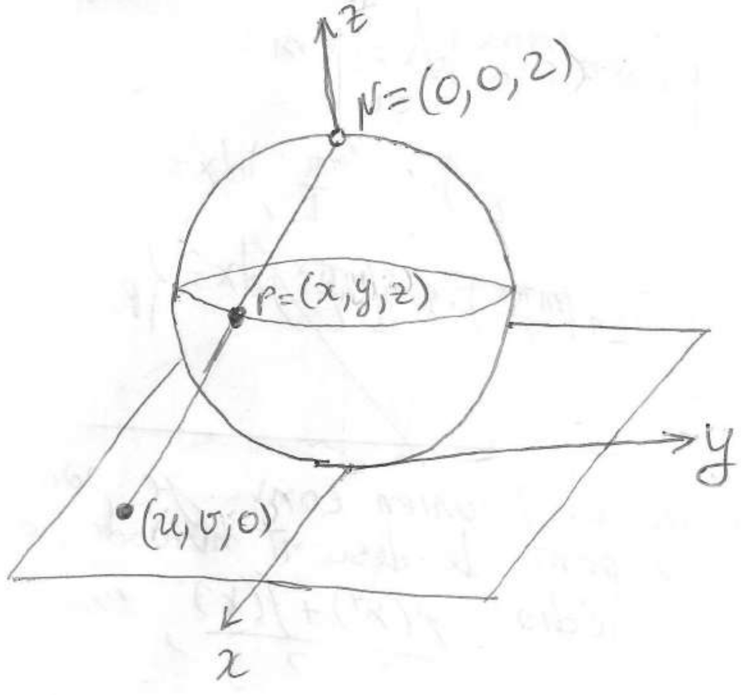


Projeção Estereográfica

Esfera:

$x^2+y^2+(z-1)^2=1$

Reta que passa por $(u,v,0)$ e $(0,0,2)$:

$r(t)=(0,0,2)+t(-u,-v,2)=(-ut,-vt,2+2t)$.

$$\Rightarrow r(t)=(-ut,-vt,2+2t).$$

Interseção da reta com a esfera:

$r(t)\in S^2\setminus\{N\} \iff (-ut)^2+(-vt)^2+(2+2t-1)^2=1$

$\iff u^2t^2+v^2t^2+1+4t+4t^2=1$

$\iff t(u^2t+v^2t+4t+4)=0$

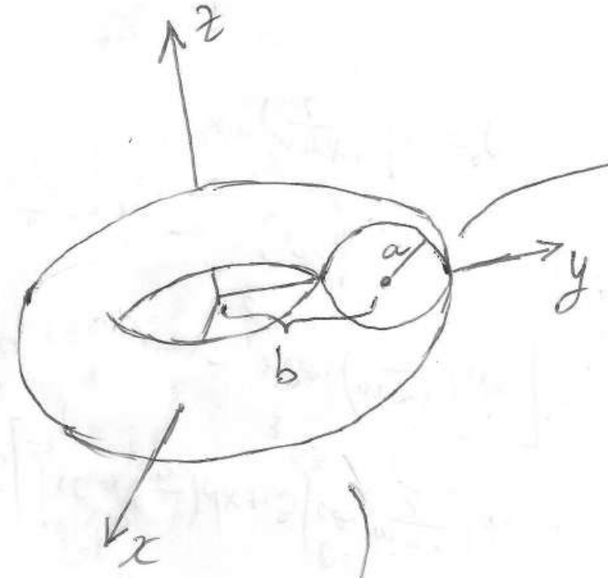
$\iff t=0 \quad$ ou $\quad t=\dfrac{-4}{u^2+v^2+4}$

- Se $t=0$, então $r(0)=N$. Porém, $r(t)\in S^2\setminus\{N\}$. Portanto, $t\neq 0$.
- Se $t=\dfrac{-4}{u^2+v^2+4}$, então $r(t)=\left(\dfrac{4u}{u^2+v^2+4},\dfrac{4v}{u^2+v^2+4},\dfrac{2(u^2+v^2)}{u^2+v^2+4}\right)$.

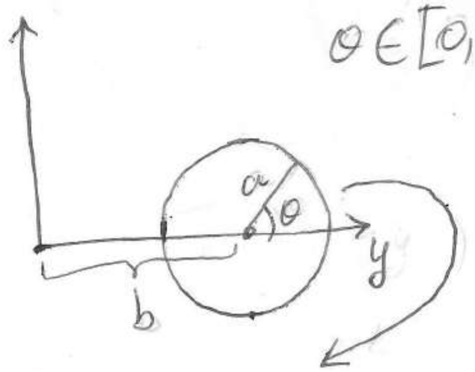
Logo, a parametrização da esfera menos o ponto N é:

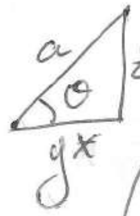
$$P(u,v)=\left(\frac{4u}{u^2+v^2+4}, \frac{4v}{u^2+v^2+4}, \frac{2(u^2+v^2)}{u^2+v^2+4}\right), \ (u,v)\in\mathbb{R}^2$$

$$P: \mathbb{R}^2 \longrightarrow S^2\setminus\{N\}$$



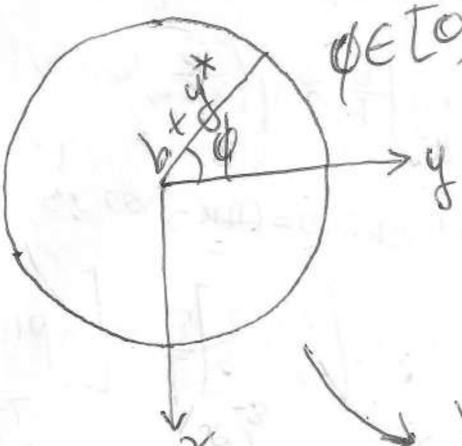
$\theta \in [0, 2\pi]$

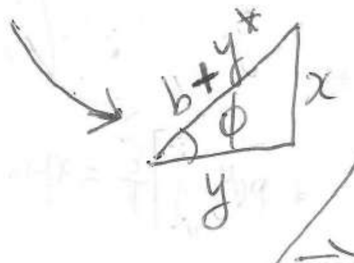




$$\begin{cases} \operatorname{sen}(\theta) = \dfrac{z}{a} \\ \Rightarrow z(\theta) = a\operatorname{sen}(\theta) \\ \cos(\theta) = \dfrac{y^*}{a} \\ \Rightarrow y^*(\theta) = a\cos(\theta) \end{cases}$$

$\phi \in [0, 2\pi]$





$$\begin{cases} \operatorname{sen}(\phi) = \dfrac{x}{b + a\cos(\theta)} \\ \Rightarrow x(\theta, \phi) = (b + a\cos(\theta))\operatorname{sen}(\phi) \\ \cos(\phi) = \dfrac{y}{b + a\cos(\theta)} \\ \Rightarrow y(\theta, \phi) = (b + a\cos(\theta))\cos(\phi) \end{cases}$$

A parametrização do toro é:

$X(\theta, \phi) = ((b + a\cos(\theta))\operatorname{sen}(\phi), (b + a\cos(\theta))\cos(\phi), a\operatorname{sen}(\theta))$,

$\theta, \phi \in [0, 2\pi]$.