Echtzeitsysteme - Übung 1

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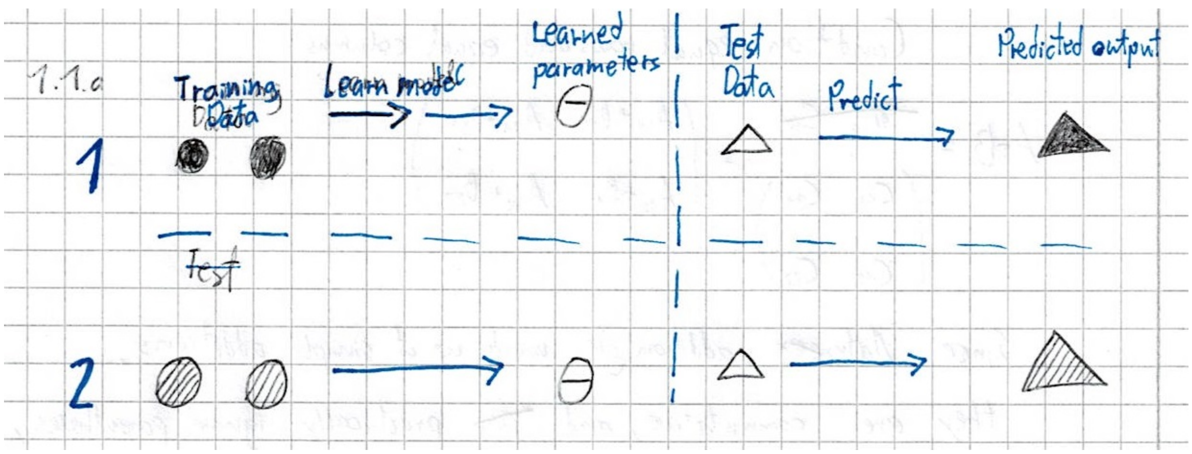
1.1a) Model Fitting



Model 1 is being trained on filled circles, and as such assumes that the triangle is also to be filled, coming to an incorrect result.
Model 2 is being trained on striped circles, and as such assumes that the triangle is also to be striped, coming to the correct conclusion.

1 1.2a) Matrix Algebra Refresher

1.1 Multiplication

$$A * B = \begin{pmatrix} A_{1,1} * B_{1,1} + A_{2,1} * B_{1,2} & A_{1,1} * B_{2,1} + A_{2,1} * B_{2,2} \\ A_{1,2} * B_{1,1} + A_{2,2} * B_{1,2} & A_{1,2} * B_{2,1} + A_{2,2} * B_{2,2} \end{pmatrix}$$

$A * B$ is defined only when Columns of B equal rows of A.

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} * \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

BUT

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Matrixmultiplication is not Commutative.
$A(B+C) = AB + AC$

$$\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix} * (\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix} + \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}) = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \begin{pmatrix}1 & 1\\0 & 0\end{pmatrix} = \begin{pmatrix}2 & 1\\0 & 0\end{pmatrix}$$

$$\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix} * \begin{pmatrix}1 & 1\\2 & 1\end{pmatrix} = \begin{pmatrix}2 & 1\\0 & 0\end{pmatrix}$$

Matrixmultiplication is Distributive.

$(A * B) * C = A * (B * C)$

$$(\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix} * \begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}) * \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = \begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}$$

$$\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix} * (\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix} * \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}) = \begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}$$

1.2 Addition

Since matrix-addition is made up of simple additions, they are commutative and practically ignore parentheses, so are also distributive and associative.
The condition for any matrix-addition is that the matrices have both equal rows and columns.

$$A_{2,2} + B_{2,2} = \begin{pmatrix}C_{1,1} & C_{2,1}\\C_{1,2} & C_{2,2}\end{pmatrix} = \begin{pmatrix}A_{1,1}+B_{1,1} & A_{2,1}+B_{2,1}\\A_{1,2}+B_{1,2} & A_{2,2}+B_{2,2}\end{pmatrix}$$

2 1.2b) Matrix Inversion

$A^{-1} =$

$$\begin{pmatrix}1 & 0 & 0\\-1 & 1 & 0\\0 & 0 & 1\end{pmatrix}$$

...Via Gauß-Jordan. Under the condition that $a = b = d = 0; c = 1$

$$\begin{pmatrix}2 & 2 & 3\\0 & 1 & 0\\8 & 3 & 12\end{pmatrix}$$

Is not invertable, since it's Determinant $Det = 0$.

3 1.2c) Matrix Pseudoinverse

Left: $A^{\#} * A = (A^T * A)^{-1} * A^T$
Right: $A * A^{\#} = A * A^T (A * A^T)^{-1}$
Since A_{2x3} has more rows than columns, the left Moore-Penrose exists.
The equation is: $A^{\#}_{3x2} * A = (A^T_{3x2} * A_{2x3})^{-1}_{2x2} * A^T_{3x2}$

4 1.2d) Basis Transformation

Vector with new Basis $v^* = T^{-1} * v$

1) $T_v = E^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; T_w = B^{-1} = \begin{pmatrix} 2 & -1.5 \\ -1 & 1 \end{pmatrix}$

2) $2 * \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 5 * \begin{bmatrix} -1,5 \\ 1 \end{bmatrix} = \begin{bmatrix} -3,5 \\ 3 \end{bmatrix} = v^*$