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# Plant Capacity and Attainability: Exploration and Remedies


Kristiaan Kerstens

CNRS (LEM-UMR 9221) and IÉSEG School of Management

Jafar Sadeghi

Kharazmi University, visitor at CNRS (LEM-UMR 9221) and IÉSEG School of Management

Ignace Van de Woestyne

KU Leuven, Research Unit MEES, Brussels, Belgium



IÉSEG School of Management

Lille Catholic University

3, rue de la Digue

F-59000 Lille

www.ieseg.fr

Tel: 33(0)3 20 54 58 92

# Plant Capacity and Attainability: Exploration and Remedies

Kristiaan Kerstens*, Jafar Sadeghi†, Ignace Van de Woestyne‡

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**Abstract**

The output-oriented plant capacity notion has been around since more than two decades. It has mainly been applied empirically in the fishery and the hospital sectors. A problem known since its introduction into the literature is that it may not be attainable, in that it presupposes potentially unlimited amounts of variable inputs to determine the maximum of outputs available. This issue of the lack of attainability has never been explored. This paper fills this void both theoretically and empirically. It finds that the attainability may be problematic, and that bounds on the amounts of variable inputs may well need to be imposed.

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*Corresponding author: CNRS-LEM (UMR 9221), IESEG School of Management, 3 rue de la Digue, F-59000 Lille, France, Tel: +33 320545892 (switchboard), Fax: +33 320574855, k.kerstens@ieseg.fr

†Department of Mathematics, Faculty of Mathematical Sciences and Computer, Kharazmi University, 50, Taleghani Ave., 15618, Tehran, Iran, std_j.sadeghi@khu.ac.ir and visitor, CNRS-LEM (UMR 9221), IESEG School of Management, 3 rue de la Digue, F-59000 Lille, France

‡KU Leuven, Research Unit MEES, Warmoesberg 26, B-1000 Brussels, Belgium, ignace.vandewoestyne@kuleuven.be

# 1 Introduction

In the economic literature a variety of capacity notions have been developed (see e.g. Johansen (1968) or Nelson (1989)). One useful taxonomy distinguishes between on the one hand technical or engineering notions and on the other hand economic capacity concepts, whereby the latter are mainly based or derived from some cost function. This paper focuses on the plant capacity notion that is part of the family of technical or engineering notions.

Johansen (1968, p. 362) defined the notion of plant capacity informally as "... the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." Färe, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989) translated this plant capacity notion into a single respectively multiple output nonparametric frontier framework in which plant capacity as well as a measure of capacity utilisation can be determined from information on observed inputs and outputs using a pair of output-oriented efficiency measures.

For over two decades empirical applications have occurred using this output-oriented plant capacity in mainly fisheries (e.g., Felthoven (2002), Pascoe, Hutton, van Putten, Dennis, Skewes, Plagányi, and Deng (2013), Tingley and Pascoe (2005) or Walden and Tomberlin (2010)) and hospital industries (e.g., Karagiannis (2015), Kerr, Glass, McCallion, and McKillop (1999), Valdmanis, Bernet, and Moises (2010) or Valdmanis, DeNicola, and Bernet (2015)). One study focuses on banking (e.g., Sahoo and Tone (2009)), and we are aware of one article describing a macro-economic application on trade barriers (e.g., Badau (2015)). But, no major methodological innovation has occurred related to this plant capacity concept. However, recently Cesaroni, Kerstens, and Van de Woestyne (2017) use the same nonparametric frontier framework to define a new input-oriented measure of plant capacity utilisation based on a couple of input-oriented efficiency measures.

Already Johansen (1968, p. 362) pointed out that the plant capacity concept need not necessarily be attainable, in that the amounts of variable inputs needed to determine the maximum potential outputs may well be unavailable at either the firm level or the sector level. To the best of our knowledge, the literature has completely ignored this issue of attainability. This paper sets as a major goal to explore this attainability problem. At the theoretical level, we will argue that there is indeed such an issue for the output-oriented plant capacity notion, but we will also show that the new input-oriented plant capacity concept does not suffer from this problem. At the empirical level, we illustrate the extent to which the amounts of variable inputs needed to determine the plant capacity output are plausible or not using a secondary data set.

It is becoming known that the axiom of convexity has a potentially large impact on the empirical analysis based on technologies (for example, Tone and Sahoo (2003)). In the context of plant capacity utilisation, for instance, Walden and Tomberlin (2010) empirically illustrate the effect of convexity on the output-oriented plant capacity notion. In a similar way, Cesaroni, Kerstens, and Van de Woestyne (2017) reveal the influence of convexity on the input-oriented plant capacity concept. Therefore, we also analyse the issue of attainability in terms of the potential effect of the convexity axiom.

The structure of this contribution is as follows. Section 2 provides the basic definitions of technology and efficiency measures representing these technologies. The next Section 3 starts out by defining both the traditional output-oriented and the new input-oriented plant capacity notions. Thereafter, we argue and illustrate that the output-oriented plant capacity notion may well fail attainability, while there is no such an issue for the input-oriented plant capacity concept. We end this section by defining an attainable output-oriented plant capacity notion that incorporates either firm or industry constraints on the availability of variable inputs. Section 4 describes the secondary data set selected for the empirical illustration and summarizes the empirical results in great detail. A final Section 5 ends with some concluding remarks.

# 2 Technology: Basic Definitions

This section introduces some basic notation and defines the technology. Given an $N$-dimensional input vector $x \in \mathbb{R}^N_+$ and an $M$-dimensional output vector $y \in \mathbb{R}^M_+$, the production possibility set or technology $T$ is defined as follows: $T = \{(x, y) | x \text{ can produce } y\}$. Associated with $T$, the input set denotes all input vectors $x$ capable of producing a given output vector $y$: $L(y) = \{x | (x, y) \in T\}$. Analogously, the output set associated with $T$ denotes all output vectors $y$ that can be produced from a given input vector $x$: $P(x) = \{y | (x, y) \in T\}$.

Throughout this contribution, technology $T$ satisfies some combination of the following standard assumptions:

(T.1) Possibility of inaction and no free lunch, i.e., $(0, 0) \in T$ and if $(0, y) \in T$, then $y = 0$.

(T.2) $T$ is a closed subset of $\mathbb{R}^N_+ \times \mathbb{R}^M_+$.

(T.3) Strong input and output disposal, i.e., if $(x, y) \in T$ and $(x', y') \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$, then $(x', -y') \geq (x, -y) \Rightarrow (x', y') \in T$.

(T.4) $T$ is convex.

Briefly discussing these traditional axioms on technology, it is useful to recall the following (see, e.g., Hackman (2008) for details). Inaction is feasible, and there is no free lunch. Technology is closed. We assume free disposal of inputs and outputs in that inputs can be wasted and outputs can be discarded. Finally, technology is convex. In our empirical analysis not all these axioms are simultaneously maintained.[1] In particular, key assumption distinguishing some of the technologies in the empirical analysis is convexity versus nonconvexity.

The radial input efficiency measure characterizes the input set $L(y)$ completely and can be defined as follows:

$$DF_i(x, y) = \min\{\lambda \mid \lambda \geq 0, \lambda x \in L(y)\}. \tag{1}$$

This radial input efficiency measure has the main property that it is smaller or equal to unity ($DF_i(x, y) \leq 1$), with efficient production on the boundary (isoquant) of $L(y)$ represented by unity, and that it has a cost interpretation (see, e.g., Hackman (2008)).

The radial output efficiency measure offers a complete characterization of the output set $P(x)$ and can be defined as:

$$DF_o(x, y) = \max\{\theta \mid \theta \geq 0, \theta y \in P(x)\}. \tag{2}$$

Its main properties are that it is larger than or equal to unity ($DF_o(x, y) \geq 1$), with efficient production on the boundary (isoquant) of the output set $P(x)$ represented by unity, and that this radial output efficiency measure has a revenue interpretation (e.g., Hackman (2008)).

In the short run, we can partition the input vector into a fixed and variable part. In particular, we denote ($x = (x^f, x^v)$) with $x^f \in R^{N_f}_+$ and $x^v \in R^{N_v}_+$ such that $N = N_f + N_v$.

Similarly, a short-run technology $T^f = \{(x^f, y) \in \mathbb{R}^{N_f}_+ \times \mathbb{R}^M_+ \mid x^f \text{ can produce } y\}$ and the corresponding input set $L^f(y) = \{x^f \in \mathbb{R}^{N_f}_+ \mid (x^f, y) \in T^f\}$ and output set $P^f(x^f) = \{y \mid (x^f, y) \in T^f\}$ can be defined. Note that technology $T^f$ is in fact obtained by a projection of technology $T \in \mathbb{R}^{N+M}_+$ into the subspace $\mathbb{R}^{N_f+M}_+$ (i.e., by setting all variable inputs equal to zero). By analogy, the same applies to the input set $L^f(y)$ and the output set $P^f(x^f)$.

Denoting the radial output efficiency measure of the output set $P^f(x^f)$ by $DF^f_o(x^f, y)$,

[1] For instance, note that the convex variable returns to scale technology does not satisfy inaction.

this output-oriented efficiency measure can be defined as follows:

$$DF_o^f(x^f, y) = \max\{\theta \mid \theta \geq 0, \theta y \in P^f(x^f)\}. \tag{3}$$

The sub-vector input efficiency measure reducing only the variable inputs is defined as follows:

$$DF_i^{SR}(x^f, x^v, y) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(y)\}. \tag{4}$$

Next, we need the following particular definition of technology: $L(0) = \{x \mid (x, 0) \in T\}$ is the input set with zero output level. The sub-vector input efficiency measure reducing variable inputs evaluated relative to this input set with a zero output level is as follows:

$$DF_i^{SR}(x^f, x^v, 0) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(0)\}. \tag{5}$$

Given data on $K$ observations ($k = 1, \ldots, K$) consisting of a vector of inputs and outputs $(x_k, y_k) \in \mathbb{R}_+^N \times \mathbb{R}_+^M$, a unified algebraic representation of convex and nonconvex nonparametric frontier technologies under the flexible or variable returns to scale assumption is possible as follows:

$$T^{\Lambda} = \left\{ (x, y) \mid x \geq \sum_{k=1}^{K} z_k x_k, y \leq \sum_{k=1}^{K} z_k y_k, z \in \Lambda, \right\}, \tag{6}$$

where

(i) $\Lambda \equiv \Lambda^{\mathrm{C}} = \left\{ z \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \geq 0 \right\}$;

(ii) $\Lambda \equiv \Lambda^{\mathrm{NC}} = \left\{ z \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \in \{0, 1\} \right\}$.

The activity vector $z$ of real numbers summing to unity represents the convexity axiom. This same sum constraint with each vector element being a binary integer is representing nonconvexity. The convex technology satisfies axioms (T.1) (except inaction) to (T.4), while the nonconvex technology adheres to axioms (T.1) to (T.3). It is now useful to condition the above notation of the efficiency measures relative to these nonparametric frontier technologies by distinguishing between convexity (convention $C$) and nonconvexity (convention $NC$).

# 3 Plant Capacity Concepts

## 3.1 Plant Capacity: Basic Definitions

Recall the informal definition of plant capacity by Johansen (1968, p. 362) as "the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." This clearly output-oriented plant capacity notion has been admirably made operational by Färe, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989) using a pair of output-oriented efficiency measures. We now recall the definition of this output-oriented plant capacity utilization ($PCU$).

**Definition 3.1.** The output-oriented plant capacity utilization ($PCU_o$) is defined as:

$$PCU_o(x, x^f, y) = \frac{DF_o(x, y)}{DF_o^f(x^f, y)}, \tag{7}$$

where $DF_o(x, y)$ and $DF_o^f(x^f, y)$ are output efficiency measures including, respectively excluding, the variable inputs as defined before in (2) and (3). Since $1 \leq DF_o(x, y) \leq DF_o^f(x^f, y)$, notice that $0 < PCU_o(x, x^f, y) \leq 1$. Thus, output-oriented plant capacity utilization has an upper limit of unity. Following the terminology introduced by Färe, Grosskopf, and Kokkelenberg (1989), Färe, Grosskopf, and Valdmanis (1989) and Färe, Grosskopf, and Lovell (1994), one can distinguish between a so-called biased plant capacity measure $DF_o^f(x^f, y)$ and an unbiased plant capacity measure $PCU_o(x, x^f, y)$. Taking the ratio of efficiency measures eliminates any existing inefficiency and yields in this sense a cleaned concept of output-oriented plant capacity.

In case of $C$, the efficiency measure $DF_o^f(x^f, y)$ is computed for observation $(x_p, y_p)$ as follows:

$$\begin{aligned} DF_o^f(x_p^f, y_p) = \quad & \max \quad \theta \\ & s.t \quad \sum_{k=1}^{K} z_k y_k \geq \theta y_p, \\ & \qquad \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\ & \qquad \sum_{k=1}^{K} z_k = 1, \\ & \qquad \theta \geq 0, z_k \geq 0, \quad k = 1, \ldots, K. \end{aligned} \tag{8}$$

In case of $NC$, the variables $z_k$ in this model need to be binary variables. In all LP models mentioned hereafter, a similar adaptation is required if $NC$ is assumed. To save space, we will not mention this again, nor formulate the corresponding models.

Observe that there are no input constraints on the variable inputs in the model (8). Note that Färe, Grosskopf, and Lovell (1994) introduce an alternative linear program (LP) with a scalar for each variable input dimension. Also note that LP (8) is equivalent to the following LP obtained by making each variable input a decision variable:

$$
\begin{aligned}
DF_o^f(x_p^f, y_p) = \quad & \max \quad \theta \\
& s.t \quad \sum_{k=1}^{K} z_k y_k \geq \theta y_p, \\
& \qquad \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\
& \qquad \sum_{k=1}^{K} z_k x_k^v \leq x^v, \\
& \qquad \sum_{k=1}^{K} z_k = 1, \\
& \qquad \theta \geq 0, z_k \geq 0, x^v \geq 0, \quad k = 1, \ldots, K.
\end{aligned}
\tag{9}
$$

Cesaroni, Kerstens, and Van de Woestyne (2017) define a new input-oriented plant capacity measure using a pair of input-oriented efficiency measures.

**Definition 3.2.** The input-oriented plant capacity utilization ($PCU_i$) is defined as:

$$
PCU_i(x, x^f, y) = \frac{DF_i^{SR}(x^f, x^v, y)}{DF_i^{SR}(x^f, x^v, 0)},
\tag{10}
$$

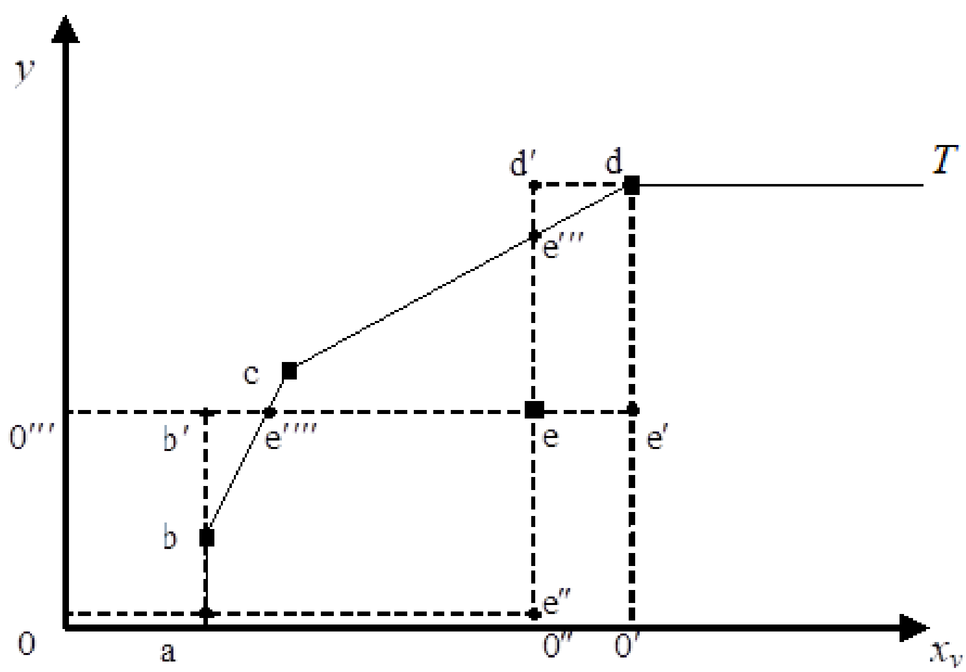
where $DF_i^{SR}(x^f, x^v, y)$ and $DF_i^{SR}(x^f, x^v, 0)$ are both sub-vector input efficiency measures reducing only the variable inputs relative to the technology, the latter efficiency measure being evaluated at a zero output level. Since $0 < DF_i^{SR}(x^f, x^v, 0) \leq DF_i^{SR}(x^f, x^v, y)$, notice that $PCU_i(x, x^f, y) \geq 1$. Thus, input-oriented plant capacity utilization has a lower limit of unity. Similar to the previous case, one can distinguish between a so-called biased plant capacity measure $DF_i^{SR}(x^f, x^v, 0)$ and an unbiased plant capacity measure $PCU_i^{SR}(x, x^f, y)$, the latter being cleaned of any prevailing inefficiency.

Now we try to clarify both these definitions with the help of Figure 1 which depicts a single variable input and output space. In particular, Figure 1 shows a total product curve for given fixed inputs as the polyline *abcd* and its horizontal extension at $d$. We focus on observation $e$. Note that observations are represented by squares and projection points by circles.

The output-oriented plant capacity measure $PCU_o(x, x^f, y)$ compares point $e$ to its vertical projection point $e'''$ on the frontier on the one hand, and the translated point $e'$ that consumes more variable inputs to its vertical projection point on the horizontal frontier segment emanating from point $d$ with maximal outputs on the other hand. Clearly, the maximal output $d$ can be labelled the plant capacity output. Thus, the unbiased plant capacity measure $PCU_o(x, x^f, y)$ is somehow linked to the distance $e'''d'$, whereby point $d'$ is simply the translation of the maximal output at point $d$ to the output level comparable with point $e$.

The input-oriented plant capacity measure $PCU_i(x, x^f, y)$ focuses on a sub-vector of variable inputs and compares point $e$ to its horizontal projection point $e''''$ on the frontier on the one hand, and the translated point $e''$ (consuming equal amounts of variable inputs but at a zero outputs level) to its horizontal projection point on the vertical frontier segment $ab$ with zero outputs on the other hand. Clearly, the minimal variable input $a$ yielding zero output can be labelled the plant capacity input. Thus, the unbiased plant capacity measure $PCU_i(x, x^f, y)$ is somehow linked to the distance $b'e''''$, whereby point $b'$ is the translation of the variable input at point $b$ to the variable input level comparable with point $e$.

Figure 1: Total product curve: Output- and input-oriented plant capacity.




We now turn to the issue of attainability of both these plant capacity concepts.

## 3.2 Plant Capacity: The Question About Attainability

While these definitions in itself are sufficiently clear, it may be useful to underscore that these concepts differ with respect to the property of attainability. As stressed by Johansen (1968, p. 362) the output-oriented plant capacity notion is not attainable in that the extra variable inputs necessary to reach the maximal plant capacity output may not be available. While the axiom of strong disposability in the inputs in principle allows for wasting infinitely many inputs to determine the maximal plant capacity outputs, in practice there may well be restrictions of various kinds that limit the availability of variable inputs.[2]

First, at the firm level there may be quasi-fixed factors like labour where firms have to invest in hiring and training activities that limit the amounts of labour that can be recruited at once. By definition, quasi-fixed factors are characterised by the fact that their supply cannot be expanded rapidly. Furthermore, depending on the nature of the labour market and the size of the firm (e.g., it may have some monopsony power), recruiting a large amount of people may well have an impact on their salaries. While this does not show up in the analytical framework of the output-oriented plant capacity notion that ignores input prices, firms may well in fact take account of these general equilibrium effects and constrain their recruitment of the quasi-fixed factor. In brief, the quasi-fixity of labour as well as other production factors may seriously impede the expansion of variable inputs and may thus prevent reaching the maximal plant capacity outputs (e.g., Oi (1962) for the seminal article in economics and Barney (2001) for the resource-based view of the firm).

Second, even if these extra variable inputs are available at the firm level, as stressed by Johansen (1968) there may be restrictions on the available extra variable inputs at the sector level that may prevent that all firms simultaneously can reach their maximal plant capacity output. For instance, quasi-fixed factors may operate at the industry level and prevent the rapid expansion of their supply in amounts needed to allow for the realization of the maximal plant capacity outputs for all firms. At the sectoral level, it is obvious that general equilibrium effects may play a role: if all firms simultaneously increase their demand for a production factor, then the price of that production factor may well increase. Again, while this does not show up in the framework of the output-oriented plant capacity notion which ignores factor prices, firms may take these general equilibrium effects into account and constrain their expansion of the production factor.

By contrast, the input-oriented plant capacity notion is always attainable in that one

[2]The idea of a kind of limited strong disposability has been pursued in the context of congestion measurement in Briec, Kerstens, and Van de Woestyne (2016).

can always reduce the amount of existing variable inputs such that one reaches an input set with zero output level. Reducing variable inputs to reach zero production levels is normally possible because of the axiom of inaction. Inaction implies that one can stop producing at all: but, in modern production facilities producing a zero output need not imply that no inputs are used.[3] Examples of zero production with positive amounts of variable inputs include critical maintenance activities at a large industrial plant impeding production, making inventories in a retailer while temporarily suspending sales, or temporarily closing a mine while keeping it exploitable with the option of reopening it as part of a real options strategy. Closing down production is therefore possible at the firm level, but it can be done as well at the sectoral level.

Therefore, attainability is a potential issue for the output-oriented plant capacity notion, while it is a priori not an issue for the new input-oriented plant capacity concept. We now turn to the modeling of constraints on the availability of variable inputs in the output-oriented plant capacity notion.

## 3.3 Attainable Output-Oriented Plant Capacity: Proposals

We now first turn to the specification of attainability constraints at the firm level. Thereafter, we explore how to model attainability constraints at the industry level.

### 3.3.1 Attainability Constraints at the Firm Level

Granting that attainability is a potential issue for the output-oriented plant capacity notion, it is important to model constraints on the availability of variable inputs in a general way. This leads us to define an attainable output-oriented efficiency measure that incorporates realistic limits on the availability of variable inputs.

**Definition 3.3.** The attainable output-oriented efficiency measure ($ADF_o$) at level $\bar{\lambda} \in \mathbb{R}_+$ is defined as:

$$ADF_o^f(x^f, y, \bar{\lambda}) = \max\{\theta \mid \theta \geq 0, 0 \leq \lambda \leq \bar{\lambda}, \theta y \in P(x^f, \lambda x^v)\}. \tag{11}$$

The amount of variable inputs is now bounded to be at most a scalar-wise multiple smaller

[3] While inaction is often phrased mathematically as $(0, 0) \in T$, the occurrence of zero outputs need not imply zero inputs. By the assumption of strong input disposability $(x, 0) \in T$ for $x > 0$. Thus, the use of positive inputs is compatible with zero outputs.

than $\bar{\lambda}$. Obviously, $ADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o^f(x^f, y)$. Note that Definition 3.3 is written in absolute terms. For instance, $\bar{\lambda} = 3$ corresponds with the impossibility of variable inputs to exceed three times the current amount of variable inputs. Alternatively, one could focus on relative comparisons to the sector aggregates ($\sum_{p=1}^{K} x_p^v$). Then, one could impose that variable inputs at the firm level cannot exceed a certain share of the total amount of variable inputs available in the sector. We opt for the first approach.

Using the attainable output-oriented efficiency measure introduced in Definition 3.3, it is natural to come up with a new attainable output-oriented plant capacity concept at the firm level.

**Definition 3.4.** An attainable output-oriented plant capacity utilization ($APCU_o$) at level $\bar{\lambda} \in \mathbb{R}_+$ is defined as:

$$APCU_o(x, x^f, y, \bar{\lambda}) = \frac{DF_o(x, y)}{ADF_o^f(x^f, y, \bar{\lambda})}, \tag{12}$$

with $DF_o(x, y)$ and $ADF_o^f(x^f, y, \bar{\lambda})$ as defined before.

By analogy with the plant capacity utilization measures introduced in Definitions 3.1 and 3.2, one can distinguish between the biased attainable plant capacity measure $ADF_o^f(x^f, y, \bar{\lambda})$ and the unbiased attainable plant capacity measure $APCU_o(x, x^f, y, \bar{\lambda})$, where the ratio of efficiency measures ensures eliminating any existing inefficiency.

Since $ADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o^f(x^f, y)$, clearly $APCU_o(x, x^f, y, \bar{\lambda}) \geq PCU_o(x, x^f, y)$. Thus, the attainable output-oriented measure of plant capacity utilization is always larger or equal to the traditional measure of output-oriented plant capacity utilization.

Modeling attainability constraints at the firm level can now be done as follows:

$$\begin{aligned}
ADF_o^f(x_p^f, y_p, \bar{\lambda}) = \quad & \max \quad \theta \\
& s.t \quad \sum_{k=1}^{K} z_k y_k \geq \theta y_p, \\
& \qquad \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\
& \qquad \sum_{k=1}^{K} z_k x_k^v \leq x^v, \\
& \qquad \sum_{k=1}^{K} z_k = 1, \\
& \qquad x^v \leq \bar{\lambda} x_p^v, \\
& \qquad \theta \geq 0, z_k \geq 0, x^v \geq 0, \quad k = 1, \ldots, K.
\end{aligned} \tag{13}$$

The constraint $x^v \leq \bar{\lambda} x_p^v$ establishes a link between the decision variable $x^v$ and the value

$x_p^v$ of the firm under observation. In the empirical analysis of Section 4, we choose $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$. Thus, we consider an increase of the variable inputs with a factor more than five or less than 0.5 (i.e., halving these variable inputs) as implausible.

In model (13), the scalar $\bar{\lambda}$ can be varied over some part of the interval $(0, \infty)$. To determine the complete feasible interval for $\bar{\lambda}$ and to classify $ADF_o^f(x^f, y, \lambda)$ and $APCU_o(x, x^f, y, \lambda)$ further on, we can define the following three critical points $L_p$, $M_p$ and $U_p$ for some observation $(x_p, y_p)$ as follows:

$$L_p = DF_i^{SR}(x_p^f, x_p^v, 0), \tag{14}$$

$$M_p = DF_i^{SR}(x_p^f, x_p^v, y), \tag{15}$$

and

$$U_p = DF_i^{SR}(x_p^f, x_p^v, DF_o^f(x^f, y)y). \tag{16}$$

Note that the critical points $L_p$ and $M_p$ make up the components of the input-oriented plant capacity measure $PCU_i(x, x^f, y)$ in Definition 3.2. To our knowledge, $U_p$ has not been described earlier in the literature. It can be interpreted as the minimal expansion of variable inputs needed to produce the maximum plant capacity outputs and can be computed as follows:

$$\begin{aligned} U_p = \quad & \min \quad \theta \\ & s.t \quad \sum_{k=1}^{K} z_k y_k \geq DF_o^f(x_p^f, y_p) y_p, \\ & \qquad \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\ & \qquad \sum_{k=1}^{K} z_k x_k^v \leq \theta x_p^v, \\ & \qquad \sum_{k=1}^{K} z_k = 1, \\ & \qquad \theta \geq 0, z_k \geq 0, k = 1, \ldots, K. \end{aligned} \tag{17}$$

These three critical points can be briefly illustrated with the help of Figure 1. First, the point $L_p$ relates to the distance from point $a$ to point $e''$: it indicates the minimal amount of variable inputs compatible with zero outputs. Second, the point $M_p$ relates to the distance from point $e''''$ to point $e$: it indicates the minimal amount of variable inputs compatible with current levels of outputs. Third, the point $U_p$ relates to the distance from point $e$ to point $e'$: it indicates the minimal amount with which variable inputs need to be expanded to be compatible with the maximal level of plant capacity outputs at point $d$.

We are now in a position to classify $ADF_o^f(x^f, y, \bar{\lambda})$ and $APCU_o(x, x^f, y, \bar{\lambda})$ in terms of these three critical points. In particular, we establish two propositions.

**Proposition 3.1.** *For the biased and unbiased attainable output-oriented plant capacity utilization in both C and NC technologies, for every observation* $(x_p, y_p)$ *we have:*

*(i) If* $\bar{\lambda} < L_p$*, then model* (13) *is infeasible.*

*(ii) If* $L_p \leq \bar{\lambda} < M_p$*, then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < 1$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > 1$.

*(iii) If* $M_p \leq \bar{\lambda}$*, then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq 1$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) \leq 1$.

*Proof.* (i) Suppose that $\bar{\lambda} < L_p$ and model (13) is feasible with optimal solution $(z_k^*, x^{v*})$. Hence, $x^{v*} \leq \bar{\lambda} x_p^v < L_p x_p^v$. Therefore, $(\hat{z}_k = z_k^*, \hat{\theta} = \bar{\lambda})$ is a feasible solution of model (14) with optimal value $\hat{\theta} = \bar{\lambda}$. But, this is contradiction since $\bar{\lambda} < L_p$.
(ii) Assume that $L_p \leq \bar{\lambda} < M_p$ and $(z_k^*, x^{v*}, \theta^*)$ is an optimal solution of model (13) such that $\theta^* \geq 1$. Hence, we have $x^{v*} \leq \bar{\lambda} x_p^v < M_p x_p^v$. So $(\hat{z}_k^* = z_k^*, \hat{\theta} = \bar{\lambda})$ is a feasible solution of model (15) with optimal value $\hat{\theta} = \bar{\lambda}$. This is contradiction since $\bar{\lambda} < M_p$.
(iii) Assume that $M_p \leq \bar{\lambda}$ and $(z_k^*, \theta^* = M_p)$ is an optimal solution of model (15). Since $M_p x_p^v \leq \bar{\lambda} x_p^v$, hence $(\hat{z}_k = z_k^*, \hat{x}^v = M_p x_p^v, \hat{\theta} = 1)$ is a feasible solution of model (13) with objective value $\hat{\theta} = 1$. Therefore, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq 1$ because the kind of model is a maximising problem. □

**Proposition 3.2.** *For the biased and unbiased attainable output-oriented plant capacity utilization in both C and NC technologies, for every observation* $(x_p, y_p)$*, we have:*

*(i) If* $L_p \leq \bar{\lambda} < U_p$*, then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < DF_o^f(x_p^f, y_p)$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > PCU_o(x_p, x_p^f, y_p)$.

*(ii) If* $\bar{\lambda} \geq U_p$*, then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$.

*Proof.* (i) Suppose that $(z_k^*, x^{v*}, \theta^*)$ is an optimal solution of model (13). This solution is also a feasible solution of model (9). Since $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \leq DF_o^f(x_p^f, y_p)$, it is sufficient to show that this solution is not an optimal solution of model (9). By contradiction, suppose that $(z_k^*, x^{v*}, \theta^*)$ is an optimal solution of model (9), since $\bar{\lambda} < U_p$, thus $x^{v*} \leq \bar{\lambda} x_p^v < U_p x_p^v$. Therefore, $(\hat{z}_k = z_k^*, \hat{\theta} = \bar{\lambda})$ is a feasible solution of model (17) with objective value $\bar{\lambda} < U_p$, which is a contradiction.
(ii) Assume that $(z_k^*, \theta^* = U_p)$ is an optimal solution of model (17). Since $\bar{\lambda} \geq U_p$, hence $\bar{\lambda} x_p^v \geq U_p x_p^v$. Therefore, $(\hat{z}_k = z_k^*, \hat{x}^v = U_p x_p^v, \hat{\theta} = DF_o^f(x_p^f, y_p))$ is a feasible solution of model

(13). Thus, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq DF_o^f(x_p^f, y_p)$. But, we also know that $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \leq DF_o^f(x_p^f, y_p)$. Hence, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$. This completes the proof. □

### 3.3.2 Attainability Constraints at the Industry Level

Similar to the firm level version, it is natural to come up with new industry attainable output-oriented plant capacity concepts. First, we introduce the industry attainable output-oriented efficiency measure as follows:

**Definition 3.5.** The industry attainable output-oriented efficiency measure ($IADF_o$) at level $\bar{\lambda} \in \mathbb{R}_+$ for observation $(x_p, y_p)$ is defined as

$$IADF_o^f(x_p^f, y_p, \bar{\lambda}) = \theta_p^*, \tag{18}$$

with $\theta_p^*$ the optimum value of $\theta_p$ in the following model:

$$\begin{array}{lll}
\max & \sum_{p=1}^{K} \theta_p & \\
s.t & \sum_{k=1}^{K} z_k^p y_k \geq \theta_p y_p, & p = 1, \dots, K, \\
& \sum_{k=1}^{K} z_k^p x_k^f \leq x_p^f, & p = 1, \dots, K, \\
& \sum_{k=1}^{K} z_k^p x_k^v \leq x_p^v, & p = 1, \dots, K, \\
& \sum_{k=1}^{K} z_k^p = 1, & p = 1, \dots, K, \\
& \sum_{p=1}^{K} x_p^v \leq \bar{\lambda} \sum_{p=1}^{K} \bar{x}_p^v, & \\
& \theta_p \geq 0, z_k^p \geq 0, x_p^v \geq 0, & k, p = 1, \dots, K.
\end{array} \tag{19}$$

Note that model (19) is a kind of central resource allocation model with $K$ LPs (one for each observation) and a bogus objective function and with a common constraint on the total amount of variable inputs available in the sector. In particular, its aim is to simultaneously determine the maximum plant capacity outputs for all observations while reallocating variable inputs among units such that a global constraint on the industry amount of variable inputs is respected. Central resource reallocation models cover a heterogeneous variety of models reallocating some inputs and/or outputs across space and/or time while eventually accounting for multiple objectives (e.g., efficiency, effectiveness, equality, etc.) simultaneously. Examples include Athanassopoulos (1998), Golany and Tamir (1995), Korhonen and

Syrjänen (2004), Lozano and Villa (2004), and Ylvinger (2000), among others. One type of central resource reallocation model which also makes use of the notion of plant capacity is the so-called short-run Johansen industry model (e.g., Kerstens, Vestergaard, and Squires (2006)).

Second, using the industry attainable output-oriented efficiency measure of Definition 3.5, the industry attainable output-oriented plant capacity utilization is defined as follows:

**Definition 3.6.** The industry attainable output-oriented plant capacity utilization ($IAPCU_o$) at level $\bar{\lambda} \in \mathbb{R}_+$ for observation $(x_p, y_p)$ is defined as

$$IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = \frac{DF_o(x_p, y_p)}{IADF_o^f(x_p^f, y_p, \bar{\lambda})}. \tag{20}$$

Since $IADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o^f(x^f, y)$, clearly $IAPCU_o(x, x^f, y, \bar{\lambda}) \geq PCU_o(x, x^f, y)$. Thus, the industry attainable output-oriented measure of plant capacity utilization is always larger or equal to the traditional measure of output-oriented plant capacity utilization. By analogy, one can distinguish between the biased industry attainable plant capacity measure $IADF_o^f(x^f, y, \bar{\lambda})$ and the unbiased industry attainable plant capacity measure $IAPCU_o(x, x^f, y, \bar{\lambda})$, where the ratio of efficiency measures ensures eliminating any existing inefficiency.

Note that the industry attainable output-oriented measure of plant capacity utilization may be smaller or larger than the attainable output-oriented measure of plant capacity utilization. This holds true for both the biased and unbiased versions. Therefore, we have $IADF_o^f(x^f, y, \bar{\lambda}) \overset{\geq}{\underset{<}{=}} ADF_o^f(x_p^f, y_p, \bar{\lambda})$ and $IAPCU_o(x, x^f, y, \bar{\lambda}) \overset{\geq}{\underset{<}{=}} APCU_o(x, x^f, y, \bar{\lambda})$.

By analogy to the firm level modelling, the scalar $\bar{\lambda}$ in model (19) can be varied over some part of the interval $(0, \infty)$. To determine this feasible interval for $\bar{\lambda}$ we can define the following two critical points $L^I$ and $U^I$. On the one hand, $L^I$ can be determined from the following LP:

$$
\begin{aligned}
L^I = \quad & \min \quad \theta \\
& s.t \quad \sum_{k=1}^{K} z_k^p x_k^f \leq x_p^f, && p = 1, \ldots, K, \\
& \qquad \sum_{k=1}^{K} z_k^p x_k^v \leq x_p^v, && p = 1, \ldots, K, \\
& \qquad \sum_{k=1}^{K} z_k^p = 1, && p = 1, \ldots, K, \\
& \qquad \sum_{p=1}^{K} x_p^v \leq \theta \sum_{p=1}^{K} \bar{x}_p^v, \\
& \qquad \theta \geq 0, z_k^p \geq 0, x_p^v \geq 0, && k, p = 1, \ldots, K.
\end{aligned}
\tag{21}
$$

On the other hand, $U^I$ is obtained solving the following LP:

$$
\begin{aligned}
U^I = \quad & \min \quad \theta \\
& s.t \quad \sum_{k=1}^{K} z_k^p y_k \geq DF_o^f(x_p^f, y_p) y_p, && p = 1, \ldots, K, \\
& \qquad \sum_{k=1}^{K} z_k^p x_k^f \leq x_p^f, && p = 1, \ldots, K, \\
& \qquad \sum_{k=1}^{K} z_k^p x_k^v \leq x_p^v, && p = 1, \ldots, K, \\
& \qquad \sum_{k=1}^{K} z_k^p = 1, && p = 1, \ldots, K, \\
& \qquad \sum_{p=1}^{K} x_p^v \leq \theta \sum_{p=1}^{K} \bar{x}_p^v, \\
& \qquad \theta \geq 0, z_k^p \geq 0, x_p^v \geq 0, && k, p = 1, \ldots, K.
\end{aligned}
\tag{22}
$$

Note that $U^I$ can be interpreted as the minimal expansion of overall variable inputs needed to produce the plant capacity outputs for all units for the industry model (19).

We are now in a position to classify $IADF_o^f(x^f, y, \bar{\lambda})$ and $IAPCU_o(x, x^f, y, \bar{\lambda})$ in terms of these two critical points in the following proposition:

**Proposition 3.3.** *For the industry biased and unbiased attainable output-oriented plant capacity utilization in both $C$ and $NC$ technologies, we have:*

*(i) If $\bar{\lambda} < L^I$, then model (19) is infeasible.*

*(ii) If $L^I \leq \bar{\lambda} < U^I$, then at least for one observed observation $(x_p, y_p)$ we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) < DF_o^f(x_p^f, y_p)$ and $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > PCU_o(x_p, x_p^f, y_p)$.*

*(iii) If $U^I \leq \bar{\lambda}$, then for every observation $(x_p, y_p)$ we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$*

*and* $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$.

*Proof.* (i) Assume that model (19) is feasible with optimal solution $(\theta_p^*, z_k^{p*}, x_p^{v*})$. Since $\bar{\lambda} < L^I$, thus

$$\sum_{p=1}^{K} x_p^{v*} \leq \bar{\lambda} \sum_{p=1}^{K} \bar{x}_p^v < L^I \sum_{p=1}^{K} \bar{x}_p^v.$$

Therefore, $(\hat{\theta} = \bar{\lambda}, \hat{z}_k^p = z_k^{p*}, \hat{x}_p^v = x_p^{v*})$ is a feasible solution of model (21) with objective value $\hat{\theta} = \bar{\lambda} < L^I$ which is a contradiction.

(ii) Let

$$IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p), \qquad p = 1, ..., K.$$

Also, $(\theta_p^*, z_k^{p*}, x_p^{v*})$ is an optimal solution of model (19) in which $\theta_p^* = IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ and

$$\sum_{p=1}^{K} x_p^{v*} \leq \bar{\lambda} \sum_{p=1}^{K} \bar{x}_p^v < U^I \sum_{p=1}^{K} \bar{x}_p^v.$$

Therefore, $(\hat{\theta} = \bar{\lambda}, \hat{z}_k^p = z_k^{p*}, \hat{x}_p^v = x_p^{v*})$ is a feasible solution of model (22) with objective value $\hat{\theta} = \bar{\lambda} < U^I$ which is a contradiction.

(iii) Assume that $(z_k^{p*}, x_p^{v*}, \theta^* = U^I)$ is an optimal solution of model (22). We have

$$\sum_{p=1}^{K} x_p^{v*} \leq \theta^* \sum_{p=1}^{K} \bar{x}_p^v \leq \bar{\lambda} \sum_{p=1}^{K} \bar{x}_p^v.$$

Therefore, $(\hat{z}_k^p = z_k^{p*}, \hat{x}_p^v = x_p^{v*}, \hat{\theta}_p = DF_o^f(x_p^f, y_p))$ is a feasible solution of model (19) in which $DF_o^f(x_p^f, y_p) \leq IADF_o^f(x_p^f, y_p, \bar{\lambda})$. But, we know that $IADF_o^f(x_p^f, y_p, \bar{\lambda}) \leq DF_o^f(x_p^f, y_p)$. Hence, $DF_o^f(x_p^f, y_p) = IADF_o^f(x_p^f, y_p, \bar{\lambda})$ and this completes the proof. □

# 4 Empirical Illustration

## 4.1 Description of the Sample

For the empirical illustration of the attainability notions introduced in previous section, we use a secondary data set from Atkinson and Dorfman (2009). The sample is based on 16 Chilean hydro-electric power generation plants observed on a monthly basis. We limit ourselves to the observations for the year 1997 and, assuming that there is no technical change, we specify an inter-temporal frontier resulting in a total of 192 units. It is well-known that Chile was one of the first countries deregulating its electricity market and that hydro-power was a dominant source of energy during the 90's. These hydro-power plants generate one output (electricity) using three inputs: labor, capital, and water. Except for the fixed input capital, the remaining flow variables are expressed in physical units. Table 1 presents basic descriptive statistics for the inputs and the single output. One can observe a large heterogeneity in terms of size among the different inputs as well as the single output.

Table 1: Descriptive Statistics for Hydro-Power Plants (1997)

| Variable | Trimmed mean[a] | Minimum | Maximum |
|---|---|---|---|
| Billions of $m^3$ of water (variable input) | 126.80 | 0.49 | 1347.47 |
| # workers (variable input) | 15.62 | 2.00 | 52.86 |
| Billions of capital (fixed input) | 0.47 | 0.04 | 5.98 |
| Thousands of kWh (output) | 46.79 | 0.40 | 353.70 |

Note: [a]10% trimming level.

## 4.2 Empirical Results for Firm Level

Tables 2 and 3 are structured in a similar way. While Table 2 reports on the biased plant capacity utilisation measures $DF_o^f(x^f, y)$ and $ADF_o^f(x^f, y, \bar{\lambda})$, Table 3 focuses on the unbiased plant capacity utilisation measures $PCU_o(x, x^f, y)$ and $APCU_o(x, x^f, y, \bar{\lambda})$. In each table, the second column reports the standard plant capacity utilisation measures, while the next ten columns describe the attainable plant capacity utilisation measures for $\bar{\lambda}$ varying between 0.5 and 5 with step size 0.5 (thus, $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$). Hence, we somewhat arbitrary assume that variable inputs can be magnified at most fivefold. Obviously, we could have selected a wider range of values to experiment with $\bar{\lambda}$. Based on Proposition 3.1, note that for 37 observations under $C$ and 41 observations under $NC$ $\bar{\lambda} = 0.5$ is too small for model (13) to be feasible. Hence, these observations are not included in the corresponding descriptive statistics computations.

Table 2: Descriptive Statistics of Biased Plant Capacity Utilisation

| | $DF_o^f(x^f,y)$ | $ADF_o^f(x^f,y,\bar{\lambda})$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Convex** | | $\bar{\lambda}=0.5$ | $\bar{\lambda}=1$ | $\bar{\lambda}=1.5$ | $\bar{\lambda}=2$ | $\bar{\lambda}=2.5$ | $\bar{\lambda}=3$ | $\bar{\lambda}=3.5$ | $\bar{\lambda}=4$ | $\bar{\lambda}=4.5$ | $\bar{\lambda}=5$ |
| Average | 13.65457 | 1.016839 | 1.663225 | 2.19119 | 2.594005 | 2.911513 | 3.153133 | 3.357557 | 3.546914 | 3.721204 | 3.876843 |
| Stand.Dev. | 77.13735 | 1.026834 | 1.721058 | 2.421282 | 3.106651 | 3.769749 | 4.349027 | 4.926613 | 5.501961 | 6.077219 | 6.644515 |
| Minimum | 1 | 0.251997 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Maximum | 884.25 | 7.732426 | 15.46485 | 21.69478 | 27.93651 | 34.3226 | 38.80662 | 43.29064 | 47.77466 | 52.25868 | 56.7427 |
| | $DF_o^f(x^f,y)$ | $ADF_o^f(x^f,y,\bar{\lambda})$ | | | | | | | | | |
| **Nonconvex** | | $\bar{\lambda}=0.5$ | $\bar{\lambda}=1$ | $\bar{\lambda}=1.5$ | $\bar{\lambda}=2$ | $\bar{\lambda}=2.5$ | $\bar{\lambda}=3$ | $\bar{\lambda}=3.5$ | $\bar{\lambda}=4$ | $\bar{\lambda}=4.5$ | $\bar{\lambda}=5$ |
| Average | 12.54116 | 0.600155 | 1.274542 | 1.507743 | 1.745801 | 1.94163 | 2.16599 | 2.367088 | 2.54667 | 2.700961 | 2.762264 |
| Stand.Dev. | 77.22638 | 0.906712 | 1.511214 | 1.818299 | 2.156781 | 2.830002 | 2.856101 | 3.501181 | 4.303416 | 4.724691 | 5.079253 |
| Minimum | 1 | 0.117647 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Maximum | 884.25 | 7.714286 | 13.5 | 19 | 21 | 33.25 | 33.25 | 33.25 | 40.28571 | 43.71429 | 45.5 |

Analysing the results in Table 2, one can draw the following conclusions. First, on average the biased plant capacity utilisation measure $DF_o^f(x^f,y)$ indicates that outputs can be magnified by at least 13.65 times under C and 12.54 times under NC. Second, there is a lot of variation in $DF_o^f(x^f,y)$ as indicated by the standard deviation and the range is even huge: the maximum increase in outputs amounts to 884.25 times under both C and NC. Third, the biased attainable plant capacity utilisation measure $ADF_o^f(x^f,y,\bar{\lambda})$ increases monotonously in $\bar{\lambda}$ and on average the output magnification under C is always higher than under NC. Fourth, for a fivefold increase in variable inputs (i.e., $\bar{\lambda}=5$), we obtain on average a 3.87 output magnification under C and a 2.76 output magnification under NC. This is ways below the average output magnification computed by the biased plant capacity utilisation measure $DF_o^f(x^f,y)$.

Table 3: Descriptive Statistics of Unbiased Plant Capacity Utilisation

| | $PCU_o(x,x^f,y)$ | $APCU_o(x,x^f,y,\bar{\lambda})$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Convex** | | $\bar{\lambda}=0.5$ | $\bar{\lambda}=1$ | $\bar{\lambda}=1.5$ | $\bar{\lambda}=2$ | $\bar{\lambda}=2.5$ | $\bar{\lambda}=3$ | $\bar{\lambda}=3.5$ | $\bar{\lambda}=4$ | $\bar{\lambda}=4.5$ | $\bar{\lambda}=5$ |
| Average | 0.521886 | 1.952362 | 1 | 0.778096 | 0.686657 | 0.637375 | 0.610698 | 0.593961 | 0.581193 | 0.571622 | 0.564426 |
| Stand.Dev. | 0.268942 | 0.705465 | 0 | 0.113308 | 0.155652 | 0.17852 | 0.192901 | 0.20318 | 0.211476 | 0.21802 | 0.223191 |
| Minimum | 0.015797 | 1 | 1 | 0.495493 | 0.331236 | 0.272089 | 0.230864 | 0.200488 | 0.177176 | 0.158721 | 0.155053 |
| Maximum | 1 | 4.915901 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | $PCU_o(x,x^f,y)$ | $APCU_o(x,x^f,y,\bar{\lambda})$ | | | | | | | | | |
| **Nonconvex** | | $\bar{\lambda}=0.5$ | $\bar{\lambda}=1$ | $\bar{\lambda}=1.5$ | $\bar{\lambda}=2$ | $\bar{\lambda}=2.5$ | $\bar{\lambda}=3$ | $\bar{\lambda}=3.5$ | $\bar{\lambda}=4$ | $\bar{\lambda}=4.5$ | $\bar{\lambda}=5$ |
| Average | 0.552863 | 2.963899 | 1 | 0.868114 | 0.781615 | 0.732702 | 0.676566 | 0.657501 | 0.646659 | 0.633939 | 0.630842 |
| Stand.Dev. | 0.304061 | 1.424327 | 0 | 0.146017 | 0.186995 | 0.204796 | 0.235747 | 0.244002 | 0.251481 | 0.258491 | 0.260398 |
| Minimum | 0.015267 | 1 | 1 | 0.458927 | 0.391808 | 0.387355 | 0.259312 | 0.098842 | 0.098842 | 0.098842 | 0.098842 |
| Maximum | 1 | 8.5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Turning to the analysis of Table 3, we can infer the following conclusions. First, on average the unbiased plant capacity utilisation measure $PCU_o(x^f,y)$ indicates that current outputs make up 52% from maximal plant capacity outputs under C and 55% under NC. Second, the heterogeneity in $PCU_o(x^f, y)$ is large as indicated by the standard deviation and the range is again huge: the minimum of 1.5% under both C and NC is simply extremely low. Third, the unbiased attainable plant capacity utilisation measure $APCU_o(x, x^f, y, \bar{\lambda})$ decreases monotonously in $\bar{\lambda}$ and on average $APCU_o(x, x^f, y, \bar{\lambda})$ is always smaller under C than under NC. Fourth, for a fivefold increase in variable inputs (i.e., $\bar{\lambda} = 5$), $APCU_o(x, x^f, y, \bar{\lambda})$ is getting close to $PCU_o(x, x^f, y)$ in the C case (a difference of only about 4%), while this gap is larger in the NC case (a difference of about 8%).

Table 4: Descriptive Statistics for Three Critical Points

| | Convex | | | | Nonconvex | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $L_p^C$ | $M_p^C$ | $U_p^C$ | $PCU_i(.)$ | $L_p^{NC}$ | $M_p^{NC}$ | $U_p^{NC}$ | $PCU_i(.)$ | $U_p^C - U_p^{NC}$ |
| Average | 0.338172 | 0.714722 | 31.58504 | 4.396554 | 0.352329 | 0.944414 | 28.75338 | 6.214177 | 2.831656 |
| Stand.Dev. | 0.301264 | 0.256362 | 106.3847 | 4.876813 | 0.313688 | 0.163897 | 106.3716 | 6.335151 | 4.894527 |
| Minimum | 0.037839 | 0.132486 | 1 | 1 | 0.037839 | 0.266667 | 0.904087 | 1 | 0 |
| 1st Quartile | 0.112802 | 0.557439 | 2.627791 | 1.271783 | 0.120576 | 1 | 1.282811 | 1.994785 | 0 |
| Median | 0.2 | 0.753519 | 4.030647 | 2.484958 | 0.245241 | 1 | 2.627116 | 3.566389 | 0.571301 |
| 3rd Quartile | 0.450583 | 0.952263 | 12.29502 | 5.732428 | 0.450583 | 1 | 5.444112 | 7.358733 | 2.6922392 |
| Maximum | 1 | 1 | 648.9984 | 26.07046 | 1 | 1 | 643.5001 | 26.42759 | 25.75867 |

Table 4 reports descriptive statistics on the three critical points $L_p$, $M_p$ and $U_p$ as defined in (14) to (16). The following conclusions can be inferred. First, the average values for $L_p$ and $M_p$ are rather moderate, whereby the values are each time lower under C than under NC. This leads to rather plausible results for the input-oriented plant capacity measure $PCU_i(x, x^f, y)$. Under C one needs on average 4.39 more variable inputs with current outputs than with zero outputs, while under NC one employs 6.21 more variable inputs with current outputs than with zero outputs.

Second, on average the critical point $U_p$ is very high: one needs 31.58 times more variable inputs than currently in use to reach maximum plant capacity outputs under C, while one can magnify variable inputs by just a factor 28.75 under NC. These amounts are huge in comparison to our prior value of allowing for only a fivefold increase in variable inputs. Third, the variation in this factor $U_p$ is huge. For instance, at the third quartile we obtain a 12.29 magnification factor under C and only a 5.44 magnification factor under NC. The maximal magnification factor of 648.99 and 643.50 under C respectively NC are very similar in magnitude and both are clearly impossible in reality. These extreme requirements on the availability of variable inputs clearly cast doubts on the plausibility of the traditional output-oriented plant capacity measure. Fourth, the last column reporting the difference $U_p^C - U_p^{NC}$ reveals that on average the variable inputs under C should be increasing at least 2.83 times more than under NC. Furthermore, there is quite a bit of heterogeneity in this difference $U_p^C - U_p^{NC}$. Thus, in short, while these magnification factors for the variable inputs are clearly implausible, it seems that the non-convex results are the least implausible.

We end this analysis with some results for certain individual observations. Each figure has two parts: the left hand side displays the biased attainable plant capacity in function of the value of $\bar{\lambda}$; the right hand side shows the unbiased attainable plant capacity in function of the value of $\bar{\lambda}$. Both figures are drawn under both the C and NC assumption. Furthermore, the same critical point $U_p$ is drawn for both C and NC in both figures.

Figure 2: Biased and Unbiased Attainable Plant Capacity for Plant 9

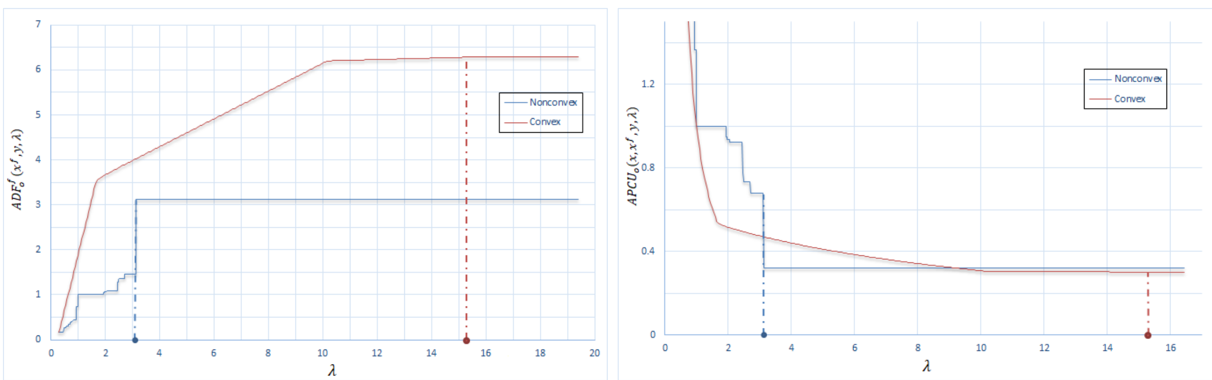
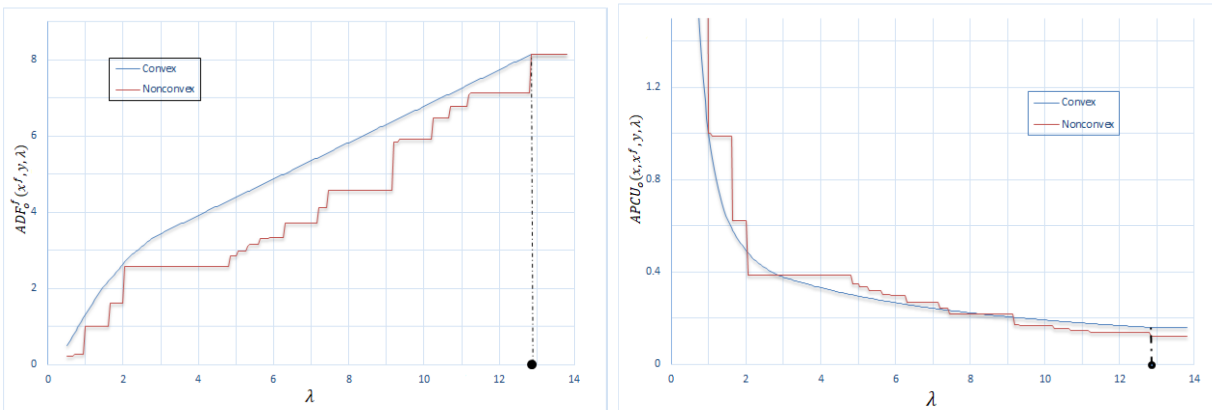



Figure 2 shows results for plant number 9. One can make the following series of observations on the LHS figure. First, the biased attainable plant capacity increases monotonously with $\bar{\lambda}$ under C and in a stepwise fashion under NC: these steps reveal the pervasive problem of slacks that is well-known under NC. Second, the maximum increase in outputs (i.e., the vertical distance between both lines) for the biased attainable plant capacity is almost double under C compared to NC. Third, the value of $U_p$ is almost four times bigger under C (15.48) compared to NC (3.11). The following observations apply to the RHS figure. First, the unbiased attainable plant capacity decreases again monotonously with $\bar{\lambda}$ under C and in a stepwise fashion under NC. Second, the unbiased attainable plant capacity under C compared to NC cross one another: only for very high values of $\bar{\lambda}$ both estimates are close to one another. Overall, this again confirms that the NC results are less implausible.

Figure 3: Biased and Unbiased Attainable plant Capacity Function in the Single Output for Hydro-power Plant 105




Finally, Figure 3 depicts the results for plant number 105. Now the value of $U_p$ under C and NC is identical (12.82). In this case, the differences between C and NC biased attainable plant capacity are rather pronounced, while these differences are mainly visible for the low range values of $\bar{\lambda}$ for the unbiased attainable plant capacity.

## 4.3 Empirical Results for Industry Level

Tables 5 and 6 are structured in a way similar to the corresponding firm level tables. While Table 5 reports on the industry biased plant capacity utilisation measure $IADF_o^f(x^f, y, \bar{\lambda})$, Table 6 focuses on the industry unbiased plant capacity utilisation measures $IAPCU_o(x, x^f, y, \bar{\lambda})$. Again, we have ten columns describing the industry attainable plant capacity utilisation measures for $\bar{\lambda}$ varying between 0.5 and 5 with step size 0.5. New is that the three last rows of Tables 5 and 6 show the number of observed units that have the amounts $ADF_o^f(.) < IADF_o^f(.)$, $ADF_o^f(.) = IADF_o^f(.)$ and $ADF_o^f(.) > IADF_o^f(.)$, respectively. Thus, these lines focus on comparing firm level and industry level results.

Table 5: Descriptive Statistics of Biased Industry Plant Capacity Utilisation

| | $IADF_o^f(x^f, y, \bar{\lambda})$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Convex** | $\bar{\lambda} = 0.5$ | $\bar{\lambda} = 1$ | $\bar{\lambda} = 1.5$ | $\bar{\lambda} = 2$ | $\bar{\lambda} = 2.5$ | $\bar{\lambda} = 3$ | $\bar{\lambda} = 3.5$ | $\bar{\lambda} = 4$ | $\bar{\lambda} = 4.5$ | $\bar{\lambda} = 5$ |
| Average | 12.09168 | 12.97306 | 13.36607 | 13.57577 | 13.64422 | 13.65457 | 13.65457 | 13.65457 | 13.65457 | 13.65457 |
| Stand.Dev. | 77.33517 | 77.23591 | 77.18058 | 77.14818 | 77.13888 | 77.13735 | 77.13735 | 77.13735 | 77.13735 | 77.13735 |
| Minimum | 0.010178 | 0.010178 | 0.317501 | 0.317501 | 0.917641 | 1 | 1 | 1 | 1 | 1 |
| Maximum | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 |
| $ADF_o^f(.) < IADF_o^f(.)$ | 109 | 112 | 128 | 145 | 130 | 119 | 110 | 99 | 89 | 82 |
| $ADF_o^f(.) = IADF_o^f(.)$ | 1 | 2 | 7 | 20 | 38 | 73 | 82 | 93 | 103 | 110 |
| $ADF_o^f(.) > IADF_o^f(.)$ | 82 | 78 | 57 | 27 | 24 | 0 | 0 | 0 | 0 | 0 |
| | $IADF_o^f(x^f, y, \bar{\lambda})$ | | | | | | | | | |
| **Nonconvex** | $\bar{\lambda} = 0.5$ | $\bar{\lambda} = 1$ | $\bar{\lambda} = 1.5$ | $\bar{\lambda} = 2$ | $\bar{\lambda} = 2.5$ | $\bar{\lambda} = 3$ | $\bar{\lambda} = 3.5$ | $\bar{\lambda} = 4$ | $\bar{\lambda} = 4.5$ | $\bar{\lambda} = 5$ |
| Average | 11.54666 | 12.22508 | 12.44996 | 12.54116 | 12.54116 | 12.54116 | 12.54116 | 12.54116 | 12.54116 | 12.54116 |
| Stand.Dev. | 77.35706 | 77.26992 | 77.23991 | 77.22638 | 77.22638 | 77.22638 | 77.22638 | 77.22638 | 77.22638 | 77.22638 |
| Minimum | 0.010178 | 0.079729 | 0.317501 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Maximum | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 | 884.25 |
| $ADF_o^f(.) < IADF_o^f(.)$ | 96 | 125 | 103 | 107 | 99 | 85 | 77 | 72 | 62 | 59 |
| $ADF_o^f(.) = IADF_o^f(.)$ | 8 | 29 | 60 | 85 | 93 | 107 | 115 | 120 | 130 | 133 |
| $ADF_o^f(.) > IADF_o^f(.)$ | 88 | 38 | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Analysing these results in Table 5, we infer the following conclusions. First, the biased industry attainable plant capacity utilisation measure $IADF_o^f(x^f, y, \bar{\lambda})$ increases almost monotonously in $\bar{\lambda}$ and on average the output magnification under C is always higher than under NC. Second, $IADF_o^f(x^f, y, \bar{\lambda})$ becomes stationary after $\bar{\lambda}$ reaches the value 3 under C, and the value 2 under NC. Third, though $IADF_o^f(x^f, y, \bar{\lambda}) \gtreqless ADF_o^f(x_p^f, y_p, \bar{\lambda})$, for the majority of observations we find $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < IADF_o^f(x^f, y, \bar{\lambda})$. Furthermore, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) > IADF_o^f(x^f, y, \bar{\lambda})$ becomes 0 when $IADF_o^f(x^f, y, \bar{\lambda})$ becomes stationary.

Table 6: Descriptive Statistics of Unbiased Industry Plant Capacity Utilisation

| | $IAPCU_o(x, x^f, y, \bar{\lambda})$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Convex** | $\bar{\lambda} = 0.5$ | $\bar{\lambda} = 1$ | $\bar{\lambda} = 1.5$ | $\bar{\lambda} = 2$ | $\bar{\lambda} = 2.5$ | $\bar{\lambda} = 3$ | $\bar{\lambda} = 3.5$ | $\bar{\lambda} = 4$ | $\bar{\lambda} = 4.5$ | $\bar{\lambda} = 5$ |
| Average | 12.69771 | 5.760532 | 0.745656 | 0.591282 | 0.525741 | 0.521886 | 0.521886 | 0.521886 | 0.521886 | 0.521886 |
| Stand.Dev. | 23.79737 | 19.19646 | 0.624234 | 0.472982 | 0.27266 | 0.268942 | 0.268942 | 0.268942 | 0.268942 | 0.268942 |
| Minimum | 0.015797 | 0.015797 | 0.015797 | 0.015797 | 0.015797 | 0.015797 | 0.015797 | 0.015797 | 0.015797 | 0.015797 |
| Maximum | 98.25 | 98.25 | 3.149599 | 3.149599 | 1.08975 | 1 | 1 | 1 | 1 | 1 |
| $APCU_o^f(.) < IAPCU_o^f(.)$ | 82 | 78 | 57 | 27 | 24 | 0 | 0 | 0 | 0 | 0 |
| $APCU_o^f(.) = IAPCU_o^f(.)$ | 1 | 2 | 7 | 20 | 38 | 73 | 82 | 93 | 103 | 110 |
| $APCU_o^f(.) > IAPCU_o^f(.)$ | 109 | 112 | 128 | 145 | 130 | 119 | 110 | 99 | 89 | 82 |
| | $IAPCU_o(x, x^f, y, \bar{\lambda})$ | | | | | | | | | |
| **Nonconvex** | $\bar{\lambda} = 0.5$ | $\bar{\lambda} = 1$ | $\bar{\lambda} = 1.5$ | $\bar{\lambda} = 2$ | $\bar{\lambda} = 2.5$ | $\bar{\lambda} = 3$ | $\bar{\lambda} = 3.5$ | $\bar{\lambda} = 4$ | $\bar{\lambda} = 4.5$ | $\bar{\lambda} = 5$ |
| Average | 10.79243 | 0.811113 | 0.672827 | 0.552863 | 0.552863 | 0.552863 | 0.552863 | 0.552863 | 0.552863 | 0.552863 |
| Stand.Dev. | 20.2639 | 1.006274 | 0.543461 | 0.304061 | 0.304061 | 0.304061 | 0.304061 | 0.304061 | 0.304061 | 0.304061 |
| Minimum | 0.015267 | 0.015267 | 0.015267 | 0.015267 | 0.015267 | 0.015267 | 0.015267 | 0.015267 | 0.015267 | 0.015267 |
| Maximum | 98.25 | 12.54255 | 3.149599 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $APCU_o^f(.) < IAPCU_o^f(.)$ | 88 | 38 | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $APCU_o^f(.) = IAPCU_o^f(.)$ | 8 | 29 | 60 | 85 | 93 | 107 | 115 | 120 | 130 | 133 |
| $APCU_o^f(.) > IAPCU_o^f(.)$ | 96 | 125 | 103 | 107 | 99 | 85 | 77 | 72 | 62 | 59 |

Turning to the results in Table 6, the following deductions emerge. First, the unbiased industry attainable plant capacity utilisation measure $IAPCU_o^f(x, x^f, y, \bar{\lambda})$ decreases almost monotonously in $\bar{\lambda}$ and on average $IAPCU_o(x, x^f, y, \bar{\lambda})$ is first smaller under NC than under C and then the reverse. Second, $IAPCU_o^f(x, x^f, y, \bar{\lambda})$ becomes stationary after $\bar{\lambda}$ reaches the value 3 under C, and the value 2 under NC. Third, while $IAPCU_o(x, x^f, y, \bar{\lambda}) \substack{\geq \\ = \\ <} APCU_o(x, x^f, y, \bar{\lambda})$, for the majority of observations we find $APCU_o(x, x^f, y, \bar{\lambda}) > IAPCU_o(x, x^f, y, \bar{\lambda})$. Furthermore, $APCU_o(x, x^f, y, \bar{\lambda}) < IAPCU_o(x, x^f, y, \bar{\lambda})$ becomes 0 when $IAPCU_o(x, x^f, y, \bar{\lambda})$ becomes stationary.

By solving models (21) and (22) we obtain the two critical points: under C, $L^{I,C} = 0.1199$ and $U^{I,C} = 2.7516$, and under NC, $L^{I,NC} = 0.1199$ and $U^{I,NC} = 1.9947$. We make three comments. First, while the lower bound is identical under C and NC, the upper bound under NC is substantially lower than under C. Second, based on Proposition 3.3, for $\bar{\lambda} \geq 2.7516$ in C case and $\bar{\lambda} \geq 1.9947$ in NC case, we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ and $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$. Thus, as can be seen in Tables 5 and 6, the five last columns in the C case and seven last columns in the NC case contain identical results. Third, it makes no sense to compare these two critical points $L^I$ and $U^I$ with, for instance, the averages of the corresponding points in the firm models $L_p$ and $U_p$.

Instead, Table 7 reports the amount of increase of aggregate variable inputs such that all units obtain the maximum of the standard plant capacity utilisation measure $DF_o^f(x_p^f, y_p)$ from both the perspective of firm and industry levels in both the C and NC cases. The second column shows the sum of observed variable inputs. The sum of needed variable inputs with the firm level model (13) under C and NC is reported in the third and fifth columns, respectively. The columns four and six present the sum of needed variable inputs with the industry level model (19) under C and NC, respectively. The second part of the table shows the magnification factors computed by taking the ratios of the sum of needed variable inputs to the sum of observed variable inputs under firm and industry models and under C and NC. The rows denote the two variable inputs: water and workers.

Table 7: Amounts of Variable Inputs Across Models

| | | Convex | | Nonconvex | |
|---|---|---|---|---|---|
| Variable inputs | $\sum_{p=1}^{K} x_p^v$ | $\sum_{p=1}^{K} U_p x_p^v$ | $\sum_{p=1}^{K} U^I x_p^v$ | $\sum_{p=1}^{K} U_p x_p^v$ | $\sum_{p=1}^{K} U^I x_p^v$ |
| Billions of $m^3$ of water | 30718.8879 | 103352.7750 | 84526.0919 | 74867.3723 | 61274.966 |
| # workers | 3203.2840 | 94183.8871 | 8814.1561 | 89220.3921 | 6389.590 |
| | | Convex | | Nonconvex | |
| Variable inputs | | $\frac{\sum_{p=1}^{K} U_p x_p^v}{\sum_{p=1}^{K} x_p^v}$ | $\frac{\sum_{p=1}^{K} U^I x_p^v}{\sum_{p=1}^{K} x_p^v}$ | $\frac{\sum_{p=1}^{K} U_p x_p^v}{\sum_{p=1}^{K} x_p^v}$ | $\frac{\sum_{p=1}^{K} U^I x_p^v}{\sum_{p=1}^{K} x_p^v}$ |
| Billions of $m^3$ of water | | 3.364 | 2.752 | 2.437 | 1.995 |
| # workers | | 29.402 | 2.752 | 27.853 | 1.995 |

Analysing the results in Table 7, one can deduce the following conclusions. First, firm models need substantially more amounts of variable inputs than industry models. Second, C models need substantially more amounts of variable inputs than NC models. Third, while the industry models with an almost doubling of variable inputs under NC and an almost tripling of variable inputs under C are not necessarily incredible, the firm models with a doubling by a factor of almost 2.5 at minimum and a thirty fold magnification at worst are clearly incredible. For the variable input workers it is simply inconceivable that one could magnify the existing amounts by a factor of 27.85 under NC and a factor of 29.40 under C.

In conclusion, we deduce the following. First, firm models necessitate unlikely amounts of variable inputs, while the results for industry models are not a priori strikingly unrealistic. Second, NC models involve less unrealistic amounts of variable input magnifications than C models.

While some may put their hope in the industry models, it is crucial to remember their limitations. First, these industry models presuppose that there is a central authority coordinating among all firms. If firms are decentralised, this clearly is no option. Second, the industry models are clearly very basic. Any more realistic industry model with additional constraints (e.g., constraints on the amounts of inefficiency that are allowed for (as in Kerstens, Vestergaard, and Squires (2006)), putting lower and upper bounds on changes in variable inputs per firm, etc.) will lead to less spectacular results.

# 5 Conclusions

The output-oriented plant capacity concept has been around for at least two decades and is quite popular for empirical applications. While it was directly inspired by the informal definition provided by Johansen (1968), the doubts of Johansen (1968) regarding the attainability of the concept have seemingly never been investigated. This paper has tried to dig deeper into this issue of attainability.

In Section 3 we have formally defined both the traditional output-oriented and the rather new input-oriented plant capacity notions. Thereafter, we have argued that the output-oriented plant capacity notion may well fail attainability in general, because the amounts of variable inputs needed to reach the maximum capacity outputs may simply not be available. There does not seem to be such an issue for the input-oriented plant capacity concept. Consequently, we have defined a new attainable output-oriented plant capacity notion that incorporates either firm or industry constraints on the availability of variable inputs. It is up to the researcher to determine plausible values limiting the upward scaling of variable inputs.

Using secondary data, we have developed an empirical illustration in Section 4. We can draw several conclusions. First, outputs need to be magnified an unreasonable amounts of times to reach traditional plant capacity outputs. Second, this phenomenon is related to the fact that variable inputs are supposed to be scalable at amounts that are unlikely to be available at either the firm or the industry level. Anyway, the amounts of scaling that need to be applied are ways above the fivefold increase with which we experimented when defining our attainable plant capacity notion. Third, while this scaling of variable inputs is probably ways beyond the reasonable, it is a fact that the computational results on a nonconvex technology are slightly less implausible than the ones obtained on a traditional convex technology. Thus, nonconvexity seems to mitigate partly the extreme results associated with the traditional output-oriented plant capacity notion. Fourth, the industry model (if applicable) leads to less incredible results than the firm model.

In conclusion, it is clear that given the fact that the traditional output-oriented plant capacity concept likely faces serious attainability problems, the new notion of an attainable output-oriented plant capacity concept merits further attention. Furthermore, since the new input-oriented plant capacity notion does not face any attainability issues, it may likely constitute an alternative framework as well.

We suggest some avenues for future research. First, our empirical analysis related to the attainability problem of the traditional output-oriented plant capacity concept needs further corroboration. In particular, it would be important to verify whether the attainability problem is equally serious when employing alternative estimators (e.g., stochastic frontier analysis as in Felthoven (2002)). Furthermore, one major limitation is that we limited our analysis to radial efficiency measures, while it is well-known that the traditional convex and especially the nonconvex technologies suffer from large amounts of unmeasured inefficiency appearing as slacks (see, e.g., De Borger, Ferrier, and Kerstens (1998)). There are some indications that slacks may also play a substantial role in the measurement of plant capacity utilisation (e.g., Dupont, Grafton, Kirkley, and Squires (2002), or Vestergaard, Squires, and Kirkley (2003)). Therefore, it could be useful to revisit the attainability problem using nonradial rather than radial efficiency measures.

Second, our attainable plant capacity notion could benefit from clarifying the amounts by which variable inputs can reasonably be magnified (i.e., the value of $\bar{\lambda}$). Expert opinion may be one source of inspiration worthwhile exploring. Otherwise, it remains a conceptual alternative for the traditional output-oriented plant capacity notion, but it has little empirical bite.

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