

Social and Information Networks

Module 1 - GraphsMatrices

Reference Book:

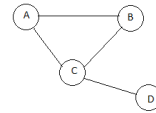
Wasserman Stanley, and Katherine Faust. (2009).
Social Network Analysis: Methods and Applications,
Structural Analysis in the Social Sciences.

Reciprocity

- The transitivity coefficient is computed for undirected networks, ignoring the direction of the edges.

– For the graph given below

- Transitivity coefficient is $T = 3 \cdot 1/5 = 0.6$



Reciprocity

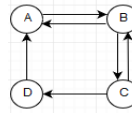
- In undirected graphs the smallest loop has length three, in a directed graph it has length two; it is a sequence x, y, x such that the directed edges (x, y) and (y, x) belong to the graph.
 - We can thus measure the frequency of loops of length two in a directed network, which is called reciprocity.
 - It measures how likely it is that a vertex that you point to also points back at you.
- Reciprocity of a directed network is the **fraction of edges that belong to a loop of length two**.

$$R = \frac{\sum_{i,j} A_{ij} A_{ji}}{\sum_{i,j} A_{ij}} = \frac{\text{Tr}(A^2)}{m}$$

where m is the number of edges and A the adjacency matrix of the graph and $\text{Tr}(X)$ is the trace of matrix X , (the sum of diagonal elements of X). And $A_{ij} A_{ji} = 1$ if and only if i links to j and vice versa.

Reciprocity

- Example:



$$R = \frac{\sum_{i,j} A_{ij} A_{ji}}{\sum_{i,j} A_{ij}} = \frac{\text{Tr}(A^2)}{m}$$

- $R = 4 / 6$

- Adjacency, $A =$

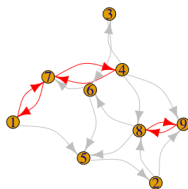
0	1	0	0
1	0	1	0
0	1	0	1
1	0	0	0

- $A^2 =$

1	0	1	0
0	2	0	1
2	0	1	1
0	1	0	0

Reciprocity

- The reciprocity of the following graph with 18 edges is $6/18 = 1/3$



Reference

- <http://users.dimi.uniud.it/~massimo.francesc/het/teaching/datascience/network/transitivity.html>