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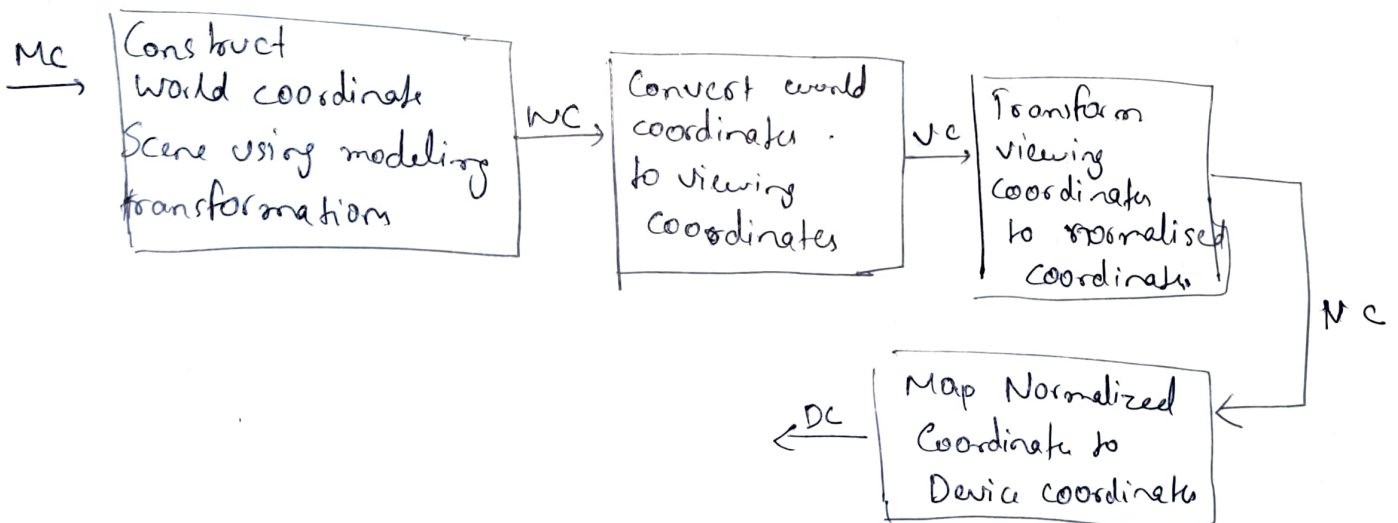
CAV Assignment

1. Build a 2D viewing transformation pipeline and also explain openGL 2D viewing functions.

= Viewing pipeline means the mapping of a two dimensional world-coordinate scene description to device coordinates is called a two dimensional viewing transformation.

This transformation is simply referred to as the window to viewport transformation or the windowing transformation.

We can describe the steps for 2D viewing pipeline as indicated in figure;



Once a world coordinate scene has been constructed, we would set up a separate two dimensional, viewing coordinate reference frame for specifying the clipping window.

To make the viewing process independent of the requirements of any output device, graphics system convert object description to normalized coordinates.

OpenGL 2D Viewing Functions

The GLU library provides a functions for specifying a two-dimensional clipping window, and we have GLUT library functions for handling display windows.

(i) OpenGL projection mode

We must set the parameters for the clipping window as part of the projection transformation function.

`glMatrixMode(GL_PROJECTION);`

We can also set the initialization as

`glLoadIdentity();`

(ii) GLU clipping window function

To define 2D clipping window, we can use the GLU function

`gluOrtho2D(xmin, xmax, ymin, ymax);`

The normalized coordinate in the range from -1 to 1 are used in the OpenGL clipping routine.

(iii) OpenGL viewport function

We specify the viewport parameter with the OpenGL function

`glViewport(xmin, ymin, vwidth, vheight);`

`glGetIntegerv(GL_VIEWPORT, vArray);`

2. Build Phong lighting Model with equation.

→ A Phong lighting model is a local model that can be computed rapidly. It's an empirical model for calculating the specular reflection range, developed by Phong.

Angle ϕ can be assigned value in the range 0° to 90° , so that $\cos \phi$ varies from 0 to 1.0.

It has 3 components:

(i) Ambient Component

The component approximates the indirect lighting by a constant

$$I = I_a + K_e$$

where, I_a = ambient light intensity (color)
 K_a = ambient reflection const (cons)

(i) Diffuse Component

It describes the diffuse reflection of rough surfaces.

$$I = I_p * K_d * \cos \theta$$

where, I_p = intensity of point light source
 K_d = diffuse reflection coefficient
 $\cos \theta$ = lambertian cosine law

(ii) Specular Component

It describes the specular reflection of smooth (shiny) surfaces.

$$I = I_p * K_s * \cos^n \alpha$$

, n = shininess

where I_p = intensity at the point light source
 K_s = specular reflection coefficient (0-1)

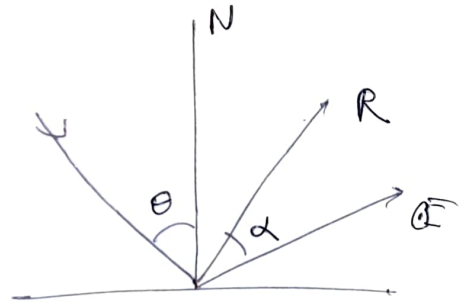
Similarly from the figure,

$$\cos \theta = N \cdot L$$

$$\cos \alpha = R \cdot V$$

So,

$$\begin{aligned} I &= I_p * K_d * NL \quad (\text{Diffusion}) \\ I &= I_p * K_s * (RV)^n \quad (\text{Specular}) \end{aligned}$$



Therefore, ray model of α^2 is

$$I = \text{Ambient} + \text{Diffusion} + \text{Specular}$$

$$\Rightarrow I = I_a K_a + I_p K_d \cos \theta + I_p K_s \cos^n \alpha$$

It also can be written in term of normal reflection & view vector as

$$I = I_a K_a + I_p K_d NL + I_p K_s (RV)^n$$

This gives the perfect ray model for a object under the illumination

3. Apply Homogeneous coordinate for translation, rotation, and scaling via matrix representation. (7)

= A 2D point P is represented in homogeneous coordinate by a 4 dim

$$\text{vec } P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \text{or} \quad P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

We use the homogeneous coordinate to eliminate addition. Hence, we use a point homogeneous coordinate, we don't do anything, it is easier to compose of everything in matrix multiplication.

(i) Translation:

$$\text{for 2D: } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} = P' \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\text{for 3D: } \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(ii) Rotation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow \text{clockwise}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow \text{anticlockwise}$$

(iii) Scaling:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

In all transformation x, y are the current points of x', y' are the resulting points after the translation, rotation & scaling.

(5)

The 3D scaling can done as below in homogeneous coordinates:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

4. Outline the difference between raster scan display and random scan displays.

Random Scan Display

1. The resolution is higher than raster scan display.

2. It is more expensive & less affordable.

3. In a random scan display, it is easy to process with modification.

4. In it, we don't prefer interlacing.

5. When it comes to image rendering, we prefer mathematical functions.

6. eg: Pen Plotter

Raster Scan Display

The resolution of raster scan is lower than random scan display.

• It is affordable as compared to random scan display.

• In it, it is difficult to do any modifications.

In raster scan display, we prefer interlacing.

When it comes to image rendering, we prefer pixel but it is good for constructing lifetime scenes.

eg: TV sets

5. Demonstrate OpenGL function for displaying window management using GLUT

2. Since we are using the OpenGL utility toolkit, we need to initialize GLUT. Steps for displaying window management using GLUT:

→ We perform the GLUT initialization with the statement:
`glutInit (&argc, argv);`

→ Next we can state that a display window is to be created on the screen with a given option for the title bar.

• `glutCreateWindow ("An example");`

→ Then the following function will provide the line segment description to the display window.

• `glutDisplayFunc (lineSegment);`

eg:- `#include <GL/glut.h>`

```
void display()
{
```

```
    glClearColor (0, 0, 0, 1);
```

```
    glClear (GL_COLOR_BUFFER_BIT);
```

```
    glBegin (GL_TRIANGLES);
```

```
    glColor3f (1, 0, 0);
```

```
    glVertex2f (-0.8, -0.8);
```

```
    glColor3f (0, 1, 0);
```

```
    glVertex2f (0.8, -0.8)
```

```
    glColor3f (0, 0, 1);
```

```
    glVertex2f (0, 0, 1);
```

```
    glEnd();
```

```
    glutSwapBuffers();
```

```
}
```

int main (int argc, char **argv)

```
{  
    glutInit(&argc, &argv);  
    glutInitDisplayMode(GLUT_SINGLE | GLUT_RGB);  
    glutInitWindowSize(500, 500);  
    glutInitWindowPosition(100, 100);  
    glutCreateWindow("GL RGB Triangle");  
    glutDisplayFunc(display);  
    glutMainLoop();  
}
```

6. Explain the OpenGL visibility detection function

= The OpenGL has the different function for the visibility detection in GLUT library. They are:

(i) OpenGL polygon culling function

This function is used to remove back face, front face or both faces of the object.

glCullFace(mode);

parameter mode is;

glEnable(GL_CULL_FACE);
glEnable(GL_CULL_FACE);

(ii) OpenGL Depth - buffer function

glutInitDisplayMode(GLUT_SINGLE | GLUT_RGB | GLUT_DEPTH);

This initialization function will request for depth buffer and refresh buffer.

glClear(GL_DEPTH_BUFFER_BIT);
glEnable(GL_DEPTH_TEST);
glEnable(GL_DEPTH_TEST);

(ii) OpenGL cusefrom surface visibility function

This function is used for min from display of the light, But display both visible and hidden edges.

`glPolygonMode(GL_FRONT_AND_BACK, GL_LINE);`

(iv) OpenGL Depth Cusing function

`glFogi(GL_FOG_MODE, GL_LINEAR);`

This function is used to vary the brightness of an object. It applies linear depth function to object color using $d_{min} = 0.0$ & $d_{max} = 1.0$ by default.

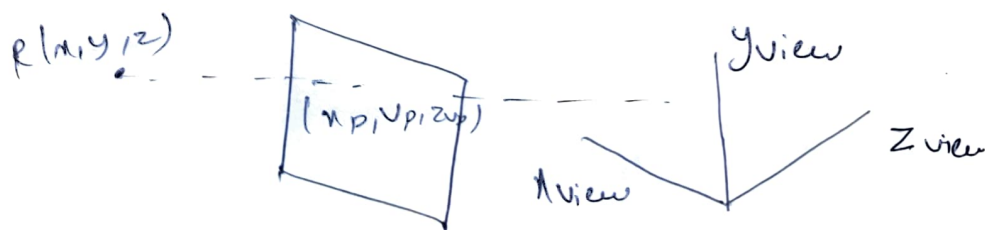
There by using above function, we can delete the visibility of objects.

7. Write the special cases that are discussed with respect to perspective projection transformation coordinate.

= The projection line intersects the view plane at the coordinates (x_p, y_p, z_p) where z_p is some selected position for the view plane on the z_{view} axis.

Fig. below show the projection path of a spatial coordinate (x, y, z) to a general projection reference point at $(x_{prp}, y_{prp}, z_{prp})$:

z_{prp} :



we can write equation describing coordinate position along the perspective projection line in parametric form as

$$x' = x - (x - x_{prp})u$$

$$y' = y - (y - y_{prp})u$$

$$z' = z - (z - z_{prp})u$$

On the view plane, $z' = z_{vp}$, we can write u as,

$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

By substituting the value of u into the equations for x' & y' we obtain the general perspective transformation equations

$$x_p = x \left(\frac{z_{prp} - z_{vp}}{z_{prp} - z} \right) + x_{prp} \left(\frac{z_{vp} - z}{z_{prp} - z} \right)$$

$$y_p = y \left(\frac{z_{prp} - z_{vp}}{z_{prp} - z} \right) + y_{prp} \left(\frac{z_{vp} - z}{z_{prp} - z} \right)$$

Special cases:

Case I: The projection reference point could be limited its position along the z -view axis, then

$$x_{prp} = y_{prp} = 0$$

$$\therefore x_p = x \left(\frac{z_{prp} - z_{vp}}{z_{prp} - z} \right), \quad y_p = y \left(\frac{z_{prp} - z_{vp}}{z_{prp} - z} \right)$$

Case II: Sometimes the projection reference point is fixed at the coordinate origin or $(x_{prp}, y_{prp}, z_{prp}) = (0, 0, 0)$ then,

$$x_p = x \left(\frac{z_{vp}}{z} \right), \quad y_p = y \left(\frac{z_{vp}}{z} \right)$$

Case III: if the view plane is the $U-V$ plane and then are no restriction on the placement of the projection reference point then we have,

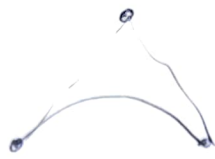
$$x_p = x \left(\frac{z_{prp}}{z_{prp} - z} \right) - x_{prp} \left(\frac{z}{z_{prp} - z} \right)$$

$$y_p = y \left(\frac{z_{prp}}{z_{prp} - z} \right) - y_{prp} \left(\frac{z}{z_{prp} - z} \right)$$

Case IV: $x_{prp} = y_{prp} = z_{vp} = 0$

$$x_p = x \left(\frac{z_{prp}}{z_{prp} - z} \right), \quad y_p = y \left(\frac{z_{prp}}{z_{prp} - z} \right)$$

8. Explain Bezier curve equation along with its properties (10)
- = Bezier curve is drawn by considering control points.
Approximate target is drawn by using control point



3 control points of Bezier curve

Simplest form of Bezier curve is connection two endpoints



→ Bezier curve \mathbb{R}^n is also known as Bezier spline curve it is developed by french engineer Pierre Bezier.

Bezier curve can be fitted to any number of control points although some graphic package limit to four control points.

→ Bezier curve equation

$$Q(u) = \sum_{k=0}^n P_k \cdot \text{BEZ}_{k,n}(u)$$

where P_k = position vector coordinate $k=0, \dots, n$

$\text{BEZ}_{k,n}(u)$ = Bezier blending function

$$\text{BEZ}_{k,n}(u) = C(n, k) u^k (1-u)^{n-k}$$

where $C(n, k) = \frac{n!}{k!(n-k)!}$

→ A set of 3 parametric equations for the individual curve coordinates can be represented as

$$x(u) = \sum_{k=0}^n x_k \cdot \text{BEZ}_{k,n}(u)$$

$$y(u) = \sum_{k=0}^n y_k \cdot \text{BEZ}_{k,n}(u)$$

$$z(u) = \sum_{k=0}^n z_k \cdot \text{BEZ}_{k,n}(u)$$

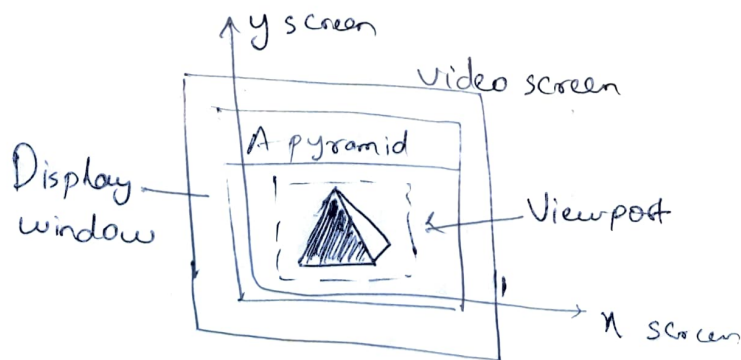
Bezier Curve properties

- depends on number of control points.
- curve will pass through end points, but not all control point.
- polynomial eqⁿ of curve depends on no. of control point
 $n \rightarrow$ control points (4)
 $n-1 \rightarrow$ degree of polynomial equation (3)

9. Explain normalization transformation for an orthogonal projection.
= Once we have established the limits of the view volume, coordinate description inside this rectangular parallelepiped are the projection coordinates and they can be mapped into a normalized view volume without any further projection processing.

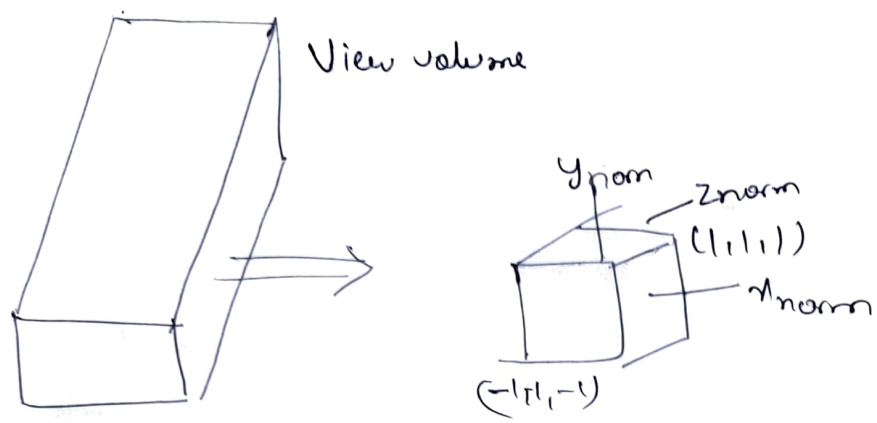
Some graphics use a unit cube for this normalized view volume with each of the x, y and z coordinates normalized in the range from 0 to 1.

Another normalization transformation approach is to use a symmetric cube, with coordinates in the range from -1 to 1.



To illustrate the normalization, we assume that the orthogonal projection view volume, is to be mapped into the symmetric normalization cube within a left handed reference frame.

Also, z-coordinate positions for the near and far planes are denoted as z_{near} and z_{far} respectively



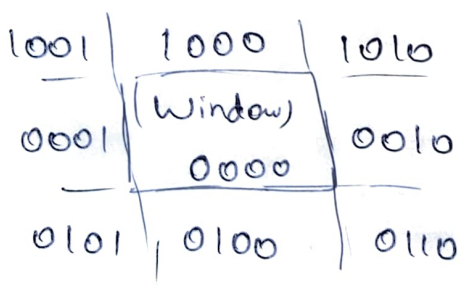
The normalization transformation for the orthogonal view volume is

$$M_{ortho, norm} = \begin{bmatrix} \frac{2}{x_{wmax} - x_{wmin}} & 0 & 0 & -\frac{x_{wmax} + x_{wmin}}{x_{wmax} - x_{wmin}} \\ 0 & \frac{2}{y_{wmax} - y_{wmin}} & 0 & -\frac{y_{wmax} + y_{wmin}}{y_{wmax} - y_{wmin}} \\ 0 & 0 & \frac{-2}{z_{wmax} - z_{wmin}} & \frac{z_{near} + z_{far}}{z_{near} - z_{far}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

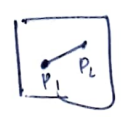
10. Explain Cohen-Sutherland line clipping algorithm.

= Cohen-Sutherland algorithm works on Region code.

Region code is 4 Bit code (ABRL; above below right left)



Case P: if both endpoint region code is zero, completely inside & visible



$P_1 = 0000$ (zero)

$P_2 = 0000$ (zero)

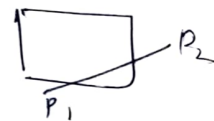
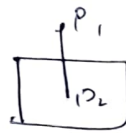
AND 0000 (zero)

Case 2: If both end point region code is non-zero, apply the logical AND and result is non-zero completely outside & invisible



$$\begin{array}{r} P_1 = 0001 \\ P_2 = 0001 \\ \hline \text{AND } 0001 \end{array}$$

Case 3: a) if one of P_1 & P_2 is zero
b) Both non-zero } logical AND is zero
Find the intersection point



Finding intersection points

(1) Find y value of vertical line.

consider a line segment (x_1, y_1) (x_2, y_2) & intersection point (x, y)

\Rightarrow Find slope of (x_1, y_1) & (x, y)

$$m = \frac{y - y_1}{x - x_1}$$

$$\text{then } (y - y_1) = m(x - x_1)$$

$$\therefore \boxed{y = y_1 + m(x - x_1)}$$

If $x = x_{\min}$ & $x = x_{\max}$

(2) Find x value of horizontal line

Find slope of (x_1, y_1) & (x, y)

$$m = \frac{y - y_1}{x - x_1}$$

$$m(x - x_1) = y - y_1$$

$$\therefore \boxed{x = x_1 + \frac{y - y_1}{m}}$$