Modulo di un vettore

$$\vec{A}(A_x, A_y, A_z)$$

$$\Rightarrow$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}|$$

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Somma di due vettori

$$\vec{A}(A_x, A_y, A_z), \quad \vec{B}(B_x, B_y, B_z)$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$\Rightarrow$$

$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$
$$C_z = A_z + B_z$$

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Es.1

$$\left(\vec{F_1}\right)_x = -3 \qquad \left(\vec{F_2}\right)_x = -3$$
$$\left(\vec{F_1}\right)_y = 0 \qquad \left(\vec{F_2}\right)_y = -3$$
$$\left(\vec{F_1}\right)_z = 0 \qquad \left(\vec{F_2}\right)_z = 0$$

Risultante

$$\vec{R}_x = \left(\vec{F_1}\right)_x + \left(\vec{F_2}\right)_x = -3 + (-3) = -6$$
$$\vec{R}_y = \left(\vec{F_1}\right)_y + \left(\vec{F_2}\right)_y = 0 + (-3) = -3$$
$$\vec{R}_z = \left(\vec{F_1}\right)_z + \left(\vec{F_2}\right)_z = 0 + (0) = 0$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(-6)^2 + (-3)^2 + 0^2} = \sqrt{45} \simeq 6.7$$

Es.2 (homework)

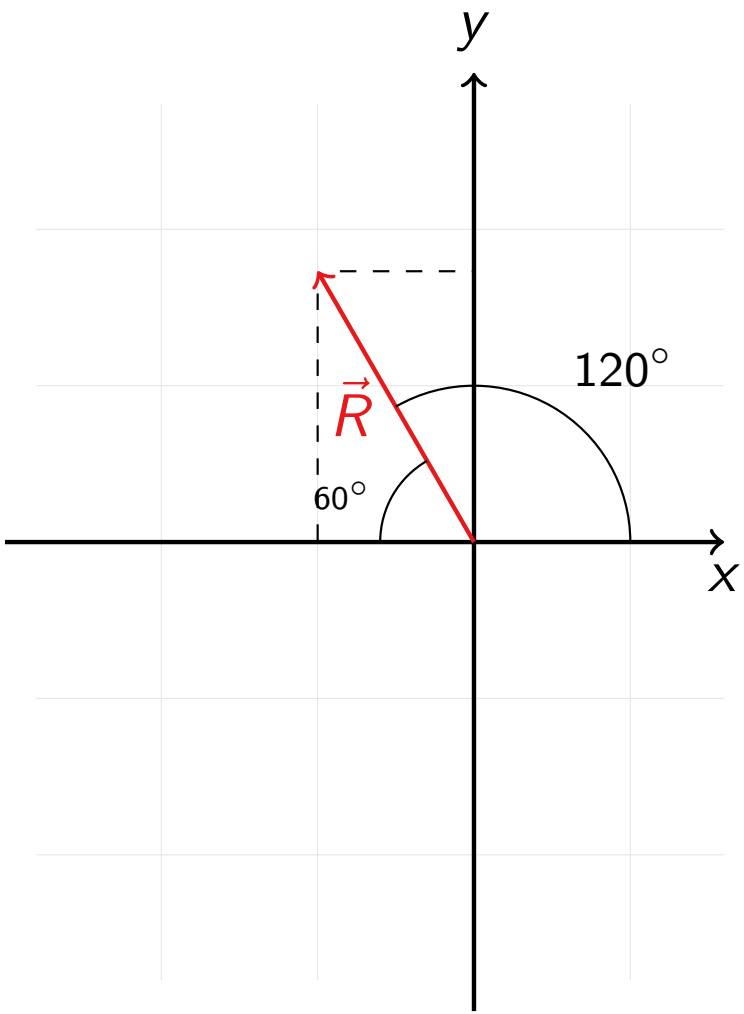
$$\left(\vec{F_1}\right)_x = 2 \qquad \left(\vec{F_2}\right)_x = -3 \qquad \left(\vec{F_3}\right)_x = -3$$
$$\left(\vec{F_1}\right)_y = 2 \qquad \left(\vec{F_2}\right)_y = 3 \qquad \left(\vec{F_3}\right)_y = -1$$
$$\left(\vec{F_1}\right)_z = 0 \qquad \left(\vec{F_2}\right)_z = 0 \qquad \left(\vec{F_3}\right)_z = 0$$

Risultante

$$\vec{R}_x = \left(\vec{F_1}\right)_x + \left(\vec{F_2}\right)_x + \left(\vec{F_3}\right)_x = ?$$
$$\vec{R}_y = \left(\vec{F_1}\right)_y + \left(\vec{F_2}\right)_y + \left(\vec{F_3}\right)_y = ?$$
$$\vec{R}_z = \left(\vec{F_1}\right)_z + \left(\vec{F_2}\right)_z + \left(\vec{F_3}\right)_z = ?$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = ?$$

Es.3



$$|\vec{R}| = 51[m]$$

Componenti

$$R_x = -|\vec{R}| \cos 60^\circ = -51 \cdot \cos 60^\circ$$

$$R_y = |\vec{R}| \sin 60^\circ = 51 \cdot \sin 60^\circ$$

Es.6

$$\left(\vec{V}_1\right)_x = 3m \qquad \left(\vec{V}_2\right)_x = -2m \qquad \left(\vec{V}_3\right)_x = m$$
$$\left(\vec{V}_1\right)_y = 2m \qquad \left(\vec{V}_2\right)_y = 4m \qquad \left(\vec{V}_3\right)_y = -2m$$
$$\left(\vec{V}_1\right)_z = 0 \qquad \left(\vec{V}_2\right)_z = 0 \qquad \left(\vec{V}_3\right)_z = 0$$

$\vec{R} = \vec{V}_1 + \vec{V}_2 + \vec{V}_3$

$$\vec{R}_x = \left(\vec{V}_1\right)_x + \left(\vec{V}_2\right)_x + \left(\vec{V}_3\right)_x = 3m - 2m + m = 2m$$
$$\vec{R}_y = \left(\vec{V}_1\right)_y + \left(\vec{V}_2\right)_y + \left(\vec{V}_3\right)_y = 2m + 4m - 2m = 4m$$

$\vec{R} = 2\vec{V}_1 - 3\vec{V}_2$

$$\vec{R}_x = 2\left(\vec{V}_1\right)_x - 3\left(\vec{V}_2\right)_x = 2 \cdot 3m - 3 \cdot (-2m) = 12m$$
$$\vec{R}_y = 2\left(\vec{V}_1\right)_y - 3\left(\vec{V}_2\right)_y = 2 \cdot 2m - 3 \cdot 4m = 8m$$

Es.5 (homework)

$$\left(\vec{V}_1\right)_x = 3 \qquad \left(\vec{V}_2\right)_x = -3 \qquad \left(\vec{V}_3\right)_x = -1$$
$$\left(\vec{V}_1\right)_y = 2 \qquad \left(\vec{V}_2\right)_y = 5 \qquad \left(\vec{V}_3\right)_y = -2$$
$$\left(\vec{V}_1\right)_z = 1 \qquad \left(\vec{V}_2\right)_y = 2 \qquad \left(\vec{V}_3\right)_y = 0$$

$\vec{R} = \vec{V}_1 - \vec{V}_2$

$$\vec{R}_x = \left(\vec{V}_1\right)_x - \left(\vec{V}_2\right)_x = ?$$
$$\vec{R}_y = \left(\vec{V}_1\right)_y - \left(\vec{V}_2\right)_y = ?$$
$$\vec{R}_z = \left(\vec{V}_1\right)_z - \left(\vec{V}_2\right)_z = ?$$

$\vec{R} = \vec{V}_2 + \vec{V}_3$

???

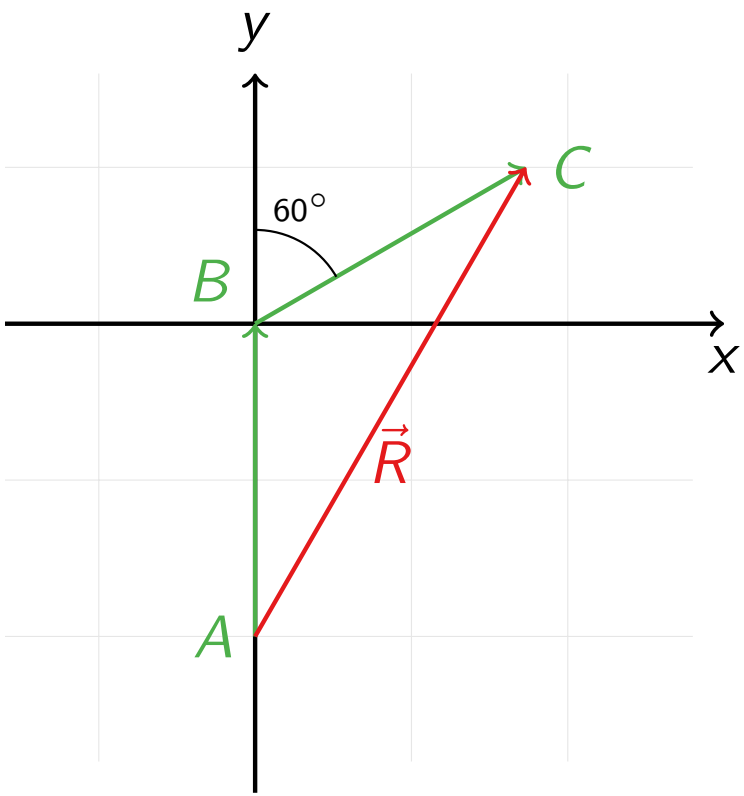
$\vec{R} = 4\vec{V}_3 - 3\vec{V}_2 + \vec{V}_1$

???

Es.5 (in C++)

```
#include <iostream>
struct Vector { double x,y,z; };

int main()
{
    Vector v1,v2,v3;
    v1.x = 3; v1.y = 2; v1.z = 1;
    v2.x =-3; v2.y = 5; v2.z = 2;
    v3.x =-1; v3.y =-2; v3.z = 0;

    Vector R;
    R.x = v1.x - v2.x;
    R.y = v1.y - v2.y;
    R.z = v1.z - v2.z;
    std::cout << "R = " << R.x << "," << R.y << "," << R.z << std::endl;

    Vector R1;
    // ???
    std::cout << "R1 = " << R1.x << "," << R1.y << "," << R1.z << std::endl;

    Vector R2;
    // ???
    std::cout << "R2 = " << R2.x << "," << R2.y << "," << R2.z << std::endl;

    return 0;
}
```

Es.7 (var 1)



$$|AB| = |BC| = 100[m]$$

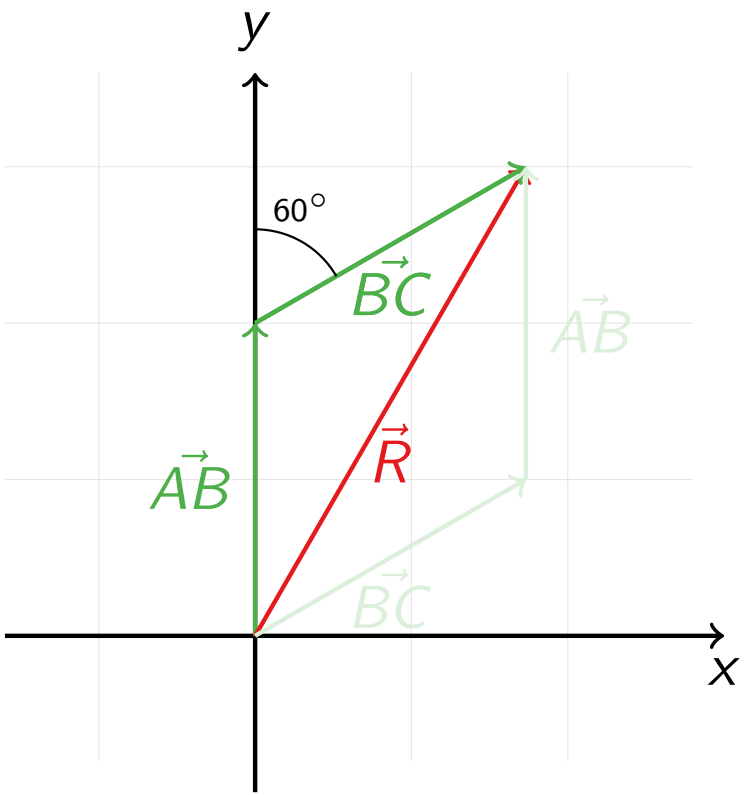
Componenti

$$R_x = |BC| \cos 60^\circ$$

$$R_y = |AB| + |BC| \sin 60^\circ$$

Es.7 (var 2)

$$|AB| = |BC| = 100[m]$$



Componenti $\vec{AB}$ e $\vec{BC}$

$$AB_x = 0, \quad AB_y = |AB|$$

$$BC_x = |BC| \sin 60^\circ$$

$$BC_y = |BC| \cos 60^\circ$$

Componenti $\vec{R}$

$$R_x = AB_x + BC_x = |BC| \sin 60^\circ$$

$$R_y = AB_y + BC_y$$

$$= |AB| + |BC| \cos 60^\circ$$

Prodotto vettoriale

$$\vec{A}(A_x, A_y, A_z), \quad \vec{B}(B_x, B_y, B_z)$$

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\Rightarrow$$

$$C_x = A_y B_z - A_z B_y$$
$$C_y = A_z B_x - A_x B_z$$
$$C_z = A_x B_y - A_y B_x$$

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Es.9

$$\vec{A}_x = 3 \qquad \vec{B}_x = -3$$
$$\vec{A}_y = 1 \qquad \vec{B}_y = 1$$
$$\vec{A}_z = 0 \qquad \vec{B}_z = 0$$

Prodotto vettoriale

$$\vec{C}_x = A_y B_z - A_z B_y = 1 \cdot 0 - 0 \cdot 1 = 0$$
$$\vec{C}_y = A_z B_x - A_x B_z = 0 \cdot (-3) - 3 \cdot 0 = 0$$
$$\vec{C}_z = A_x B_y - A_y B_x = 3 \cdot 1 - 1 \cdot (-3) = 6$$

Es.9 (in C++)

```
#include <iostream>

struct Vector { double x,y,z; } ;

int main()
{
    Vector a,b;
    a.x = 3; a.y = 1; a.z = 0;
    b.x =-3; b.y = 1; b.z = 0;

    Vector C;
    C.x = a.y * b.z - a.z * b.y;
    C.y = a.z * b.x - a.x * b.z;
    C.z = a.x * b.y - a.y * b.x;

    std::cout << "C = " << C.x << "," << C.y << "," << C.z << std::endl;

    return 0;
}
```

Es.10 (homework)

$$\vec{A}_x = 1 \qquad \vec{B}_x = -1 \qquad \vec{C}_x = -4$$
$$\vec{A}_y = 2 \qquad \vec{B}_y = 3 \qquad \vec{C}_y = 1$$
$$\vec{A}_z = 1 \qquad \vec{B}_z = 2 \qquad \vec{C}_z = 1$$

Prodotto $\vec{L_1} = \vec{A} \times \vec{B}$

$$\left(\vec{L}_1\right)_x = A_y B_z - A_z B_y = \ ?$$
$$\left(\vec{L}_1\right)_y = A_z B_x - A_x B_z = \ ?$$
$$\left(\vec{L}_1\right)_z = A_x B_y - A_y B_x = \ ?$$

Prodotto $\vec{L_2} = \vec{A} \times \vec{C}$

???

Prodotto $\vec{L_3} = \vec{C} \times \vec{B}$

???