Lightning project

Theorems

1 Reduce Maximum Betweenness problem with a channel capacity parameter to MBI problem

Suppose a new graph G with the same set of nodes as the original network. There are $exp(exp(sc))$ edges between nodes u and v in G, if there is an edge with the capacity c between them in the original network. The MBI problem in G is equivalent to maximizing our betweenness except each pair (u, v) has a weight (importance) in the objective function which is described below.

$$\exp(-\exp(-\min_{(i,j)\text{in shortest path of } u, v}\{c_{i,j}\}))$$

for large enough c.

In the theorem above, we wanted the importance of a pair of nodes (u, v) and our presence in the shortest path between them being related to the minimum capacity of the edges along the shortest path. That is because the minimum capacity of the edges between them limits the amount of transactions being passed through this path.

2 Reduce the weighted MBI problem to MBI

Assume that a pair of nodes u and v pays a fee proportional to the value $\mu_u\mu_v$ of the transaction, where μ is a vector whose dimension is equal to the number of nodes in the network. In this case, there exists a graph such that solving the MBI problem on it is equivalent to maximizing the following expression:

$$\sum_{u,v} \mu_u\mu_v \mathbf{1}_{\{\text{being in shortest path of}(u,v)\}}$$

This graph is a copy of the graph of the lightning network itself, except each node in the new graph is attached to a number of new nodes. For each node in the original graph, we introduce a number of new nodes proportional to μ_u and these new nodes are only connected to their corresponding node from the original graph.