The pp-Chain vs CNO-Cycle

A Descent into Nuclear Reaction Turmoil

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Intro

STARS, RIGHT?

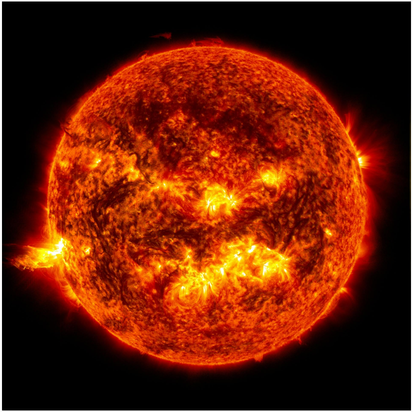
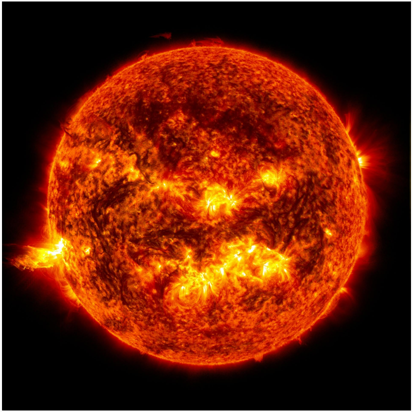


Figure 1: A close-by star,
credit: NASA, GSFC

STARS, RIGHT?

They're all around, they provide heat and occasionally, light.



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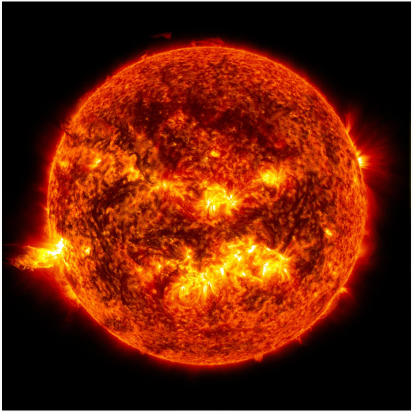
Stars, right?

They're all around, they provide heat and occasionally, light.

How? Through nuclear fusion!



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Thermonuclear Reactions

$$A + B \rightarrow C$$

C generation $=$

Thermonuclear Reactions

$$\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C}$$

Abundance of A

C generation $=$

Thermonuclear Reactions

$$\text{A} + \text{B} \rightarrow \text{C}$$

$$\text{C generation} = \begin{array}{l} \text{Abundance of A} \\ \text{Abundance of B} \end{array}$$

Thermonuclear Reactions

$$A + B \rightarrow C$$

C generation = Abundance of A
Abundance of B
Reaction Rate

Thermonuclear Reactions

$$\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C}$$

C generation =

Abundance of A
Abundance of B
Reaction Rate
Mass Density

Hydrogen Fusion

$$^{1}\mathrm{H} + {}^{1}\,\mathrm{H} \rightarrow {}^{2}\mathrm{H}$$

Hydrogen Fusion

$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H}$$

$$\frac{dY_{^1\mathrm{H}}}{dt} = -Y_{^1\mathrm{H}} Y_{^1\mathrm{H}} \lambda_{^1\mathrm{H} \rightarrow {}^2\mathrm{H}} \rho$$

Hydrogen Fusion

$$ {}^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} $$

$$ \frac{dY_{^1\mathrm{H}}}{dt} = -Y_{^1\mathrm{H}} Y_{^1\mathrm{H}} \lambda_{^1\mathrm{H}\rightarrow {}^2\mathrm{H}} \rho $$

$$ \frac{dY_{^2\mathrm{H}}}{dt} = Y_{^1\mathrm{H}} Y_{^1\mathrm{H}} \lambda_{^1\mathrm{H}\rightarrow {}^2\mathrm{H}} \rho $$

Hydrogen Fusion + Deuterium Fusion

$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H}$$

$$^{1}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He}$$

Hydrogen Fusion + Deuterium Fusion

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$$^{1}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He}$$

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Hydrogen Fusion + Deuterium Fusion

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$$\frac{dY_{^3\mathrm{He}}}{dt} = Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} \lambda_2 \rho$$

Hydrogen Fusion + Deuterium Fusion

$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H}$$
$$^{1}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He}$$

$$\frac{dY_{^1\mathrm{H}}}{dt} = -Y_{^1\mathrm{H}} Y_{^1\mathrm{H}} \lambda_1 \rho - Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} \lambda_2 \rho$$

$$\frac{dY_{^2\mathrm{H}}}{dt} = Y_{^1\mathrm{H}} Y_{^1\mathrm{H}} \lambda_1 \rho - Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} \lambda_2 \rho$$

$$\frac{dY_{^3\mathrm{He}}}{dt} = Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} \lambda_2 \rho$$

PP-CHAIN VS CNO-CYCLE

$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H}$$
$$^{1}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He}$$
$$^{3}\mathrm{He} + {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He}$$

4 species $\rightarrow$ 4 diff. equations

PP-CHAIN VS CNO-CYCLE

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4 species $\rightarrow$ 4 diff. equations

$$^{12}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{13}\mathrm{N}$$
$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C}$$
$$^{13}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N}$$
$$^{14}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{15}\mathrm{O}$$
$$^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N}$$
$$^{15}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$$

8 species $\rightarrow$ 8 diff. equations

PP-CHAIN VS CNO-CYCLE

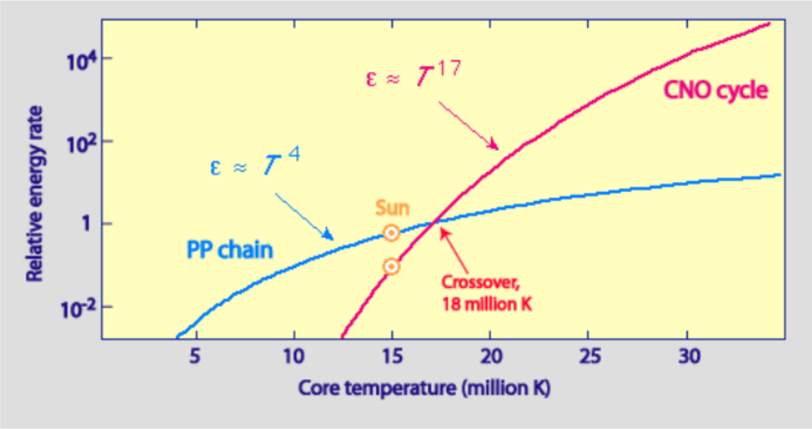

Figure 2: Source: Mike Guidry, University of Tennessee

Which one is faster

Two things control the efficiency

Core Temperature $\rightarrow$ Reaction Rate

Metallicity $\rightarrow$ Catalyst Abundances

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Two things control the efficiency

Core Temperature → Reaction Rate

Metallicity → Catalyst Abundances

Which is the dominant fusion pathway, as a function of core temperature and metallicity?

Methods

The Stiffness Problem

^{1}H Fusion λ:

$7.90 \times 10^{-20} \frac{\text{cm}^3}{\text{mol s}}$

^{2}H Fusion λ:

$1.01 \times 10^{-1} \frac{\text{cm}^3}{\text{mol s}}$

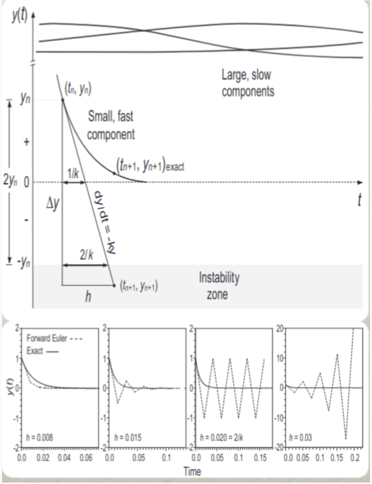

Figure 3: Explicit methods destroy solutions for stiff equactions
Source: W.R. Hix

Implicit Euler & Newton-Raphson

$$\frac{Y_{n+1} - Y_n}{\Delta t} = \dot{Y}_{n+1}$$

Implicit Euler & Newton-Raphson

$$\frac{Y_{n+1} - Y_n}{\Delta t} = \dot{Y}_{n+1}$$
$$\frac{Y_{n+1} - Y_n}{\Delta t} - \dot{Y}_{n+1} = 0$$

IMPLICIT EULER & NEWTON-RAPHSON

$$\frac{Y_{n+1} - Y_n}{\Delta t} = \dot{Y}_{n+1}$$

$$\frac{Y_{n+1} - Y_n}{\Delta t} - \dot{Y}_{n+1} = 0$$

$$F(Y_{n+1}) = 0 \rightarrow \text{Solve!}$$

Implicit Euler & Newton-Raphson

$$\frac{Y_{n+1} - Y_n}{\Delta t} = \dot{Y}_{n+1}$$
$$\frac{Y_{n+1} - Y_n}{\Delta t} - \dot{Y}_{n+1} = 0$$
$$F(Y_{n+1}) = 0 \rightarrow \text{Solve!}$$

Through Newton-Raphson!

$$Y_{n+1} = Y_n - [J_F(Y_n)]^{-1} F(Y_n), \quad (J_F)_{ij} = \frac{\partial \dot{Y}_i}{\partial Y_j} - \frac{\delta_{ij}}{\Delta t}$$

Implicit Euler & Newton-Raphson

$$
J_{\mathrm{pp}} = \begin{bmatrix}
-4\lambda_1 Y_{\mathrm{H}} - \lambda_2 Y_{\mathrm{D}} - h & -\lambda_2 Y_{\mathrm{H}} & 2\lambda_3 Y_{^3\mathrm{He}} & 0 \\
2\lambda_1 Y_{\mathrm{H}} - \lambda_2 Y_{\mathrm{D}} & -\lambda_2 Y_{\mathrm{H}} - h & 0 & 0 \\
\lambda_2 Y_{\mathrm{D}} & \lambda_2 Y_{\mathrm{H}} & -4\lambda_3 Y_{^3\mathrm{He}} - h & 0 \\
0 & 0 & 2\lambda_3 Y_{^3\mathrm{He}} & -h
\end{bmatrix}
$$

$$
J_{\mathrm{CNO}} = \begin{bmatrix}
-\lambda_1 Y_{^{12}\mathrm{C}} - \lambda_3 Y_{^{13}\mathrm{C}} - \lambda_4 Y_{^{14}\mathrm{N}} - \lambda_6 Y_{^{15}\mathrm{N}} - h & -\lambda_1 Y_{\mathrm{H}} & 0 & \lambda_3 Y_{\mathrm{H}} & -\lambda_4 Y_{\mathrm{H}} & 0 & -\lambda_6 Y_{\mathrm{H}} & 0 \\
-\lambda_1 Y_{^{12}\mathrm{C}} + \lambda_6 Y_{^{15}\mathrm{N}} & -\lambda_1 Y_{\mathrm{H}} - h & 0 & 0 & 0 & 0 & \lambda_6 Y_{\mathrm{H}} & 0 \\
\lambda_1 Y_{^{12}\mathrm{C}} & \lambda_1 Y_{\mathrm{H}} & -\lambda_2 - h & 0 & 0 & 0 & 0 & 0 \\
-\lambda_3 Y_{^{13}\mathrm{C}} & 0 & \lambda_2 & -\lambda_3 Y_{\mathrm{H}} - h & 0 & 0 & 0 & 0 \\
\lambda_3 Y_{^{13}\mathrm{C}} - \lambda_4 Y_{^{14}\mathrm{N}} & 0 & 0 & \lambda_3 Y_{\mathrm{H}} & -\lambda_4 Y_{\mathrm{H}} - h & 0 & 0 & 0 \\
\lambda_4 Y_{^{14}\mathrm{N}} & 0 & 0 & 0 & \lambda_4 Y_{\mathrm{H}} & -\lambda_5 - h & 0 & 0 \\
-\lambda_6 Y_{^{15}\mathrm{N}} & 0 & 0 & 0 & 0 & \lambda_5 & -\lambda_6 Y_{\mathrm{H}} - h & 0 \\
\lambda_6 Y_{^{15}\mathrm{N}} & 0 & 0 & 0 & 0 & 0 & \lambda_6 Y_{\mathrm{H}} & -h
\end{bmatrix}
$$

Figure 4: Jacobians, by hand. Yeah...

Flow chart

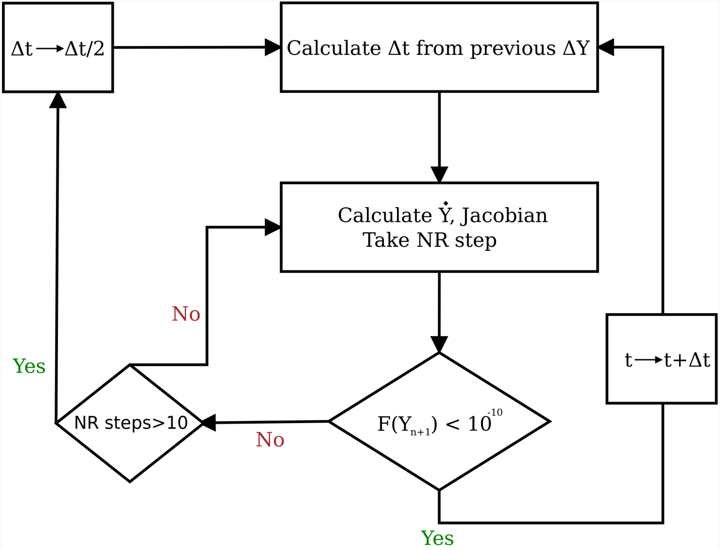

Figure 5: Flow chart of our NRN. Based on SkyNet Lippuner & Roberts ’18

Initial Conditions & Assumptions

- T_{core}, ρ are constant. No mixing.
  MESA simulations to get typical values.

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Initial Conditions & Assumptions

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  MESA simulations to get typical values.
- $\rho = f(T_{\rm core})$
- Constant λ.
  Thermonuclear from Angulo+'99, Beta decays Kondev+'21
- Initial Y from ISM, unless provided explicitly.
  Full citations in the README.

Results

The pp-chain

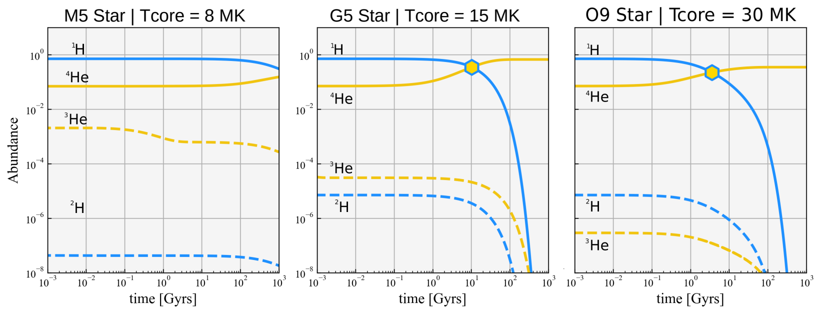
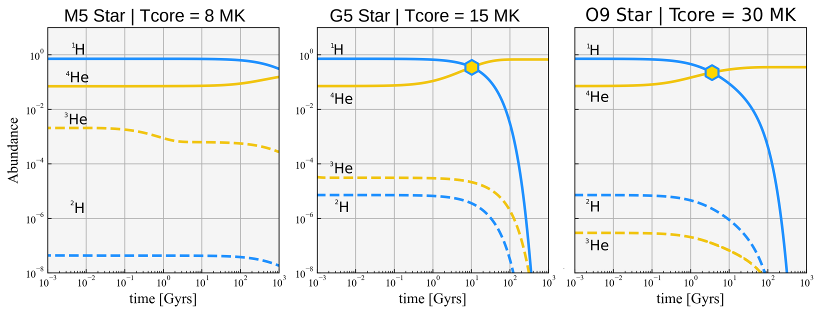


Figure 6: Nuclear fusion via the pp-chain for a M5, G5, and O9 star.

The pp-chain



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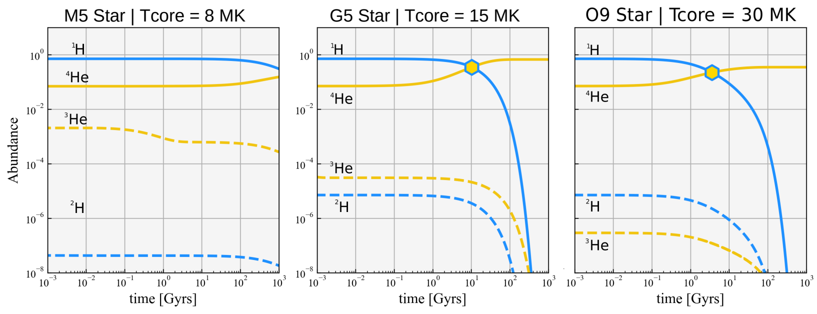
- Higher $T \rightarrow$ more fusion $\Rightarrow$ Hotter stars live shorter

The pp-chain



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The pp-chain

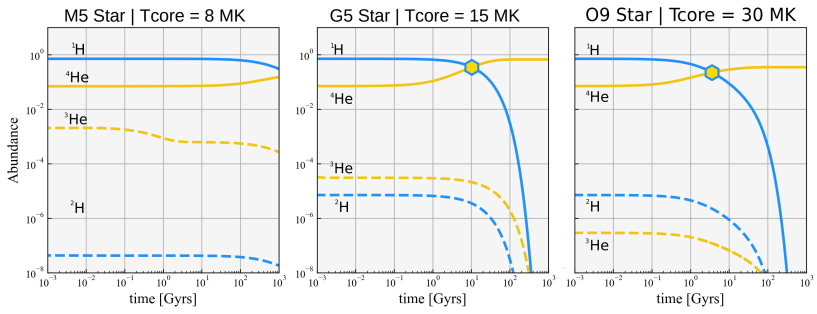


Figure 6: Nuclear fusion via the pp-chain for a M5, G5, and O9 star.

- Higher $T \rightarrow$ more fusion $\Rightarrow$ Hotter stars live shorter
- Abundances reach an equilibrium
- ^{1}H-^{4}He equality roughly correspond to star lifetime

The CNO-cycle

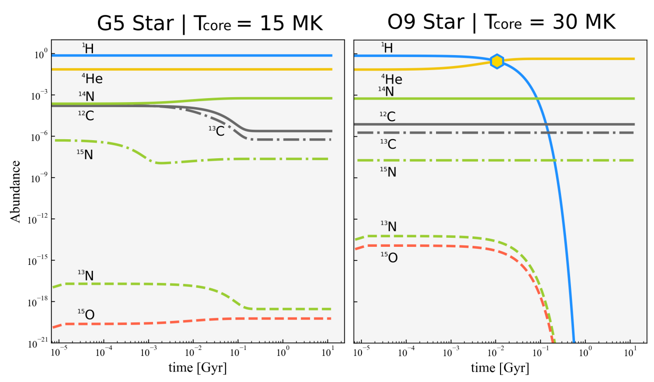
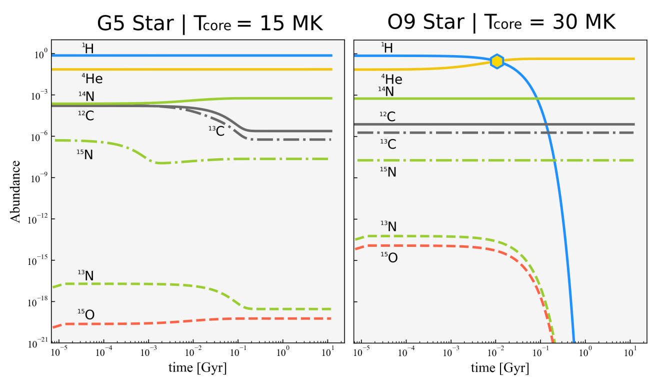


Figure 7: Nuclear fusion via the CNO-cycle for a G5 and O9 star.

The CNO-cycle



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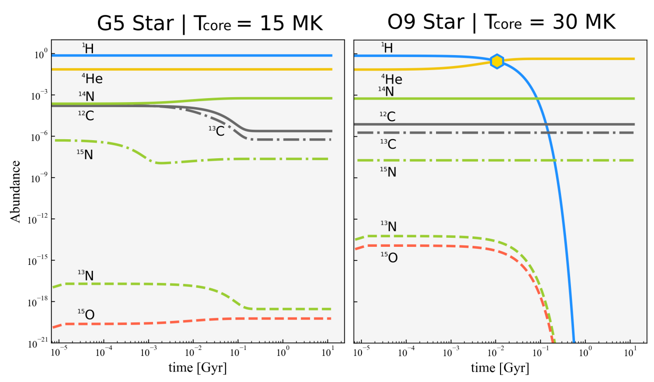
Similarities to pp-chain

- Higher $T \rightarrow$ more fusion $\Rightarrow$ Hotter stars live shorter
- Abundances reach an equilibrium

THE CNO-CYCLE



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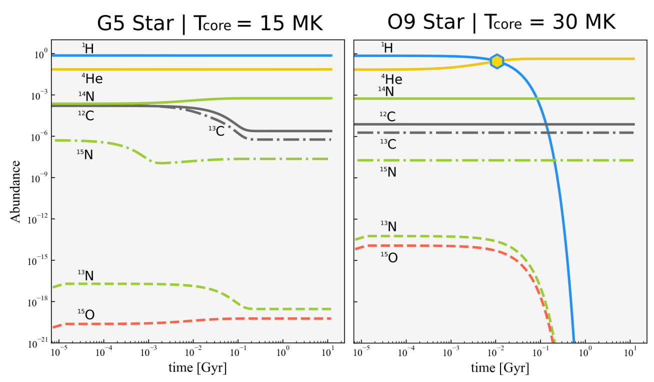
$$^{13}N \rightarrow {}^{13}C$$
$$^{13}C + {}^{1}H \rightarrow {}^{14}N$$
$$^{14}N + {}^{1}H \rightarrow {}^{15}O$$
$$^{15}O \rightarrow {}^{15}N$$
$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$$

The CNO-cycle



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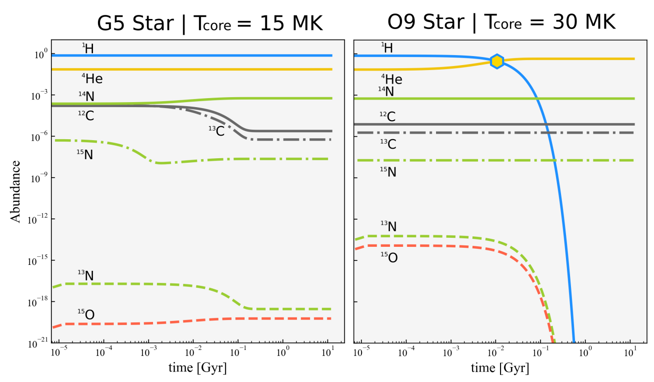
- Once ^{1}H runs out, only decay reactions occur

The CNO-cycle



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- Once ^{1}H runs out, only decay reactions occur
- Catalysts become constant in the long run

PP-CHAIN VS CNO-CYCLE

pp-chain vs CNO-cycle

Which one dominates when?

PP-CHAIN VS CNO-CYCLE

What we expect:

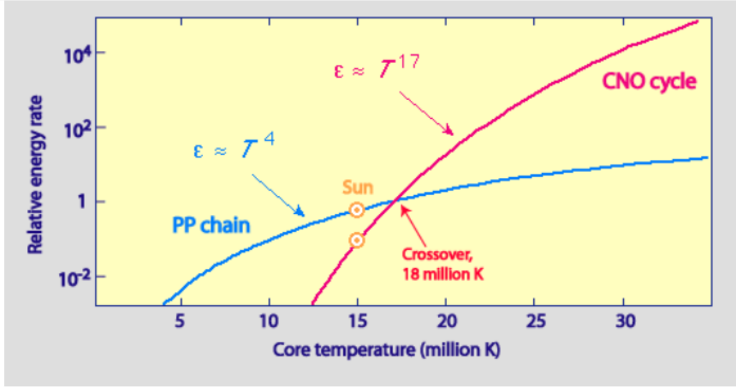

Figure 8: Source: Mike Guidry, University of Tennessee

PP-CHAIN VS CNO-CYCLE

What we got:

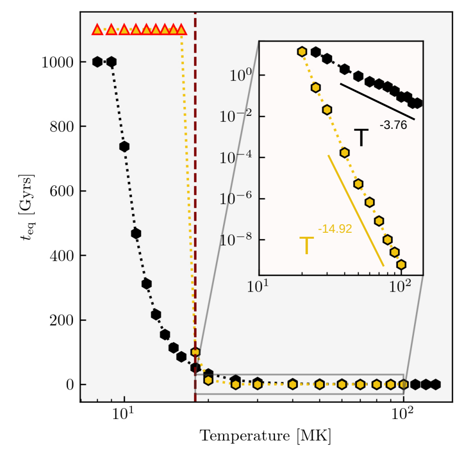

Figure 8: ^{1}H-^{4}He equality time for pp-chain and CNO-cycle across temperatures

TEMPERATURE AND METALLICITY

That was for ISM abundances

How does metallicity change the game?

Temperature and Metallicity

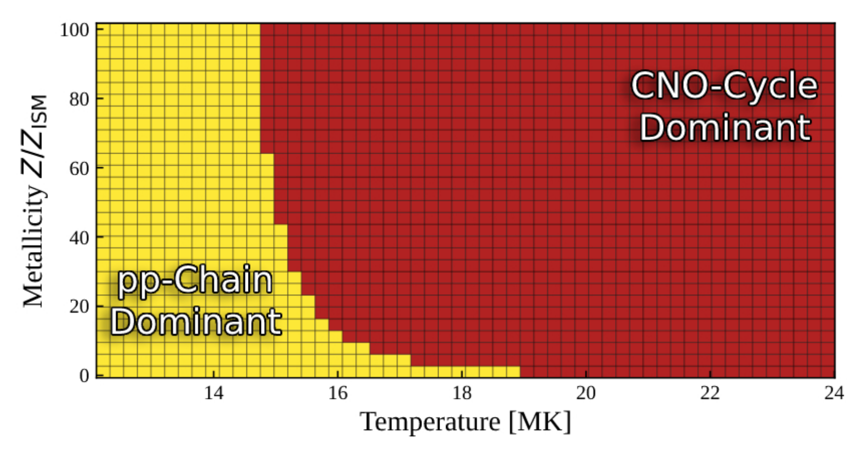

Figure 9: Which fusion pathway dominates at which $(T_{\rm core}, Z)$

Summary

SUMMARY

- Modelling nuclear fusion in a star
  - pp-chain and CNO-cycle
- Temperature and metallicity dependence
  - Which fusion pathway dominates?
- Set of differential equations
  - Implicit Euler & Newton-Raphson root finder
  - Assumptions: T_{core}, ρ, and λ constant (unrealistic)
- Results align well with what we expect!
- For future: less idealised (changing T_{core}, ρ)
  more reactions (pp-branches, cold/hot CNO-cycles)

Luigi Picture

Thank you for your time!

As is tradition, you win a picture of Luigi.

Figure 10: Luigi is very happy you sat through that and thanks you for your attention

Extra Slides

maybe talk about deuterium

Reaction Rates