Module2_Transitivity and Structural balance

- **Reference Book:**
- **Wasserman Stanley, and Katherine Faust. (2009), Social Network Analysis: Methods and Applications, Structural Analysis in the Social Sciences.**

Balance Theory

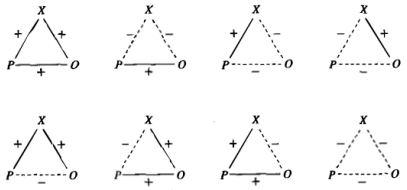
- Concerned with **how an individual's attitudes or opinions coincide with those of others** in a network
- balance vs dissonance
  - if two actors who are "friends" have the same "attitude" toward a third entity, there is balance
  - if two friends have different attitudes toward a third entity, there is dissonance

Structural Balance: Representation

- Signed graph with positive or negative edges
  - positive is "liking" negative is "not liking"
  - Edges
    - nondirectional, i.e., mutual
    - directional, i.e., i -> j is distinct from j -> i

Structural Balance

- Example: P-O-X triples (top 4 triples are balanced, bottom 4 triples are unbalanced)



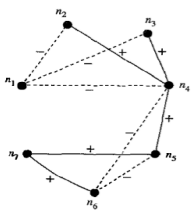
The eight possible *P-O-X* triples

Structural Balance: Signed Nondirectional Graphs

- Characterize a graph by its cycles
- Sign of a cycle is the product of signs of its edges
- Balanced cycle has a positive sign
- Simplest cycle is a triple (three edges)
  - zero or two negative edges is balanced
  - one negative edge is unbalanced
  - If all triples in a graph have positive signs, it is balanced

Structural Balance: Signed Nondirectional Graphs

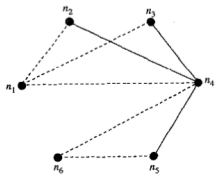
- Sign of n-length cycle with
  - zero or even number of negative edges is balanced
  - odd number of negative edges is unbalanced
- **A signed graph is balanced if and only if all cycles have positive signs.**
- A graph with no cycles is vacuously balanced: neither balanced nor unbalanced
- Example:



An unbalanced signed graph

Structural Balance: Signed Nondirectional Graphs

- **A signed graph is balanced if and only if all cycles have positive signs.**
- Example



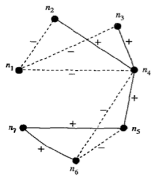
A balanced signed graph

Structural Balance: Directional Graphs

- Cycles in a directional graph (digraph) require all arcs to "point in the same direction"
- A digraph may not contain any cycles
- Use semicycles, which ignore arc direction
- **A signed digraph is balanced if and only if all semicycles have positive signs.**

Structural Balance: Metric

- To measure how unbalanced a graph or digraph is, use the cycle index for balance
- PC = number of positive (semi)cycles
- TC = total number of (semi)cycles
- **Cycle index for balance = PC/TC**
- **PC/TC=3/4 for the graph here**



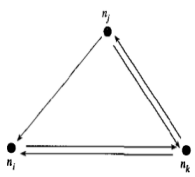
An unbalanced signed graph

Transitivity

- Transitivity is a key structural property in social network data.
- For unsigned digraphs Directed edges are either present or null
- A triad of nodes i, j, and k is transitive if whenever i -> j and j -> k, then i -> k.
- **A triad is vacuously transitive if either condition is not met.**
- **A digraph is transitive if every triad it contains is transitive.**

Transitivity

- Example



Triple #1 : $n_i\ n_j\ n_k$			
$n_i \not\to n_j$	$n_j \to n_k$	$n_i \to n_k$	Vacuously transitive
Triple #2 : $n_i\ n_k\ n_j$			
$n_i \to n_k$	$n_k \to n_j$	$n_i \not\to n_j$	Intransitive
Triple #3 : $n_j\ n_i\ n_k$			
$n_j \to n_i$	$n_i \to n_k$	$n_j \to n_k$	Transitive
Triple #4 : $n_j\ n_k\ n_i$			
$n_j \to n_k$	$n_k \to n_i$	$n_j \to n_i$	Transitive
Triple #5 : $n_k\ n_i\ n_j$			
$n_k \to n_i$	$n_i \not\to n_j$	$n_k \to n_j$	Vacuously transitive
Triple #6 : $n_k\ n_j\ n_i$			
$n_k \to n_j$	$n_j \to n_i$	$n_k \to n_i$	Transitive