

Social and Information Networks

Module 1 - Notations

Reference Book:

Wasserman Stanley, and Katherine Faust. (2009).
Social Network Analysis: Methods and Applications,
Structural Analysis in the Social Sciences.

NOTATIONS FOR SOCIAL NETWORK DATA

- Three network schemas to represent a wide range of social network data

- Graph Theoretic
- Sociometric
- Algebraic

Graph:

- Consists of a number of nodes and a number of lines connecting the nodes
- Symbol **N** is used to indicate a set of actors/nodes in a graph, denoted as $\{N_1, N_2, \dots, N_n\}$

Graph

- Suppose we have a collection of 6 actors
 $N = \{N_1 = \text{Allison}; N_2 = \text{Drew}; N_3 = \text{Eliot}; N_4 = \text{Keith}; N_5 = \text{Ross}; N_6 = \text{Sarah}\}$
- Relation: **Defined to see how each actor is related to other actors on this relation**
 - **Dichotomous relations** mean either the relation among actors/nodes present or absent in a graph
 - **Directional ties** mean relationship from actor A to B is distinctive from the relationship from actor B to A.

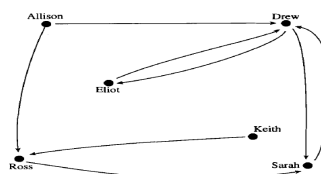
Graph

- Ordered pair denotes a pair of actors/nodes related from actor I to actor J on a relationship.
- If the ordered pair under consideration is $\langle n_i, n_j \rangle$, and if there is a tie present, represent the tie as $n_i \rightarrow n_j$
 - Example: $\langle \text{Allison}, \text{Drew} \rangle, \langle \text{Allison}, \text{Ross} \rangle$
 - Allison views Drew as a friend, Allison also views Ross as a friend
- How many ordered pair can be present in a directional graph with g nodes/actors?

Graph

Sociogram:

- **Graphic representation of social links** that a person has with others
- Plots the structure of **interpersonal relations in a group**
- **Developed by Jacob L. Moreno** to analyze choices or preferences within a group
- Example : Graph with Single Relation

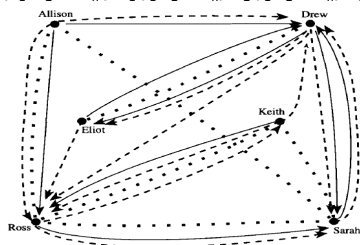


Graph

- $G(N\{n_1, n_2, \dots, n_n\}, L\{l_1, l_2, \dots, l_n\})$
- **Valued graphs** have one more factor of values attached to each line
 - $G(N\{n_1, n_2, \dots, n_n\}, L\{l_1, l_2, \dots, l_n\}, W\{w_1, w_2, \dots, w_n\})$

Graph

- Graph with Multiple Relations
 - $G(N, L_r), r = 1, 2, \dots, R$
 - Example:
 - $G(N\{n_1, n_2, \dots, n_n\}, L_1\{l_1, l_2, \dots, l_n\}, L_2\{l_1, l_2, \dots, l_n\}, L_3\{l_1, l_2, \dots, l_n\})$



Classification of Graphs

- Four types of graphs

	Non-directed	Directed
Binary	Binary, non-directional graphs	Binary Directional Graphs
Valued	Valued, non-directional graphs	Valued, directional graphs

Sociometric Notation

- Sociometry is the **study of positive and negative affective relations**
 - Example: liking/disliking and friends/enemies, among a set of people.
- A social network data set **consisting of people and measured affective relations between people is often referred to as sociometric.**
- Relational data are often **presented in two-way matrices termed sociomatrixes**
- Two dimensions of a sociomatrix are indexed by the **sending actors** (the rows) and the **receiving actors** (the columns).

Sociometric Notation

- In a one-mode network, the sociomatrix will be square
- A sociomatrix for a dichotomous relation is the adjacency matrix for the graph (or sociogram) quantifying the ties between the actors for the relation
- An entry X_{ij} in the sociomatrix is
 X_{ij} = the value of the tie from n_i to n_j
- To capture multiple relationship among actors
 X_{ijr} = the value of the tie from n_i to n_j on relation X_r

Sociometric Notation

- Multiple Relations
 - If g is the number of nodes and R is the number of relations then there exists $R \text{ } g \times g \text{ } R$ sociomatrices
- R sociomatrices can be viewed as layers in a three-dimensional matrix of size $g \times g \times R$
 - Rows index the sending actors, columns index the receiving actors and the layers index the relations
 - Matrix is referred to as a **super-sociomatrix**, representing the information in a multi-relational network

Sociometric Notation

- Sociomatrices

Friendship at Beginning of Year							Lives Near						
	Allison	Drew	Eliot	Keith	Ross	Sarah		Allison	Drew	Eliot	Keith	Ross	Sarah
Allison	-	1	0	0	1	0	Allison	-	0	0	0	1	1
Drew	0	-	1	0	0	1	Drew	0	-	1	0	0	0
Eliot	0	1	-	0	0	0	Eliot	0	1	-	0	0	0
Keith	0	0	0	-	1	0	Keith	0	0	0	-	1	1
Ross	0	0	0	0	-	1	Ross	1	0	0	1	-	1
Sarah	0	1	0	0	0	-	Sarah	1	0	0	1	1	-

Friendship at End of Year						
	Allison	Drew	Eliot	Keith	Ross	Sarah
Allison	-	1	0	0	1	0
Drew	0	-	1	0	1	1
Eliot	0	0	-	0	1	0
Keith	0	1	0	-	1	0
Ross	0	0	0	1	-	1
Sarah	0	1	0	0	0	-

Algebraic Notation

- Most **useful for multirelational networks**
- Relations are **represented with distinct capital letters**
- **Example:** Use F to denote the relation "is a friend of" and E for the relation "is an enemy of"
- Record the presence of a tie from **actor i to actor j on relation F as iFj**
- In general, $X_{ij}F = 1$ if $ni \rightarrow nj$ on the relation labeled F