Space-time codes in wireless communications

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The need to provide reliable high data rate communication over the wireless channel has led to the development of efficient modulation and coding schemes. The wireless channel suffers from time-varying impairments like multi-path fading, interference and noise. Diversity (time/frequency/space/ polarization/angle) is an effective method to combat the fading of the wireless channel. Thus, link reliability is improved. The downside is that time and frequency diversity lead to loss in bandwidth efficiency. However, by employing multiple antennas at the transmitter and/or at the receiver, spatial diversity mitigates fading without sacrificing the precious bandwidth resource. As a result, this concept is gaining popularity.

Receive diversity techniques—wherein multiple receiver antennas along with suitable combining are used—have already been implemented to improve the performance in the uplink. But, it is difficult to implement receive diversity in the downlink because of the size/power limitations on the portable/mobile terminal. This has motivated the use of transmit diversity schemes wherein multiple antennas are used at the transmitter for the downlink transmission from the base station to the portable terminal.

In general, a wireless communication system in which multiple antennas are used at the transmitter and at the receiver is referred to as a multiple input multiple output (MIMO) system as shown in Fig 1. Higher channel capacities can be realized using MIMO wireless systems when compared to single antenna systems.

Space-time coding

In space-time coding, multiple antennas are employed at the transmitter. There the intelligent coding of symbols across space and time can be done to reap the advantages due to coding and diversity. The coding in space is obtained by using multiple antennas at the transmitter. We shall denote a few terms before proceeding with the explanation. Let N_{TX} and N_{RX} denote the number of transmit and receive antennas, respectively. Let Q represent the number of symbol periods that the space-time code spans.

Typically, a space-time code can be represented as a Q x N_{TX} matrix C. Each entry $c^k{}_i$ represents the complex baseband symbol. This is typically a single symbol or a linear combination of symbols from the constellation set, which is transmitted from the i^{th} antenna during the k^{th} symbol period.

$$C = \begin{bmatrix} c_1^1 & c_2^1 & c_3^1 & .. & c_{N_{TX}}^1 \\ c_1^2 & c_2^2 & c_3^2 & . & c_{N_{TX}}^2 \\ c_1^3 & c_2^3 & . & . & . \\ . & . & . & . & . \\ c_1^Q & c_2^Q & . & . & c_{N_{TX}}^Q \end{bmatrix} \tag{1}$$

Most space-time code designs work for any number of receive antennas. But some designs, which are out of the scope of this article, also take the number of receive antennas into account. The extra receive antennas provide receive diversity gain in addition to the transmit diversity gain due to the space-time coding. However, the receive diversity gain comes at the cost of extra hardware due to the need for more processing chains at the receiver.

In Fig. 1, the multiple transmit and receive antennas are shown. The wireless channel between any pair of transmit and receive antenna is modeled in the complex baseband notation by a parameter. We have considered flat fading channels initially wherein the channel parameter from the i^{th} transmit antenna to the j^{th} receive antenna is given by a complex

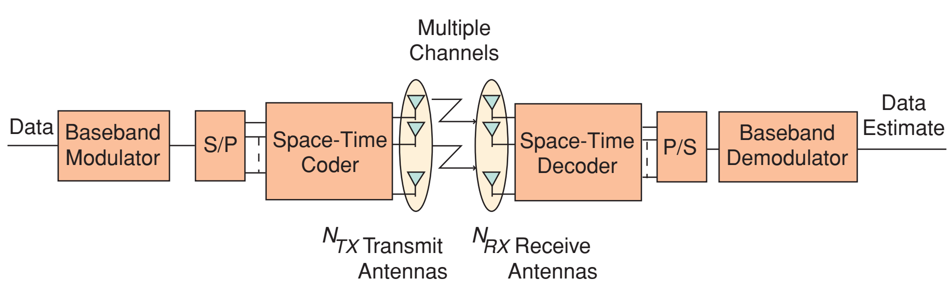

Fig. 1 Illustration of a general MIMO system

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number h_{ij}. Each of the channel parameters is modeled using a complex Gaussian random variable with zero mean and unit variance so as to simulate Rayleigh fading. In total, there are N_{TX}*N_{RX} channel parameters and they are modeled to be independent of each other. This assumption is true if the separation between the multiple antennas is greater than the wavelength of the carrier used for transmission.

The sampled received signal received during any symbol period can be represented as

$$r_j^k = \sum_{i=1}^{M} h_{ij} c_i^k + n_j^k \qquad (2)$$

where r_j^k is the received signal at the j^{th} antenna and the k^{th} symbol period. The received signal matrix can then be written as

$$R = SH + N \qquad (3)$$

where R is a Q x N_{RX} matrix, and its element $r^k{}_j$ is the entry in the j^{th} row and the k^{th} column.

The matrix N in (3) represents the additive noise at the N_{RX} receive antennas. It is modeled as a spatially and temporally white and Gaussian random process with zero mean and variance equal to the power of the noise.

Probability of error for space-time coding

Let C be the transmitted code matrix and let it be decoded erroneously as E. We define the code difference matrix as $B = C\text{-}E$ and matrix $A = BB+$, where $B+$ denotes the Hermitian transpose of matrix B. The pairwise error probability (PEP) gives an upper bound on the probability of the code matrix C being erroneously decoded as E. The PEP thus directly affects the Bit Error Rate (BER) of the digital communication system. When the Signal-to-Noise Ratio (SNR) at the receiver is high, the PEP for a Rayleigh fading channel is given by

$$P(C \rightarrow E) \leq \left(\frac{1}{E_s/N_0} \right)^{rN} \left(\prod_{i=1}^{r} \lambda_i \right) \qquad (4)$$

where E_s is the average symbol energy, $N_o/2$ is the power spectral density of the additive white Gaussian noise (AWGN) per dimension at the receiver, where r is the number of non-zero eigen values **A**, and λ_i's are the non-zero eigen values of **A**.

The first term on the RHS of equation (4) contains the effect of thediversity gain where rN_{RX} is the diversity order that is dependent on the rank of the code difference matrix. The second term is called the coding gain which depends on the product of the non-zero eigen values of **A**.

Design criteria

The design of space-time codes is an active area of research and the following steps are important in the design process.

1. *Rank criterion:* The code difference matrix, taken over all possible combinations of code matrices, should be full rank. This criterion maximizes the diversity gain obtained from the space-time code. The maximum diversity order that can be achieved is

$$r = \min(Q, N_{TX})$$

This criterion ensures that the space-time code gives the maximum *diversity gain*.

2. *Determinant criterion:* The minimum determinant of **A**, taken over all possible combinations of code matrices, should be maximized. This maximizes the *coding gain* from the space-time code. From equation (1), it can be seen that the diversity gain term dominates the error probability at high SNR. So the diversity gain should be maximized before the coding gain while designing a space-time code.

One important property of a space-time code is its "rate." The rate of a space-time code is defined as the ratio of the number of independent data symbols transmitted through the code matrix to the number of symbol time units that the code matrix spans. For instance, if 4 data symbols are transmitted in 4 time slots (which correspond to 4 symbol periods), then the code rate is unity. There are several types of space-time codes and they can be broadly classified as: 1) Space-Time Block Codes (STBC) and 2) Space-Time Trellis Codes (STTC) .

Space-time block codes

In STBC, a block of data symbols are buffered and a code matrix is formed from these data symbols. These are transmitted from the multiple antennas as mentioned earlier. The data symbols are detected using suitable techniques at the receiver. The challenge in STBC is designing the code matrix such that diversity gain, coding gain, and channel capacity can be maximized. In addition, simple detection techniques at the receiver are a key in the transmitter code design.

Space-time block codes from orthogonal designs

As the name suggests, this sub-class of STBCs derives its code matrix structure from orthogonal matrices. One of the main advantages of these codes is the simple processing at the receiver for detecting data. This linear processing detection scheme exhibits the same performance as maximum likelihood joint detection. But, it is computationally less complex. However, such STBCs employing complex symbols and with unit rate exist for N_{TX}=2 only.

Example:

Alamouti proposed a STBC for two transmit antennas encompassing two symbol periods. The code matrix is given by

$$C = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$$

with $s_1, s_2 \in$ QAM/PSK constellation. The sampled received signal for the 2 symbol periods is given by

$$r^1 = h_1 s_1 + h_2 s_2 + n^1$$
$$r^2 = -h_1 s_2^* + h_2 s_1^* + n^2$$

These two equations can be represented in matrix notation as,

$$\underbrace{\begin{bmatrix} r^1 \\ r^{2*} \end{bmatrix}}_{R} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}}_{S} + \underbrace{\begin{bmatrix} n^1 \\ n^{2*} \end{bmatrix}}_{N}$$

Define,

$$Z = H^{+}R = \alpha I_2 S + N(k)$$

where H^+ denotes the Hermitian transpose of H and I_2 is the second order identity matrix. As a result of this operation, data symbols s_1 and s_2 are decoupled and can be decoded independently. Note that this operation does not introduce any correlation between the entries of N matrix. It can be proved that r=2 for this code. Thus, this STBC gives a diversity order $2N_{RX}$ but does not offer any additional coding gain.

Space-time block codes from algebraic designs

Let $S = \{1, w, w^2, w^3, w^4, w^5\}$ be the symbol constellation set where $w = e^{(j2\Pi/6)}$

One example of such STBC is

$$C = \begin{bmatrix} a & b \\ wb & a \end{bmatrix}$$

where $a, b \in S$.

There is no simple decoding algorithm for such STBC. Also, the maximum likelihood (ML) decoding has to be performed. The complexity of this operation is prohibitively large when the order of the symbol constellation is very large. This code also gives a diversity order of $2N_{RX}$.

Space-time trellis codes (STTC)

In STTC, an encoding trellis is used in a manner similar to trellis coded modulation (TCM) for single antenna systems. The structure of the trellis and the number of states in it determines the coding gain of the encoding trellis. A soft Viterbi decoder is used at the receiver to recover the transmitted symbols. As in STBC, a block of data symbols is fed to the input of a Space-Time Trellis encoder. The output symbols are transmitted from the multiple antennas. A tail of zeros is appended to the input stream to bring the encoder into the zero state at the end of a burst (Fig. 2).

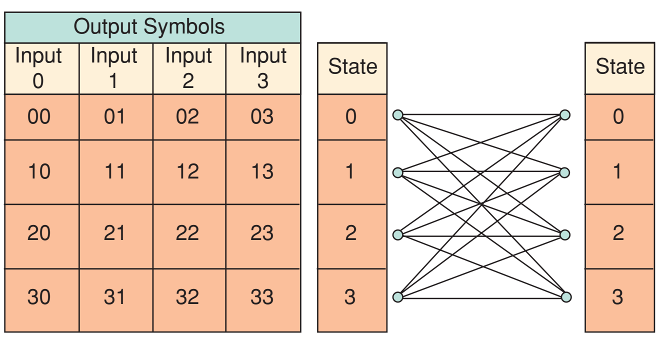

Fig. 2 Illustration of a STTC

Consider the above trellis for a quadrature phase shift keying (QPSK) constellation. The state transitions and the corresponding outputs are shown. Depending on the current encoder state and the current input, the symbols to be transmitted are decided. If the encoder is in state 'p' and the input symbol is 'q', then the symbol corresponding to 'p' is transmitted from first antenna. The symbol corresponding to 'q' is transmitted from the second antenna and the encoder goes into a 'q' state. If the input symbol stream to the encoder is [1 3 2 1 0..], then the resultant code matrix is

$$C = \begin{bmatrix} 0 & 1 & 3 & 2 & 1 & 0 & . \\ 1 & 3 & 2 & 1 & 0 & . & . \end{bmatrix}$$

Note that the same symbol is transmitted through both antennas at different time instants to get the necessary diversity advantage. At the receiver, a soft Viterbi decoder is employed. The order of diversity in the previous example is $2N_{RX}$. In addition to the diversity gain, STTC gives coding gain also due to the trellis encoding.

Special features of some space-time codes

1. *Linear processing:* A key feature of the STBC—obtained from orthogonal designs—is that linear processing can be performed at the receiver to recover the data symbols. For instance, in the Alamouti STBC, the detection of the two data symbols is decoupled. What's more, they do not have to be detected jointly as in a brute force, maximum likelihood detection scheme. This linear processing feature is attractive from the computational complexity aspect especially for a mobile or portable terminal.

2. *Coding gain:* The diversity gain obtained from a STTC is equal to the diversity gain from a STBC for the same number of transmit and receive antennas. The STTC has enhanced coding gain. As a result, it can enable better error performance. However, the additional coding gain comes at the cost of increased decoding complexity at the receiver because a soft Viterbi decoder has to be employed. The complexity of the decoder increases exponentially with the number of states in the trellis and number of transmit antennas.

3. *Channel capacity attained by the space-time code:* As mentioned earlier, space-time codes can enable increased channel capacities as compared to the single antenna systems. A trivial STBC, called V-BLAST, achieves the highest channel capacity by transmitting symbols without any coding across space or time. Attempts are underway to design codes that can approach optimum channel capacity.

Implementation issues

1. *Imperfect channel knowledge at receiver.* In most systems where space-time coded transmission is used, accurate channel knowledge is assumed at the receiver for decoding the data symbols. In practice, the channel matrix 'H' (as in equation (3)) has to be estimated using training sequences. In MIMO systems, noise and interference from multiple transmissions can influence the channel estimate and efficient channel estimation schemes have to be designed.

2. *Frequency selective fading channels.* We have discussed the operation of space-time codes in a flat-fading channel environment. However, space-time codes can be applied to frequency selective fading channels by combining them with Orthogonal Frequency Division Multiplexing (OFDM) transmission. Thus by using space-time coded OFDM transmissions, gains can be obtained in frequency selective channels.

Conclusion

We have discussed aspects of space-time coding. This is an important research topic in the design of emerging wireless systems. Research and development work is underway globally in this field to enhance wireless systems, so that they can satisfy users' aspirations. Deployment of wireless systems using space-time codes is expected in the near future.

About the authors

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