Multi-Input Multi-Output Communication Lab Report

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***Abstract*—Contemporary wireless systems are supposed to achieve high efficiency, capacity and reliability simultaneously, and using multiple transmission antennas is a feasible solution that has been applied ubiquitously. This paper analyses a typical 2-by-2 multi-input multi-output (MIMO) communication system. We investigate the influence of fading phenomenon and utilise it to achieve transmission diversity. Capacities of generated ergodic channels on different signal-to-noise (SNR) were simulated with Monte Carlo method (MCM), and quadrature amplitude modulation (QAM) together with space-time coding are employed for symbol mapping. Detection schemes of maximum-likelihood (ML), zero-forcing (ZF) and minimum-mean-square-error (MMSE) are examined and compared regarding bit error rate (BER) and complexity. It is proved that the ML detector is optimal to recover the signal but with high complexity, while the MMSE one achieves a balance between reliability and complexity. Results also show that the reliability can be further enhanced by combining the conventional detectors with Alamouti code.**

***Index Terms*—MIMO, diversity, array processing, estimation and detection, space-time coding.**

I. Introduction

Wireless communication systems are ubiquitous almost everywhere. They are essential for the study, business, and entertainment purposes. With more applications in emerging techniques as virtual reality and blockchain, the growing demands of capacity and reliability are challenging existing networks again. The popularity of MIMO has been witnessed for decades mainly because this scheme can utilise fading to achieve signal diversity. Fading indicates the transmitted signal travels through diverse paths before reaching the receiver, resulting in the received copies with different delays, attenuation and phase shifts. Conventional systems based on single transmitter and receiver aim to mitigate the influence by fading, but the randomness of time-varying channels cannot be eliminated; thus the performance is not satisfying. This leads to the introduction of multiple antennas. The idea of MIMO is exactly on the opposite: to utilise the randomness of fading. If several antennas are far spaced, communication channels can be regarded as parallel. Therefore signals on them experience independent fading, which brings in diversity. At the receiver end, we can use detection algorithms to remove fading effect and recover the transmitted signal.

Advantages of MIMO are obvious: capacity, reliability and efficiency. By using multiple antennas, we can transmit more bits in a fixed time. If the transmitters split the data stream and distribute the sub-streams on antennas, the channel capacity can be increased multiple times without using extra bandwidth. Therefore, the spectrum efficiency is advanced. On the other hand, if the sender employs multiple antennas to encode the stream with strategies as space-time code [2], the BER can be reduced, and system reliability can be reinforced. The cost of extra antennas is acceptable compared with the enhancement in performance, but the complexity of coding and detection should be controlled.

This article investigates the system construction, coding strategies, and detection schemes of a 2-by-2 MIMO system on capacity and BER. It is assumed that channels are complex Gaussian distributed with zero mean and unit variance. Also, the data stream is split and mapped to QPSK symbol before transmission. First, the MIMO capacity is simulated for systems with a different number of antenna elements under various SNR. Then, detection algorithms of ML, ZF and MMSE are compared on BER with same stream received. Finally, the BER of space-time coding in forms of Alamouti code is contrasted with that of plain QPSK code.

II. Theory and Methods

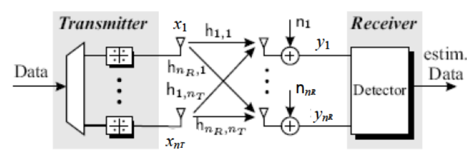

Fig. 1. MIMO system diagram

A typical MIMO system is illustrated by Figure 1 [1]. It indicates that the diagram can be divided into three major components: transmitter, channel, and receiver. The system equation can be written as

$$\underline{y} = \sqrt{\frac{\rho}{n_T}}\underline{\underline{H}}\,\underline{x} + \underline{n} \tag{1}$$

where $\underline{x}$ is the transmitted signal, $\underline{y}$ is the received signal, ρ is the SNR value, n_T is the number of transmitter, $\underline{\underline{H}}$ is the channel matrix, $\underline{n}$ is the noise vector. Before transmission, error detection and correction code may be employed before mapping to ensure fewer errors in signal recovery. Then, raw bits should be modulated to symbols with a specific coding scheme. Conventional mapping as amplitude, phase, frequency shift keying, and quadrature amplitude modulation can be applied here. The data stream can be fed into transmitters after those steps. As mentioned in Introduction, for multiple transmission antennas, the two possible strategies of data processing are:

- split data into sub-sections and assign each to one antenna

- transmit all information with every antenna

The first option increases capacity while the other ensures reliability by redundancy.

In this simulation, we consider the fundamental case of a 2-by-2 system. It is assumed no error detection or correction strategy, and we only care about the baseband signal. Plain QPSK modulation with Gray mapping was used in the second simulation, corresponding to the first strategy above. For the investigation of space-time coding, we combined QPSK with Alamouti code, which implies strategy two. Alamouti code [3] for two antennas is

$$\underline{\underline{X}} = \begin{pmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{pmatrix} \tag{2}$$

where x_1 and x_2 are two consecutive symbols. Rows correspond to antennas, and columns indicate time slots, which means element one transmit x_1 and $-x_2^*$ and element two take the rest. For every matrix corresponding to two QPSK symbols, there are $2^4 = 16$ possible combinations. The streams are orthogonal thus can be separated at the receiver. Based on the explanation, it is predicted that space-time coding leads to a lower BER, but the cost is the data rate.

Fading channel models are employed in MIMO systems. There is a route between every transmitter-receiver pair where the signal can travel through. If the space between elements is far enough, we regard fading on channels as independent ones. Those parallel channels introduce signal diversity that can be utilised for recovery.

We checked the channel capacity for the count of transmitter and receiver antennas varies from five to ten, for SNR from zero to thirty dB. The capacity is given by

$$C_{\underline{\underline{H}}} = \log_2 \left| \underline{\underline{I}} + \frac{\rho}{n_T} \underline{\underline{H}}\underline{\underline{H}}^H \right| \tag{3}$$

where $C_{\underline{\underline{H}}}$ is the instantaneous capacity, and $\underline{\underline{I}}$ is the unit matrix. Shannon capacity of ergodic channels is equal to the mean value of it.

The responsibility of the receiver is to collect the delayed, attenuated, and phase-shifted versions of the transmitted signal and attempt to estimate the original bit sequence. Due to the randomness of channels, some bits can be misinterpreted from the received copies, especially when the channel status is below standard. Different detectors are available based on a various criterion. Three representative detectors, namely ML, ZF, and MMSE [3], were analysed on the performance on BER.

As the name suggests, the standard maximum-likelihood detector divides the received signal into separate symbols, enumerates all possible patterns, and regard the original symbol as the one that is most likely to produce the result segment. It can be interpreted as the following equation

$$\underline{\widehat{x}} = \arg\min_{\underline{x} \in C} \left\| \underline{y} - \sqrt{\frac{\rho}{n_T}} \underline{\underline{H}}\underline{x} \right\|^2 \tag{4}$$

where C is the constellation set. As a one-by-one matching, it is expected that ML can be accurate but time-consuming.

It can be easily extended to the system with Alamouti coding

$$\underline{\underline{Y}} = \sqrt{\frac{\rho}{n_T}} \underline{\underline{H}}\underline{\underline{X}} + \underline{\underline{N}} \tag{5}$$

where the situation is similar except every symbol is mapped to a pair. The recovered symbol stream is now

$$\widehat{x} = \arg \min_{(x_1, x_2) \in C} \left\| \underline{\underline{Y}} - \sqrt{\frac{\rho}{n_T}} \underline{\underline{H}}\underline{\underline{X}} \right\|_F^2 \tag{6}$$

where C is the constellation set of two consecutive symbols and $\|.\|_F$ is the Frobenius norm.

Zero-forcing is a traditional detection approach that intends to suppress the influence of the channel on transmitted signal component to zero. Equation 7 shows that the scalar factor on the original signal is $\sqrt{\frac{\rho}{n_T}}\underline{\underline{H}}$, therefore the weight at the receiver should be $\left(\sqrt{\frac{\rho}{n_T}}\underline{\underline{H}}\right)^{-1}$. The overall signal z become

$$\underline{z} = \underline{x} + \left(\sqrt{\frac{\rho}{n_T}} \underline{\underline{H}} \right)^{-1} \underline{n} \tag{7}$$

and the recovered signal is

$$\underline{\widehat{x}} = Q \left\{ \left(\sqrt{\frac{\rho}{n_T}} \underline{\underline{H}} \right)^{-1} \underline{y} \right\} \tag{8}$$

where $Q\{.\}$ means quantisation. It is obvious that although the impact on signal component is removed, the noise is multiplied by $\left(\sqrt{\frac{\rho}{n_T}}\underline{\underline{H}}\right)^{-1}$ as well. If the channel is in poor condition, the effect of noise can be amplified and result in high BER.

An advanced algorithm aims to minimise the noise component called minimum mean-square error can mitigate the problem above as long as the noise variance is known. In our system, assume unit variance, and we have

$$\underline{\widehat{x}} = Q \left\{ \left(\frac{\rho}{n_T} \underline{\underline{H}}^H \underline{\underline{H}} + \underline{\underline{I}} \right)^{-1} \sqrt{\frac{\rho}{n_T}} \underline{\underline{H}}^H \underline{y} \right\} \tag{9}$$

where the variance matrix is denoted by $\underline{\underline{I}}$. This approach suppresses the impact of noise and brings about more similarity between the received signal and the transmitted one, which is especially suitable for ill-conditioned channels.

III. Result and Analysis

A. MIMO Capacity

MIMO capacity for systems with 5 to 10 transmitting and receiving antennas were simulated based on MCM. We generated 10,000 channels for each case and averaged the instantaneous capacity to approximate the Shannon capacity for SNR from 0 to 30 dB.

Figure 2 indicates that there exists a positive correlation between capacity and SNR. For all MIMO systems, the capacity

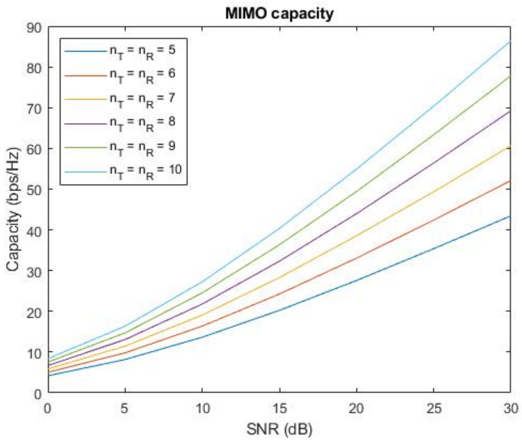

Fig. 2. MIMO capacity for system with 5-10 transmit and receive antennas for $SNR = 0 - 30$ dB

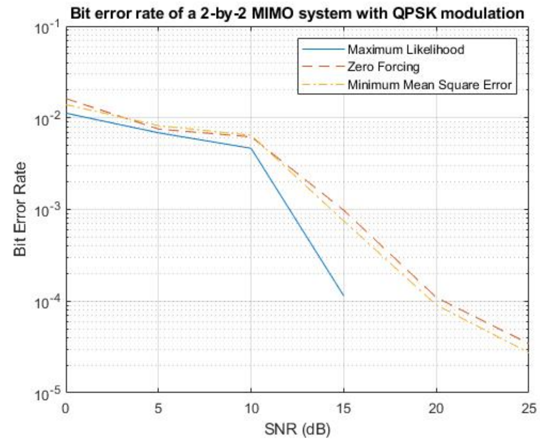

Fig. 3. Average BER of different detectors for 2-by-2 MIMO system with $SNR = 0 - 35$ dB

enlarges when SNR is increased. For example, the capacity of a 5T5R system is 4.1891, 8.1757, and 13.6476 bps/Hz for $SNR = 0, 5, 10$ dB, which are almost in a linear relationship. The plot with independent variable in dB suggests that the relationship in between is almost logarithmic, as estimated from Equation 3.

It can be interpreted from another dimension, by the number of elements. For a fixed SNR, it appears that MIMO capacity is proportional to the count of antenna pairs. An obvious comparison between 5 and 10 antenna system indicates that the capacity of the former is around half of the latter. When $SNR = 0, 10, 20$ dB, the capacity are 4.1891, 13.6476, 27.5785 and 8.3841, 27.2373, 54.9246 bps/Hz respectively. It denotes that adding antennas can lead to a linear increase in system capacity.

Findings above agree with the theory, although there are minor deviations. The reason is that we only simulated several cases of MIMO systems with a limited number of channels being generated. The error can be further reduced by averaging more instantaneous channels.

The result also suggests that a MIMO system can be utilised to increase spectrum efficiency. When the available bandwidth is fixed, and the higher data rate is demanded (as for existing communication systems), multiple-antenna transmission is a feasible solution.

B. Transmission and Reception

BER of ML, ZF, MMSE detectors were compared for a 2-by-2 MIMO system with plain QPSK modulation. The result is averaged over 30 channels each carrying 1,000,000 bits for SNR from 0 to 35 dB.

Figure 3 suggests that the BER tends to decrease with the increase of SNR as expected. Only cases of $SNR = 0-25$ dB were shown because when the channel SNR is high enough, the impact of noise on the signal can be negligible. Therefore, the transmitted signal can be completely recovered, and the BER is 0.

ML is the optimal algorithm concerning BER, especially for high SNR cases. The BER drops almost exponentially when the SNR is increased in dB scale. In this case, error-free communication can be achieved when the SNR is around 20 dB with ML detector, without the requirement of error detection code. Nevertheless, limitations of this approach restrict its application. As mentioned in the theory session, the detection is based on a one-by-one symbol comparison that examines the target function of every possible element. In this plain QPSK case, to estimate one symbol, we need to calculate deviation function for four cases ($0 + 0i$, $0 + i$, $1 + i$, $1 + 0i$). In other words, the complexity of the system is high, and the efficiency is low. Therefore, it is unsuitable for the transmission of long data stream or systems with complicated modulation schemes. This leads to the invention of sub-optimal detectors as ZF and MMSE.

In sharp contrast, the BER trends of ZF and MMSE are sort of linear with SNR in dB. Although their performance difference with the optimal ML algorithm is small for low SNR case (0-10), both detectors can still produce some errors for SNR greater than 15 dB. It is also demonstrated that MMSE defeat ZF in most cases but the gap is narrow. Only when $SNR = 5$ dB ZF is slightly better (7.527×10^{-3} vs. 8.218×10^{-3}), and this is regard as an edge case due to the limitation of sample. On average, the BER performance of MMSE is around 15% better than ZF.

One significant advantage of the two sub-optimal algorithms is complexity. Instead of 'divide-and-conquer', they handle the entire received stream directly then quantise the result. The processing time can be reduced significantly, making them more appropriate for long streams, complex constellations and real-time applications.

As a consequence, there is a trade-off between system reliability and complexity, and specific requirements should determine the choice of detector. ZF and MMSE are feasible for cases as video call where users demand real-time communications while the constraint of BER is not as high. For trustworthy systems as military communication, ML can be utilised to guarantee fewer errors.

C. Space-Time Coding

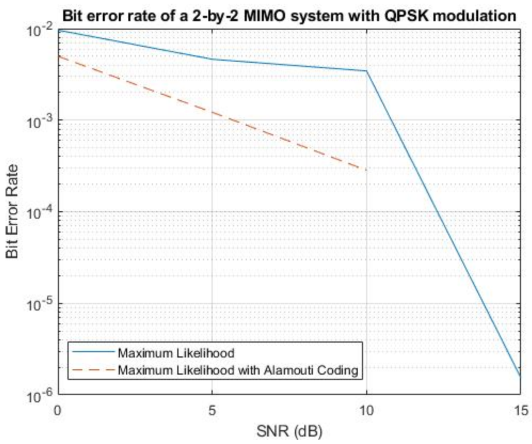

Fig. 4. Average BER of plain and Alamouti detectors for 2-by-2 MIMO system with $SNR = 0 - 35$ dB

Space-time coding improves BER with the cost of the data rate. Figure 4 compares the performance of ML with plain QPSK modulation and another one with Alamouti coding. Note that they conveyed the same raw bits, so the stream length of the latter one was twice than the former.

ML with Alamouti coding demonstrated its advantage on reliability. For $SNR = 0, 5, 10$ dB, the BER are 4.992, 1.219, 0.2835 $\times 10^{-3}$ for Alamouti and 9.622, 4.627, 3.446 $\times 10^{-3}$ for non-Alamouti, which suggests the improvement on BER is more than twice and the gap continues enlarging when SNR increases. Also, the space-coding reached error-free communication for our MIMO system with SNR less than 15 dB. It can be interpreted that space-time coding 'optimise' the SNR by more than 5 dB.

Space-time coding enhanced BER further by transmission redundancy or diversity. In contrast, the cost is data rate since every symbol should be sent more than once. Nevertheless, this can be easily solved by adding the number of antennas. As a consequence, modern communication systems utilise more and more antenna elements to keep the BER below a threshold while maintaining the data rate.

This simulation has several limitations. First, we only investigated a 2-by-2 MIMO system that is impractical for real-world applications. The primary purpose is to investigate and compare conventional detection algorithms regarding performance and complexity. Second, the simulation result is not very accurate, due to the limitation of time. The step size of SNR should have been 2.5 rather than 5 dB to create a better plot. Nevertheless, the simulation takes quite a long time thus the current version is used.

IV. Conclusion

In this article, we explored the influence of antenna elements count on system capacity and examined the system construction, encoding strategies, detection schemes a 2-by-2 MIMO system. It has been proved that MIMO capacity is proportional to the number of antenna pairs. Also, ML is the optimal detection scheme on BER criteria, but the overall complexity is high. In comparison, MMSE and ZF detectors are with larger error rate but easy to implement, thus suitable for real-time applications. Moreover, space-time code as Alamouti code can enhance the BER of existing systems significantly, with the cost of the data rate. Adding the number of antennas can be a practical approach to ensure low BER while maintaining system capacity. The simulation results may be not very accurate due to the limitation of time. The resolution can be improved, and the system can be extended to more complicated cases, with the code provided in Appendices.

V. Appendix: MATLAB code

The source code can be accessed via https://github.com/SnowzTail/multi-input-multi-output-communication.

References

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