Université libre de Bruxelles

École polytechnique de Bruxelles

ELEC-H415

Communication channels
Exercises

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1 Preliminaries

1.1 Decibels

1. Transform the following quantities to the correct decibel unit.
   (a) 100 mW
   (b) 0.75 W
   (c) 0.15 V/m
2. Transform the following decibel quantities to the correct linear unit.
   (a) -6 dB
   (b) 65 dBμV/m
   (c) 20 dBm $+$ 10 dB $-$ 2.5 dB $-$ 4 dB
3. For an input signal with power 250 mW, a set of different systems yields powers at the output of 75 mW, 2.5 W, and 0.5 W. What is their respective gain/attenuation in dB?
   What is the output power when this input signal is sent through an amplifier with gain 12 dB? Give the answer both in W and decibels.
4. Considering that 10 dB corresponds to a linear multiplication by 10 and that 3 dB corresponds approximately to a multiplication by 2, determine the approximate linear factor corresponding to the following decibel quantities.

-23 dB	0 dB	5 dB	10 dB
-12 dB	1 dB	6 dB	13 dB
-3 dB	2 dB	7 dB	21 dB
-2 dB	3 dB	8 dB	37 dB
-1 dB	4 dB	9 dB	40 dB

5. Transform the following formulas such that they can be used with decibel values:
   (a) Effective Isotropic Radiated Power (EIRP):
   $$\text{EIRP} = G_{Tx}(\theta, \phi) P_{Tx}$$
   (b) Over-the-ground propagation (3.25):
   $$P_{RX}(d) = G_{Tx} G_{Rx} P_{Tx} \frac{h_{Tx}^2 h_{Rx}^2}{d^4}$$

1.2 Plane Waves

1. Assume an electromagnetic plane wave at a frequency of 3.6 GHz propagating in the direction $\frac{1}{\sqrt{2}}\vec{1}_x - \frac{1}{\sqrt{2}}\vec{1}_y$, with an electric field polarized along $\frac{1}{\sqrt{2}}\vec{1}_x + \frac{1}{\sqrt{2}}\vec{1}_y$ and an amplitude $E_0 = 1$ V/m.
   (a) Determine the wave vector amplitude and direction.
   (b) Determine the magnetic field amplitude, phase and the unit vector describing its polarization, in Cartesian coordinates.
   (c) What is the power flux density amplitude and direction? Verify that (2.9) and (2.10) yield the same result.

1.1 Decibels

1. Transform the following quantities to the correct decibel unit.

   (a) 100 mW

   (b) 0.75 W

   (c) 0.15 V/m

(a) $100\,mW = 10^{-1}\,W \Rightarrow 10\log\frac{10^{-1}W}{1W} = -10dB = 10\log\frac{100mW}{1mW} = 20dBm$

(b) $0.75W \Rightarrow 10\log\frac{0.75W}{1W} = -1.25dB = 28.75\,dBm$

(c) $0.15V/m \Rightarrow 20\log 0.15 = -16.47\,dBV/m$

$= 20\log\frac{0.15}{1uV} = 103.5\,dBuV/m$

2. Transform the following decibel quantities to the correct linear unit

(a) -6 dB

(b) 65 dBμV/m

(c) 20 dBm + 10 dB − 2.5 dB − 4 dB

(a) $-6\,dB = 10\log 10^{-0.6} \Rightarrow 10^{-0.6} = 0.25$

(b) $65\,dB\mu V/m = 20\cdot\log 10^{3.25} \Rightarrow 10^{3.25} = 1778\,\mu V/m$

$\approx \boxed{1.78\,mV/m}$

(c) $20\,dBm + 10\,dB - 2.5\,dB - 4\,dB$

$= 23.5\,dBm$

$= 10\log 10^{2.35}$

$\Rightarrow 10^{2.35} \approx 224\,mW \approx \boxed{0.224\,W}$

3. For an input signal with power 250 mW, a set of different systems yields powers at the output of 75 mW, 2.5 W, and 0.5 W. What is their respective gain/attenuation in dB?
What is the output power when this input signal is sent through an amplifier with gain 12 dB? Give the answer both in W and decibels.

① $250\,mW = 0.25\,W = -6.02\,dB$ (input)

$Gain(dB) = P_{out} - P_{in}$

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a) $75\,mW = 0.075\,W = 10\log 0.075 = -11.24\,dB$

$Gain = -11.24 - (-6.02) \approx \boxed{-5\,dB}$

b) $2.5\,W = 10\log 2.5 = 3.97\,dB$

$Gain = 3.97 - (-6.02) \approx \boxed{10\,dB}$

c) $0.5\,W = 10\log 0.5 = -3.01 \Rightarrow Gain = -3.01 + 6.02 \approx \boxed{3\,dB}$

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② $G = 12\,dB \rightarrow P_{out} = P_{in} + G = -6.02 + 12 = \boxed{6\,dBW} \Rightarrow 10\log 10^{0.6} \Rightarrow 10^{0.6} = \boxed{4W}$

$4W \Rightarrow 10\log\frac{4W}{1mW} = \boxed{36\,dBm}$

4. Considering that 10 dB corresponds to a linear multiplication by 10 and that 3 dB corresponds approximately to a multiplication by 2, determine the approximate linear factor corresponding to the following decibel quantities.

-23 dB	0 dB	5 dB	10 dB
-12 dB	1 dB	6 dB	13 dB
-3 dB	2 dB	7 dB	21 dB
-2 dB	3 dB	8 dB	37 dB
-1 dB	4 dB	9 dB	40 dB

$-23\,dB = -10dB - 10dB - 3dB$

$= 10\log(10\cdot 10\cdot 2)^{-1}$

$\boxed{\Rightarrow (10\cdot 10\cdot 2)^{-1} = 2e^{-2}}$

$-12\,dB = -3dB \cdot 4$

$= 10\log 2^{-4}$

$\boxed{\Rightarrow 2^{-4} = \frac{1}{16}}$

Same proceder for the rest of decibel quantites ...

$5\,dB = \frac{1}{2}\cdot 10dB$

$= 10\log 10^{\frac{1}{2}}$

$10^{\frac{1}{2}} = \boxed{3.16}$

5. Transform the following formulas such that they can be used with decibel values:

(a) Effective Isotropic Radiated Power (EIRP):

$$\mathrm{EIRP} = G_{Tx}(\theta, \phi) P_{Tx}$$

(b) Over-the-ground propagation (3.25):

$$P_{RX}(d) = G_{Tx} G_{Rx} P_{Tx} \frac{h_{Tx}^2 h_{Rx}^2}{d^4}$$

a) $\mathrm{EIRP} = G_{Tx}(\theta, \phi) P_{Tx} \Rightarrow \mathrm{EIRP}[\mathrm{dBm}] = \underbrace{10 \log G_{Tx}}_{G_{Tx}[\mathrm{dB}]} + P_{Tx}[\mathrm{dBm}]$

b)

$$P_{Rx}(d)[\mathrm{dBm}] = \underbrace{10 \log G_{Tx} + 10 \log G_{Rx}}_{G_{Tx}[\mathrm{dB}] + G_{Rx}[\mathrm{dB}]} + \underbrace{10 \log \frac{P_{Tx}}{1\mathrm{mW}}}_{P_{Tx}[\mathrm{dBm}]} + 20 \log(h_{Tx} h_{Rx}) - 4 \cdot 10 \log d$$

- **Plane wave:** uniform field in each plane perpendicular to direction of propagation
  - e.g. propagation along $\vec{1}_z$:
    - $\vec{E}(x, y, z, t) = \vec{E}(z, t)$
    - $\vec{B}(x, y, z, t) = \vec{B}(z, t)$
  - Good far-field, local area approximation
- Propagation along $\vec{1}_\beta$:
  - Wave vector: $\vec{\beta} = \beta \vec{1}_\beta = \frac{\omega}{c} \vec{1}_\beta$
  - Electric field: $\underline{\vec{E}}(\vec{r}) = E_0 e^{-j\vec{\beta}.\vec{r}} \vec{1}_E, \qquad \vec{1}_\beta \perp \vec{1}_E$
  - Magnetic field: $\underline{\vec{B}}(\vec{r}) = B_0 e^{-j\vec{\beta}.\vec{r}} \vec{1}_B = \dfrac{1}{c}\left(\vec{1}_\beta \times \underline{\vec{E}}(\vec{r})\right), \qquad \vec{1}_\beta \perp \vec{1}_B$

$$\vec{r} \perp \vec{1}_\beta \Leftrightarrow \vec{\beta}.\vec{r} = 0 \quad \Longrightarrow \quad \underline{\vec{E}}(\vec{r}), \underline{\vec{B}}(\vec{r}) = \text{cte}$$

$$\vec{1}_\beta \perp \vec{1}_E \perp \vec{1}_B$$

  - Flux density (Poynting vector): $\vec{S}(\vec{r}) = \dfrac{1}{2\mu} \text{Re}\left(\underline{\vec{E}} \times \underline{\vec{B}}^*\right) = \dfrac{1}{2\mu c} |\underline{E}(\vec{r})|^2 \vec{1}_\beta$

1.2 Plane Waves

a) $$\beta = \frac{w}{c} = \frac{2\pi \cdot f}{3 \cdot 10^8} = \frac{2\pi \cdot 6 \cdot 10^7}{3 \cdot 10^8} = 75.4 \text{ rad/m}$$

$$\vec{1}_\beta = \frac{1}{\sqrt{2}} \vec{1_x} - \frac{1}{\sqrt{2}} \vec{1_y}$$

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b) $$\vec{E}(\vec{r}) = E_0 \cdot e^{-j\vec{\beta}\cdot\vec{r}} \cdot \vec{1_E}$$

$$\vec{\beta} \cdot \vec{r} = \left(\frac{\beta}{\sqrt{2}} \vec{1_x} - \frac{\beta}{\sqrt{2}} \vec{1_y} + 0 \cdot \vec{1_z}\right) \cdot \left(x \cdot \vec{1_x} + y \cdot \vec{1_y} + z \cdot \vec{1_z}\right)$$

$$= \frac{\beta}{\sqrt{2}} x - \frac{\beta}{\sqrt{2}} \cdot y$$

$$\Rightarrow \vec{E}(\vec{r}) = E_0 \cdot e^{-j \cdot \frac{\beta}{\sqrt{2}}(x-y)} \cdot \left(\frac{1}{\sqrt{2}} \vec{x} + \frac{1}{\sqrt{2}} \vec{y}\right)$$

$$\vec{B}(\vec{r}) = \frac{1}{c} \cdot (\vec{1}_\beta \times \vec{E}(\vec{r}))$$

$$= \frac{E_0}{c} \cdot e^{-j \cdot \frac{\beta}{\sqrt{2}}(x-y)} (\vec{1}_\beta \times \vec{1}_E)$$

$$\vec{1}_\beta \times \vec{1}_E = \left(\frac{1}{\sqrt{2}}\vec{1}_x - \frac{1}{\sqrt{2}}\vec{1}_y\right) \times \left(\frac{1}{\sqrt{2}}\vec{1}_x + \frac{1}{\sqrt{2}}\vec{1}_y\right) = \left(\frac{1}{2}+\frac{1}{2}\right) \cdot \vec{1}_z$$

$$= \vec{1}_z$$

$$\Rightarrow \vec{B}(\vec{r}) = \frac{E_0}{c} \cdot e^{-j(x-y) \cdot \frac{\beta}{\sqrt{2}}} \cdot \vec{1}_z$$

$$\Rightarrow \begin{cases} B_0 = E_0/c = \frac{1}{8} \cdot 10^{-8} \frac{Vs}{m^2} \\ \angle \vec{B}(\vec{r}) = (y-x) \cdot \frac{\beta}{\sqrt{2}} \\ \vec{1}_B = \vec{1}_z \end{cases}$$

c) $\vec{S}(\vec{r}) = \frac{1}{2\mu_0} \cdot Re\{\vec{E} \times \vec{B}^*]$

$= \frac{1}{2\mu_c} |\vec{E}|^2 \, \vec{1}_\beta$

from 1st rule:

$$\vec{S}(\vec{r}) = \frac{1}{2\mu_0} \cdot Re\left\{ \left[\underbrace{E_0 \cdot e^{-j \cdot \frac{\beta}{\sqrt{2}}(x-y)} \cdot \frac{E_0}{c} \cdot e^{j(x-y) \cdot \frac{\beta}{\sqrt{2}}}}_{= e^0 = 1} \cdot \begin{bmatrix} \vec{x} & \frac{1}{\sqrt{2}} & 0 \\ \vec{y} & \frac{1}{\sqrt{2}} & 0 \\ \vec{z} & 0 & 1 \end{bmatrix} \right] \right.$$

$$= \frac{E_0^2}{2\mu_0 c} \cdot \left(\frac{1}{\sqrt{2}} \vec{1}_x - \frac{1}{\sqrt{2}} \vec{1}_y \right)$$

$$= \frac{E_0^2}{2\mu_0 c} \cdot \vec{1}_\beta$$

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from 2^{nd} rule assuming far field

$$\vec{S}(\vec{r}) = \frac{1}{2\mu_0 c} |\vec{E}|^2 \cdot \vec{I}_\beta$$

$$|\vec{E}|^2 = E_0^2$$

$$\Rightarrow \vec{S}(\vec{r}) = \frac{1}{2\mu_0 c} \cdot E_0^2 \cdot \vec{I}_\beta \quad (2)$$

$$(1) = (2)$$

2 Wireless Channel Models

2.1 Single path channel

Consider a wireless link in free-space with a transmitter and receiver separated by 100m, operating at a carrier frequency of 3.6GHz. Any impact from the transmitter and receiver antennas can be neglected. Calculate the necessary channel parameters (attenuation, channel phase and delay) to define:

1. The physical channel impulse response.
2. The channel transfer function. Does the received power depend on the baseband frequency ?
3. The TDL impulse response for a bandwidth of 100 MHz. In which tap does the ray arrive and what is the effect of applying the uncorrelated scattering assumption on this impulse response ?

2.2 Two-path channel

Let a 5G small cell base station be located at the same height as a mobile phone, at a distance of 30m. The mobile receives two waves: the direct Line-of-Sight wave and a reflected wave propagating in the inverse direction and having propagated 50m more (assume perfect reflection and ignore again any antenna impact). The 5G communication system bandwidth is 100MHz at a carrier frequency of 3.6GHz. Calculate the channel parameters and define:

1. The physical impulse response and transfer function. Does the received power depend on the baseband frequency?
2. The wideband TDL impulse response under the uncorrelated scattering assumption.
3. The TDL narrowband impulse response and transfer function.
4. How do the wideband and narrowband TDL impulse response vary as a function of the receiver position between the transmitter and reflector? For which distance r between transmitter and receiver do both rays fall in the same tap of the wideband TDL impulse response?

2.1 Single path channel

Consider a wireless link in free-space with a transmitter and receiver separated by 100m, operating at a carrier frequency of 3.6GHz. Any impact from the transmitter and receiver antennas can be neglected. Calculate the necessary channel parameters (attenuation, channel phase and delay) to define:

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$$h(\tau) = a_0 \cdot e^{j\phi_0} \cdot e^{-j2\pi f_c \tau_0} \delta(\tau - \tau_0)$$

since $\phi_0 = 0 \Rightarrow h(\tau) = a_0 \cdot e^{-j2\pi f \tau_0}$

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2. $H(f) = \alpha_0 \cdot e^{-j2\pi f \tau_0} \Rightarrow |H(f)|^2 = \text{cte}$, $\boxed{\text{No}}$

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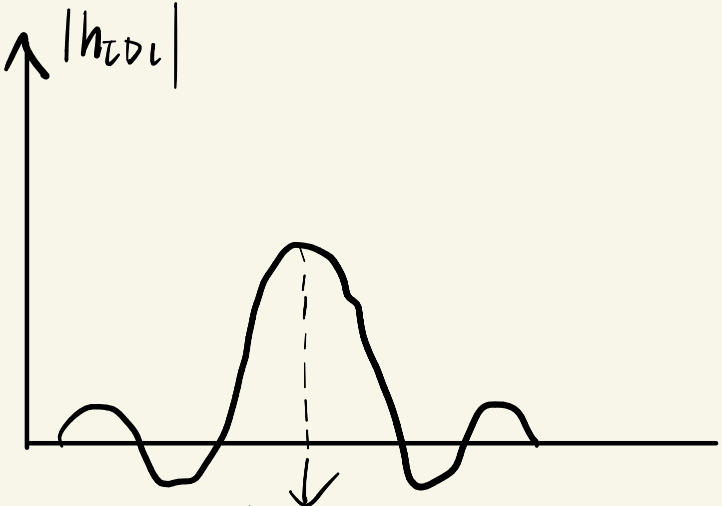
3. $h_{TDL}(\tau) = \sum_{l=0}^{L} h_l \, \delta(\tau - l\Delta\tau)$

$\Delta\tau = \frac{1}{2B} = \frac{1}{100\text{MHz}} = 10\text{ ns}$

$$h_l = a_0 \cdot e^{j\phi_0} \cdot e^{-j2\pi \cdot f_c \cdot \tau_0} \text{sinc}[2B(\tau_0 - l\Delta\tau)]$$



$\tau_0 = l\Delta\tau$

$f_c = 3.6\,GHz \qquad \lambda = \frac{c}{f_c} = \frac{1}{12}$

$\tau_0 = \frac{d}{c} = \frac{100}{3\cdot 10^8} = 333\,ns$,

$y(t) = h(\tau) * x(t)$

$= a_0 \cdot e^{j\phi_0} \cdot e^{-j2\pi f_c \tau_0} \cdot x(t-\tau_0)$

$G_{Tx} = G_{Rx} = L_{Rx} = L_{Tx} = 1$

$\Rightarrow a_0 = \frac{|y(t)|}{|x(t-\tau_0)|} = \sqrt{\frac{P_{Rx}}{P_{Tx}}} = \sqrt{\frac{P_{Tx}\left(\frac{\lambda}{4\pi d}\right)^2}{P_{Tx}}} = \frac{\lambda}{4\pi d}$

$\boxed{a_0 = 6.63\cdot 10^{-5}}$

$\Rightarrow l = \frac{\tau_0}{\Delta\tau} = \frac{333\,ns}{10\,ns} = 33.3$

Uncorrelated Scattering: $h_l = \begin{cases} a_0 \cdot e^{j\phi_0} \cdot e^{-j2\pi f_c \tau_0} & l = 33.3 \\ 0 & \text{elsewhere} \end{cases}$

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Let a 5G small cell base station be located at the same height as a mobile phone, at a distance of 30m. The mobile receives two waves: the direct Line-of-Sight wave and a reflected wave propagating in the inverse direction and having propagated 50m more (assume perfect reflection and ignore again any antenna impact). The 5G communication system bandwidth is 100MHz at a carrier frequency of 3.6GHz. Calculate the channel parameters and define:

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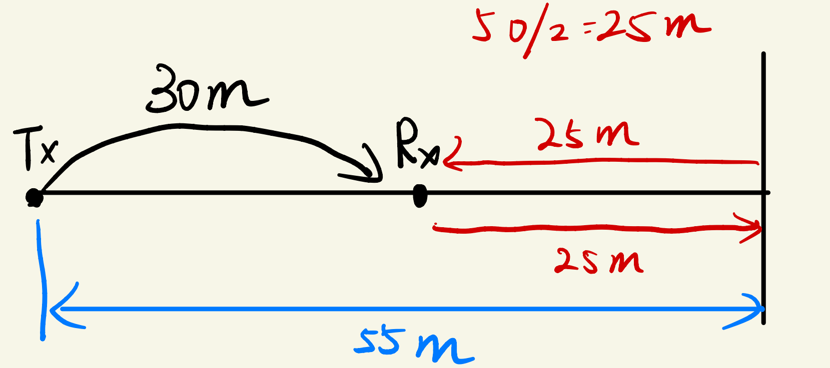

$$\tau_0 = \frac{d_1}{c} = \frac{30}{3\cdot 10^8} = \boxed{100\,ns}$$

$$\tau_1 = \frac{d_2}{c} = \frac{80}{3\cdot 10^8} = \boxed{267\,ns}$$

$$c = 3\cdot 10^8\,m/s \quad d_1 = 30m \quad d_2 = 80m$$

$$2B = 100\,MHz \Rightarrow \boxed{\Delta\tau = \frac{1}{2B} = 10\,ns}$$

$$f_c = 3.6\,GHz \Rightarrow \lambda = \frac{c}{f} = \frac{1}{12}$$

$$\phi_0 = 0 = \phi_1$$

$$a_0 = \frac{|y(t)|}{|x(t-\tau_0)|} = \sqrt{\frac{P_{Rx_1}}{P_{Tx}}} = \frac{\lambda}{4\pi d_1} = \frac{1}{12\cdot 4\cdot\pi\cdot 30} = \boxed{2.21\cdot 10^{-4}}$$

$$a_1 = \frac{|y(t)|}{|x(t-\tau_1)|} = \sqrt{\frac{P_{Rx_2}}{P_{Tx}}} = \frac{\lambda}{4\pi d_2} = \frac{1}{12\cdot 4\cdot\pi\cdot 80} = 8.29\cdot 10^{-5} \xrightarrow{\text{opposite direction}} \boxed{a_1 = -8.29\cdot 10^{-5}}$$

1. $h(\tau) = a_0 \cdot e^{-j2\pi f \tau_0} + a_1 \cdot e^{-j2\pi f \tau_1}$

$|H(f)|^2 \neq cte \Rightarrow$ multiple wave arriving will cause $\boxed{\text{fading}}$

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2. $h^{WB}_{TDL}(\tau) = h_{10}\,\delta(\tau - 10\Delta\tau) + h_{27}\,\delta(\tau - 27\Delta\tau)$

$h_{10} = a_0 \cdot e^{-j\phi_0} \cdot e^{-j2\pi f_c \tau_0}$

$h_{27} = a_1 \cdot e^{-j\phi_1} \cdot e^{-j2\pi f_c \tau_1}$

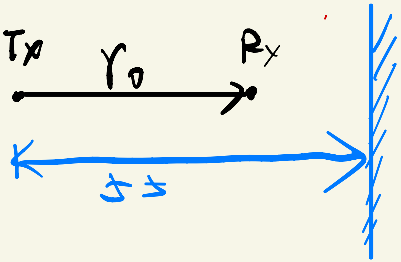
$\Delta\tau = 10\,ns$

3. $\sigma_\tau << \Delta\tau$

$$h^{NB}_{TDL}(\tau) = (h_{10} + h_{27}) \cdot \delta(\tau)$$

$$= (a_0 \cdot e^{-j\phi_0} \cdot e^{-j2\pi f_c \tau_0} + a_1 \cdot e^{-j\phi_1} \cdot e^{-j2\pi f_c \tau_1}) \, \delta(\tau)$$

4.



$r_1 = 110 - r_0$

$l_0 = \left[\frac{r_0/c}{\Delta\tau}\right]$, $l_1 = \left[\frac{(110-r_0)/c}{\Delta\tau}\right]$ $\Rightarrow$

$l_0 = l_1 \quad r_0 = 110 - r_0$

$\rightarrow r_0 = 55 \rightarrow l_0 = [18,3]$

$\rightarrow l_0 = 19$

$l_0 = 18 \Longleftrightarrow r = 18 \cdot \Delta\tau \cdot c$

$l_1 = 19 \Longleftrightarrow (110 - r) = 19 \cdot \Delta\tau c$

$\Rightarrow l_0 max = l_1 min = 19$

$l_0^{max} = 19$ for $r_0 \in \,]54, 55]$ m

$l_1^{min} = 19$ for $r_0 \in \,]53, 55]$ m

3 Physical Propagation Models

3.1 Cellular link

Let a cellular macrocell base station have the following characteristics: $P_{TX} = 10W$, $f = 900$MHz, $G_{TX} = 10$dB. It communicates to a mobile phone located 200m away, equipped with an antenna with $G_{RX} = 0$dB. We suppose free-space propagation. Calculate:

1. The base station EIRP.
2. The received power (in dBW and dBm).
3. The safety distance to the base station according to ICNIRP guidelines, and according to Brussels government guidelines (6 V/m) ?

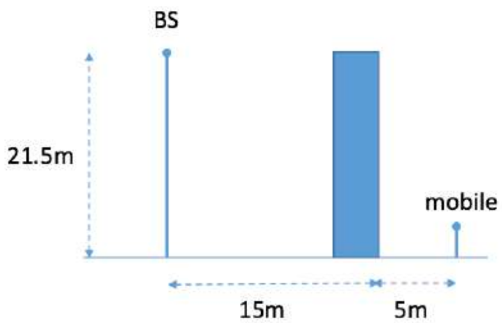
3.2 Coverage of a macrocell in the countryside

Let a GSM macrocell base station have the following characteristics: $h_{TX} = 20m$, $P_{TX} = 10W$, $f = 900$MHz, $G_{TX} = 10$dB. It communicates to a mobile phone located at 1.5m above the ground, equipped with an antenna with $G_{RX} = 0$dB. We suppose that the environment is typical of the countryside, without any obstacle.

1. What is the cell radius if the free-space approximation is made ?
2. Write (3.25) in a canonical form.
3. From which distance from the base station onwards is this law accurate ?
4. Deduce the cell radius from the over-the-ground propagation law

3.3 NLOS propagation

Let us consider a base station located 21.5m above the ground. A building with the same height is located between the BS and the mobile (whose height is 1.5m), as shown in the figure. Consider that the other transmit parameters are equal to the exercise above.



What is the attenuation due to diffraction by the building at the two frequencies foreseen for 5G: 3.6GHz and 26GHz? What is the resulting received power at the mobile?

3.1 Cellular link

Let a cellular macrocell base station have the following characteristics: $P_{TX} = 10W$, $f = 900\text{MHz}$, $G_{TX} = 10\text{dB}$. It communicates to a mobile phone located 200m away, equipped with an antenna with $G_{RX} = 0\text{dB}$. We suppose free-space propagation. Calculate:

1. The base station EIRP.
2. The received power (in dBW and dBm).
3. The safety distance to the base station according to ICNIRP guidelines, and according to Brussels government guidelines (6 V/m) ?

$d = 200\ m$

$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{9 \cdot 10^8} = \frac{1}{3}$

$L_{Rx} = L_{Tx} = 1$

1. $EIRP = G_{Tx} \cdot P_{Tx} = 10W \cdot 10db = 100$

2. $P_{Rx}(d) = P_{Tx} \cdot \frac{G_{Rx} \cdot G_{Tx}}{L_{Rx} \cdot L_{Tx}} \cdot \left(\frac{\lambda}{4\pi d}\right)^2$

$G_{Rx} = 10dB = 10\log\frac{10W}{1W} = 10W$

$G_{TX} = 0dB = 10\log\frac{1W}{1W} = 1W$

$P_{TX} = 10dB = 10W$

$\Rightarrow P_{Rx} = 10 \cdot \frac{10}{1} \cdot \left(\frac{\frac{1}{3}}{4 \cdot \pi \cdot 200}\right)^2 = 1.6 \cdot 10^{-6}\ W$

3,

$$\text{Rms}\begin{cases} \text{ICNIRP} \rightarrow E_L = 1.375\sqrt{f} = 41.25\,V/m \\ \text{Brussels} \rightarrow E_L = 6V/m \end{cases}$$

$$|\vec{E}(d)| = \frac{\sqrt{60EIRP}}{d} = E_{pk}$$

$$E_{L(Rms)} = \frac{E_{pk}}{\sqrt{2}}$$

$$\Rightarrow d_{ICNIRP} = \frac{\sqrt{60EIRP}}{\sqrt{2}\cdot E_{L(ICNIRP)}} = 1.3m$$

$$d_{BXL} = \frac{\sqrt{60EIRP}}{\sqrt{2}\cdot E_{L(BXL)}} = 9.1m$$

3.2 Coverage of a macrocell in the countryside

Let a GSM macrocell base station have the following characteristics: $h_{TX} = 20m$, $P_{TX} = 10W$, $f =$ 900MHz, $G_{TX} = 10$dB. It communicates to a mobile phone located at 1.5m above the ground, equipped with an antenna with $G_{RX} = 0$dB. We suppose that the environment is typical of the countryside, without any obstacle.

1. What is the cell radius if the free-space approximation is made ?
2. Write (3.25) in a canonical form.
3. From which distance from the base station onwards is this law accurate ?
4. Deduce the cell radius from the over-the-ground propagation law

$$P_{Tx} = 10W = 30 dBm$$

$$P_{RX_{min}} = -102 dBm$$

$$\lambda = \frac{c}{f} = \frac{1}{3}$$

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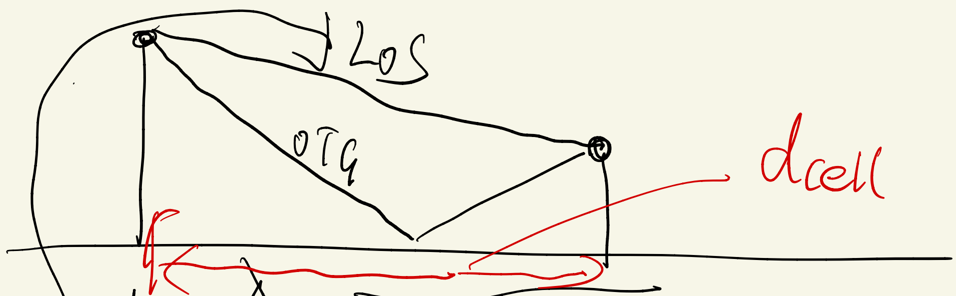


$$d_{los} = \frac{\lambda}{4\pi}\sqrt{\frac{P_{Tx}}{P_{RX}} G_{Tx} G_{RX}} = 1056 km$$

$$d_{Los} >> h_{Tx} \cdot h_{RX} \Rightarrow d_{cell} \simeq d_{los}$$

2. $$P_{Rx}(d) = G_{Tx}\cdot G_{Rx}\cdot P_{Tx}\,\frac{h_{Tx}^2\cdot h_{Rx}^2}{d^4}$$

$$\Rightarrow P_{Rx}(d)_{dB} = G_{Tx} + G_{Rx} + P_{Tx} + 2\cdot 10\log_{10}(h_{Tx}\cdot h_{Rx}) - \boxed{10\cdot 4\log d}$$

$$+\boxed{10\cdot 4\cdot \log d_0} - 10\cdot 4\log d_0$$

$$\begin{cases} P_{Rx}(d_0) = P_{Rx}(d_0) - 10\cdot 4\log\left(\frac{d}{d_0}\right) \\ L(d) = L_0(d_0) + 10\cdot 4\cdot\log\left(\frac{d}{d_0}\right) \end{cases}$$

3. $P_{RX}^{Friis} = P_{RX}^{OTG}$

$$\cancel{P_{TX}} \cdot \cancel{G_{TX}} \cdot \cancel{G_{RX}} \cdot \left(\frac{\lambda}{4\pi d}\right)^2 = \cancel{G_{TX}} \cdot \cancel{G_{RX}} \cdot \cancel{P_{TX}} \, \frac{h_{TX}^2 \cdot h_{RX}^2}{d^4}$$

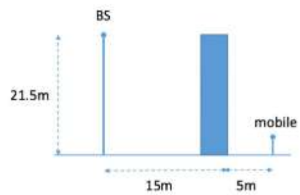
$$\Rightarrow d = \frac{h_{TX} h_{RX}}{\lambda / 4\pi} = 1131\,m$$

---

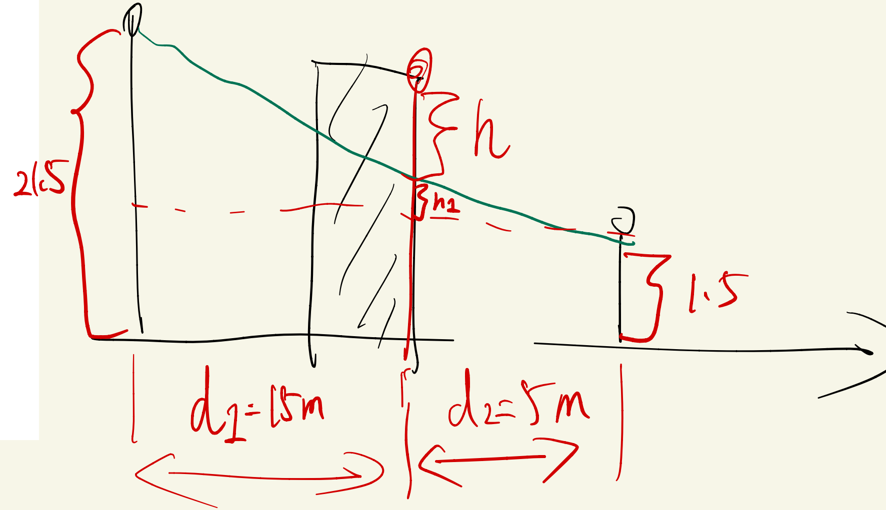
4. $$d = \sqrt[4]{\frac{P_{TX}}{P_{RX}} G_{TX} G_{RX} \, h_{TX}^2 \cdot h_{RX}^2} = 35\,km$$

3.3 NLOS propagation

Let us consider a base station located 21.5m above the ground. A building with the same height is located between the BS and the mobile (whose height is 1.5m), as shown in the figure. Consider that the other transmit parameters are equal to the exercise above.



What is the attenuation due to diffraction by the building at the two frequencies foreseen for 5G: 3.6GHz and 26GHz? What is the resulting received power at the mobile?



$$\lambda_1 = \frac{c}{f} = \frac{3.0 \cdot 10^8}{26 \cdot 10^9} = \frac{1}{12}$$

$$\frac{(21.5-1.5)}{(15+5)} = \frac{h_1}{5} \Rightarrow h_1 = 5 \Rightarrow h = (21.5-1.5) - h_1 = 15$$

$$v_1 = h \cdot \sqrt{\frac{2}{\lambda}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)} = 15 \cdot \sqrt{24 \cdot \frac{4}{15}} \simeq 37.5$$

$$|F(v_1)|^2 = \frac{1}{2\pi^2 v_1^2} = 3.6 \cdot 10^{-5}$$

$$L_{re_1} = 10 \cdot \log_{10} |F(v)|^2 = -44.5\,\mathrm{dB}$$ (3.6 GHz)

$$\lambda_2 = \frac{c}{f} = \frac{3{,}0 \cdot 10^8}{26 \cdot 10^9} = \frac{3}{260}$$

$$V_2 = h \cdot \sqrt{\frac{2}{\lambda}\left(\frac{d_1+d_2}{d_1 \cdot d_2}\right)} = 15 \cdot \sqrt{\frac{520}{3} \cdot \frac{4}{15}}$$

$$\approx 101{,}98$$

$$|F(V_2)|^2 \approx 4{,}8 \cdot 10^{-6}$$

$$L_{Re_2} = 10 \cdot \log_{10} |F V_2|^2 = \underbrace{-53{,}12\,dB}_{(26GHz)}$$

$$\lambda_1 = \frac{1}{12} \qquad d_{los} = \sqrt{20^2 + 20^2} = 28,28$$

$$P_{Rx}^{Fris}(d) = G_{Tx} \cdot G_{Rx} \cdot P_{Tx} \cdot \left(\frac{\lambda}{4\pi d}\right)^2$$

$$G_{TX} = 10\,dB = 10W, \quad G_{Rx} = 0\,dB = 1W$$

$$P_{RX}^{Fris} = 10^2 \cdot 1 \cdot \left(\frac{\frac{1}{12}}{4\pi \cdot 28,28}\right)^2 = 5,49 \cdot 10^{-6}$$

$$= -52,6\,dB$$

3,6GHz

$$P_{Rx}^{NLos} = L_{RE} \cdot P_{Rx}^{Fris} = -97,1\,dB = -67,1\,dBm$$

$$\lambda_2 = \frac{3}{260}$$

$$P_{RX}^{Friis} = 10^{2} \cdot \left(\frac{\frac{3}{260}}{4\pi \cdot 28.28} \right)^{2} = -69.77 \text{ dB}$$

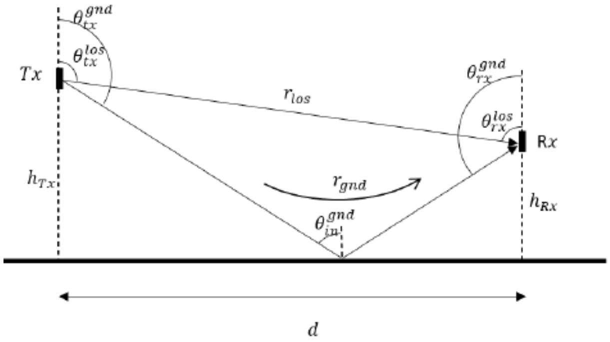
$$P_{RX}^{NLOS} = -53.12 - 69.77 = -122.89 \text{ dB}$$

$$\simeq \underline{-92.89 \text{ dBm}}$$

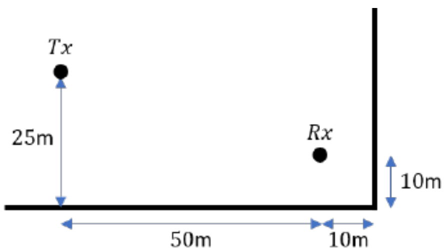
26GHz

4 Ray-Tracing

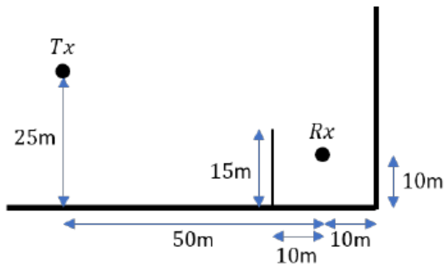
1. Given the geometry below, write the theoretical expression for the electric field of the LOS and ground reflected waves at the receiver and the voltages they induce at the receiving antenna. Consider *lossless vertically polarized half wavelength dipole antennas.*



2. The figures below show the top view of a communication environment. Use the image method and ray tracing theory to identify the valid propagation paths and their corresponding propagation distance and delay, and potential incidence angles, as well as any reflection or diffraction coefficients, if applicable.
Calculate the electric field of each ray at the receiver and the total electric field. Consider up to 2 reflections, as well as the ground reflection and diffraction where necessary.
The carrier frequency is 3.6GHz and the transmitted power is 1W. Assume polarization perpendicular to the figure planes. All walls and the ground can be considered as being uniformly made of concrete with relative permittivity of 4.5. The transmitter and receiver are at the same height above the ground (2m). Antennas are omni-directional with unit gain, all other antenna effects can be ignored.



(a)



(b)

1.

$$\vec{E}(r) = \hat{j} Z_0 I_{in} \frac{\beta}{4\pi} \frac{e^{-j\beta r}}{r} \vec{h}_e^{Tx}(\theta, \phi)$$

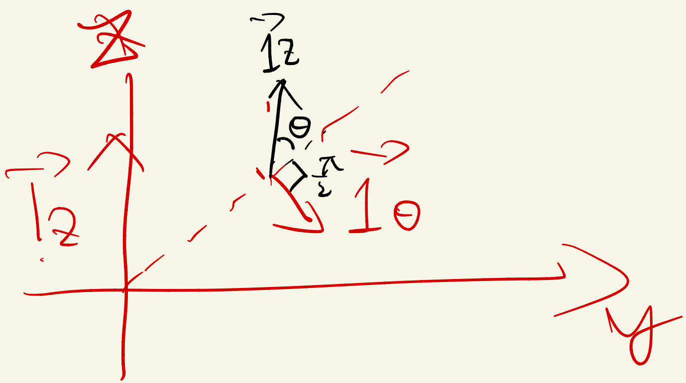
$$\vec{h}_e = -\frac{\lambda}{\pi} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \vec{1}_z$$

$$\vec{h}_{e(\theta,\phi)} = \underbrace{h_{e,\theta}}_{\vec{h}_e \cdot \vec{1}_\theta} \cdot \vec{1}_\theta + \underbrace{h_{e,\phi} \cdot \vec{1}_\phi}_{\vec{h}_e \cdot \vec{1}_\phi} = 0$$

Since $\vec{1}_\phi \perp \vec{1}_z \Rightarrow h_{e,\phi} = 0$

$$\Rightarrow h_e(\theta,\phi) = \left(\vec{h}_e \cdot \vec{1}_\theta\right) \cdot \vec{1}_\theta$$

$$= \left(\underbrace{\left(-\frac{\lambda}{\pi}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\cdot \vec{1}_z\right)}_{\vec{h}_e} \cdot \vec{1}_\theta\right) \vec{1}_\theta$$



$$\vec{1}_z \cdot \vec{1}_\theta = \|\vec{1}_z\| \cdot \|\vec{1}_\theta\| \cdot \underbrace{\cos\left(\frac{\pi}{2}+\theta\right)} = -\sin\theta$$

$$\Rightarrow \underbrace{h_e(\theta,\phi) = \frac{\lambda}{\pi} \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \cdot \vec{1}_\theta}$$

$$\because \vec{E}(r) = j Z_0 I_{in} \frac{\beta}{4\pi} \frac{e^{-j\beta r}}{r} \underbrace{\vec{h}_e(\theta,\phi)}$$

$$\Rightarrow \vec{E}^{los}(r) = j \frac{Z_0 I_{in}}{2\pi} \frac{\cos\left(\frac{\pi}{2}\cos\theta_{tx}^{los}\right)}{\sin\theta_{tx}^{los}} \frac{e^{-j\beta r^{los}}}{r^{los}} \vec{1}_\theta$$

$$V_{oc}^{los} = \underbrace{\vec{E}^{Los}} \cdot \vec{h}_e^{Rx}$$

$$= j \frac{Z_0 I_{in}}{\pi\beta} \frac{\cos\left(\frac{\pi}{2}\cos\theta_{tx}^{los}\right)}{\sin\theta_{tx}^{los}} \frac{\cos\left(\frac{\pi}{2}\cos\theta_{rx}^{los}\right)}{\sin\theta_{rx}^{los}} \frac{e^{-j\beta r^{los}}}{r^{los}}$$

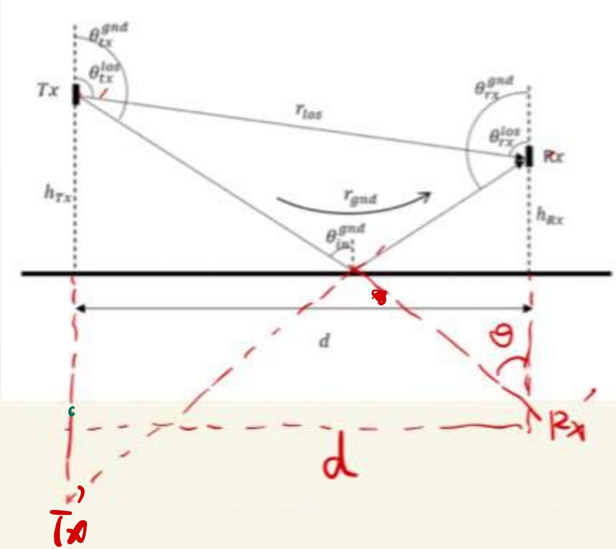
$$r_{los} = \sqrt{d^2 - (h_{Tx} - h_{Rx})^2}$$

$$\theta_{Tx}^{los} = \frac{\pi}{2} + \mathrm{atan}\left(\frac{h_{Tx} - h_{Rx}}{d}\right)$$

$$\theta_{Rx}^{los} = \frac{\pi}{2} - \mathrm{atan}\left(\frac{h_{Tx} - h_{Rx}}{d}\right)$$

$$r_{gnd} = \sqrt{d^2 + (h_{Tx} + h_{Rx})^2}$$

$$\theta_{in}^{gnd} = \frac{\pi}{2} - \mathrm{atan}\frac{h_{Tx} + h_{Rx}}{d} \qquad \theta^{gnd} = \theta_{Tx}^{gnd} = \theta_{rx}^{gnd} = \frac{\pi}{2} + \mathrm{atan}\frac{h_{Tx} + h_{Rx}}{d}$$

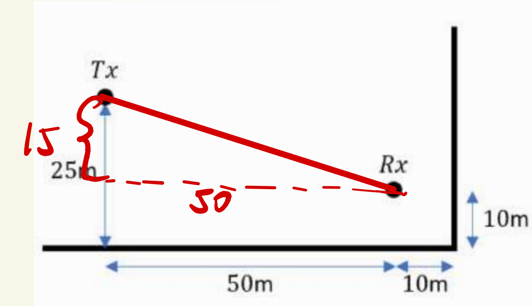


$$\vec{E}^{GND} = \Gamma_{\parallel}(\theta_{in}^{GND}) \cdot j \cdot \frac{Z_0 I_{in}}{2\pi} \frac{\cos(\frac{\pi}{2}\cos\theta_{Tx}^{GND})}{\sin\theta_{Tx}^{GND}} \frac{e^{-j\beta \cdot r^{GND}}}{r^{gnd}} \vec{I}_{\theta}$$

$$V_{oc}^{gnd} = \vec{E}^{GND} \cdot \vec{h}_e^{Rx}$$

$$= \Gamma_{\parallel}(\theta_{in}^{GND}) j \frac{Z_0 I_{in}}{\pi\beta} \left(\frac{\cos(\frac{\pi}{2}\cos\theta^{gnd})}{\sin\theta^{gnd}}\right)^2 \cdot \frac{e^{-j\beta r^{gnd}}}{r^{gnd}}$$

4.2 (a) $d_{LOS} = \sqrt{50^2 + 15^2} = 52.2m$



i) LOS

$T_{LOS} = \frac{d_{LOS}}{c} = 174ns$

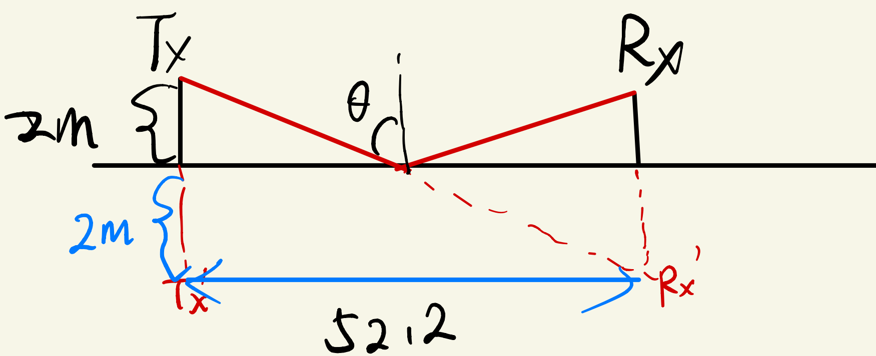
$E_{LOS} = \sqrt{60 P_{Tx} G_{Tx}} \cdot \frac{e^{-j\beta d^{LOS}}}{d_{LOS}}$

$P_{Tx} = 1W, \; G_{Tx} = 1, \; \beta = \frac{2\pi}{\lambda} = 24\pi$

$\Rightarrow E_{LOS} = 0.148 \cdot e^{-j0.83\pi}$

---

ii) Ground



$r_{gnd} = \sqrt{(52.2)^2 + (2+2)^2} = 52.4m$

$T_{gnd} = \frac{52.4 \, m}{3 \cdot 10^8 \, m/s} = 175ns$

$\theta_{in}^{gnd} = \frac{\pi}{2} - \operatorname{atan}\left(\frac{4}{52.2}\right)$

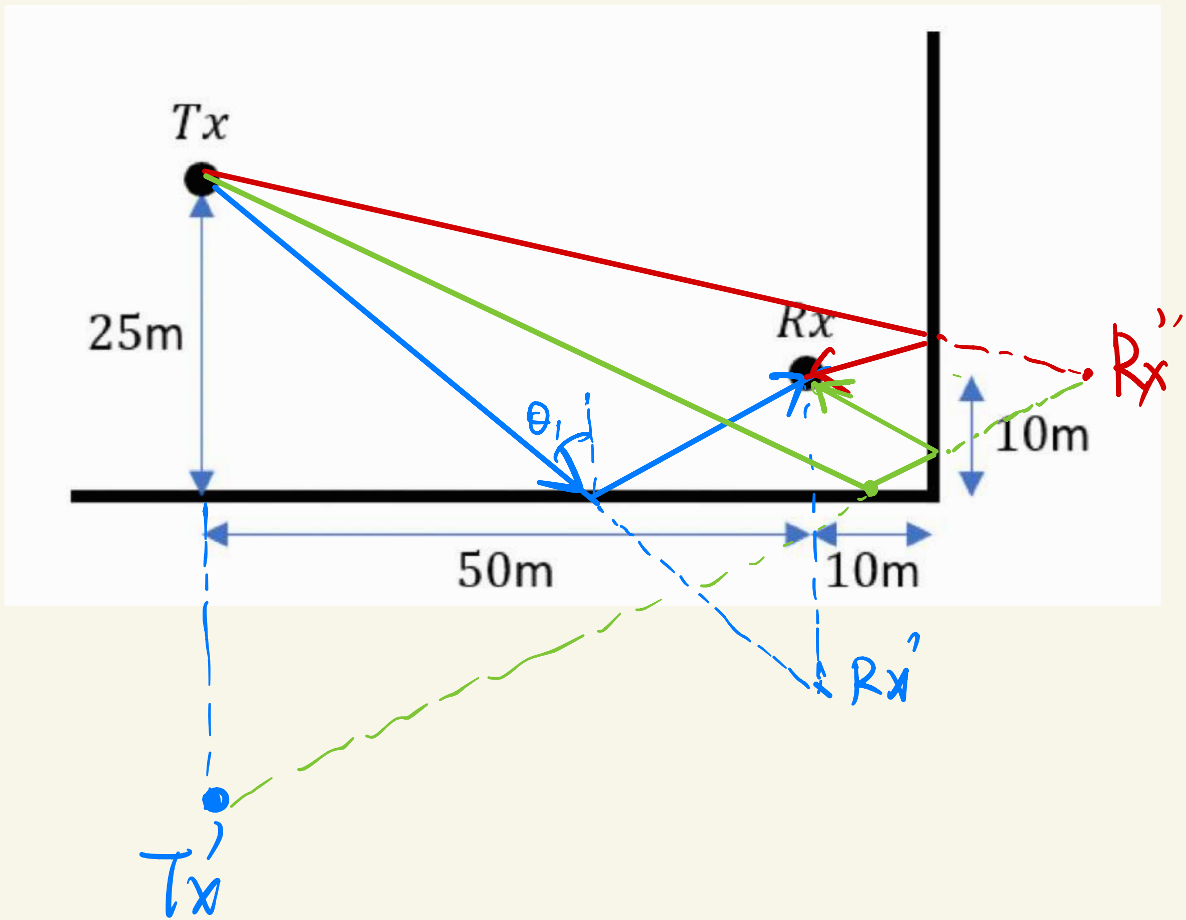
$= 85.6°$

$$\Gamma_{\parallel gnd} = \frac{\cos(85.6^\circ) - \frac{1}{\sqrt{4.5}}\sqrt{1 - \frac{1}{4.5}\cdot \sin^2(85.6)}}{\cos(85.6^\circ) + \frac{1}{\sqrt{4.5}}\sqrt{1 - \frac{1}{4.5}\cdot \sin^2(85.6)}}$$

$$= \frac{0.076 - 0.41}{0.076 + 0.41} = -0.687$$

Since polarization is perpendicular to planes

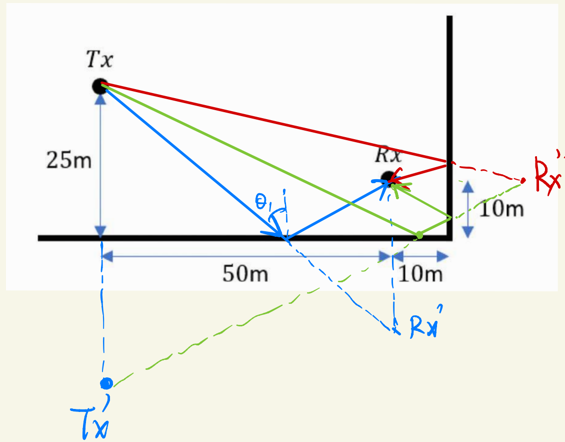
$$E_{gnd} = \Gamma_{\parallel gnd} \sqrt{60 P_{Tx} G_{Tx}} \frac{e^{-j\beta r_{gnd}}}{r_{gnd}} = -0.1 \cdot e^{-j\, 0.6\pi}$$



Tx
Rx
25m
50m
10m
10m
θ1
Rx''
Rx'
Tx'

iii) one-time reflection #1

$$r_1 = \sqrt{(25-10)^2 + (10+60)^2} = 71.6\,m$$



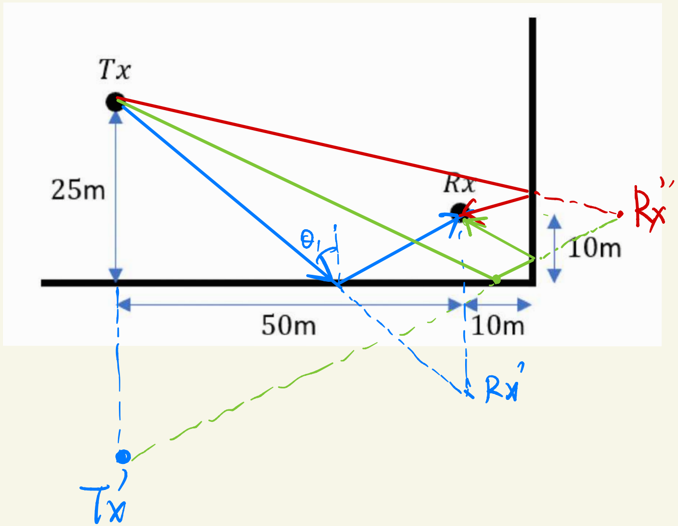
$$T_1 = \frac{71.6\,m}{3 \cdot 10^8\,m/s} = 239\,ns$$

$$\theta_{in1} = \text{actan}\left(\frac{15}{70}\right) = 12.1^\circ$$

$$\Gamma_\perp^1 = \frac{\cos(12.1) - \sqrt{4.5}\sqrt{1 - \frac{1}{4.5}\cdot \sin^2(12.1)}}{\cos(12.1) + \sqrt{4.5}\sqrt{1 - \frac{1}{4.5}\cdot \sin^2(12.1)}} = -0.37$$

$$\Rightarrow E_1 = \Gamma_\perp^1 \cdot \sqrt{60 P_{Tx} G_{Tx}}\,\frac{e^{-j\beta r_1}}{r_1} = 0.04 \cdot e^{j0.6\pi}$$

iv) one - time reflection #2



$$r_2 = \sqrt{(50-15)^2 + 50^2} = 61\,m.$$

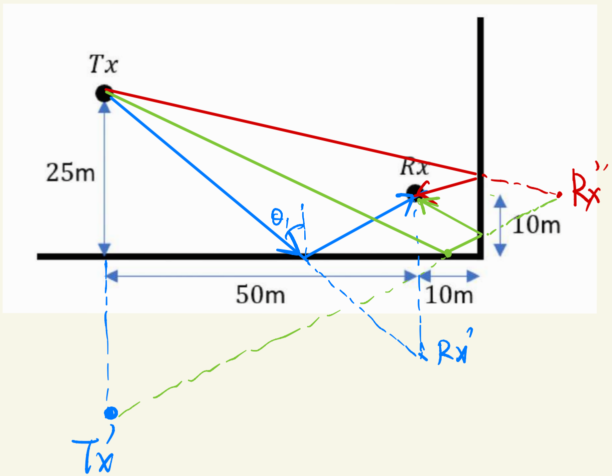
$$T_2 = \frac{r_1}{c} = 203\,ns$$

$$\theta_{in_2} = \frac{\pi}{2} - \operatorname{atan}\left(\frac{h_{Tx}+h_{Rx}}{50}\right) = 55^\circ$$

$$\Gamma_\perp^2 = \frac{\cos(55) - \sqrt{4.5}\sqrt{1-\frac{1}{4.5}\cdot\sin^2(55)}}{\cos(55) + \sqrt{4.5}\sqrt{1-\frac{1}{4.5}\cdot\sin^2(55)}} = -0.55$$

$$\Rightarrow E_2 = \Gamma_\perp^2 \cdot \sqrt{60 P_{Tx} G_{Tx}}\, \frac{e^{-j\beta r_2}}{r_2} = 0.07 \cdot e^{-j\pi}$$

V) Two-time reflection #2



$$r_3 = \sqrt{(50-15)^2 + (10+60)^2} = 78.3\,m$$

$$\theta_{in_3}^{1} = \frac{\pi}{2} - a\tan\left(\frac{25}{50}\right) = 63.4^\circ$$

$$\tau_3' = \frac{78.3}{3\cdot 10^8} \doteq 261\,ns$$

$$\Gamma_{\perp 3}^{1} = \frac{\cos(63.4) - \sqrt{4.5}\sqrt{1-\frac{1}{4.5}\cdot\sin^2(63.4)}}{\cos(63.4) + \sqrt{4.5}\sqrt{1-\frac{1}{4.5}\cdot\sin^2(63.4)}} = -0.62$$

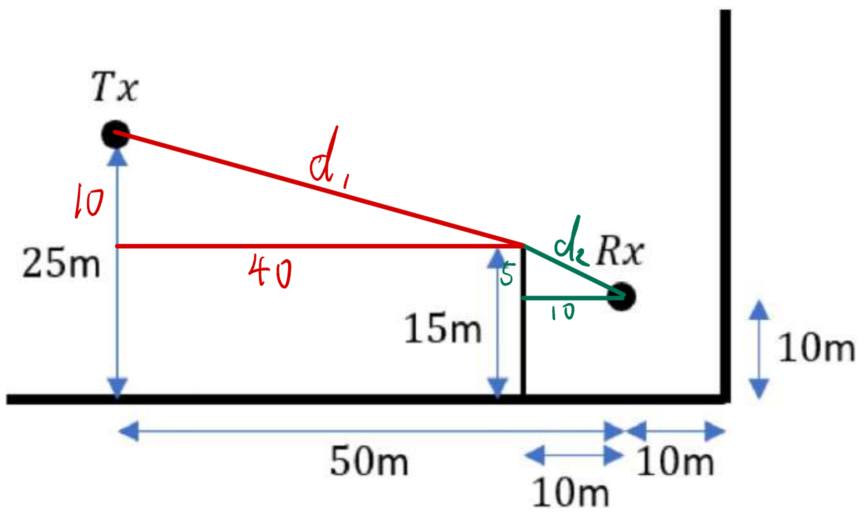
$$\theta_{in_3}^{2} = a\tan\left(\frac{35}{70}\right) = 26.6^\circ$$

$$\Gamma_{\perp 3}^{2} = \frac{\cos(26.6) - \sqrt{4.5}\sqrt{1-\frac{1}{4.5}\cdot\sin^2(26.6)}}{\cos(26.6) + \sqrt{4.5}\sqrt{1-\frac{1}{4.5}\cdot\sin^2(26.6)}} = -0.4$$

$$E_3 = \Gamma^{1}_{\perp 3} \cdot \Gamma^{2}_{\perp 3} \cdot \sqrt{60 P_{Tx} G_{Tx}} \frac{e^{-j\beta r_3}}{r_3} = 0.024 \cdot e^{j0.8\pi}$$

$$E_{total} = E_{Los} + E_{gnd} + E_1 + E_2 + E_3 = 0.29 \cdot e^{-j0.85\pi}$$

(b)



(b)

$$r_{ke} = \sqrt{10^2 + (50-10)^2} + \sqrt{5^2 + 10^2} = 52.4\,m$$

$$d_{Los} = \sqrt{50^2 + 15^2} = 52.2\,m$$

$$\Delta r = r_{ke} - d_{Los} = 0.2\,m \quad \tau_n = \frac{52.4}{3 \cdot 10^8} = 174.7\,ns$$

Knife Edge Model:

$$v = \sqrt{\frac{2}{\pi} \beta \Delta r} = \sqrt{\frac{2}{\pi} \cdot \frac{2\pi}{\lambda} \cdot 0.2} = \sqrt{\frac{0.8}{\lambda}} \simeq 3.1$$

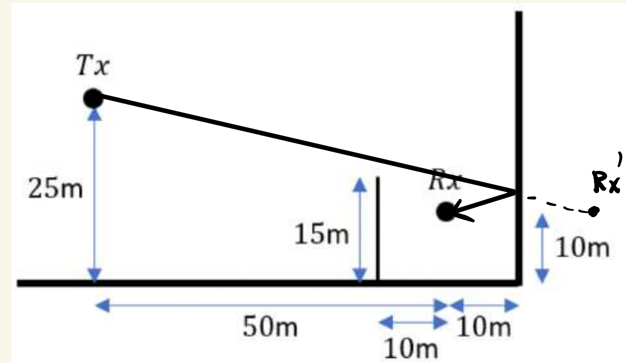
$$\Rightarrow \begin{cases} |F(v)|^2 [dB] \simeq -6.9 - 20\log\left(\sqrt{(v-0.1)^2 + 1} + v - 0.1\right) \\ \angle F(v) = -\frac{\pi}{4} - \frac{\pi}{2} v^2 \end{cases}$$

$$\Rightarrow \begin{cases} |F(v)|^2 [dB] = -6.9 - 20\log(\sqrt{10} + 3) = -22.7\,dB \\ \angle F(v) = 0.95\pi \end{cases}$$

$$E_{ke} = F(v) \cdot \sqrt{60 \cdot P_{Tx} \cdot G_{Tx}} \cdot \frac{e^{-j\beta \cdot r_{ke}}}{r_{ke}} = 0.011 \cdot e^{-j\, 0.65\pi}$$

One-time reflection

$$r_1 = \sqrt{(60+10)^2 + (25-10)^2} = 71.6m$$



$$T_1 = \frac{r_1}{c} = 239ns$$

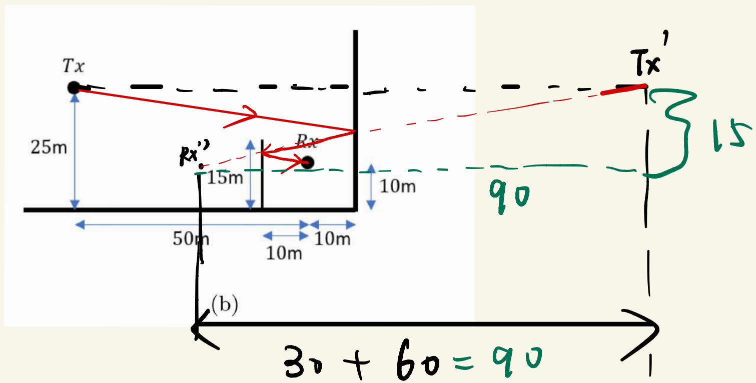
$$\theta = 12.1^\circ$$

$$\Gamma_{\perp}^{1} = -0.37$$

$$E_1 = 0.04 \cdot e^{j\, 0.6\pi}$$

Double Reflection

$$r_2 = \sqrt{90^2 + 15^2} = 91.2\,m$$



$$\tau_2 = \frac{91.2}{3 \cdot 10^8} = 304\,ns$$

$$\theta_2^1 = \theta_2^2 = \operatorname{atan}\left(\frac{15}{90}\right) = 9.5^\circ$$

$$\Rightarrow \Gamma_{\perp 2}^{1} = \Gamma_{\perp 2}^{2} = \frac{\cos(9.5) - \sqrt{4.5}\cdot\sqrt{1 - \frac{1}{4.5}\sin^2(9.5)}}{\cos(9.5) + \sqrt{4.5}\cdot\sqrt{1 - \frac{1}{4.5}\sin^2(9.5)}} = -0.36$$

$$\Rightarrow E_2 = 0.011 \cdot e^{-j0.8\pi}$$

$$E_{total} = E_1 + E_2 + E_{ke} = 0.034 \cdot e^{j0.78\pi}$$

5 Shadowing – Narrowband Fast Fading

5.1 Coverage of an urban macrocell

Let a GSM macrocell base station have the following characteristics: $h_{TX} = 30m$, $P_{TX} = 10W$, $f =$ 900MHz, $G_{TX} = 10$dB. It communicates to a mobile phone located at 1.5m above the ground in a urban environment, equipped with an antenna with $G_{RX} = 0$dB. Use the Okumura model.

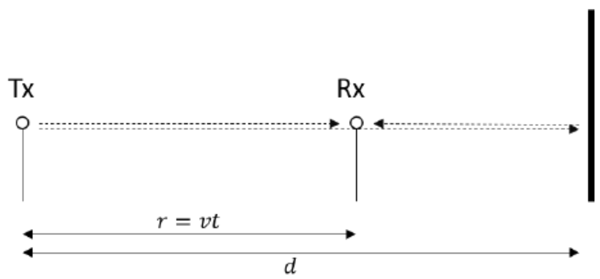
1. What is the cell radius ?
2. What is the cell radius if a 90% probability of communication is required at the cell edge ?
3. What is the communication probability through the whole cell area in this case ?

Note that to evaluate the erfc functions, following approximation can be made:

$$\mathrm{erfc}(x) \sim \frac{3\, e^{-x^2}}{2\sqrt{\pi x^2} + \sqrt{9 + \pi x^2}}$$

5.2 Narrowband fast fading

1. Let a base station be located at the same height as a mobile antenna. As shown in the figure below, the mobile receives two waves, the line-of-sight wave and a wave reflected off a wall located at distance d from the base station. We suppose that both waves have the same complex amplitude α. The mobile is moving at a speed v.



   (a) Give the general expression of the corresponding time-variant channel transfer function h in a local area around the mobile.

   (b) How does the received power vary over time?

2. What should be the average SNR in a Rayleigh channel if the instantaneous SNR must be above 10 dB with a probability 0.999 ?

5.3 Link Budget

1. Assume an 4G-LTE base station transmitting in a 20 MHz bandwidth at a power of 60 W, with gain of 15 dBi. For a receiver with noise figure 10 dB and unit gain antenna:

   (a) what is the maximal allowed path loss for a target SNR of 2dB and total margin of 17dB?

   (b) what should the minimal EIRP of the base station be to ensure a target data rate (capacity) of 40Mbits/s?

5.1. ① $P_{Tx} = 10\cdot\log\frac{10W}{0.0001} = 30\,dBm$

$P_{Rx_{min}} = -102\,dBm$

$\Rightarrow L_{max}[dB] = P_{Tx} - P_{Rx\,min} = 142$

$L = L_0 - G_{Tx} - G_{Rx}$

$\Rightarrow L_0 = L + G_{Tx} + G_{Rx} = 142 + 10 = 152\,dB$

OH:

$$L_0(d) = 69.55 + 10\cdot n\cdot\log\frac{d}{d_0} + 26.16\log f - 13.82\log h_{Tx} - a(h_{Rx})$$

$n = 4.49 - 0.655\log h_{Tx} \approx 3.5$

$\Rightarrow d = d_0\cdot 10^{\frac{L_0(d)-126.4}{10\cdot n}} \Rightarrow d_{max} = 5.3\,km$

② $G_L = 0.65 \cdot \log^2 f - 1.3 \log f + 5.2$

$= 5.67 - 3.84 + 5.2$

$= 7.02\,dB$

$Pr[L_{G_L} > \sigma] = 1 - 0.9 = 0.1 = \frac{1}{2} erfc\left(\frac{\sigma}{\sigma_L \sqrt{2}}\right)$

$\Rightarrow \sigma = 9\,dB$

OA $= \quad \frac{L_0(d_0) - \sigma}{10 \cdot n} - 126.4$

$d_{max} = d_0 \cdot 10$

$d_{max} \approx 3\,km$

③

$$a = \frac{\gamma}{\sqrt{2}\,\sigma_L} = \frac{9}{\sqrt{2}\cdot 7.02} \approx 0.9$$

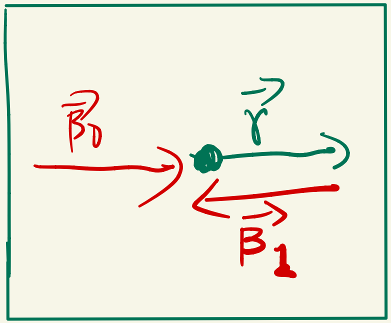
$$b = \frac{1}{\sqrt{2}\,\sigma_L}\cdot 10 \cdot n \cdot \log e$$

$$= 1.53$$

$$F_u = 1 - \frac{1}{2}\mathrm{erfc}(a) + \frac{1}{2}e^{2a/b + 1/b^2}\cdot \mathrm{erfc}\left(a + \frac{1}{b}\right)$$

$$= 1 - 0.1 + 2.46 \cdot 0.028 \approx 0.968$$

5.2 1. a)



$$H = \underbrace{a_0 \cdot e^{j\phi_0}}_{\alpha_1} \cdot e^{-j\vec{\beta}_0 \cdot \vec{r}(t)} + \underbrace{a_1 e^{j\phi_1}}_{\alpha_2} \cdot e^{-j\vec{\beta}_1 \cdot \vec{r}(t)}$$

$$\alpha_1 = \alpha_2 \Rightarrow H = \alpha\left(e^{-j\vec{\beta}_0 \cdot \vec{r}(t)} + e^{-j\vec{\beta}_1 \cdot \vec{r}(t)}\right)$$

$$\vec{r}(t) = v \cdot t \cdot \vec{1}_x, \quad \vec{\beta}_0 = \beta \cdot \vec{1}_x, \quad \vec{\beta}_1 = -\beta \cdot \vec{1}_x$$

$$\Rightarrow H(t) = \alpha\left[e^{-j\beta v t} + e^{j\beta v t}\right]$$

$$= 2\alpha \cos(\beta v t)$$

b) $P_{Rx} \sim \frac{|H(t)|^2}{2} = 2\alpha^2 \cos^2(\beta v t)$

2.

$$\Pr(\gamma < \gamma_T] = 1 - e^{-\gamma_T/\Gamma}$$

$$\Rightarrow \Pr[\gamma > \gamma_T] = e^{-\gamma_T/\Gamma} = 0.999$$

$$\gamma_T = 10\,dB$$

$$\Gamma = \frac{-\gamma_T}{\log_e 0.999} = 9995$$

$$\simeq 40\,dB$$

5.3 $\quad SNR[dB] = P_{Rx}[dBW] - F_{dB} - 10\log(kTB)$

a) $\Rightarrow P_{min_{Rx}}[dBm] = SNR[dB] + F_{dB} + 10\log\left(\frac{kTB}{1mW}\right)$

$SNR = 2dB, \quad F_{dB} = 10dB, \quad k = 1.379 \cdot 10^{-23} W \cdot Hz^{-1} \cdot K^{-1}$

$T = 298.15K \quad B = 20 \cdot 10^{6} Hz$

$= 2dB + 10dB - 100.85dBm$

$= 12dB - 100.85dBm$

$\approx -89\, dBm$

$EIRP = P_{Tx} + G_{Tx} = 10 \cdot \log\frac{60}{1mW} + 15dBi$

$\approx 62.78(dBm)$

$M_T = 17\,dB$

$L_{max} = EIRP - P_{Rx\,min} - M_T$

$= 62.78 - (-89) - 17\,dB$

$= 135\,dB$

b) $C = B \cdot \log_2(1 + SNR)$

$C = 4 \cdot 10^7$ bits/s

$B = 2 \cdot 10^7$ Hz

$\Rightarrow SNR = 4 - 1 = 3 \approx 5$ dB

$P_{min} = -86$ dBm

$\Rightarrow EIRP = L_{max} + P_{Rx_{min}} + M_T = 66$ dBm

6 Long-range Propagation – Optical Fibers

6.1 Microwave links

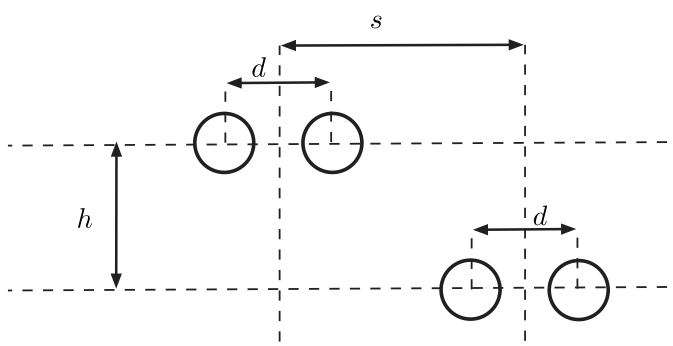
1. What is the horizon distance when you are standing (i) at sea-level (ii) on top of the Eiffel tower (324m) ?
2. A microwave link of length $d = 30$km is established in Belgium between a mast A with height $h_A = 50\,$m, and a mast B with height $h_B = 100\,$m. The working frequency is $f = 6$GHz, the antennas are vertically polarized with a gain of 20dB each.
   (a) What could be the maximal communication distance between these antennas (i) without tropospheric refraction (ii) taking tropospheric refraction into account ?
   (b) What should be the clearance distance to ensure LOS propagation ?
   (c) If this clearance distance is satisfied, compute the path loss.
   (d) Which margin should be added to the path loss to ensure 99.99% availability of the link in rainy weather ?

6.2 Optical Fibers

1. A multi-mode fiber is chosen to operate a LAN with maximal communication distance equal to 1km. Give an estimation of the maximal bit rate if (i) $n_1 = 1.5$ and $n_2 = 1$, (ii) $n_1 = 1.5$ and $\Delta = 2\,10^{-3}$.
2. For the operating wavelength range 1.3-1.6 μm, the fiber is generally designed to become single mode for $\lambda >$1.2 μm. Which core radius has to be chosen for monomode operation if $n_1 = 1.45$ and $\Delta = 5\,10^{-3}$? What should be the value of Δ if the fiber has the usual core radius $a = 4\mu$m ?
3. A 2 Gb/s long range optical communication must be designed @ 1.3μm. The estimated laser spectral width at this wavelength is $\Delta\lambda$ =4 nm. Give an estimation of the maximal distance between repeaters.

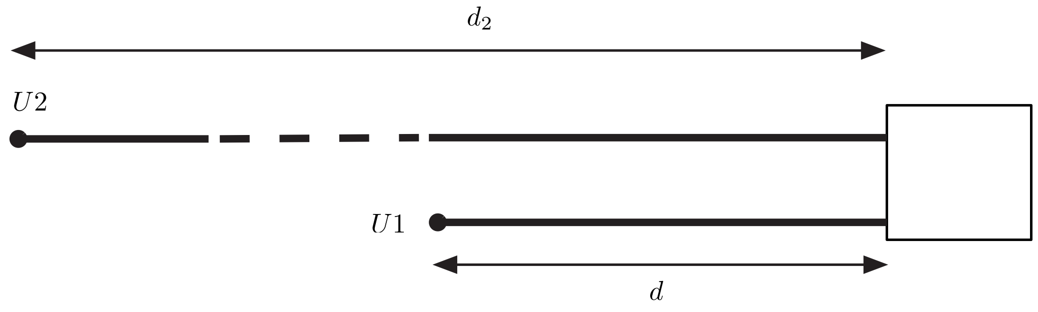
6.3 Two-wire Transmission Lines

1. What is the mutual inductance between the two transmission lines drawn below ? If $h = d$, when is the crosstalk maximum ? Again for $h = d$, at which distance(s) s does the crosstalk vanish, and what is the physical explanation ?



2. Let us consider a DSL network whose topology is drawn below, with $d << d_2$. All signals sent on the lines are supposed to have the same power spectral density $S(f)$. How does the SNR vary with the line length d on both ends of the line for a communication between User 1 and the DSL

router ? To calculate the SNR, suppose that the noise is dominated by the NEXT and/or FEXT crosstalks (which one and why ?). What is the impact of d on the narrowband capacity in the high SNR range?



6.1 1. $R_e = 6.371 \cdot 10^6 (m)$

(1) $d = \sqrt{2 k_e R_e}\left(\sqrt{h_A} + \sqrt{h_B}\right)$, $h_B = 0$

$n = 1 + 10^{-6} \cdot N_t \quad N_t \approx 315 - \frac{315}{H} \cdot h$

$H = 7.35 \cdot 10^3 (m) \quad \frac{dn}{dn} = -10^{-6} \cdot \frac{315}{H} = -42.85 \cdot 10^{-9}$

$$k_e \cdot R_e = \frac{R_e}{1 + R_e \cdot \frac{dn}{dn}} = \frac{6.371 \cdot 10^6}{1 - 0.273} = 8.76 \cdot 10^6$$

$h_A = 1{,}75\ m$

$d = \sqrt{2k_e R_e} \cdot \sqrt{h_A}$

$= 4186{,}5 \cdot 1{,}32$

$\boxed{\approx 5{,}538\ km}$

(2) $h_A = 324\ m$

$d = \sqrt{2k_e R_e} \cdot \sqrt{h_A} = \boxed{75{,}357\ km}$

2.

2. A microwave link of length $d = 30\text{km}$ is established in Belgium between a mast A with height $h_A = 50\,\text{m}$, and a mast B with height $h_B = 100\,\text{m}$. The working frequency is $f = 6\text{GHz}$, the antennas are vertically polarized with a gain of 20dB each.

(a) What could be the maximal communication distance between these antennas (i) without tropospheric refraction (ii) taking tropospheric refraction into account ?

(b) What should be the clearance distance to ensure LOS propagation ?

(c) If this clearance distance is satisfied, compute the path loss.

(d) Which margin should be added to the path loss to ensure 99.99% availability of the link in rainy weather ?

a)

$d = 30\text{km}$, $h_A = 50\text{m}$ $h_B = 100\text{m}$ $G_{Tx} = G_{Rx} = 20\text{dB}$

$f = 6\text{GHz} \Rightarrow \lambda = \frac{c}{f} = \frac{1}{20}$

① $d_{max} = \sqrt{2R_e} \cdot (\sqrt{h_A} + \sqrt{h_B}) = 3569 \cdot (7.07 + 10)$
$= 60.93\text{ km}$

② $d_{max} = \sqrt{2k_eR_e} \cdot (\sqrt{h_A} + \sqrt{h_B}) \simeq 70\text{ km}$

b) $r_{cl} = \frac{3}{10}\sqrt{\lambda d} = 11.6\,m$

c) $L = \frac{P_{Tx}}{P_{Rx}} = \frac{L_{Tx} \cdot L_{Rx}}{G_{Tx} \cdot G_{Rx}} \left(\frac{4\pi d}{\lambda}\right)^2 = 97.6\,dB$

d)

$R = 22\,mm/h$

$k = 2 \cdot 10^{-3}$

$\alpha = 1.25$

$\gamma_R\,[dB/km] = k \cdot R^{\alpha}$

$= 0.095\,dB/km$

$L_{rain} = d \cdot \gamma_R = 30\,km \cdot 0.095 = 2.85\,dB$

6.2 Optical Fibers

1. A multi-mode fiber is chosen to operate a LAN with maximal communication distance equal to 1km. Give an estimation of the maximal bit rate if (i) $n_1 = 1.5$ and $n_2 = 1$, (ii) $n_1 = 1.5$ and $\Delta = 2\,10^{-3}$.

2. For the operating wavelength range 1.3-1.6 μm, the fiber is generally designed to become single mode for $\lambda > 1.2$ μm. Which core radius has to be chosen for monomode operation if $n_1 = 1.45$ and $\Delta = 5\,10^{-3}$? What should be the value of Δ if the fiber has the usual core radius $a = 4\mu$m ?

3. A 2 Gb/s long range optical communication must be designed @ 1.3μm. The estimated laser spectral width at this wavelength is $\Delta\lambda = 4$ nm. Give an estimation of the maximal distance between repeaters.

1.

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.5 - 1}{1.5} = \frac{1}{3}$$

a)

$$B < \frac{c}{\Delta} \cdot \frac{n_2}{n_1^2} \cdot \frac{1}{L} = \frac{3 \cdot 10^8}{\frac{1}{3}} \cdot \frac{1}{\frac{9}{4}} \cdot \frac{1}{1 \cdot 10^3} = 4 \cdot 10^5 \text{ b/s}$$

$$B < 0.4 \text{ Mb/s}$$

b)

$$\Delta = \frac{n_1 - n_2}{n_1} \Rightarrow n_2 = n_1 (1 - \Delta) = 1.5 \cdot (1 - 0.002) \approx 1.497$$

$$B < \frac{3 \cdot 10^8}{2 \cdot 10^{-3}} \cdot \frac{1.497}{(1.5)^2} \cdot \frac{1}{1 \cdot 10^3} = 0.998 \cdot 10^8 \Rightarrow 99.8 \text{ Mb/s}$$

2. $\Delta \Rightarrow n_2 = n_1(1-\Delta) = 1.45 \cdot (1-0.005) \approx 1.44275$

$\lambda = 1.2$

$$\frac{a}{\lambda} < \frac{2.405}{2\pi\sqrt{n_1^2 - n_2^2}} \Rightarrow a < \frac{2.405 \cdot \lambda}{2\pi\sqrt{n_1^2 - n_2^2}} = 3.17\,\mu m$$

$$n_2 < \sqrt{-\left[\left(\frac{2.405 \cdot \lambda_2}{2\pi a}\right)^2 - n_1^2\right]} = 1.4454$$

$$\Rightarrow \Delta = \frac{n_1 - n_2}{n_1} \approx 3.17 \cdot 10^{-3}$$

3) $|D| \approx 2\,\mathrm{ps}/(\mathrm{km}\cdot\mathrm{nm})$ $\quad \Delta\lambda = 4\,\mathrm{mm}$ $\quad B = 2\cdot 10^9\,\mathrm{b/s}$

$$L < \frac{1}{B\,|D|\,\Delta\lambda} = 62.5\,\mathrm{km}$$

Part II

Solutions

1 Preliminaries — Solutions

1.1 Decibels

1. (a) 100 mW $\longrightarrow -10$ dBW = 20 dBm
   (b) 0.75 W $\longrightarrow -1.25$ dBW = 28.75 dBm
   (c) 0.15 V/m $\longrightarrow -16.5$ dBV/m = 103.5 dBμV/m
2. (a) -6 dB $\longrightarrow 0.25$
   (b) 65 dBμV/m $\longrightarrow$ 1.78 mV/m
   (c) 20 dBm + 10 dB − 2.5 dB − 4 dB $\longrightarrow$ 0.224 W
3.
   - $P_{out} = 250$ mW $\longrightarrow G = -5$ dB
   - $P_{out} = 2.5$ W $\longrightarrow G = 10$ dB
   - $P_{out} = 0.5$ W $\longrightarrow G = 3$ dB
   - $G = 12$ dB $\longrightarrow P_{out} = 4$ W = 36 dBm = 6 dBW
4.

-23 dB → 2e-2	0 dB → 1	5 dB → 16/5	10 dB → 10
-12 dB → 1/16	1 dB → 5/4	6 dB → 4	13 dB → 20
-3 dB → 1/2	2 dB → 8/5	7 dB → 5	21 dB → 125
-2 dB → 5/8	3 dB → 2	8 dB → 32/5	37 dB → 5e3
-1 dB → 4/5	4 dB → 5/2	9 dB → 8	40 dB → 1e4

5. (a)
$$EIRP[\text{dBm}] = \underbrace{10\log G_{Tx}(\theta,\phi)}_{G_{Tx}(\theta,\phi)[\text{dB}]} + P_{Tx}[\text{dBm}]$$
   (b)
$$P_{Rx}(d)[\text{dBm}] = \underbrace{10\log G_{Tx}}_{G_{Tx}[\text{dB}]} + \underbrace{10\log G_{Rx}}_{G_{Rx}[\text{dB}]} + P_{Tx}[\text{dBm}] + 20\log(h_{Tx}h_{Rx}) - 40\log d$$

1.2 Plane Waves

2

1. (a)
   - $\beta = 75.4$ rad/m
   - $\vec{1}_\beta = \frac{1}{\sqrt{2}}\vec{1}_x - \frac{1}{\sqrt{2}}\vec{1}_y$

   (b) $\underline{\vec{B}}(\vec{r}) = \dfrac{1}{c}E_0 e^{-j(x-y)\frac{\beta}{\sqrt{2}}}\vec{1}_z$
   - $B_0 = E_0/c = \dfrac{1}{3}.10^{-8}\dfrac{Vs}{m^2}$
   - $\angle\underline{\vec{B}}(\vec{r}) = (y-x)\frac{\beta}{\sqrt{2}}$
   - $\vec{1}_B = \vec{1}_z$

   (c)
   - $|\vec{S}(\vec{r})| = 1.3\dfrac{\text{mW}}{\text{m}^2}$
   - $\vec{1}_S = \frac{1}{\sqrt{2}}\vec{1}_x - \frac{1}{\sqrt{2}}\vec{1}_y = \vec{1}_\beta$

2 Wireless Channel Models — Solutions

2.1 Single path channel

1. $h(\tau) = a_0 e^{j\phi_0} e^{-j2\pi f_c \tau_0} \delta(\tau - \tau_0) = \alpha_0 \delta(\tau - \tau_0)$
2. $H(f) = a_0 e^{j\phi_0} e^{-j2\pi(f_c+f)\tau_0} = \alpha_0 e^{-j2\pi f \tau_0}$
   $|H(f)|^2 = \text{cte}$
3.
   - $h_{TDL}(\tau) = \sum_{l=-\infty}^{\infty} h_l \delta(\tau - l\Delta\tau)$
   - $h_l = a_0 e^{j\phi_0} e^{-j2\pi f_c \tau_0} \text{sinc}[2B(\tau_0 - l\Delta\tau)]$
   - Uncorrelated Scattering: $h_l = \begin{cases} a_0 e^{j\phi_0} e^{-j2\pi f_c \tau_0} & l = 34 \\ 0 & \text{elsewhere} \end{cases}$

with: $a_0 = 6.63.10^{-5}$, $\phi_0 = 0$, $\tau_0 = 333$ ns, and $\Delta\tau = 10$ ns

2.2 Two-path channel

1. $h(\tau) = a_0 e^{j\phi_0} e^{-j2\pi f_c \tau_0} \delta(\tau - \tau_0) + a_1 e^{j\phi_1} e^{-j2\pi f_c \tau_1} \delta(\tau - \tau_1) = \alpha_0 \delta(\tau - \tau_0) + \alpha_1 \delta(\tau - \tau_1)$
   $H(f) = a_0 e^{j\phi_0} e^{-j2\pi(f_c+f)\tau_0} + a_1 e^{j\phi_1} e^{-j2\pi(f_c+f)\tau_1} = \alpha_0 e^{-j2\pi f \tau_0} + \alpha_1 e^{-j2\pi f \tau_1}$
   $|H(f)|^2 \neq \text{cte}$
2. $h_{TDL}^{WB}(\tau) = h_{10} \delta(\tau - 10\Delta\tau) + h_{27} \delta(\tau - 27\Delta\tau)$
   $h_{10} = a_0 e^{j\phi_0} e^{-j2\pi f_c \tau_0}$
   $h_{27} = a_1 e^{j\phi_1} e^{-j2\pi f_c \tau_1}$
3. $h_{TDL}^{NB}(\tau) = (a_0 e^{j\phi_0} e^{-j2\pi f_c \tau_0} + a_1 e^{j\phi_1} e^{-j2\pi f_c \tau_1}) \delta(\tau)$
4. $54\text{m} < r < 55\text{m}$

with: $a_0 = 2.21.10^{-4}$, $\phi_0 = 0$, $\tau_0 = 100$ ns, $a_1 = -8.29.10^{-5}$, $\phi_1 = 0$, $\tau_1 = 267$ ns, and $\Delta\tau = 10$ ns

3 Physical Propagation Models — Solutions

3.1 Cellular link

1. $EIRP = 50$ dBm $= 20$ dBW $= 100$ W
2. $P_{Rx} = -28$ dBm $= -58$ dBW $= 1.6$e-6 W
3. $d_{ICNIRP} = 1.3$ m, $d_{BXL} = 9.1$ m

3.2 Coverage of a macrocell in the countryside

1. $d_{max} = 1056$ km
2. $L_0(d) = L_0(d_0) + 10.4 \log(d/d_0)$
3. $d_{BP} = 1131$ m
4. $d_{max} = 35$ km

3.3 NLOS propagation

- (3.6 GHz) $L_{ke} = -44.5$ dB , $P_{Rx,nlos} = -71.9$ dBm
- (26 GHz) $L_{ke} = -52.9$ dB , $P_{Rx,nlos} = -95.3$ dBm

4 Ray-tracing — Solutions

1. - $\vec{E}^{los}(r^{los}) = j\dfrac{Z_0 I_{in}}{2\pi}\dfrac{\cos(\frac{\pi}{2}\cos\theta_{tx}^{los})}{\sin\theta_{tx}^{los}}\dfrac{e^{-j\beta r^{los}}}{r^{los}}\vec{1}_\theta$
   - $V_{oc}^{los} = j\dfrac{Z_0 I_{in}}{\pi\beta}\dfrac{\cos(\frac{\pi}{2}\cos\theta_{tx}^{los})}{\sin\theta_{tx}^{los}}\dfrac{\cos(\frac{\pi}{2}\cos\theta_{rx}^{los})}{\sin\theta_{rx}^{los}}\dfrac{e^{-j\beta r^{los}}}{r^{los}}$
   - $\vec{E}^{gnd}(r^{gnd}) = \Gamma_{\parallel}(\theta_{in}^{gnd}) j\dfrac{Z_0 I_{in}}{2\pi}\dfrac{\cos(\frac{\pi}{2}\cos\theta_{tx}^{gnd})}{\sin\theta_{tx}^{gnd}}\dfrac{e^{-j\beta r^{gnd}}}{r^{gnd}}\vec{1}_\theta$
   - $V_{oc}^{gnd} = \Gamma_{\parallel}(\theta_{in}^{gnd}) j\dfrac{Z_0 I_{in}}{\pi\beta}\left(\dfrac{\cos(\frac{\pi}{2}\cos\theta^{gnd})}{\sin\theta^{gnd}}\right)^2\dfrac{e^{-j\beta r^{gnd}}}{r^{gnd}}$
   - $r^{los} = \sqrt{d^2 + (h_{Tx} - h_{Rx})^2}$
   - $\theta_{Tx}^{los} = \dfrac{\pi}{2} + \arctan\dfrac{h_{Tx} - h_{Rx}}{d}$
   - $\theta_{Rx}^{los} = \dfrac{\pi}{2} - \arctan\dfrac{h_{Tx} - h_{Rx}}{d}$
   - $r^{gnd} = \sqrt{d^2 + (h_{Tx} + h_{Rx})^2}$
   - $\theta_{in}^{gnd} = \dfrac{\pi}{2} - \arctan\dfrac{h_{Tx} + h_{Rx}}{d}$
   - $\theta^{gnd} = \theta_{tx}^{gnd} = \theta_{rx}^{gnd} = \dfrac{\pi}{2} + \arctan\dfrac{h_{Tx} + h_{Rx}}{d}$

2. (a)
   - $r_{los} = 52.2\text{m}$, $\tau_{los} = 174\text{ns}$
     $\underline{E}_{los} = 0.15e^{-j0.8\pi}$
   - $r_{gnd} = 52.4\text{m}$, $\tau_{gnd} = 175\text{ns}$
     $\theta_{in,gnd} = 85.6°$, $\Gamma_{gnd} = -0.69$
     $\underline{E}_{gnd} = 0.10e^{-j0.6\pi}$
   - $r_1 = 71.6\text{m}$, $\tau_1 = 239\text{ns}$
     $\theta_{in,1} = 12.1°$, $\Gamma_1 = -0.37$
     $\underline{E}_1 = 0.040e^{j0.6\pi}$
   - $r_2 = 61.0\text{m}$, $\tau_2 = 203\text{ns}$
     $\theta_{in,2} = 55.0°$, $\Gamma_2 = -0.55$
     $\underline{E}_2 = 0.070e^{-j\pi}$
   - $r_3 = 78.3\text{m}$, $\tau_3 = 261\text{ns}$
     $\theta_{in,3}^1 = 63.4°$, $\Gamma_3^1 = -0.62$
     $\theta_{in,3}^2 = 26.6°$, $\Gamma_3^2 = -0.40$
     $\underline{E}_3 = 0.025e^{j0.8\pi}$
   - $\underline{E}_{tot} = 0.29e^{-j0.85\pi}$

   (b)
   - $r_1 = 71.6\text{m}$, $\tau_1 = 239\text{ns}$
     $\theta_{in,1} = 12.1°$, $\Gamma_1 = -0.37$
     $\underline{E}_1 = 0.040e^{j0.6\pi}$
   - $r_2 = 91.2\text{m}$, $\tau_2 = 304\text{ns}$
     $\theta_{in,2}^1 = 9.5°$, $\Gamma_2^1 = -0.36$
     $\theta_{in,2}^2 = 9.5°$, $\Gamma_2^2 = -0.36$
     $\underline{E}_2 = 0.011e^{-j0.8\pi}$
   - $r_{ke} = 52.4\text{m}$, $\tau_{ke} = 175\text{ns}$
     $|F(\nu)| = -22.7\text{dB}$, $\angle F(\nu) = 0.95\pi$
     $\underline{E}_{ke} = 0.011e^{-j0.65\pi}$
   - $\underline{E}_{tot} = 0.034e^{j0.78\pi}$

5 Shadowing – Narrowband Fast Fading — Solutions

5.1 Coverage of an urban macrocell

1. $d_{max} = 5.3$ km
2. $d_{max} = 3.0$ km
3. $F_u = 0.97$

5.2 Narrowband fast fading

1. (a) $h(t) = \alpha e^{-j\bar{\beta}_0 \cdot \bar{r}} + \alpha e^{-j\bar{\beta}_1 \cdot \bar{r}} = 2\alpha \cos(\beta v t)$
   (b) $P_{Rx}(t) = |h(t)|^2/2 = 2\alpha^2 \cos^2(\beta v t)$
2. $\Gamma = 40$ dB

5.3 Link Budget

1. (a) $L_{max} = 135$ dB
   (b) $EIRP = 66$ dBm

6 Long-range Propagation – Optical Fibers — Solutions

6.1 Microwave links

1. (i) $d = 5.45$ km , (ii) $d = 74.2$ km
2. (a) (i) $d = 61$ km , (ii) $d = 70$ km
   (b) $r_{cl} = 11.6$ m
   (c) $L = 97.6$ dB
   (d) $L_{rain} = 2.86$ dB

6.2 Optical Fibers

1. (i) $B < 0.4$ Mb/s
   (ii) $B < 99.8$ Mb/s
2. $a < 3.17\mu$m
   $\Delta = 3.17.10^{-3}$
3. $L < 62.5$ km

6.3 Two-wire Transmission Lines

1. $M_{12} = \dfrac{\mu}{2\pi} \ln \left(\dfrac{1 + (s/h)^2}{\sqrt{1 + (s/h - d/h)^2}\sqrt{1 + (s/h + d/h)^2}} \right)$
   Maximal crosstalk: $s = 0$
   Zero crosstalk: $s = \sqrt{3/2}h = \sqrt{3/2}d$ and $s = \infty$
2. $h(f) = e^{-\alpha(f)d}e^{-j\beta(f)d}$
   $C = B \log_2(1 + SNR) \overset{SNR\to\infty}{\approx} B \log_2(SNR)$
   - $U_1 \to$ Router:
     - NEXT dominant
     - $SNR = \dfrac{e^{-2\alpha(f)d}}{K_N f^{3/2}}$, with K_N the proportionality constant for $|h_{NEXT}|^2$
     - linear capacity increase for decreasing distance (and v.v.)
   - Router $\to U_1$:
     - FEXT dominant
     - $SNR = \dfrac{1}{K_F f^2 d}$, with K_F the proportionality constant for $|h_{FEXT}|^2$
     - logarithmic capacity increase for decreasing distance (and v.v.)