Chapter 19 电荷与电场

- 题型1：运用积分解决连续电场分布的场强问题（复习的时候手动积分）
  - 一、基本步骤：
    - Continuous charge distribution

      The electric field can be calculated by integral

      ① Divide it into infinitesimal charges dQ

      ② Contribution from dQ: $d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}\hat{r}$

      ③ Consider all the components dE_x, dE_y, dE_z

      列分量式子

      ④ Finish the integration:

      $$E_x = \int dE_x,\ E_y = \int dE_y,\ E_z = \int dE_z$$
    - 1.建立坐标系
    - 2.写出场强的微分，写出dQ
      - 点电荷微元
      - 圆圈电荷微元
      - 无限（有限）长直线微元
      - （无限大平面微元）
    - 3.写出场强分量式子 Ex Ey Ez
    - 4.将三个分量积分
    - 5.场强是矢量，有方向，不要忘记写方向
  - 二、求场强过程中可能用到的方法
    - 对称性（轴对称，旋转对称）
    - 割补法
    - 化多重积分为一重积分—本质上是运用一重积分计算的结果
  - 三、典型形状的积分与推导（背）
    - 1.有限长直线
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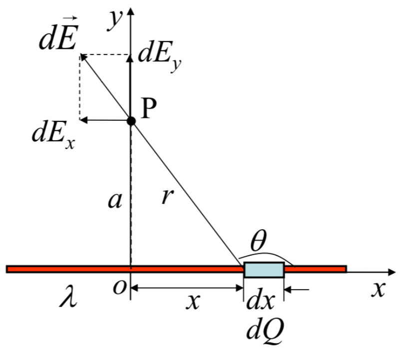
Solution: ① x-y axes

② $dQ = \lambda dx$

③ $dE = \dfrac{\lambda dx}{4\pi\varepsilon_0 r^2}$

④ $dE_x = dE \cdot \cos\theta$

$dE_y = dE \cdot \sin\theta$



⑤ $E_x = \int \dfrac{\lambda dx}{4\pi\varepsilon_0 r^2}\cos\theta$

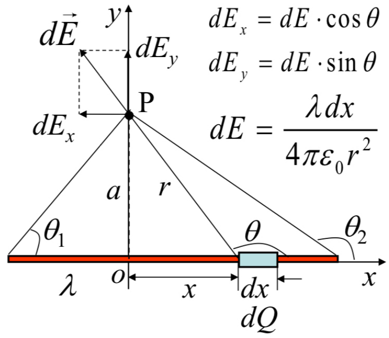
$= \dfrac{\lambda}{4\pi\varepsilon_0 a}\int_{\theta_1}^{\theta_2}\cos\theta d\theta$

$= \dfrac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1)$

$E_y = \int \dfrac{\lambda dx}{4\pi\varepsilon_0 r^2}\sin\theta$

$= \dfrac{\lambda}{4\pi\varepsilon_0 a}\int_{\theta_1}^{\theta_2}\sin\theta d\theta$

$= \dfrac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2)$



$dE_x = dE \cdot \cos\theta$

$dE_y = dE \cdot \sin\theta$

$dE = \dfrac{\lambda dx}{4\pi\varepsilon_0 r^2}$

$r = a/\sin\theta$, $x = -a\cdot\cot\theta$,

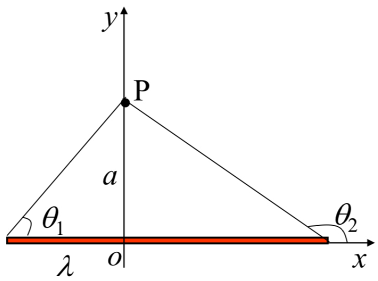
$dx = a d\theta/\sin^2\theta$

$\vec{E} = E_x\vec{i} + E_y\vec{j}$

$E_x = \dfrac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1)$

$E_y = \dfrac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2)$

Discussion:



- 过程当中运用的技巧：化长度为角度进行积分
- 由有限长直线引申出的结论：无限长直线

1) If it is very long or infinite, $\theta_1 = 0$, $\theta_2 = \pi$

$E_x = 0$, $\boxed{E_y = \dfrac{\lambda}{2\pi\varepsilon_0 a}}$ → useful result

- 2.无限大平面推导
  - 基本方法：运用无限长直线在垂直方向进行一重积分
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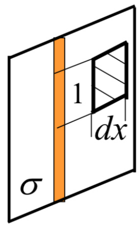
$$E = E_y = \int_{-\infty}^{\infty} \frac{\sigma\, dx}{2\pi\varepsilon_0 r}\cos\theta \qquad \lambda=\sigma\cdot dx$$

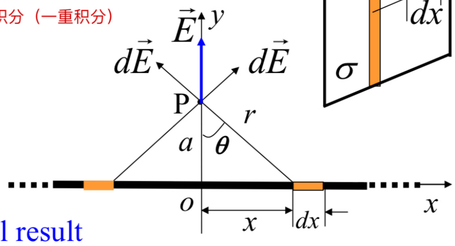
将微元设置为无限长的直线，从垂直方向向积分（一重积分）

$$= \frac{\sigma}{2\pi\varepsilon_0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta$$

$$= \frac{\sigma}{2\varepsilon_0}$$

→ useful result





- 另外方法：二重积分（最好采用极坐标）
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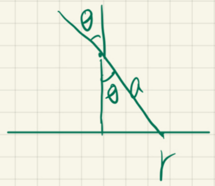

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2+a^2}\cos\theta$$

$$\cos\theta = \frac{a}{\sqrt{a^2+r^2}} \qquad dQ = \sigma r\,dr\,d\theta$$

$$\Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{a}{(a^2+r^2)^{\frac{3}{2}}}dQ = \frac{a}{4\pi\varepsilon_0}\sigma\int_0^{2\pi}d\theta\int_0^{\infty} r\frac{1}{(a^2+r^2)^{\frac{3}{2}}}dr$$

$$= \frac{\sigma a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{1}{a}d\theta = \frac{\sigma}{2\varepsilon_0}$$

- 3.圆圈微元推导（旋转对称性）
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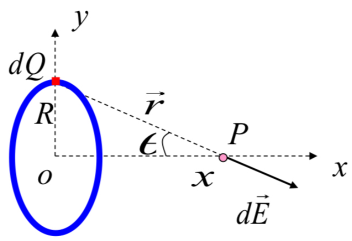
Solution: $$dE = \frac{dQ}{4\pi\varepsilon_0 r^2}$$

$$E = E_x = \int_{Ring}\frac{dQ}{4\pi\varepsilon_0 r^2}\cos\theta$$

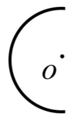
$$= \frac{Q\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{Q\cdot x}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$



  - 注意：
  - 半圆产生的场强和无限长直线相等
  - **注意，半球不是等价的，最下方红字是错误的

Discussion:

1) $x=0$ or $x>>R$, $E=?$ $\frac{Q}{4\pi\varepsilon_0 x^2}$



2) At what position along the axis, $E=E_{max}$? $\frac{dE}{dx}=0$

3) If there is a small gap in the circle, $E_o=?$ 割补

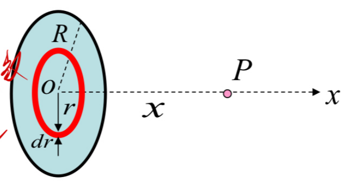
4) If there is only a semi-circle, $E_o=?$ $\frac{\lambda}{2\pi\varepsilon_0 R}$

再延伸，如果是一个半球，那么和无限长的平面也是等价的哦

- 4.圆盘推导
  - 方法1:运用圈状微元

Solution: $dQ = \sigma 2\pi r dr$

取圈状微元



$$dE = \frac{x dQ}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}}$$

$$E = \int_o^R \frac{\sigma \cdot x \cdot 2\pi r dr}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}} = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

  - 方法2:二重积分（极坐标）
- 5.无限大平行板电容器
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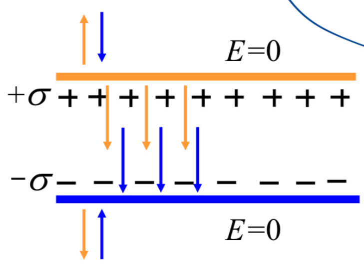
When $R\to\infty$: $\boxed{E=\frac{\sigma}{2\varepsilon_0}}$ infinite plane $\to \times 2$

Parallel-plate capacitor



$E=\frac{\sigma}{\varepsilon_0}$

- 题型二：多电荷系统平衡问题（简单看）
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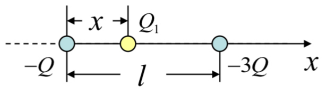
Electric equilibrium

平衡位置问题从两个维度考虑
1.位置维度（某个位置合场强为0）
2.电荷维度 （对另一电荷列方程，合力为0）
本质上，就是对两个位置合场强为0

Example1: Two charges, $-Q$ and $-3Q$, are a distance l apart. How can we place a third charge nearby to reach an equilibrium?



Solution: Position?

$$\frac{1}{4\pi\varepsilon_0}\frac{-Q}{x^2}-\frac{1}{4\pi\varepsilon_0}\frac{-3Q}{(l-x)^2}=0 \Rightarrow x=\frac{\sqrt{3}-1}{2}l=0.366l$$

How much is the Charge?

$$-\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{x^2}-\frac{1}{4\pi\varepsilon_0}\frac{-3Q}{l^2}=0 \Rightarrow Q_1=\frac{6-3\sqrt{3}}{2}Q=0.402Q$$

- 题型三：电偶极矩问题
  - 1.电偶极矩p=Ql，方向由负电荷指向正电荷
  - 2.力矩=p×E