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UNIT-1

Mathematical Logic

Statement (or) Preposition :- A declarative sentence or mathematical equation or expression which is either true or false but not both is called statement (or) Preposition.

In General we denote the Prepositions p, q, r, s, ... etc

$p: 4+5=9$

$q: 9\times9=81$

$r: 5^2>4^2$

→ Every Preposition has unique value called as truth value.

→ If the Preposition is true its truthvalue is denoted by '1' (or) 'T'

→ If the Preposition is false its truthvalue is denoted by '0' (or) 'F'

the truthvalue of p is "1"

the truthvalue of q is "1"

the truthvalue of r is "1"

Connectives:

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1. Negation (NOT):-

let 'p' be any given Preposition then the negation of p is denoted by $\sim p$ / $\neg p$ (Read as negation p or Not p) and it is obtained by inserting the word "NOT" at an appropriate place in p.

Rule:

If P is true then $\sim P$ is false

If P is false then $\sim P$ is true.

Truth table for Negation

P	$\sim P$
1	0
0	1

Ex: $P: 5+4=9$

$\sim P: 5+4 \neq 9$

q: New Delhi is capital of India

$\sim q$: New Delhi is not capital of India.

2. Conjunction (AND):-

Let P, q be two prepositions then the conjunction of P, q is denoted by $P \wedge q$ (Read as P AND Q) and it is obtained by inserting the word "AND" between P, q.

Rule:

$P \wedge q$ is true only when both P, q are true in all other remaining cases $P \wedge q$ is false.

Truth table:-

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

Ex: $p: 4+3=7$

$q: 2-5=0$

$r: 5+6=11$

$s: 6+3=12$

then the truthvalue of $p \wedge q$ is "0"

the truthvalue of $r \wedge p$ is "1"

the truthvalue of $s \wedge q$ is "0".

3. Disjunction (OR):-

let p, q be two Prepositions then the disjunction of p, q is denoted by $p \vee q$ (Read as p OR q) and it is obtained by inserting the word "OR" between p, q.

Rule:

$p \vee q$ is false only when both p, q are false.

In all other remaining cases $p \vee q$ is true.

Truthtable

P	q	$P \vee q$
1	1	1
1	0	1
0	1	1
0	0	1

Ex: $P : 4+3=7$

$q : 2-5=0$

$r : 6+5=11$

$s : 6+3=12$

then the truthvalue of $P \vee q$ is "1".

the truth value of $r \vee P$ is "1".

the truth value of $s \vee q$ is "0".

4. Implication or Conditional [If then]

let P, q be two PrePositions. then the statements If P then q is called Implication or Conditional statement and it is denoted $P \rightarrow q$. Read as P implies q.

In $P \rightarrow q$, P is called Antecedent first term

q is called Consequent second term.

Rule:

the Conditional statement $P \rightarrow q$ is false only when P is true and q is false. In all other remaining cases $P \rightarrow q$ is true.

Truthtable

P	q	$P \rightarrow q$	$q \rightarrow P$
1	1	1	1
1	0	0	1
0	1	1	0
0	0	1	1

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5. Bi-implication or Bi-conditional [if and only if] :-

Let P, q be two Prepositions then the satatements P if and only if q is called bi-implication or Bi-Conditional Statement and it is denoted by $P \leftrightarrow q$ [Read as P implies and implied by q]

Rule

$P \leftrightarrow q$ is true if both P, q have same truth values and in all other cases $P \leftrightarrow q$ is false

Truthtable

P	q	$P \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

Compound Preposition:-

One or more Prepositions combined with Connecti[ves] is called a compound Prepositions.

Ex:

$P \rightarrow (q \wedge r)$

$P \rightarrow q$

Compound Prepositions are classified into three types.

1. Tautology:

A compound Preposition which is true in all cases is called Tautology. and it is denoted by 'T_0'

2. Contradiction:

A compound Preposition which is false in all cases is called contradiction and it is denoted by 'F_0'

3. Contingency:

A Compound Preposition which is neither tautology nor Contradiction is called contigency.

Note:

1. A compound Preposition contains different 'n' variable then its truthtable must contain 2^n rows.

1. Show that $(P \to q) \leftrightarrow (\sim P \vee q)$ is tautology.

P	q	$P \to q$	$\sim P$	$\sim P \vee q$	$(P \to q) \leftrightarrow (\sim P \vee q)$
1	1	1	0	1	1
1	0	0	0	0	1
0	1	1	1	1	1
0	0	1	1	1	1

The compound Preposition is true in all cases then it is tautology.

2. Construt a touth table for $P \to (q \wedge r)$

P	q	r	$q \wedge r$	$P \to (q \wedge r)$
1	1	1	1	1
1	1	0	0	0
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1
0	1	0	0	1
0	0	1	0	1
0	0	0	0	1

The Compound Preposition is Contingency.

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Well-formed-formula

A compound preposition represented the symbolic form which can't be intrapreted in more than one way is called well-formed-format.

Appropriate brackets are to be used in well-formed formulas.

Ex: 1. $(P \to Q) \to r$ — well-formed formula
2. $P \to Q \to r$ — Not well-formed-formula.

The following are considered as well-formed formulas,

1. Premitive statements
2. The Negation of well-formed-formula
3. The conjunction, Disjunction, implication, Bi-implication statements whose components are well-formed-formulas.

Construct a truth table for $(\sim P) \to (P \to Q)$

P	Q	$P \to Q$	$\sim P$	$(\sim P) \to (P \to Q)$
1	1	1	0	1
1	0	0	0	1
0	1	1	1	1
0	0	1	1	1

The given compound preposition is tautology.

Construct a truth table for $[P \wedge (P \rightarrow q)] \rightarrow q$.

P	q	$P \rightarrow q$	$P \wedge (P \rightarrow q)$	$[P \wedge (P \rightarrow q)] \rightarrow q$
1	1	1	1	1
1	0	0	0	1
0	1	1	0	1
0	0	1	0	1

The given Compound Preposition is tautology.

3. $[(P \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(P \vee q) \rightarrow r]$

P	q	r	$P \rightarrow r$	$q \rightarrow r$	S	$P \vee q$	$(P \vee q) \rightarrow r$	$S \rightarrow M$
1	1	1	1	1	1	1	1	1
1	1	0	0	0	0	1	0	1
1	0	1	1	1	1	1	1	1
1	0	0	0	1	0	1	0	1
0	1	1	1	1	1	1	1	1
0	1	0	1	0	0	1	0	1
0	0	1	1	1	1	0	1	1
0	0	0	1	1	1	0	1	1

The given Compound Preposition is tautology.

4. $p \wedge (q \wedge \sim q)$

p	q	$\sim q$	$q \wedge \sim q$	$p \wedge (q \wedge \sim q)$
1	1	0	0	0
1	0	1	0	0
0	1	0	0	0
0	0	1	0	0

the given compound Preposition is Contradiction.

5. $p \rightarrow (q \rightarrow r)$

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$
1	1	1	1	1
1	1	0	0	0
1	0	1	1	1
1	0	0	1	1
0	1	1	1	1
0	1	0	0	1
0	0	1	1	1
0	0	0	1	1

The given compound Preposition is Contingency.

Logical Equivalence :-

let A, B are two compound prepositions if $A \leftrightarrow B$ is a tautology then we say that A, B are logically equivalent and denoted as $A \cong B$ / $A \Leftrightarrow B$.

If A, B are not logical equivalent then we denoted as $A \ncong B$.

Verify $p \to q$, $(\sim p \vee q)$ are logically equivalent or not?

verify $(p \to q) \cong (\sim p \vee q)$ or not.

let $A = p \to q$
$B = \sim p \vee q$

p	q	$p \to q$	$\sim p \vee q$	$(p \to q) \leftrightarrow (\sim p \vee q)$
1	1	1	1	1
1	0	0	0	1
0	1	1	1	1
0	0	1	1	1

the given compound $A \leftrightarrow B$ is tautology

$\therefore A \cong B$.

Verify $\sim(p \vee q) \cong (\sim p \wedge \sim q)$

let $A = \sim(p \vee q)$, $B = (\sim p \wedge \sim q)$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$A \leftrightarrow B$
1	1	1	0	0	0	0	1
1	0	1	0	0	1	0	1
0	1	1	0	1	0	0	1
0	0	0	1	1	1	1	1

$\therefore A \leftrightarrow B$ is tautology.
$\therefore A \cong B$.

Verify $P \cong \sim(\sim P)$

let $A = P$ $\quad B = \sim(\sim P)$

P	$\sim P$	$\sim(\sim P)$	$P \leftrightarrow (\sim(\sim P))$
1	0	1	1
0	1	0	1

$\therefore P \leftrightarrow \sim(\sim P)$ is tautology.

$\therefore A \cong B.$

$$\boxed{\Rightarrow P \cong \sim(\sim P).}$$

Verify $[P \rightarrow (q \wedge r)] \cong [(P \rightarrow q) \wedge (P \rightarrow r)]$

let $A = P \rightarrow (q \wedge r)$ $\quad B = (P \rightarrow q) \wedge (P \rightarrow r)$

P	q	r	$q \wedge r$	A	$P \rightarrow q$	$P \rightarrow r$	B	$A \leftrightarrow B$
1	1	1	1	1	1	1	1	1
1	1	0	0	0	1	0	0	1
1	0	1	0	0	0	1	0	1
1	0	0	0	0	0	0	0	1
0	1	1	0	1	1	1	1	1
0	1	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	0	0	0	1	1	1	1	1

$\therefore A \leftrightarrow B$ is tautology.

$A \cong B.$ $\quad \Rightarrow [P \rightarrow (q \wedge r)] \cong [(P \rightarrow q) \wedge (P \rightarrow r)]$

Verify $p \to (q \to r) \cong (p \to q) \to r$ (or) not.

p	q	r	$q \to r$	A	$p \to q$	B	$A \leftrightarrow B$
1	1	1	1	1	1	1	1
1	1	0	0	0	1	0	1
1	0	1	1	1	0	1	1
1	0	0	1	1	0	1	1
0	1	1	1	1	1	1	1
0	1	0	0	1	1	0	0
0	0	1	1	1	1	1	1
0	0	0	1	1	1	0	0

$A \leftrightarrow B$ is not tautology

$\therefore A \not\cong B$

$p \to (q \to r) \not\cong (p \to q) \to r$

Verify $[(p \to q) \wedge (p \to \sim q)] \cong \sim p$

p	q	$p \to q$	$\sim q$	$(p \to \sim q)$	A	$\sim p$	$A \leftrightarrow B$.
1	1	1	0	0	0	0	1
1	0	0	1	1	0	0	1
0	1	1	0	1	1	1	1
0	0	1	1	1	1	1	1

$A \leftrightarrow B$ is tautology.

$\therefore A \cong B$.

Verify $(p \to q) \cong (\sim q \to \sim p)$

P	Q	$P \to Q$	$\sim P$	$\sim Q$	$\sim Q \to \sim P$	$A \leftrightarrow B$
1	1	1	0	0	1	1
1	0	0	0	1	0	1
0	1	1	1	0	1	1
0	0	1	1	1	1	1

$A \leftrightarrow B$ is tautology.

$\therefore A \cong B$

$\therefore (P \to Q) \cong (\sim Q \to \sim P)$

Converse, Inverse, Contrapositive of an Implication:-

let $P \to Q$ is an implication then.

1. "$Q \to P$" is called Converse of "$P \to Q$"
2. "$\sim P \to \sim Q$" is called Inverse of '$P \to Q$'
3. "$\sim Q \to \sim P$" is called Contrapositive of "$P \to Q$."

Laws of Logic:-

Set	logic.
$\cup$	$\vee$
$\cap$	$\wedge$
$'$	$\sim$
$=$	$\cong$
μ	T_0
$\emptyset$	F_0

1. Double-Negation law:-

$\sim(\sim P) \cong P$ $\qquad (A')' = A$

2. Idempotent law:-

$P \vee P \cong P$ $\qquad A \cup A = A$

$P \wedge P \cong P$ $\qquad A \cap A = A$

3. Commutative law :-

- $p \vee q \cong q \vee p$
- $p \wedge q \cong q \wedge p$

$A \cup B = B \cup A$
$A \cap B = B \cap A$

4. Associative law:-

- $p \vee (q \vee r) \cong (p \vee q) \vee r$
- $p \wedge (q \wedge r) \cong (p \wedge q) \wedge r.$

- $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (B \cap C) = (A \cap B) \cap C$

5. Distributive law:

$p \vee (q \wedge r) \cong (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \cong (p \wedge q) \vee (p \wedge r).$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. Identity laws:

$p \vee F_0 \cong p$
$p \wedge T_0 \cong p$

$A \cup \phi = A$
$A \cap \mu = A$

7. Inverse laws:-

$p \vee (\sim p) \cong T_0$
$p \wedge (\sim p) \cong F_0.$

$A \cup A' = \mu$
$A \cap A' = \phi$

8. Domination laws:-

$p \vee T_0 \cong T_0$
$p \wedge F_0 \cong F_0$

$A \cup \mu = \mu$
$A \cap \phi = \phi$

9. Demorgan's law:

$\sim (p \vee q) \cong \sim p \wedge \sim q$
$\sim (p \wedge q) \cong \sim p \vee \sim q.$

$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

10. Absorption law:-

$P \vee (P \wedge Q) \cong P$ $\qquad$ $A \cup (A \cap B) = A$

$P \wedge (P \vee Q) \cong P$. $\qquad$ $A \cap (A \cup B) = A$

11. Implication law:-

$(P \leftrightarrow Q) \cong (P \rightarrow Q) \wedge (Q \rightarrow P)$

$(P \rightarrow Q) \cong \sim Q \rightarrow \sim P$

$(P \rightarrow Q) \cong \sim P \vee Q$.

Definitions of logical Equivalence without using truth tables.

we use the following Procedure to verify the logical equivalence without truthtables.

1. Elimation of $\leftrightarrow$, if exists
2. Elimination of $\rightarrow$, if exists.
3. Apply Demorgan's law if exists.
4. Apply Distributive or Associative laws, if exists.
5. Apply any other law is required.

1. P.T $P \vee [P \wedge (P \vee Q)] \cong P$.

$P \vee [P \wedge (P \vee Q)]$

$\cong P \vee P$ [By Absorption law]

$\cong P$ [Idempotent law]

$\therefore P \vee [P \wedge (P \vee Q)] \cong P$

2. S.T $p \to (q \to r) \cong (p \wedge q) \to r$

$p \to (q \to r)$

$\cong p \to (\sim q \vee r)$ [Implication law]

$\cong \sim(p) \vee (\sim q \vee r)$ [" "]

$\cong (\sim p \vee \sim q) \vee r$ [Associative law]

$\cong \sim(p \wedge q) \vee r$ [Demorgan's law]

$\cong (p \wedge q) \to r.$ [Implication law].

$p \to (q \to r) \cong (p \wedge q) \to r$

3. S.T $\sim [p \vee (\sim p \wedge q)] \cong (\sim p \wedge \sim q)$

$\cong [p \vee (\sim p \wedge q)]$

$\cong \sim p \wedge \sim (\sim p \wedge q)$

$\cong \sim p \wedge (\sim(\sim p) \vee \sim q)$

$\cong \sim p \wedge (p \vee \sim q)$

$\cong (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$\cong F_0 \vee (\sim p \wedge \sim q)$

$\cong \sim p \wedge \sim q.$

$\therefore \sim [p \vee (\sim p \wedge q)] \cong \sim p \wedge \sim q.$

4. $[\sim p \wedge (\sim q \wedge r)] \vee [q \wedge r] \vee [p \wedge r] \cong r.$

consider

$[\sim p \wedge (\sim q \wedge r)] \vee [q \wedge r] \vee [p \wedge r]$

$\cong [(\sim p \wedge \sim q) \wedge r] \vee [q \wedge r] \vee (p \wedge r]$

$\cong [\sim(p \lor q) \land r] \lor [r \land q] \lor (r \land p)$

$\cong [\sim(p \lor q) \land r] \lor \{r \land (q \lor p)\}$

$\cong [r \land (\sim(p \lor q))] \lor [r \land (p \lor q)]$

$\cong r \land [\sim(p \lor q) \lor (p \lor q)]$

$\cong r \land T_0$

$\cong r.$

$\therefore [\sim p \land (\sim q \land r)] \lor [q \land r] \lor [p \land r] \cong r$

S.T $([p \lor q] \land [p \lor \sim q]) \lor q \cong p \lor q.$

$([p \lor q] \land [p \lor \sim q]) \lor q$

$\cong [p \overset{\lor}{\land} [q \land \sim q]] \lor q$

$\cong (p \overset{\lor}{\land} F_0) \lor q$

$\cong p \lor q.$

$[[p \lor q] \land (p \lor \sim q)] \lor q \cong p \lor q.$

S.T $(p \rightarrow r) \land (q \rightarrow r) \cong (p \lor q) \rightarrow r.$

$(p \rightarrow r) \land (q \rightarrow r)$

$\cong (\sim p \lor \overset{r}{q}) \land (\sim q \lor r)$ [∵ Implication law]

$\cong (r \lor \sim p) \land (r \lor \sim q)$ [∵ Commutative law]

$\cong r \lor (\sim p \land \sim q)$ [∵ Distributive (Associative) law]

$\cong r \lor \sim(p \lor q)$ [∵ Demorgan's law]

$\cong \sim(P \vee q) \vee r$ ∵ [Commutative law]

$\cong (P \vee q) \rightarrow r$ [∵ Implication law]

$\therefore (P \rightarrow r) \wedge (q \rightarrow r) \cong (P \vee q) \rightarrow r.$

S.T $P \rightarrow (q \vee r) \cong (P \rightarrow q) \vee (P \rightarrow r)$

L.H.S. $P \rightarrow (q \vee r)$

$\cong \sim P \vee (q \vee r)$ [∵ Implication law]

$\cong (\sim P \vee q) \vee r$. [∵ Associative law]

$\cong (P \rightarrow q) \vee r$

R.H.S $(P \rightarrow q) \vee (P \rightarrow r)$

$\cong (\sim P \vee q) \vee (\sim P \vee r)$

$\cong (q \vee \sim P) \vee (\sim P \vee r)$

$\cong q \vee (\sim P \vee \sim P) \vee r$

$\cong (q \vee \sim P) \vee r$

$\cong (\sim P \vee q) \vee r$

$\cong (P \rightarrow q) \vee r$

$\therefore$ L.H.S = R.H.S

$\therefore P \rightarrow (q \vee r) \cong (P \rightarrow q) \vee (P \rightarrow r).$

Normal forms

Elimentary Sums :-

In a logical expression the disjunction of Prepositions and their negations is called elimentary sums / sums.

Ex:

$p \vee q$

$p \vee q \vee r$

$p \vee q \vee \sim r$

$\sim p \vee \sim q \vee \sim r \vee p$.

Elimentary Products

In a logical expression the conjunction of Prepositions and their negations is called elimentary Products / Products.

Ex:- $p \wedge q$

$p \wedge q \wedge r$

$p \wedge q \wedge \sim r$

$\sim p \wedge \sim q \wedge \sim r \wedge p$.

Disjunctive Normal form (DNF)

A logical expression is said to be in DNF if it is Sum of Products (SOP)

Ex. (Pro) v (Prod) v (Pro)

$(p \wedge q) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q)$

$(\sim p \wedge \sim q) \vee (\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge \sim r)$

Conjunctive Normal form :-

A logical expression is said to be in CNF if it is Product of sums (Pos)

$(sum) \wedge (sum) \wedge (sum).$

$(P \vee Q) \wedge (P \vee r) \wedge (P \vee Q \vee r)$

$(\sim P \vee \sim Q) \wedge (\sim P \vee \sim r) \wedge (\sim P \vee \sim Q \vee r)$

Procedure for finding DNF/CNF :-

1. Eliminate $\longleftrightarrow$ if exists.
2. Eliminate $\longrightarrow$ if exists.
3. Apply Demorgan's law if necessary.
4. Apply Distributive (or) Associative law if necessary.
5. Apply any other law to get the result.

1. find DNF and CNF of $P \leftrightarrow Q$.

$P \leftrightarrow Q$

$= (P \to Q) \wedge (Q \to P)$

C.N.F $= (\sim P \vee Q) \wedge (\sim Q \vee P)$

$= [(\sim P \vee Q) \wedge \sim Q] \vee [(\sim P \vee Q) \wedge P]$

$= [(\sim Q \wedge \sim P) \vee (\sim Q \vee Q)] \vee [(P \wedge \sim Q) \vee (P \wedge Q)]$

D.N.F $= (\sim Q \wedge \sim P) \vee (\sim Q \vee Q) \vee (P \wedge \sim Q) \vee (P \wedge Q)$

2. find DNF and CNF of $\cdot P \vee [\sim P \to (Q \vee (Q \to \sim r))]$

$P \vee [\sim P \to (Q \vee (\sim Q \vee \sim r))]$

$P \vee [P \vee (Q \vee (\sim Q \vee \sim r))]$

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Principle disjunctive Normal form (PDNF):-

A logical expression is said to be in PDNF form if it is sum of min. terms.

Ex: 1. $(P \wedge Q) \vee (P \wedge \sim Q)$

2. $(P \wedge Q \wedge \sim r) \vee (\sim P \wedge \sim Q \wedge r)$

Principle Conjunctive Normal form (PCNF)

A logical expression is said to be ~~(PDCNF)~~ PCNF form if it is Product of max. terms

Ex: 1. $(P \vee Q) \wedge (P \vee \sim Q)$

2. $(P \vee Q \vee \sim r) \wedge (\sim P \vee \sim Q \vee r)$

Note: 1. for any given compound preposition PDNF/PCNF is always unique.

Procedure to find PDNF using truthtables

1. Construct truth table for given compound preposition.
2. For each truth value 1 of given C.P write min.terms
3. While writing min terms, the variable has truth value 1 write the preposition. and the variable has truth value 0 write its negation.

4. The sum of min terms so obtained is the required PDNF.

1. Find PDNF of $P \leftrightarrow Q$.

P	Q	$P \leftrightarrow Q$	
1	1	1	$\longrightarrow$ ① $P \wedge Q$
1	0	0	
0	1	0	
0	0	1	$\longrightarrow$ ② $\sim P \wedge \sim Q$.

The C.P is true in two cases: case ①, case ②

The min terms in each two cases are $P \wedge Q, \sim P \wedge \sim Q$.

The sum of min terms is $(P \wedge Q) \vee (\sim P \wedge \sim Q)$

2. Find PDNF of $\sim P \vee Q$.

P	Q	$\sim P$	$\sim P \vee Q$	
1	1	0	1	$\longrightarrow$ ① $Q \wedge P$
1	0	0	0	
0	1	1	1	$\longrightarrow$ ② $Q \wedge \sim P$
0	0	1	1	$\longrightarrow$ ③ $\sim P \wedge \sim Q$

The C.P is true in three case : case ①, case ②, case ③

the min terms in each case is $Q \wedge P, \sim P \wedge Q, \sim P \wedge \sim Q$

The sum of min terms is $(P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$

3. Find PDNF of $[P \rightarrow (q \wedge r)] \wedge [(\sim p \rightarrow (\sim q \wedge \sim r))]$

p	q	r	$q \wedge r$	A	$\sim p$	$\sim q$	$\sim r$	$(\sim q \wedge \sim r)$	B
1	1	1	1	1	0	0	0	0	1
1	1	0	0	0	0	0	1	0	1
1	0	1	0	0	0	1	0	0	1
1	0	0	0	0	0	1	1	1	1
0	1	1	1	1	1	0	0	0	0
0	1	0	0	1	1	0	1	0	0
0	0	1	0	1	1	1	0	0	0
0	0	0	0	1	1	1	1	1	1

$A \cap B$.

1 $\longrightarrow$ case ① $\sim P \wedge q \wedge r$

0

0

0

0

0

0

1 $\longrightarrow$ case ② $P \wedge \sim q \wedge \sim r$

The c.p is true in two cases: case ①, case ②

the min terms in each two cases are: $(\sim P \wedge q \wedge r)$; $(P \wedge \sim q \wedge \sim r)$

the sum of min terms is $(\sim P \wedge q \wedge r) \vee (P \wedge \sim q \wedge \sim r)$

4. find PDNF of $(p\wedge q)\vee(\sim p\wedge r)\vee(q\wedge r)$

p	q	r	$p\wedge q$	$\sim p$	$\sim p\wedge r$	$q\wedge r$	$A\vee B\vee C$
1	1	1	1	0	0	1	1 → ①
1	1	0	1	0	0	0	1 → ②
1	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0
0	1	1	0	1	1	1	1 → ③
0	1	0	0	1	0	0	0 –
0	0	1	0	1	1	0	1 → ④
0	0	0	0	1	0	0	0

the C.P is true in 4 cases: ①, ②, ③, ④

the min terms in each 4 cases are $(p\wedge q)\wedge(p\wedge\sim r)\wedge(q\wedge r)$

$(p\wedge q)\wedge(p\wedge\sim r)\wedge(\sim q\vee\sim r)$

$(\sim p\vee\sim q)\wedge(\sim p\wedge r)\wedge(q\wedge r)$

$(\sim p\vee\sim q)\wedge(\sim p\wedge r)\wedge(\sim q\vee\sim r)$.

Procedure to find PCNF

Construct a truth

For each truth value 0 of given C.P, write max terms

Find PDNF of $\sim(P\vee Q)\leftrightarrow(P\wedge Q)$

$\sim(P\vee Q)\leftrightarrow(P\wedge Q)$

$\cong [\sim(P\vee Q)\rightarrow(P\wedge Q)]\wedge[(P\wedge Q)\rightarrow[\sim(P\vee Q)]]$

$\cong [(P\vee Q)\vee(P\wedge Q)]\wedge[(\sim(P\wedge Q))\vee(\sim(P\vee Q))]$

$\cong [(P\vee Q)\vee(P\wedge Q)]\wedge[(\sim P\vee\sim Q)\vee(\sim P\wedge\sim Q)]$

$\cong [[(P\vee Q)\vee P]\wedge((P\vee Q)\vee Q)]\wedge[[(\sim P\vee\sim Q)\vee\sim P]\wedge[(\sim P\vee\sim Q)\vee\sim Q]]$

$\cong (P\vee Q)\wedge(P\vee Q)\wedge(\sim P\vee\sim Q)\wedge(\sim P\vee\sim Q)$

$\cong (P\vee Q)\wedge(\sim P\vee\sim Q)$

$\cong [(P\vee Q)\wedge\sim P]\wedge[(P\vee Q)\wedge\sim Q]$

$\cong (\sim P\wedge P)\vee(\sim P\wedge Q)\vee(\sim Q\wedge P)\vee(\sim Q\wedge Q)$

$\cong [F_0\vee(\sim P\wedge Q)]\vee[(\sim Q\wedge P)\vee F_0]$

$\cong (\sim P\wedge Q)\vee(\sim Q\wedge P).$

Procedure to find PCNF

1. Find CNF. (Product of sums for given Preposition)
   (sum) $\wedge$ (sum) $\wedge$ (sum)

2. If all the sums in CNF are max.terms then the CNF is itself PCNF.

3. Otherwise, Convert those terms sums which are not max terms into max.terms as follows.

i, If the sum contains $P \vee \sim P$ replaced with T_0 and then

ii, If in a sum 'α' the preposition P is missing then write 'α' as $\alpha \vee F_0$ and then write $\alpha \vee (P \wedge \sim P)$ and then $[(\alpha \vee P) \wedge (\alpha \vee \sim P)]$

4. Repeat step 3 untill all the sums are converted into Max. terms in CNF

5. Any max.term is repeated write its only

1. find PCNF $P \leftrightarrow Q$

$$P \leftrightarrow Q$$

$$\cong (P \rightarrow Q) \wedge (Q \rightarrow P)$$

$$\cong (\sim P \vee Q) \wedge (\sim Q \vee P)$$

2 find the PCNF of $(P \rightarrow Q) \wedge (Q \leftrightarrow r)$

$$(P \rightarrow Q) \wedge (Q \leftrightarrow r)$$

$$\cong (\sim P \vee Q) \wedge (Q \rightarrow r) \wedge (r \rightarrow Q)$$

$$\cong (\sim P \vee Q) \wedge (\sim Q \vee r) \wedge (\sim r \vee Q)$$

$$\cong [(\sim P \vee Q) \vee F_0] \wedge [(\sim Q \vee r) \vee F_0] \wedge [(\sim r \vee Q) \vee F_0]$$

$$\cong [(\sim P \vee Q) \vee (r \wedge \sim r)] \wedge [(\sim Q \vee r) \vee (P \wedge \sim P)] \wedge [(\sim r \vee Q) \vee (P \wedge \sim P)]$$

A V B∧C

$$\cong [(\sim P \vee Q \vee r) \wedge (\sim P \vee Q \vee \sim r)] \wedge [(\sim Q \vee r \vee P) \wedge (\sim Q \vee r \vee \sim P)]$$

$$\wedge [(\sim r \vee Q \vee P) \wedge (\sim r \vee Q \vee \sim P)]$$

find PCNF $(p\wedge q)\vee(\sim q\wedge r)$.

$(p\wedge q)\vee(\sim q\wedge r)$

$(p\wedge q\vee\sim q)\wedge(p\wedge q\vee r)$

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Duality :-

Let "U" be a c.p which doesn't Contain the Connectives $\leftrightarrow$, $\rightarrow$

replace each occurence of $\vee$ — $\wedge$, $\wedge$ — $\vee$, T_0 — F_0, F_0 — T_0

the resulting new c.p is denoted by u^d / u^* and is called dual of 'u'.

Ex:

$u: (\sim p\vee q)\wedge[(\sim q\vee\sim p)\wedge F_0]$

$u^*: (\sim p\wedge q)\vee[(\sim q\wedge\sim p)\vee T_0]$

Procedure to find Dual

1. Eliminate $\leftrightarrow$, if exists

2. Eliminate $\rightarrow$, if exists

3. Replace $\vee \to \wedge$, $\wedge \to \vee$, $T_0 - F_0$, $F_0 - T_0$

Find dual of $p \to (q \wedge r)$

$A: p \to (q \wedge r)$

$\sim p \vee (q \wedge r)$

$A^*: \sim p \wedge (q \vee r)$

Find dual of $p \leftrightarrow [(q \to r) \wedge F_0]$

$A: p \leftrightarrow [(q \to r) \wedge F_0]$

$p \to [(q \to r) \wedge F_0] \wedge [(q \to r) \wedge F_0] \to p$

$[\sim p \vee [(q \to r) \wedge F_0]] \wedge [\sim[(q \to r) \wedge F_0] \vee p]$

$[\sim p \vee (\sim q \vee r) \wedge F_0] \wedge [\sim [(\sim q \vee r) \wedge F_0] \vee p]$

$[\sim p \wedge (\sim q \wedge r) \vee T_0] \wedge [q \vee \sim r \vee T_0 \vee p]$

T_0 $[\sim p \wedge (\sim q \wedge r) \vee T_0] \vee [q \wedge \sim r \wedge F_0 \wedge p]$;

Laws of dual

1. If U is any C.P then $(U^d)^d \cong U$

2. If $U \cong V$ then $U^d \cong V^d$. The second law is the Principal of duality.

Verify the Principal of duality for the logical equivalence

$(p \vee q) \vee [\sim p \wedge (\sim p \wedge q)] \cong (\sim p \wedge q)$

let $u: (p \vee q) \wedge [\sim p \wedge (\sim p \wedge q)]$

$v: \sim p \wedge q$

$u^d: (p \wedge q) \vee [\sim p \vee (\sim p \vee q)]$

$v^d: \sim p \vee q.$

If $u^d \cong v^d$

$u^d: (p \wedge q) \vee [\sim p \vee (\sim p \vee q)]$

$\cong (p \wedge q) \vee \underline{(\sim p \vee q)}_{A}$

$\cong (\sim p \vee q \vee p) \wedge (\sim p \vee q \vee q)$

$\cong (\sim p \vee p \vee q) \wedge (\sim p \vee q)$

$\cong (T_0 \vee q) \wedge (\sim p \vee q)$

$\cong T_0 \wedge (\sim p \vee q)$

$\cong \sim p \vee q$

$\cong v^d$

$\therefore u^d \cong v^d$ //

Rules of Inference

Argument:

Consider a set of Prepositions $P_1, P_2, P_3, \ldots\ldots P_n$, C.

then the C.P $\boxed{[P_1 \wedge P_2 \wedge P_3 \wedge \ldots\ldots \wedge P_n] \longrightarrow C}$ is called an "argument."

Here $P_1, P_2, P_3 \ldots\ldots P_n$ are called Premises of the argument and 'C' is called "Conclusion" of the argument.

In General, we denote the argument as

$$\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \\ \hline C \\ \hline \end{array}$$

* Arguments are classified into two types

1. Valid Argument
2. Invalid Argument.

An argument with Premises $P_1, P_2 \cdots P_n$ and a conclusion 'c' is said to be Valid Argument if each $P_1, P_2 \cdots P_n$ are true then 'c' is true. Otherwise the argument is said to be Invalid Argument

In Argument Problems, by default we consider all the Premises are always true.

1. Rule of Conjunction :-

$$\begin{array}{l} P \wedge q \\ \hline \therefore P \\ \hline \end{array} \qquad \begin{array}{l} P \wedge q \\ \hline \therefore q \\ \hline \end{array}$$

2 Rule of disjunction :-

$$\begin{array}{l} q \\ \hline \therefore P \vee q \\ \hline \end{array} \qquad \begin{array}{l} P \\ \hline \therefore P \vee q \\ \hline \end{array}$$

syllogism :-

$$\begin{array}{l} P \to q \\ q \to r \\ \hline \therefore P \to r \\ \hline \end{array}$$

Modus Pones :-

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Modus Tollens:-

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

Disjunctive Syllogism.

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array} \qquad \begin{array}{l} p \vee q \\ \sim q \\ \hline \therefore p \end{array}$$

→ If You work hard then You will Pass the exam.

p : work hard

q : Pass the exam.

$p \to q$.

→ You didn't Pass the exam, You didn't work hard.

$\sim q$.

$p \to q$

$\sim q$

$\sim p$

C : $\sim p$

→ If sachin gets 100 then he gets man of the match.

sachin gets man of the match ... sachin gets 100.

P: sachin gets 100

q: gets man of the match.

$p \rightarrow q$

q

C: p

there is no rule to say if $p \rightarrow q$ is true, q is true then p is true

3

P: study well

q: Pass the exam

r: Don't watch TV

the given Problem is

1. $p \rightarrow q$. (Premises)
2. $r \rightarrow p$ (")
3. $\sim q$. (")

C: $\sim r$.

4. $r \rightarrow q$. (Syllogism from 2,1)
5. $\sim r$ m. Tollens to 4,3

4.

1. $P \to r$ (Premises)

2. $\sim P \to q$ (")

3. $q \to s$ (")

4. $\sim P \to s$ syllogism to 2,3

5. $\sim r \to \sim P$ Contrapositive of 1.

6. $\sim r \to s$ Syllogism to 4,5

C: $\sim r \to s$.

5. $P \to q, \sim q, \sim r$ Conclusion is $\sim(P \vee r)$

$P \to q$. (Premises)

$\sim q$ (")

$\sim r$ (")

$\sim P$

$\sim P \wedge \sim r$

$\sim(P \vee r)$ Demorgan's law.

6 S.T 's' can be derived from the Premises $P, P \to q, s \vee r, r \to \sim q$.

P (Premises)

$P \to q$ (")

$s \vee r$ (")

$r \to \sim q$ (")

q (modulus Pones to 1,2)

$\sim r$ (modulus tollens to 4,5)

s (modus tollens to 3,6)

7. $p\wedge q,\ q\to r,\ \sim p\to m,\ \sim m.$

1. $p\wedge q$ (Premises)

2. $q\to r$ (")

3. $\sim p\to m$ (")

4. $\sim m$ (")

$p\wedge q = q$, q

$q\to r$

5 p (conjunction)

6. q (")

7. r (modus Pones to 2,6)

8. $p\vee q$ (

9. $r\wedge(p\vee q)$

Predicate logic:-

A Declarative sentence which contains one (or) more variables is called an open sentences.

Open sentences are not Prepositions.

the variables in open sentences are called free variables.

The Part of open Sentences which describes the Property of a variable (or) relation among variables is called a Predicate.

In general, we denote open sentences with "$P(x)$, $Q(x,y)$"...

Open sentences are not Prepositions but once we assign values to variable open sentence and they become Prepositions and they have truth value.

The set from which the variable at open sentence for assigning values is called universal set

$P(x): x+5=9$

$Q(x,y): x+y>9$

Quantifiers:-

* The words/phrase "for all" is called Universal Quantifier and it is denoted by $\forall$.

* The word "there exists" is called existential Quantifier and it is denoted by $\exists$

Quantifier statements:-

$\forall x, P(x)$ is called universal Quantifier statement

$\exists x, P(x)$ is called Existential Quantifier statement.

The variables in Quantifier satatements are called "bound variables"

Ex:

1. All girls are wise → $\forall x, P(x)$ where 'u' is set of Girls is universal set

$\forall x$

U = set of Girls

$P(x)$: x is wise

2. There are some dangerous animals → $\exists x, P(x)$

$\exists x$

U = animals.

$P(x)$ = x is danagerous

Rules for Quantifier sentences :-

Quantifier sentences are Prepositions

1. $\forall x, P(x)$ is true if $P(x)$ is true for each small 'x', in 'u'

2. $\forall x, P(x)$ is false if $P(x)$ is false for (each) atleast one small 'x', in 'u'

3. $\exists x, P(x)$ is true if $P(x)$ is true for one atleast one 'x' in 'u'.

4. $\exists x, P(x)$ is false $P(x)$ is false for each 'x' in u

Negation of Quantifier sentence :-

$$\sim[\forall x, P(x)] \cong \exists x, \sim P(x)$$

$$\sim[\exists x, P(x)] \cong \forall x, \sim P(x)$$

Rules of Inference for Predicative logic :-

1. U.I (Universal Instantiation):

$$\begin{array}{l} \forall x; P(x) \\ c \in U \\ \hline \therefore P(c) \end{array}$$

2. U.G (Universal Generalization) :-

$$\begin{array}{l} P(x_1) \\ P(x_2) \\ \vdots \\ P(x_n) \\ \hline \therefore \forall x, P(x) \end{array} \qquad U = \{x_1, x_2, \cdots x_n\}$$

3. E.I (Existential Instantiation):-

$$\exists x \; P(x)$$

$$\therefore P(c)$$

4. E.G (Existential Generalisation):-

$$P(c) \qquad c \in U$$

$$\therefore \exists x, P(x)$$

1. All Mathematicians think logically.
2. Ramanujan is a Mathematician.

∴ Ramanujan thinks logically.

2. 1) All Tigers are dangerous animals
2) They are some tigers.

∴ there are some dangerous animals.