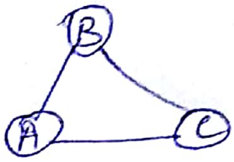
CSE3021 - Module - 2

Eigen Vector Centrality Measure

Example 1:



Here $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Solving for Eigen values:

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$-\lambda(\lambda^2 - 1) - 1(-\lambda - 1) + 1(1 + \lambda) = 0$$

$$-\lambda^3 + \lambda + \lambda + 1 + 1 + \lambda = 0$$

$$-\lambda^3 + 3\lambda + 2 = 0$$

$$\lambda^3 - 3\lambda - 2 = 0$$

$$(\lambda - 2)(\lambda + 1)^2 = 0$$

Hence $\lambda_1 = 2$, $\lambda_2 = \lambda_2 = -1$

Solving for Eigen Vector corresponding to the largest Eigen Value, (i.e) $\lambda = 2$

Hence $(A-\lambda I)\cancel{V} = 0$

$$\Rightarrow \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = 0$$

$$\Rightarrow \left.\begin{array}{l} -2V_1 + V_2 + V_3 = 0 \\ V_1 - 2V_2 + V_3 = 0 \\ V_1 + V_2 - 2V_3 = 0. \end{array}\right\} \Rightarrow$$ System of linear equations in 3 variables.

Solving for V_1, V_2 and V_3:

$$\begin{array}{r} 2V_1 - 4V_2 + 2V_3 = 0 \\ 2V_1 \underset{(-)}{+} 2V_2 \underset{(+)}{-} 4V_3 = 0 \\ \hline -6V_2 + 6V_3 = 0 \\ \hline \end{array}$$

$$\Rightarrow \boxed{V_2 = V_3}$$

Hence $V_1 + V_3 - 2V_3 = 0$

$$\boxed{V_1 = V_3}$$

$$\Rightarrow \boxed{V_1 = V_2 = V_3}$$

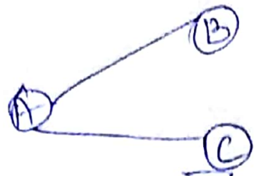
If $V_1 = t$

Eigen Vector $V = \begin{bmatrix} t \\ t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$

Hence a solution for $V = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow$ All nodes have the same measure.

Example 2:



Here $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Solving for Eigen Values:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$$

$$-\lambda(\lambda^2) - 1(-\lambda) + 1(\lambda) = 0$$

$$-\lambda^3 + 2\lambda = 0$$

$$\lambda^3 - 2\lambda = 0$$

$$\lambda(\lambda^2 - 2) = 0$$

Here $\lambda_1 = 0$, $\lambda_2 = +\sqrt{2}$, $\lambda_3 = -\sqrt{2}$

Solving for Eigen Vector with $\lambda = +\sqrt{2}$

$$(A - \lambda I)V = 0$$

$$\begin{pmatrix} -\sqrt{2} & 1 & 1 \\ 1 & -\sqrt{2} & 0 \\ 1 & 0 & -\sqrt{2} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = 0$$

$\Rightarrow$

$$-\sqrt{2}V_1 + V_2 + V_3 = 0$$

$$V_1 - \sqrt{2}V_2 = 0$$

$$V_1 - \sqrt{2}V_3 = 0$$

$\Rightarrow \quad V_1 = \sqrt{2}V_2$

$V_1 = \sqrt{2}V_3$

$\Rightarrow \quad V_2 = V_3 = t$ (say)

Hence $V_1 = \sqrt{2}t$

$\therefore$ Eigen Vector, $V = \begin{bmatrix} \sqrt{2}t \\ t \\ t \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ 1 \\ 1 \end{bmatrix} t$

Taking $t=1$ $V = \begin{bmatrix} \sqrt{2} \\ 1 \\ 1 \end{bmatrix}$

$\therefore$ Eigen Vector Centrality measure for

actor A is $\sqrt{2}$

actor B is 1 and

actor C is 1

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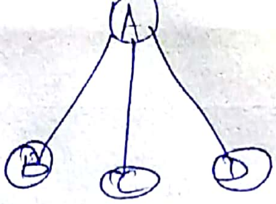
Example 3:



Here $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Solving for Eigen values:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 & 1 & 1 \\ 1 & -\lambda & 0 & 0 \\ 1 & 0 & -\lambda & 0 \\ 1 & 0 & 0 & -\lambda \end{vmatrix} = 0$$

$$-\lambda \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 & 0 \\ 1 & -\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} + 1 \begin{vmatrix} 1 & -\lambda & 0 \\ 1 & 0 & 0 \\ 1 & 0 & -\lambda \end{vmatrix} + $$

$$-1 \begin{vmatrix} 1 & -\lambda & 0 \\ 1 & 0 & -\lambda \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$-\lambda\left(-\lambda(\lambda^2)\right) - 1\left(1(\lambda^2)\right) + 1\left(1(0) + \lambda(-\lambda)\right)$$

$$-1\left(1(0) + \lambda(\lambda)\right) = 0$$

$$\lambda^4 - \lambda^2 - \lambda^2 - \lambda^2 = 0$$

$$\lambda^4 - 3\lambda^2 = 0$$

$$\lambda^2(\lambda^2 - 3) = 0$$

$$\Rightarrow \lambda = [0, 0, +\sqrt{3}, -\sqrt{3}]$$

Solving for Eigen Vector with $\lambda = \sqrt{3}$:

$(A - \lambda I)v = 0$

$$\begin{pmatrix} -\sqrt{3} & 1 & 1 & 1 \\ 1 & -\sqrt{3} & 0 & 0 \\ 1 & 0 & -\sqrt{3} & 0 \\ 1 & 0 & 0 & -\sqrt{3} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = 0$$

$\Rightarrow$ $-\sqrt{3}v_1 + v_2 + v_3 + v_4 = 0$

$v_1 - \sqrt{3}v_2 + 0 = 0 \Rightarrow v_1 = \sqrt{3}v_2$

$v_1 - \sqrt{3}v_3 = 0 \Rightarrow v_1 = \sqrt{3}v_3$

$v_1 - \sqrt{3}v_4 = 0 \Rightarrow v_1 = \sqrt{3}v_4$

$\Rightarrow$ $v_2 = v_3 = v_4 = t$ (say) $\Rightarrow v_1 = \sqrt{3}t$

$\cancel{\Rightarrow -\sqrt{3}v_1 + v_2 + v_2 + v_2 = 0}$.

$\therefore$ Eigen Vector, $V = \begin{bmatrix} \sqrt{3}t \\ t \\ t \\ t \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ 1 \\ 1 \\ 1 \end{bmatrix} t$

If $t = 1$, $V = \begin{bmatrix} \sqrt{3} \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$\therefore$ Eigen Vector Centrality measure for

actor A is $\sqrt{3}$

actor B is 1

actor C is 1

actor D is 1

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