Linear algebra
Exercises

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2022

Contents

1 Week 36

1.1 Which equation are linear in x_1, x_2 and x_3

1.1.1 $x_1 + 5x_2 - \sqrt{2}x^3 = 1$

This is a linear equation

1.1.2 $x_1 = -7x_2 + 3x_3$

This is a linear equation

1.1.3 $x_1^{3/5} - 2x_2 + x_3 = 4$

This is not a linear equation with x_1 having a power

1.2 Convert from matrix form to equation form

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$2x_1 = 0$
$3x_1 - 4x_2 = 0$
$x_2 = 1$

1.3 Convert from equations to matrix

$-6x_1 - x_2 + 3x_3 = 4$
$5x_2 - x_3 = 1$

$$\begin{bmatrix} -6 & -1 & 3 & 4 \\ 0 & 5 & -1 & 1 \end{bmatrix}$$

1.4 Determine if the solution hold in the following system

$(5, 8, 1)$
$x + 2y - 2z = 3$
$3x - y + z = 1$
$-x + 5y - 5z = 5$

$5 + 2(8) - 2(1) = 3$
$19 = 3$
By the first equation the solution does not hold

1.5 Determine if the following matrices are in echoleon form or reduced echelon form

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reduced echelon form

$$\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Echelon form

2 Week 38

2.1 Show that the determinant og the matix is 0

$$\begin{bmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{bmatrix}$$

Column 2 and 4 are proportional to eachother therefore making the det=0

2.2 Is the following matrix invertible

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}$$

The determinant is -6 and therefore not zero and therefore invertible

2.3 Find the standard matrix for the transformation defined by the equations

$$\begin{aligned} w_1 &= 7x_1 + 2x_2 - 8x_3 \\ w_2 &= -x_2 + 5x_3 \\ w_3 &= 4x_1 + 7x_2 - x_3 \end{aligned}$$

$$\begin{bmatrix} 7 & 2 & -8 \\ 0 & -1 & 5 \\ 4 & 7 & -1 \end{bmatrix}$$

2.4 Is a linear function a transformation of R

Yes the linear function can be a transform of the space R

2.5 The images of the standard basis vectors for R^3 are given for a linear trasnformation $T : R^3 \to R^3$. Find the standard matrix for the transformation and find $T(x)$

$$T(e_1) = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

$$T(e_2) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$T(e_3) = \begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}$$

$$x = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 0 & -3 \\ 0 & 1 & -1 \end{bmatrix}$$

$$T(x) = \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$$

2.6 Find the standard matrix A for the linear transformation $T : R^2 \to R^2$ for which

$$T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, T(\begin{bmatrix} 2 \\ 3 \end{bmatrix}) = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix}1\\0\end{bmatrix} = c_1 \begin{bmatrix}1\\1\end{bmatrix} + c_2 \begin{bmatrix}2\\3\end{bmatrix}$$

$$\begin{bmatrix}0\\1\end{bmatrix} = k_1 \begin{bmatrix}1\\1\end{bmatrix} + k_2 \begin{bmatrix}2\\3\end{bmatrix}$$

c variable found by

$$\begin{bmatrix}1 & 2 & 1\\1 & 3 & 0\end{bmatrix}$$

k variable found by

$$\begin{bmatrix}1 & 2 & 0\\1 & 3 & 1\end{bmatrix}$$

Gauss jordian magic!

$$c_1 = 3, c_2 = -1, k_1 = -2, k_2 = 1$$

$$T(\begin{bmatrix}1\\0\end{bmatrix}) = 3T(\begin{bmatrix}1\\1\end{bmatrix}) + (-1)T(\begin{bmatrix}2\\3\end{bmatrix})$$

$$= \begin{bmatrix}3\\-6\end{bmatrix} - \begin{bmatrix}-2\\5\end{bmatrix} = \begin{bmatrix}5\\-11\end{bmatrix}$$

$$T(\begin{bmatrix}0\\1\end{bmatrix}) = -2T(\begin{bmatrix}1\\1\end{bmatrix}) + 1T(\begin{bmatrix}2\\3\end{bmatrix})$$

$$= \begin{bmatrix}-2\\4\end{bmatrix} + \begin{bmatrix}-2\\5\end{bmatrix} = \begin{bmatrix}-4\\9\end{bmatrix}$$

$$A = \begin{bmatrix}5 & -4\\-11 & 9\end{bmatrix}$$