

Projeção Estereográfica

Esfera:

$$x^2 + y^2 + (z-1)^2 = 1$$

Reta que passa por  $(u, v, 0)$  e  $(0, 0, 2)$ :

$$n(t) = (0, 0, 2) + t(-u, -v, 2) = (-ut, -vt, 2+2t).$$

$$\Rightarrow n(t) = (-ut, -vt, 2+2t).$$

Inteção da reta com a esfera:

$$n(t) \in S^2 \setminus \{N\} \Leftrightarrow (-ut)^2 + (-vt)^2 + (2+2t-1)^2 = 1$$

$$\Leftrightarrow u^2 t^2 + v^2 t^2 + 1 + 4t + 4t^2 = 1$$

$$\Leftrightarrow t(u^2 t + v^2 t + 4t + 4) = 0$$

$$\Leftrightarrow t = 0 \quad \text{ou} \quad t = \frac{-4}{u^2 + v^2 + 4}$$

• Se  $t=0$ , então  $n(0) = N$ . Porém,  $n(t) \in S^2 \setminus \{N\}$ ,

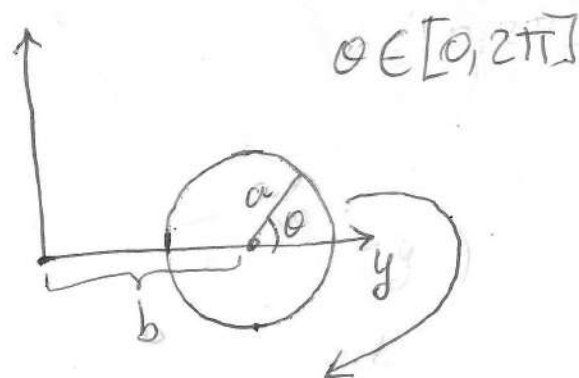
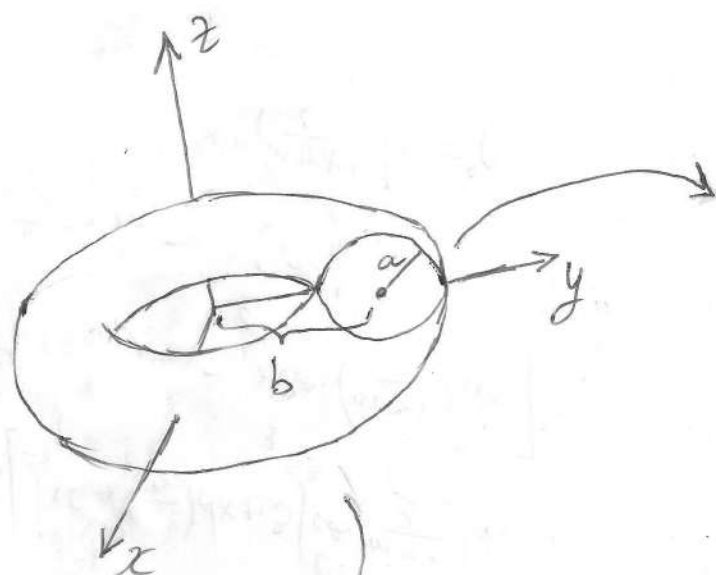
Portanto,  $t \neq 0$ .

• Se  $t = \frac{-4}{u^2 + v^2 + 4}$ , então  $n(t) = \left( \frac{4u}{u^2 + v^2 + 4}, \frac{4v}{u^2 + v^2 + 4}, \frac{2(u^2 + v^2)}{u^2 + v^2 + 4} \right)$ .

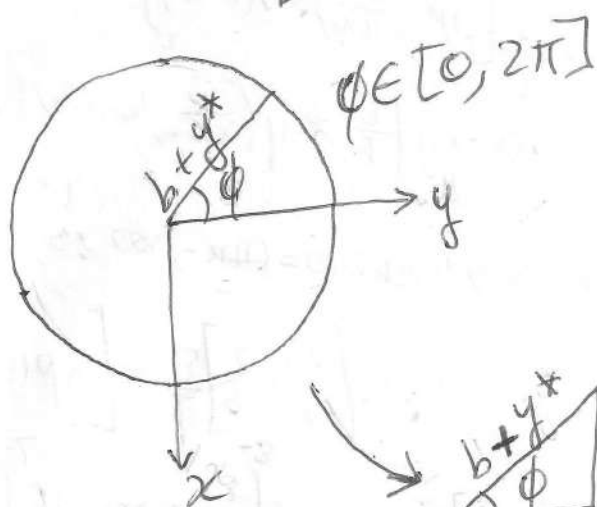
Logo, a parametrização da esfera menos o ponto  $N$  é:

$$P(u, v) = \left( \frac{4u}{u^2 + v^2 + 4}, \frac{4v}{u^2 + v^2 + 4}, \frac{2(u^2 + v^2)}{u^2 + v^2 + 4} \right), (u, v) \in \mathbb{R}^2.$$

$$P: \mathbb{R}^2 \rightarrow S^2 \setminus \{N\}.$$



$$\begin{aligned} \text{From the first diagram: } \begin{cases} \text{Hypotenuse} = a \\ \text{Angle} = \theta \\ \text{Opposite} = z^* \\ \text{Adjacent} = y^* \end{cases} & \begin{cases} \sin(\theta) = \frac{z^*}{a} \\ \Rightarrow z(\theta) = a \sin(\theta) \\ \cos(\theta) = \frac{y^*}{a} \\ \Rightarrow y^*(\theta) = a \cos(\theta) \end{cases} \end{aligned}$$



$$\begin{aligned} \text{From the second diagram: } \begin{cases} \text{Hypotenuse} = b + y^* \\ \text{Angle} = \phi \\ \text{Opposite} = x \\ \text{Adjacent} = y \end{cases} & \begin{cases} \sin(\phi) = \frac{x}{b + y^*} \\ \Rightarrow x(\theta, \phi) = (b + a \cos(\theta)) \sin(\phi) \\ \cos(\phi) = \frac{y}{b + a \cos(\theta)} \\ \Rightarrow y(\theta, \phi) = (b + a \cos(\theta)) \cos(\phi) \end{cases} \end{aligned}$$

A parametrização do toro é:

$$X(\theta, \phi) = ((b + a \cos(\theta)) \sin(\phi), (b + a \cos(\theta)) \cos(\phi), a \sin(\theta)),$$

$$\theta, \phi \in [0, 2\pi].$$