Linear algebra

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1 Systems of linear Systems of Linear Equations

A system of linear equations are multiple equations containing unknows which are shared. Ex.

$$\begin{aligned} 2x + \ \ y + 3z &= 10 \\ x + \ \ y + \ \ z &= \ \ 6 \\ x + 3y + 2z &= 13 \end{aligned}$$

The same system may be wirtten in form of a matrix

$$\begin{bmatrix} 2 & 1 & 3 & 10 \\ 1 & 1 & 1 & 6 \\ 1 & 3 & 2 & 13 \end{bmatrix}$$

When solving a system their may be

- No solutions - No possible value can be assigned to the variable such all equation are true
- One solution - A combination of values can be assigned to make every equation true
- Infinite solutions - One or more unknows may have an infinite amount of possible assignable values

By transforming a system of equations to a matrice, the following operations can be performed:

- Multiply a row through by a nonzero constant.
- Interchange two rows.
- Add a constant times one row to another.

1.1 Gaussian Elimination

Solving a system of equation in a matrice, can be done such the matrice has the following requirements

- If the row contain nothing but zeroes the first number should be a 1, called the leading 1.

- If a row is made of nothing but zeroes it should be grouped at the bottom
- In two rows the top row should contain a leading 1 further to the left than the bottom
- Each column which contains a leading 1, every number in the same column below should be 0

This form is called row echelon form.
The solution may also be written as:

$$x_1 = 4, x_2 = 6, x_3 = t, x_4 = v, x_5 = 1$$

Where variables means they can be any possible asignment or a function with given restrictions.
In case of the leading 1 column is zero both above and underneath the matrice is in reduced echlon form.

To make a matrice into echoleon form the following algorithm can be used:

1. Locate the leftmost column that does not consist entruely of zeros and exhange it to the top
2. Multiply the top row by $\frac{1}{a}$ where a is the leading number in the row
3. subtract top row from every other below row, such the top row is the only non zero value in the column
4. Repeat 2 and 3 but ignore the top row and let the second top row be the top

To reduce the echloen form, from the bottom the bottom row is added to the above rows until the leading 1 is the only in the column. This is repeated until the top is reached.
A homogeneous linear system are systems which all constant (right part of equal) are 0 and the trivial solution (all variables are assigned 0) are a possible solution.
A free variable is the term for a variable which can be assigned multiple values. The number of free variables wil lbe equal to the number of variables minus zero rows.
In a homogenous linear system if the number of unknows exceed the number of equation there will be an infinite amount of solutions.

Back substitution is a method taking the echoloen form and starting from the bottom isolating the leading variable and substituting it upwards.
A echoloen form is not unique to a system but a reduced echoloen form is unique and the number of zero rows will be unique.