(5,5) \end{Bmatrix} + 0.8$$

$$= \min \begin{Bmatrix} 1.35 \\ 1.35 \\ 1.70 \end{Bmatrix} = 1.35$$

$j - i = 4$

$4 - 0 = 4 \ (0,4)\ (1,4)$

$5 - 1 = 4 \ (1,5)\ (2,5)$

$$\text{cost}(0,4) = \min \begin{Bmatrix} 0+1.20 \\ 0.05+0.90 \\ 0.25+0.25 \\ 0.85+0 \end{Bmatrix} = 0.85$$

$$= \min \begin{Bmatrix} 2.05 \\ 1.80 \\ 1.35 \\ 1.77 \end{Bmatrix} = 1.35$$

$$\text{cost}(1,5) = \min_{K=2,3,4,5} \begin{Bmatrix} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{Bmatrix} + 0.95$$

$$= \min \begin{Bmatrix} 0+1.35 \\ 0.15+1.35 \\ 0.70+0.15 \\ 1.20+0 \end{Bmatrix} + 0.95$$

$$= \min \begin{Bmatrix} 2.30 \\ 2.45 \\ 1.80 \\ 2.15 \end{Bmatrix} = 1.80$$

$$\text{cost}(0,5) = \min_{K=1,2,3,4,5} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,5) \\ \text{cost}(0,1) + \text{cost}(2,5) \\ \text{cost}(0,2) + \text{cost}(3,5) \\ \text{cost}(0,3) + \text{cost}(4,5) \\ \text{cost}(0,4) + \text{cost}(5,5) \end{Bmatrix} + 1$$

$$= \min \begin{Bmatrix} 2.80 \\ 2.65 \\ 1.80 \\ 2.00 \\ 2.35 \end{Bmatrix} = 1.80$$

(10.) Given a distance matrix for 6 cities, find the shortest path using the nearest neighbor heuristic.

A : [0, 10, 8, 9, 7, 5]

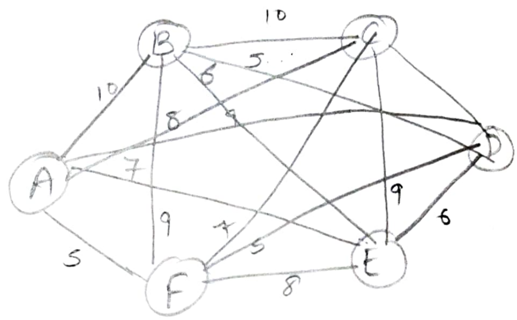
B : [10, 0, 10, 5, 6, 9]

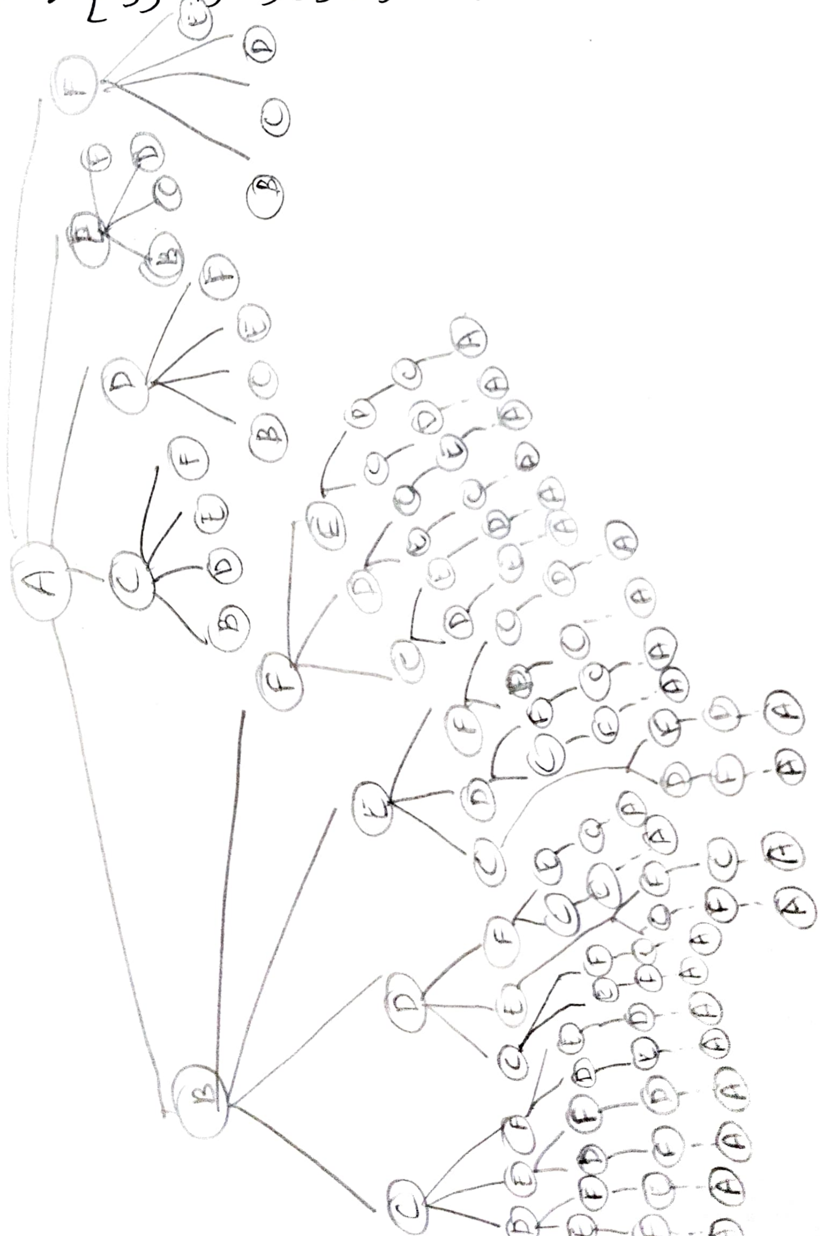
C : [8, 10, 0, 8, 9, 7]

D : [9, 5, 8, 0, 6, 5]

E : [7, 6, 9, 6, 0, 8]

F : [5, 9, 7, 5, 8, 0].





o/p — A → F → E → D → C → B → A = 47

11. Solve the fractional knapsack problem for a knapsack with a capacity of 60 units.

Item 1 : W = 20 , V = 100

Item 2 : W = 30 , V = 120

Item 3 : W = 10 , V = 60 .

Sol :

Item	weight	Value (P)
1.	20	100
2.	30	120
3.	10	60

w = 60 units.

v/w	0	10	20	30	40	50	60
0	0	0	0	0	0	0	0
1	0	0	100	100	100	100	100
2	0	0	100	120	120	220	220
3	0	10	100	120	180	220	280

Formula –

$$V[i, w] = \max \{ V[i-1, w], V[i-1, w-w[i]] + value[i] \}.$$

(12.) consider a directed graph with 5 vertices V_1, V_2, V_3, V_4, V_5 v_1, v_2, v_3, v_4, v_5 v_1, v_2, v_3, v_4, V_5 and the following edges with weights:

$v_1 \rightarrow v_2$ v_1) rightarrow $v_2 v_1 \rightarrow v_2$ with w-3.

$v_1 \rightarrow v_3 v_1$ w-8

$v_2 \rightarrow v_3 v_2$ w-2

$v_2 \rightarrow v_4 v_2$ w-5

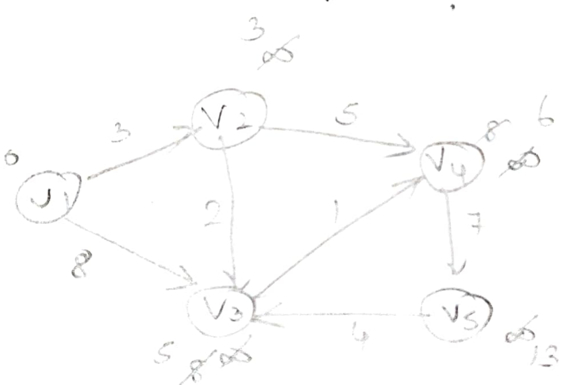
$v_3 \rightarrow v_4 v_3$ w-1

$v_4 \rightarrow v_5 v_4$ w-7

$v_5 \rightarrow v_3 v_5$ w-4.

Apply Bellman-Ford algorithm.

Sol:



$v_1 \rightarrow v_2$ 3

$v_1 \rightarrow v_3$ 8

$v_2 \rightarrow v_3$ 2

$v_2 \rightarrow v_4$ 5

$v_3 \rightarrow v_4$ 1

$v_4 \rightarrow v_5$ 7

$v_5 \rightarrow v_3$ 4.

Initialize:

V	v_1	v_2	v_3	v_4	v_5
d	0	∞	∞	∞	∞
π	-	-	-	-	-

(1)

V	v_1	v_2	v_3	v_4	v_5
d	0	3	8	∞	∞
p	-	v_1	v_1	v_2	v_4

(5)

v	v_1	v_2	v_3	v_4	v_5
d	0	3	5	6	13
p	-	v_1	v_2	v_3	v_4

(4)

v	v_1	v_2	v_3	v_4	v_5
d	0	3	5	6	13
p	-	v_1	v_2	v_3	v_4

O/P $\rightarrow v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5$

Path	Shortest distance	Shortest path
$v_1 - v_2$	3	$v_1 - v_2$
$v_1 - v_3$	5	$v_1 - v_2 - v_3$
$v_1 - v_4$	6	$v_1 - v_2 - v_3 - v_4$
$v_1 - v_5$	13	$v_1 - v_2 - v_3 - v_4 - v_5$

(13) Given two eight-sided dice, compute the No. of ways to achieve a sum of 10. Then, extend this to three dice & find the new number of ways to get the same sum.

Sol: we need to count the pairs (x, y) such that $x+y=10$ where $1 \le x, y \le 8$.

Possible pairs-

$(x,y) = (2,8)$
$(x,y) = (3,7)$
$(x,y) = (4,6)$
$(x,y) = (5,5)$
$(x,y) = (6,4)$
$(x,y) = (7,3)$
$(x,y) = (8,2)$

No. of ways to achieve a sum of 10 is : 7. we need to count the no. of triples (x,y,z) such that $x+y+z=10$ where $1 \le x, y, z \le 8$.

(3)

V	v_1	v_2	v_3	v_4	v_5
d	0	3	5	6	15
p	-	v_1	v_2	v_3	v_4

(4)

V	v_1	v_2	v_3	v_4	v_5
d	0	3	5	6	13
p	-	v_1	v_2	v_3	v_4

O/P $\Rightarrow v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5$

Path	Shortest distance	Shortest Path
$v_1 - v_2$	3	$v_1 - v_2$
$v_1 - v_3$	5	$v_1 - v_2 - v_3$
$v_1 - v_4$	6	$v_1 - v_2 - v_3 - v_4$
$v_1 - v_5$	13	$v_1 - v_2 - v_3 - v_4 - v_5$

(13) Given two eight-sided dice, compute the No. of ways to achieve a sum of 10. Then, extend this to three dice & find the new number of ways to get the same sum.

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Possible pairs-

$(x, y) = (2, 8)$
$(x, y) = (3, 7)$
$(x, y) = (4, 6)$
$(x, y) = (5, 5)$
$(x, y) = (6, 4)$
$(x, y) = (7, 3)$
$(x, y) = (8, 2)$

No. of ways to achieve a sum of 10 is : 7. we need to count the no. of triples (x, y, z) such that $x+y+z=10$ where $1 \le x, y, z \le 8$.

1. $x=1$:

$y+z=9$:

$(1,1,8)$, $(1,2,7)$, $(1,3,6)$, $(1,4,5)$, $(1,5,4)$, $(1,6,3)$

$(1,7,2)$, $(1,8,1)$

2. $x=2$:

$y+z=8$:

$(2,1,7)$, $(2,2,6)$, $(2,3,5)$, $(2,4,4)$, $(2,5,3)$, $(2,6,2)$,

$(2,7,1)$.

3. $x=3$:

$y+z=7$:

$(3,1,6)$, $(3,2,5)$, $(3,3,4)$, $(3,4,3)$, $(3,5,2)$, $(3,6,1)$.

4. $x=4$:

$y+z=6$:

$(4,1,5)$, $(4,2,4)$, $(4,3,3)$, $(4,4,2)$, $(4,5,1)$

5. $x=5$:

$y+z=5$:

$(5,1,4)$, $(5,2,3)$, $(5,3,2)$, $(5,4,1)$

6. $x=6$:

$y+z=4$:

$(6,1,3)$, $(6,2,2)$, $(6,3,1)$

7. $x=7$:

$y+z=3$:

$(7,1,2)$, $(7,2,1)$

8. $x=8$:

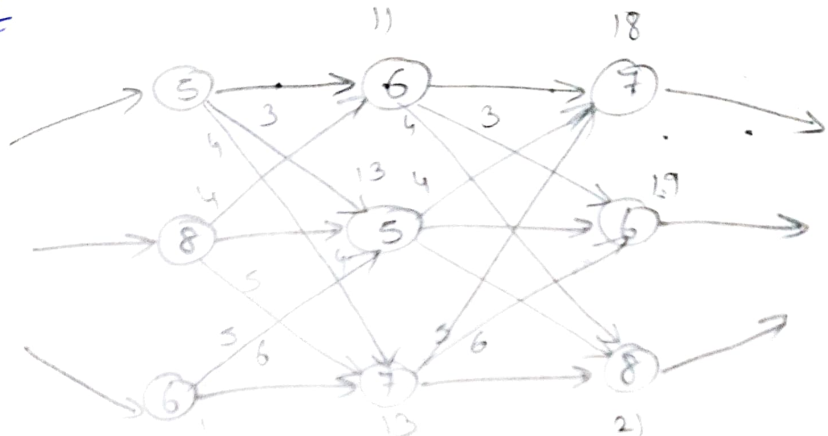
$y+z=2$: $(8,1,1)$

Sum $= 8+7+6+5+4+3+2+1 = 36$.

So, the no. of ways to a sum of $10 = 36$ //.

(14) Given station times for L1 : [5, 6, 7], Line 2: [8, 5, 6], & Line 3: [6, 7, 8] and transfer times b/w lines: [3, 4], [4, 5] and [5, 6], calculate the minimum time required to complete the product assembly.

Sol:



	1	2	3
$F_1[j]$	5	11	18
$F_2[j]$	8	13	19
$F_3[j]$	6	13	21

	1	2	3
$L_1[j]$	1	1	1
$L_2[j]$	2	2	2
$L_3[j]$	3	3	3

$F_1[j] = \min\{(f_1(j-1) + a_1 j), (f_2(j-1) + (t_2, j-1) + a_1 j),$
$(f_3(j-1) + (t_3, j-1) + a_1 j)\}$

$= \min\{11, 18, 17\} = 11$

(15) Given keys {5, 15, 25, 35, 45, 55} with access probabilities {0.1, 0.05, 0.2, 0.25, 0.3, 0.1}, use dp to find the OBST, show the steps of your calculation and the resulting cost.

Sol:

{5, 15, 25, 35, 45, 55}

{0.1, 0.05, 0.2, 0.25, 0.3, 0.1}

	0	1	2	3	4	5	6
0	0	0.1	[1] 0.2	[3] 0.55	[3] 1.05	[4] 1.75	[4] 2.05
1		0	0.05	[3] 0.3	[3,4] 0.8	[4] 1.4	[4] 1.7
2			0	0.2	[4] 0.65	[4] 1.25	[4] 1.55
3				0	0.25	[5] 0.8	[5] 1
4					0	0.3	[5] 0.5
5						0	0.1
6							0

$j - i = 1$

$1 - 0 = 1$

$2 - 1 = 1$

$3 - 2 = 1$

$4 - 3 = 1$

$5 - 4 = 1$

$6 - 5 = 1$

$j - i = 2$

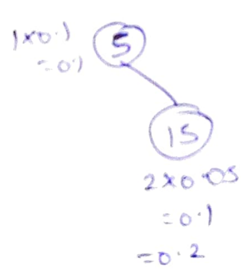
$2 - 0 = 2$ (0, 2) (1, 2)

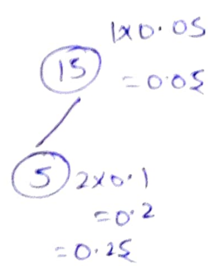
$3 - 1 = 2$ (1, 3) (2, 3)

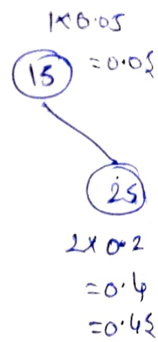
$4 - 2 = 2$ (2, 4) (3, 4)

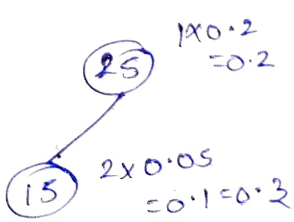
$5 - 3 = 2$ (3, 5) (4, 5)

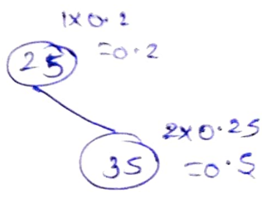
$6 - 4 = 2$ (4, 6) (5, 6)

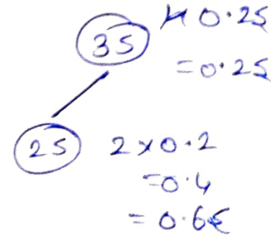


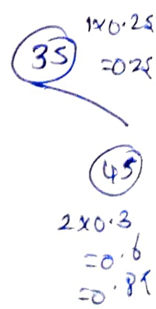


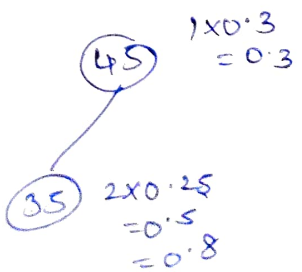


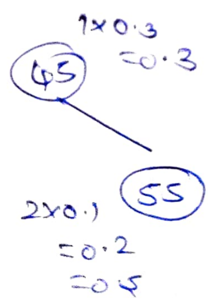


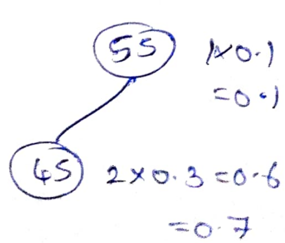












$j - i = 3$

$3 - 0 = 3 \quad (0,3)\ (1,3)$

$4 - 1 = 3 \quad (1,4)\ (2,4)$

$5 - 2 = 3 \quad (2,5)\ (3,5)$

$6 - 3 = 3 \quad (3,6)\ (4,6)$

$$cost(i,j) = \min \{ cost(i, k-1) + cost(k, j) \} + w_i$$

$$cost(0,3) = \min_{k=1,2,3} \left\{ \begin{array}{l} cost(0,0) + cost(1,3) \\ cost(0,1) + cost(2,3) \\ cost(0,2) + cost(3,3) \end{array} \right\} + 0.35$$

$$= \min \left\{ \begin{array}{l} 0 + 0.3 \\ 0.1 + 0.65 \\ 0.2 + 0 \end{array} \right\} + 0.35 = 0.55$$

$$cost(1,4) = \min_{k=2,3,4} \left\{ \begin{array}{l} cost(1,1) + cost(2,4) \\ cost(1,2) + cost(3,4) \\ cost(1,3) + cost(4,4) \end{array} \right\} + 0.5$$

$$= 0.8$$

$$cost(2,5) = \min_{k=3,4,5} \left\{ \begin{array}{l} cost(2,2) + cost(3,5) \\ cost(2,3) + cost(4,5) \\ cost(2,4) + cost(5,5) \end{array} \right\} + 0.75$$

$$= \min \left\{ \begin{array}{l} 1.55 \\ 1.25 \\ 1.4 \end{array} \right\} = 1.25$$

$$cost(3,6) = \min_{k=4,5,6} \left\{ \begin{array}{l} cost(3,3) + cost(4,6) \\ cost(3,4) + cost(5,6) \\ cost(3,5) + cost(6,6) \end{array} \right\} + 0.65$$

$$= \min \left\{ \begin{array}{l} 1.15 \\ 1 \\ 1.45 \end{array} \right\} = 1$$

$j - i = 4$

$4 - 0 = 4 \quad (0,4)\ (1,4)$

$5 - 1 = 4 \quad (1,5)\ (2,5)$

$6 - 2 = 4 \quad (2,6)\ (3,6)$

$$\text{cost}(0,4) = \min_{k=1,2,3,4} \left\{ \begin{array}{l} \text{cost}(0,0) + \text{cost}(1,4) \\ \text{cost}(0,1) + \text{cost}(2,4) \\ \text{cost}(0,2) + \text{cost}(3,4) \\ \text{cost}(0,3) + \text{cost}(4,4) \end{array} \right\} + 0.6$$

$$= \min \left\{ \begin{array}{l} 1.4 \\ 1.35 \\ 1.05 \\ 1.15 \end{array} \right\} = 1.05.$$

$$\text{cost}(1,5) = \min_{k=2,3,4,5} \left\{ \begin{array}{l} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{array} \right\} + 0.8$$

$$= \min \left\{ \begin{array}{l} 2.05 \\ 1.65 \\ 1.4 \\ 1.6 \end{array} \right\} = 1.4$$

$$\text{cost}(2,6) = \min_{k=3,4,5,6} \left\{ \begin{array}{l} \text{cost}(2,2) + \text{cost}(3,6) \\ \text{cost}(2,3) + \text{cost}(4,6) \\ \text{cost}(2,4) + \text{cost}(5,6) \\ \text{cost}(2,5) + \text{cost}(6,6) \end{array} \right\} + 0.85$$

$$= \min \left\{ \begin{array}{l} 1.85 \\ 1.55 \\ 1.6 \\ 2.1 \end{array} \right\} = 1.55.$$

$j - i = 5$

$5 - 0 = 5 \ (0,5)\ (1,5)$

$6 - 1 = 5 \ (1,6)\ (2,6)$

$$\text{cost}(0,5) = \min_{k=1,2,3,4,5} \left\{ \begin{array}{l} \text{cost}(0,0) + \text{cost}(1,5) \\ \text{cost}(0,1) + \text{cost}(2,5) \\ \text{cost}(0,2) + \text{cost}(3,5) \\ \text{cost}(0,3) + \text{cost}(4,5) \\ \text{cost}(0,4) + \text{cost}(5,5) \end{array} \right\} + 0.9.$$

$$= \min \begin{Bmatrix} 2.3 \\ 2.25 \\ 1.9 \\ 1.75 \\ 1.95 \end{Bmatrix} = 1.75.$$

$$\text{cost}(0,6) = \min_{k=1,2,3,4,5,6} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,6) \\ \text{cost}(0,1) + \text{cost}(2,6) \\ \text{cost}(0,2) + \text{cost}(3,6) \\ \text{cost}(0,3) + \text{cost}(4,6) \\ \text{cost}(0,4) + \text{cost}(5,6) \\ \text{cost}(0,5) + \text{cost}(6,6) \end{Bmatrix} + 1$$

$$= \min \begin{Bmatrix} 2.7 \\ 2.65 \\ 2.2 \\ 2.05 \\ 2.15 \\ 2.75 \end{Bmatrix} = 2.05.$$

$$\text{cost}(1,6) = \min_{k=2,3,4,5,6.} \begin{Bmatrix} \text{cost}(1,1) + \text{cost}(2,6) \\ \text{cost}(1,2) + \text{cost}(3,6) \\ \text{cost}(1,3) + \text{cost}(4,6) \\ \text{cost}(1,4) + \text{cost}(5,6) \\ \text{cost}(1,5) + \text{cost}(6,6) \end{Bmatrix} + 0.9.$$

$$= \min \begin{Bmatrix} 2.45 \\ 2 \\ 1.7 \\ 1.8 \\ 2.3 \end{Bmatrix}$$

$$= 1.7.$$

16. Extend the following distance matrix to 7 cities.

A: [0, 12, 10, 19, 8, 16]

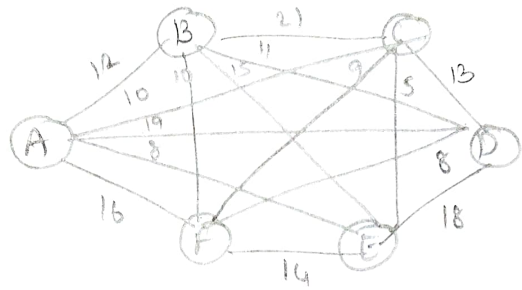
B: [12, 0, 21, 11, 15, 10]

C: [10, 21, 0, 13, 3, 7]

D: [19, 11, 13, 0, 18, 8]

E: [8, 15, 5, 18, 0, 14]

F: [16, 10, 7, 8, 14, 0].



O/P → A – B – C – D – E – F = 83 //

(17) A Knapsack capacity of 70 units and the following items:

Item 1: W = 25, V = 80

Item 2: W = 35, V = 90

Item 3: W = 45, V = 120

Item 4: W = 30, V = 70.

V\W	0	25	35	45	30	70
0	0	0	0	0	0	0
1	0	80	80	80	80	80
2	0	80	90	90	90	90
3	0	80	90	120	120	90
4	0	80	90	80	120	150.

(18.) For a graph

A → BA, w-1

A → CA, w-4

B → CB, w-3

B → DB, w-2

D → BD, w-1

D → CD, w-5

E → DE, w-3

use Bellman-Ford & solve it.

Sol: A → B = -1   B → D = 2   D → C = 5

A → C = 4   B → E = 2   E → D = -3.

B → C = 3   D → B = 1

V	A	B	C	D	E
d	0	∞	∞	∞	∞
P	–	–	–	–	–

(1)

V	A	B	C	D	E
d	0	-1	4	∞	∞
P	–	A	A	–	–

(2)

V	A	B	C	D	E
d	0	-1	4	∞	1
P	–	A	A	–	B

(3)

V	A	B	C	D	E
d	0	-1	4	3	1
P	–	A	A	E	B

(4)

V	A	B	C	D	E
d	0	-1	4	3	1
P	–	A	A	E	B

Path	distance	shortest path
A	0	A
B	-1	A → B
C	4	A → C
D	3	A → E → D
E	1	A → B → C

(19.) Find the expected value of the sum of outcomes when rolling 3 four-sided dice. Show your calculation & reasoning.

Sol: Sum - 3 (1+1+1)

Sum

$4 = \frac{3}{64}$ (1+1+2, 1+2+1, 2+1+1)

$5 = \frac{6}{64}$ (1+1+3, 1+2+2, 1+3+1, 2+1+2, 2+2+1, 3+1+1)

$6 = \frac{10}{64}$ (1+2+3, 1+3+2, 2+1+3, 2+2+2, 2+3+1, 3+1+2, 3+2+1, 1+4+1, 2+2+2, 2+3+1)

7 = 7 (1+3+3, 2+2+3, 2+3+2, 3+1+3, 3+2+2, 3+3+1, 1+4+2, 2+3+2, 2+4+1, 3+2+2, 3+3+1, 4+1+2)

$8 = \frac{12}{64}$, $9 = \frac{10}{64}$, $10 = \frac{6}{64}$, $11 = \frac{3}{64}$, $12 = \frac{1}{64}$.

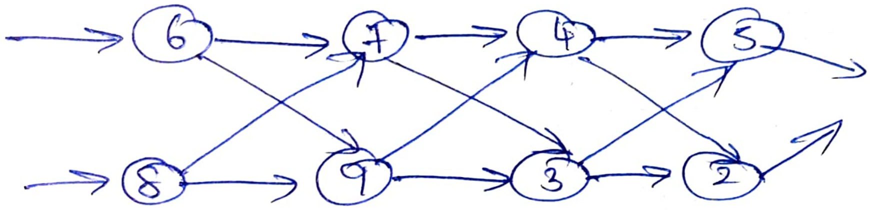
Σ (sum * probability)

$$= \left(3 \times \frac{1}{64}\right) + \left(4 \times \frac{3}{64}\right) + \left(5 \times \frac{6}{64}\right) + \left(6 \times \frac{10}{64}\right) + \left(7 \times \frac{12}{64}\right) + \left(8 \times \frac{12}{64}\right) + \left(9 \times \frac{10}{64}\right) + \left(10 \times \frac{6}{64}\right) + \left(11 \times \frac{3}{64}\right) + \left(12 \times \frac{1}{64}\right)$$

$$= \frac{480}{64} = 7.5$$

(20.) calculate min. time for Line 1: [6, 7, 4, 5], Line 2: [8, 9, 3, 2] with transfer lines [4, 5, 6] 1 to 2 & [6, 5, 4] 2 to 1.

Sol:



	1	2	3	4
f_1 [j]	6	13	17	12
f_2 [j]	8	17	20	22

	1	2	3	4
l_1 [j]	1	1	1	1
l_2 [j]	2	2	2	2

(21) keys {10, 20, 30} have probabilities {0.2, 0.5, 0.3} do OBST.

Sol:

K = {10, 20, 30}.

V = {0.2, 0.5, 0.3}

$j - 1 = 3$

$3 - 0 = (0.3)$

$\text{cost}(0,3) = \min \begin{Bmatrix} 2.1 \\ 1.5 \\ 1.1 \end{Bmatrix}$

	0	1	2	3
0	0	0.2	0.7	1.1
1		0	0.5	1.1
2			0	0.3.
3				0

(22) using 5 cities:

A : [0, 14, 4, 10, 20]

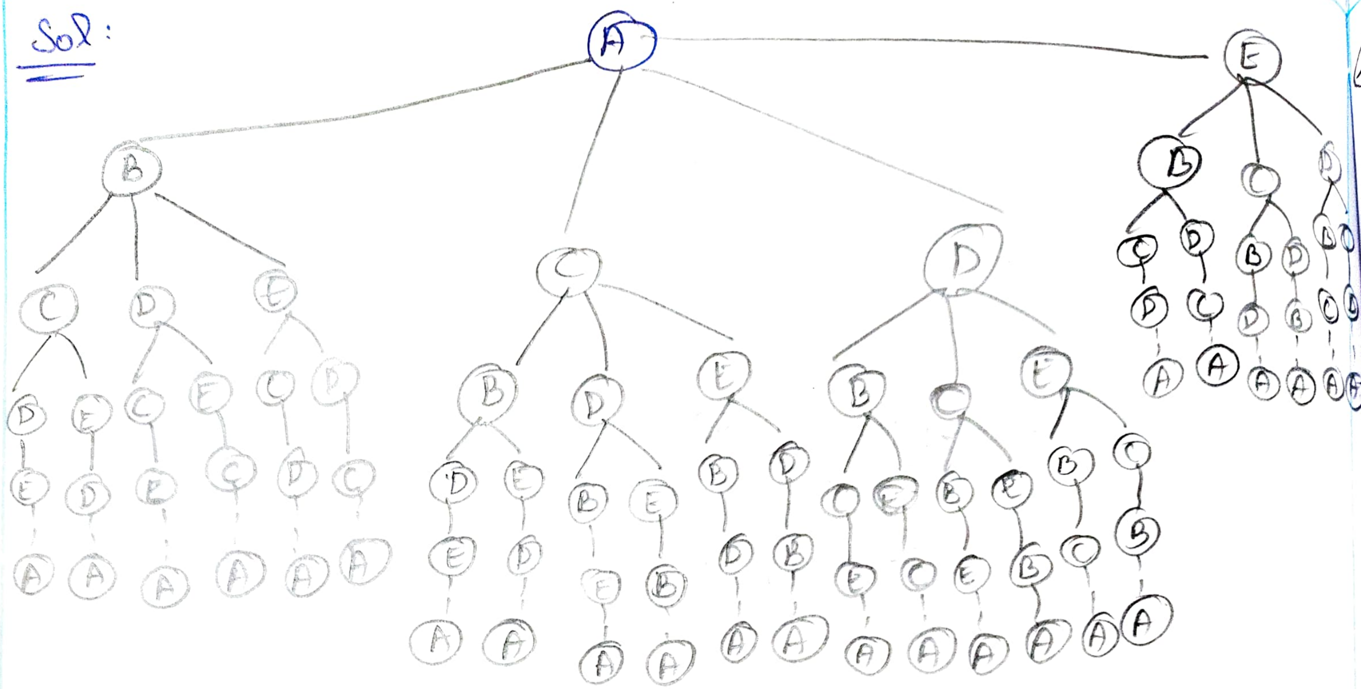
B : [14, 0, 7, 8, 7]

C : [4, 7, 0, 12, 6]

D : [10, 8, 12, 0, 15]

E : [20, 7, 6, 15, 0].

Sol:



(23) Knapsack 0/1 50 units.

I-1 = w=10, v=50

I-2 = w=20, v=70

I-3 = w=30, v=90

I-4 = w=25, v=60

I-5 = w=15, v=40.

Sol:

	0	10	20	30	25	15	50
0	0	0	0	0	0	0	0
1	0	50	50	50	50	50	50
2	0	50	70	70	70	70	70
3	0	50	70	90	90	90	160
4	0	50	70	90	90	90	160
5	0	50	70	90	90	90	160

24) Bellman - Ford.

$1 \to 21, \ w = 4$

$1 \to 31, \ w = 5$

$2 \to 32, \ w = -2$

$3 \to 43, \ w = 3$

$4 \to 24, \ w = -10$.

V	1	2	3	4
d	0	∞	∞	∞
p	-	-	-	-

V	1	2	3	4
d	0	4	5	∞
p	-	1	1	-

Sol:

$1 \to 2 \quad -4$

$1 \to 3 \quad -5$

$2 \to 3 \ - 3$

$4 \to 2 = -10$

V	1	2	3	4
d	0	4	2	∞
p	-	1	2	-

V	1	2	3	4
d	0	4	2	5
p	-	1	2	3

Vertex	dist	Path
1	0	1
2	4	$1 \to 2$
3	2	$1 \to 2 \to 3$
4	5	$1 \to 2 \to 3 \to 4$

25) Roll six six-sided dice. Determine the no. of ways to get a sum of 18, ensuring that at least one die shows a 6.

Sol: $x + x^2 + x^3 + x^4 + x^5 + x^6$.

$\Rightarrow x(1 + x + x^2 + x^3 + x^4 + x^5)$

$= \dfrac{x(1+x^6)}{1-x}$

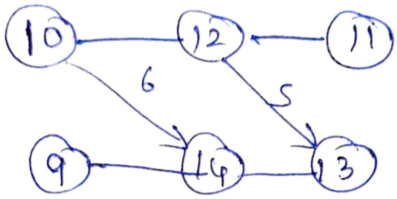
for six dice.

$\left(\dfrac{x(1-x^6)}{1-x}\right)^6 = x^6 (1-x^6)^6 (1-x)^{-6} = (x^{18})$

$= 340$

(26.) Given L1 : [10, 12, 11], Line 2 : [9, 14, 13],

Transfer lines [6, 5], by 2 units.



Before: Reduction:

L1	6	5
L2	30	30

After: reduction (2)

L1	4	5
L2	28	27

(27.) For keys (8, 12, 16, 20, 24) with access possibilities

{0.2, 0.05, 0.4, 0.25, 0.1}. Determine OBST using dp.

Sol:

(8, 12, 16, 20, 24)

(0.2, 0.05, 0.4, 0.25, 0.1)

	0	1	2	3	4	5
0	0	0.2	0.3	0.7	1.45	1.8
1		0	0.05	0.5	1	1.3
2			0	0.4	0.9	1.2
3				0	0.25	0.05
4					0	0.1
5						0

$j - i = 0$

$j - i = 1$

$j - i = 2$

$2 - 0 = [0, 2]$

$3 - 1 = [1, 3]$

$4 - 2 = [2, 4]$

$5 - 3 = [3, 5]$

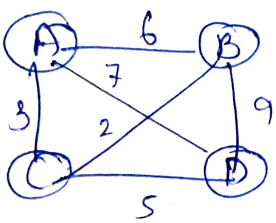
$$= \min \left\{ \begin{array}{l} 0 + 1.2 \\ 0.05 + 0.45 \\ 0.5 + 0.1 \\ 1 + 0 \end{array} \right\} + 0.8 = 1.3$$

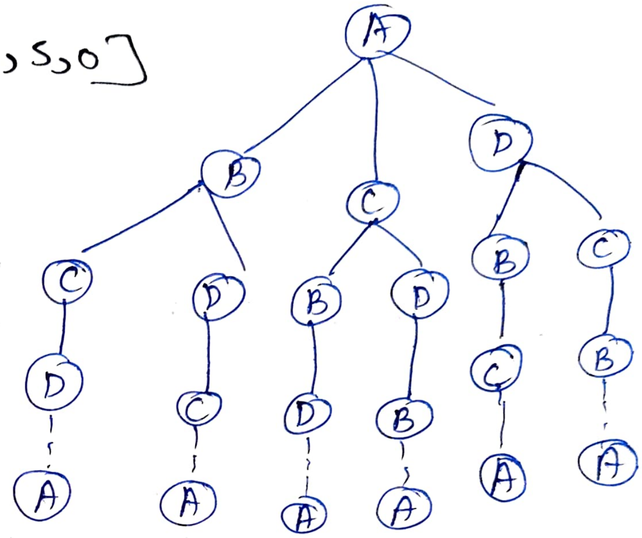
(28.) Solve TSP 4 cities :

A : [0, 6, 3, 7] C : [3, 2, 0, 5]

B : [6, 0, 2, 9] D : [7, 9, 5, 0]

Sol :





A - B - C - D - A - 20
A - D - C - B - A - 20 } min optimal path.

(29.) Knapsack 0/1 50 units.

I_1, w = 10, v = 60

I_2, w = 20, v = 100

I_3, w = 30, v = 120

I_4, w = 40, v = 200.

v|w	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	180
4	0	60	100	120	200	260

30 Bellman - Ford :

A → BA, w = 6
A → DA, w = 7
B → CB, w = 5
B → EB, w = 4
F → CF, w = 2

B → DB, w = 8
C → BC, w = -2
D → CD, w = -3
D → ED, w = -9
E → FE, w = 7

Sol:

V	A	B	C	D	E	F
d	0	∞	∞	∞	∞	∞
P	-	-	-	-	-	-

①

V	A	B	C	D	E	F
d	0	6	4	7	2	9
P	-	A	D	A	B	E

②

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

③

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

Vertex	dist	Path
A	0	A
B	2	A - D - C - B
C	4	A - D - C
D	7	A - D
E	2	A - D - C - B - E
F	9	A - D - C - B - E - F