Л9

Вериги с индуктивни връзки 2

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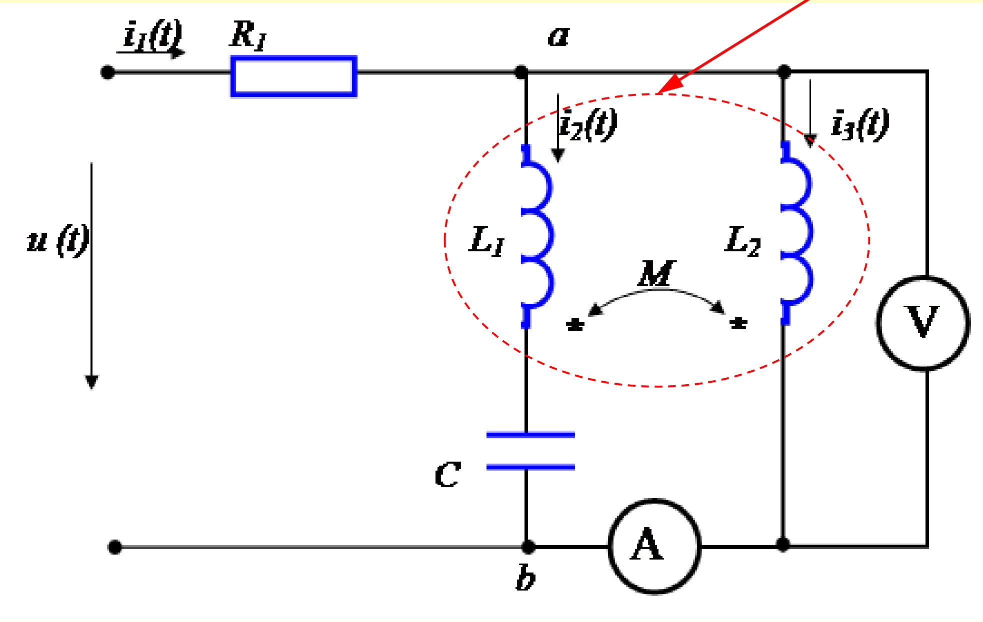
Вериги с индуктивни връзки

В някои случаи между отделните части на ел.верига може да има не само електрическа, но и **магнитна връзка**.

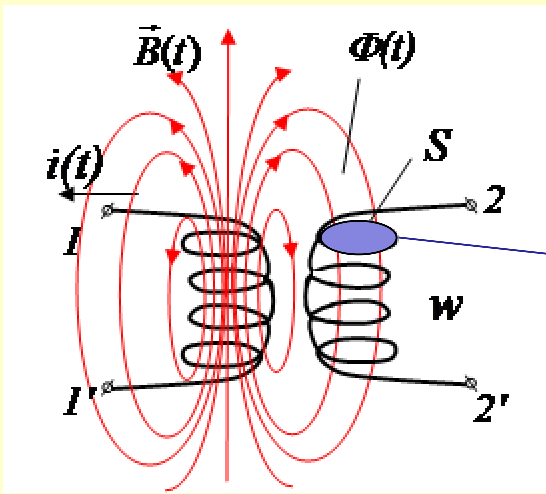
Това означава, че във веригата са включени **магнитносвързани бобини**, т.е. преминаването на ток през едната създава магнитно поле, което обхваща навивките на другата и съгласно закона за електромагнитната индукция **индуктира** в нея **напрежение**.



- В този случай приемаме, че между бобините има индуктивна връзка, а преминаването на **ток през едната** води до появата на **напрежение в другата** и обратно
- при анализа на вериг с индуктивни връзки се **отчитат** и тези **допълнителни напрежения**

Вериги с индуктивни връзки

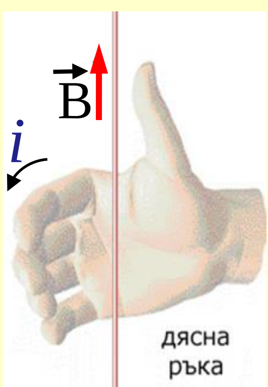
Във веригата са включени **магнитносвързани бобини**, т.е. преминаването на ток през едната създава магнитно поле, което обхваща навивките на другата и съгласно закона за електромагнитната индукция **индуктира** в нея **напрежение**.



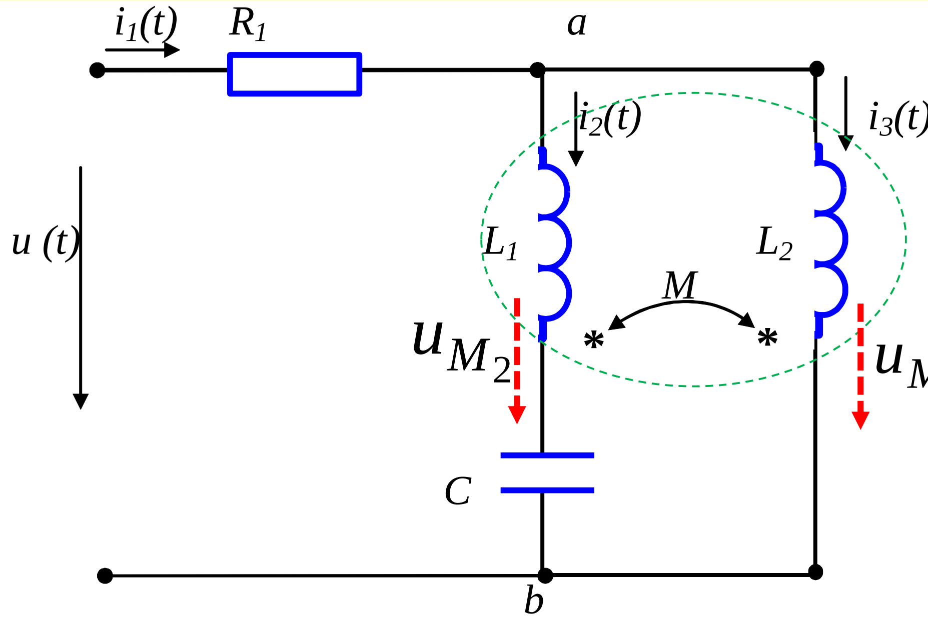
$$i(t) \longrightarrow \vec{B}(t)$$

$$\Phi(t) = \oiint_S \vec{B}(t)\,d\vec{S}$$

$$\Psi(t) = w.\Phi(t)$$



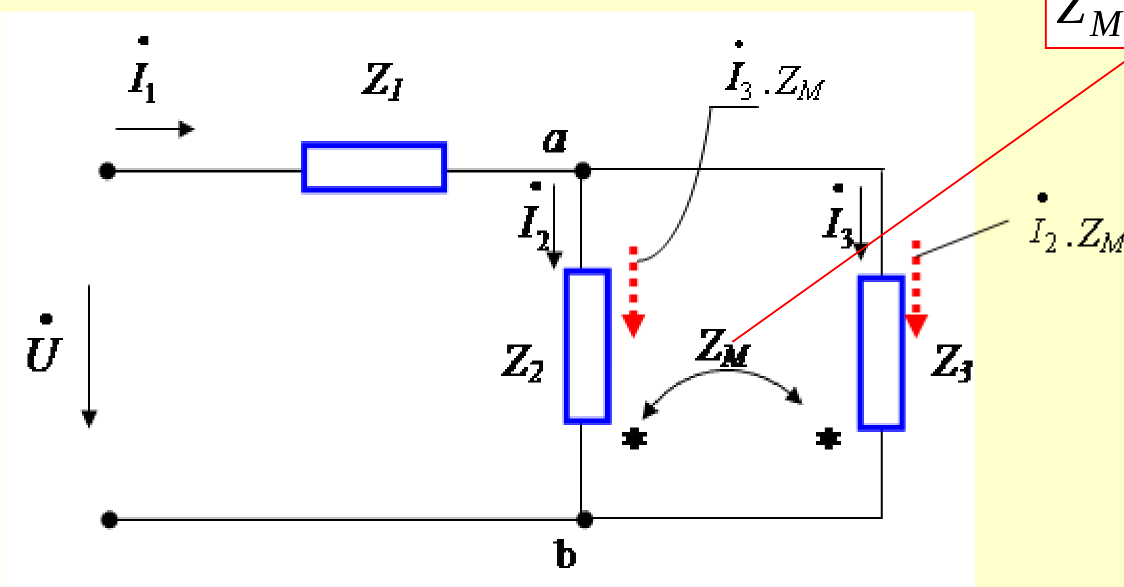
$$\boxed{e(t) = -\frac{d\Psi}{dt}}$$



$$u_{M_2} = M\frac{di_3}{dt}$$

$$u_{M_3} = M\frac{di_2}{dt}$$

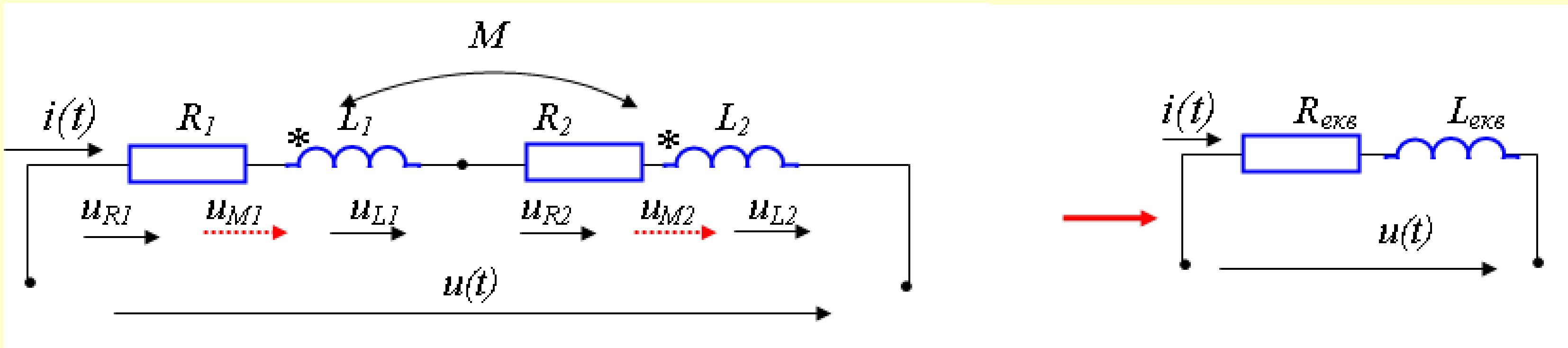
$$Z_M = j\omega M$$



$$\begin{cases} \dot{I}_1 - \dot{I}_2 - \dot{I}_3 = 0 \\ \dot{I}_1 Z_1 + \dot{I}_2 Z_2 + \dot{I}_3 Z_M = \dot{U} \\ \dot{I}_3 Z_3 - \dot{I}_2 Z_2 + \dot{I}_2 Z_M - \dot{I}_3 Z_M = 0 \end{cases}$$

Последователно съединение на два индуктивно свързани елемента.

1.Съгласувано свързване



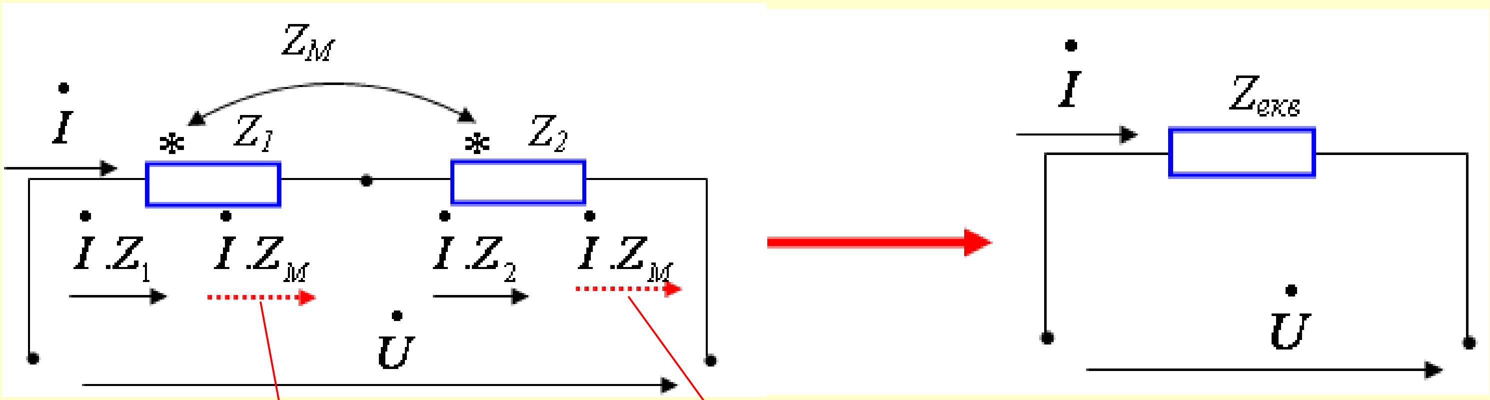
$$u(t)=u_{R_1}(t)+u_{L_1}(t)+\boxed{u_{M_1}(t)}+u_{R_2}(t)+u_{L_2}(t)+\boxed{u_{M_2}(t)}=$$

$$R_1 i(t)+L_1\frac{di}{dt}+M\frac{di}{dt}+R_2 i(t)+L_2\frac{di}{dt}+M\frac{di}{dt}=$$

$$(R_1+R_2).i(t)+(L_1+L_2+2M).\frac{di}{dt}=R_{екв}.i(t)+L_{екв}.\frac{di}{dt}$$

$$\boxed{\begin{aligned} R_{екв}&=R_1+R_2\\ L_{екв}&=L_1+L_2+2M\end{aligned}}$$

Съгласувано свързване – синусоидален ток



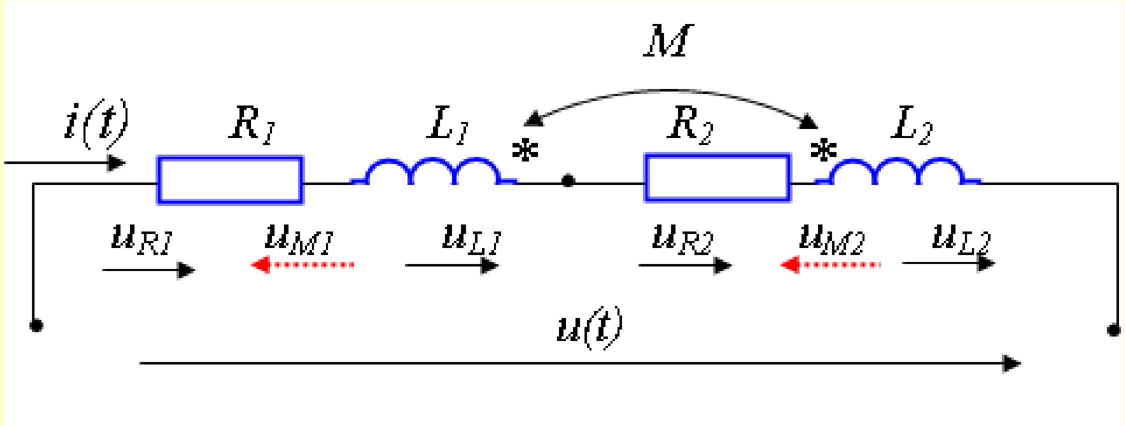
$$Z_1 = R_1 + j\omega L_1; \qquad Z_2 = R_2 + j\omega L_2; \qquad Z_M = j\omega M$$

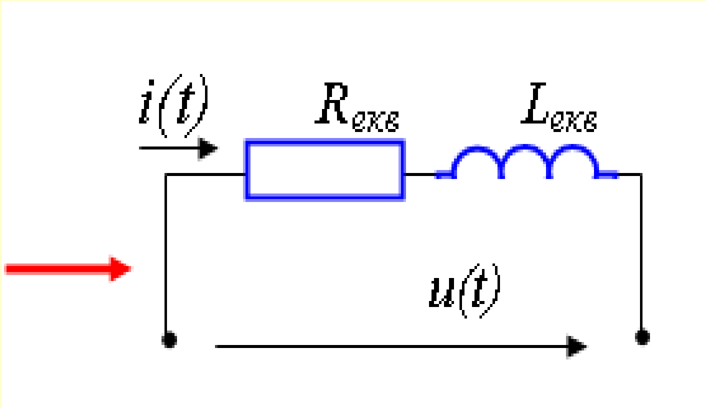
$$\dot U = \dot I(R_1 + j\omega L_1) + \dot I.j\omega M + \dot I(R_2 + j\omega L_2) + \dot I.j\omega M = \dot I Z_1 + \dot I.Z_M + \dot I Z_2 + \dot I.Z_M$$

$$\Rightarrow \dot U = \dot I(Z_1 + Z_2 + 2Z_M) = \dot I Z_{екв}$$

$$\boxed{Z_{екв} = Z_1 + Z_2 + 2Z_M}$$

2. Несъгласувано свързване.





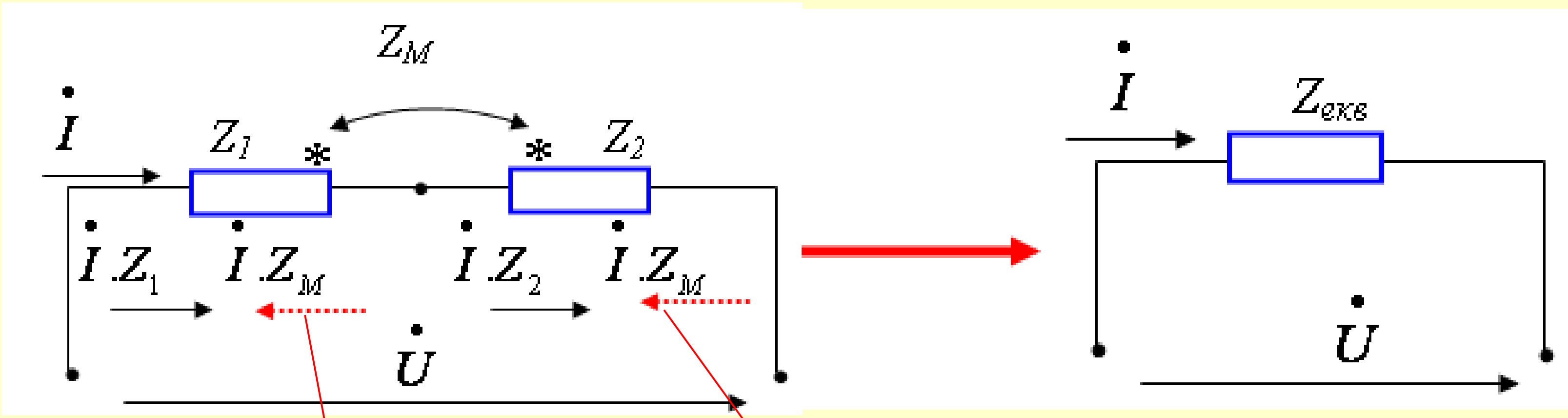
$$u(t) = u_{R_1}(t) + u_{L_1}(t) - \boxed{u_{M_1}(t)} + u_{R_2}(t) + u_{L_2}(t) - \boxed{u_{M_2}(t)} =$$

$$R_1 i(t) + L_1 \frac{di}{dt} - M\frac{di}{dt} + R_2 i(t) + L_2 \frac{di}{dt} - M\frac{di}{dt} =$$

$$(R_1 + R_2).i(t) + (L_1 + L_2 - 2M).\frac{di}{dt} = R_{екв}.i(t) + L_{екв}.\frac{di}{dt}$$

$$\boxed{\begin{aligned} R_{екв} &= R_1 + R_2 \\ L_{екв} &= L_1 + L_2 - 2M \end{aligned}}$$

2. Несъгласувано свързване – синусоидален ток



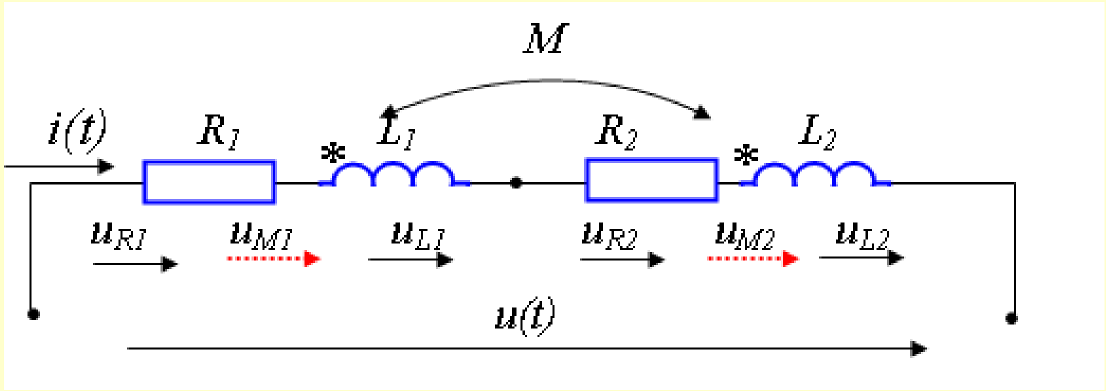
$$Z_1 = R_1 + j\omega L_1; \qquad Z_2 = R_2 + j\omega L_2; \qquad Z_M = j\omega M$$

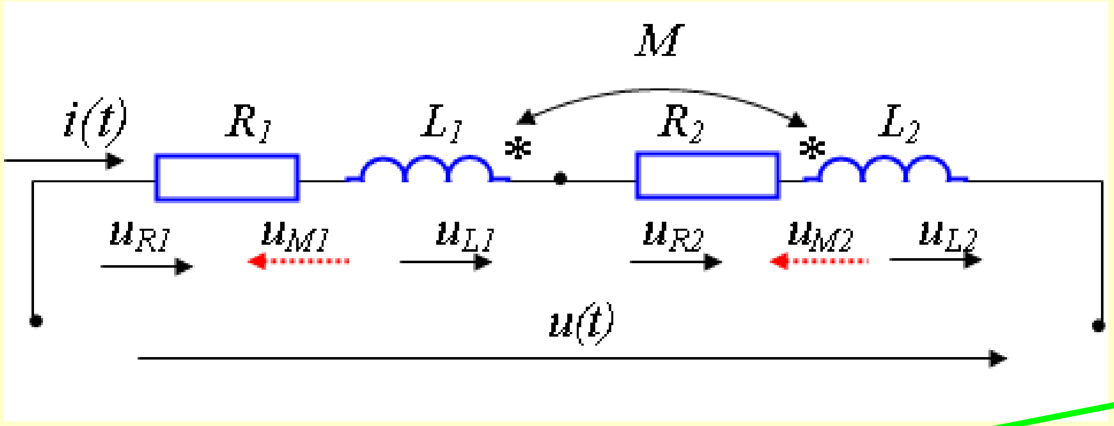
$$\dot{U} = \dot{I}(R_1 + j\omega L_1) - \dot{I}.j\omega M + \dot{I}(R_2 + j\omega L_2) - \dot{I}.j\omega M = \dot{I}\,Z_1 - \dot{I}.Z_M + \dot{I}\,Z_2 - \dot{I}.Z_M$$

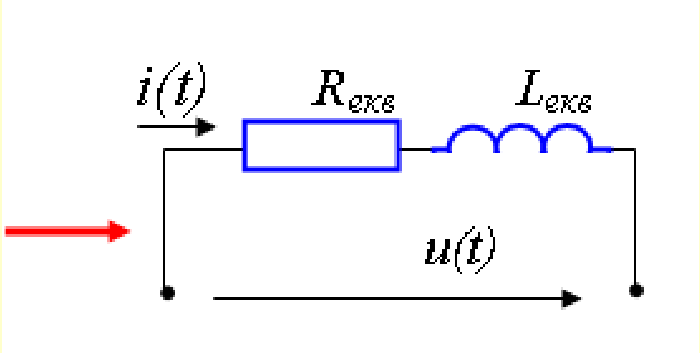
$$\Rightarrow \dot{U} = \dot{I}(Z_1 + Z_2 - 2Z_M) = \dot{I}\,Z_{екв}$$

$$\boxed{Z_{екв} = Z_1 + Z_2 - 2Z_M}$$

Извод







$$R_{екв} = R_1 + R_2$$
$$L_{екв} = L_1 + L_2 \pm 2M$$

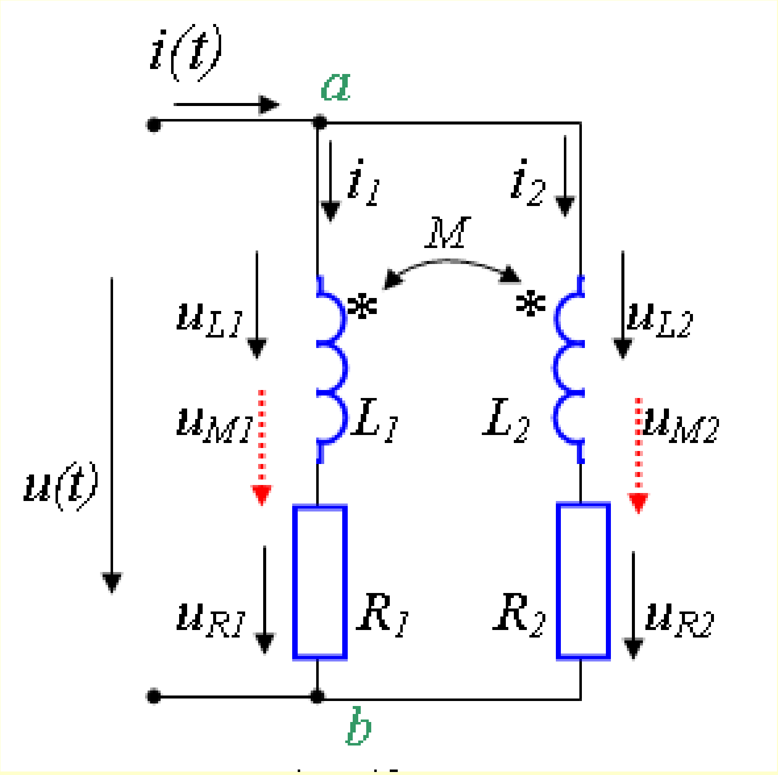
Знакът пред 2М се определя от начина на свързване:

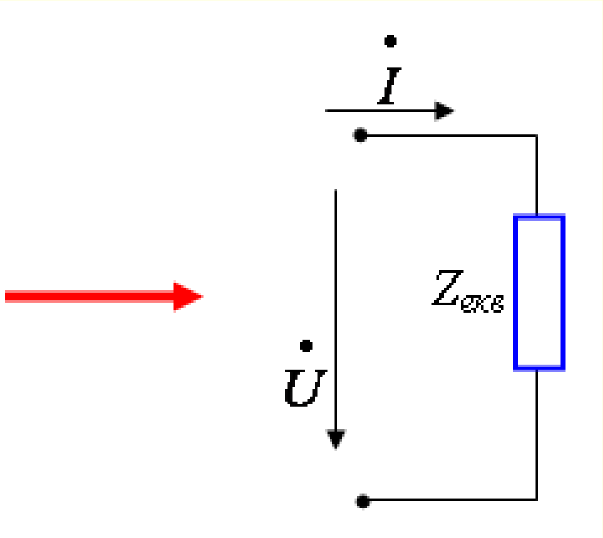
<u>"+" при съгласувано</u> свързване

<u>"-" при несъгласувано</u> свързване.

Паралелно съединение на два индуктивно свързани елемента.

Едноименни изводи в общата точка





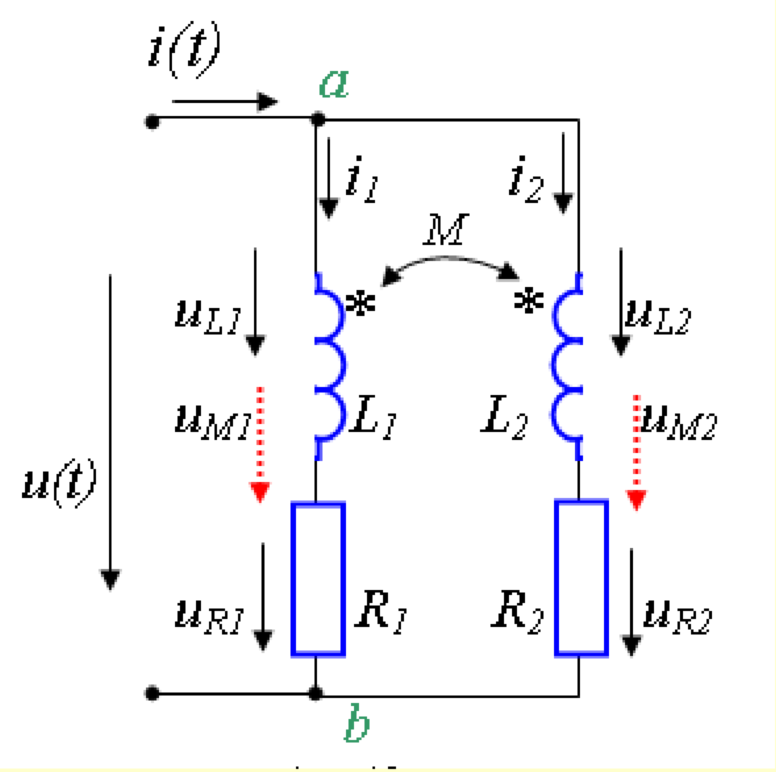
Паралелно съединение на два индуктивно свързани елемента.

Едноименни изводи в общата точка

$$i(t) = i_1(t) + i_{2_1}(t)$$

$$u(t) = u_{R_1}(t) + u_{L_1}(t) + u_{M_1}(t)$$

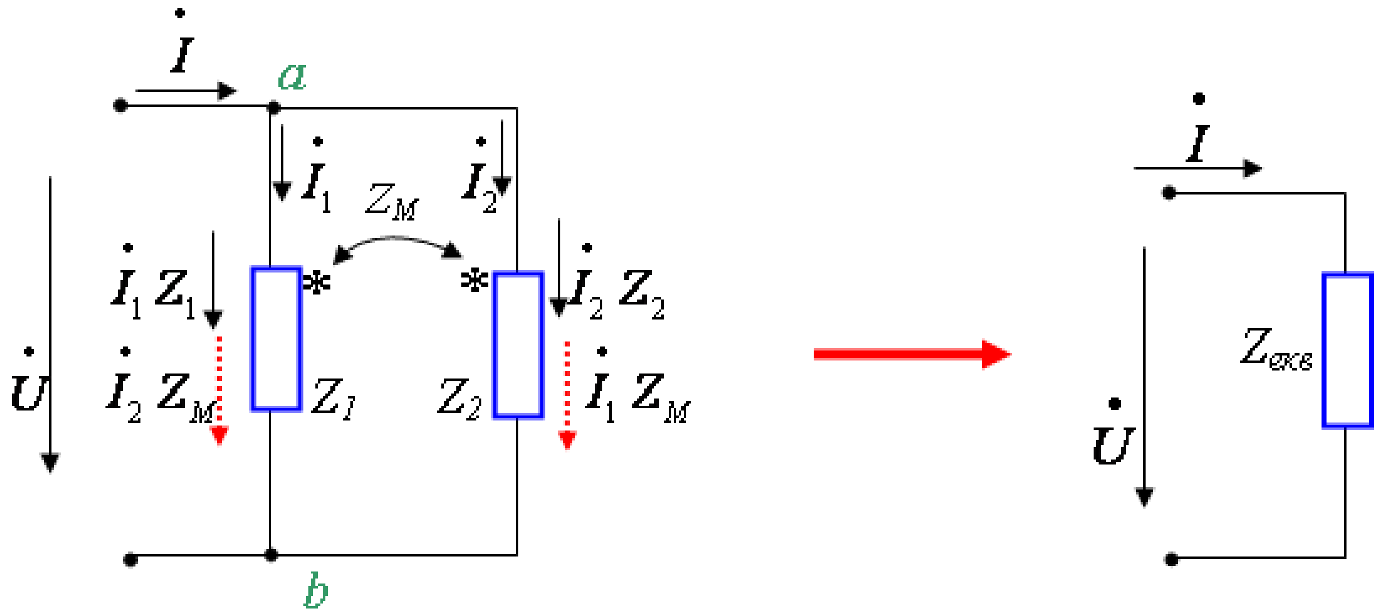
$$u(t) = u_{R_2}(t) + u_{L_2}(t) + u_{M_2}(t)$$



$$i(t) = i_1(t) + i_{2_1}(t)$$

$$u(t) = R_1 i_1(t) + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u(t) = R_2 i_2(t) + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$



$$Z_1 = R_1 + j\omega L_1; \qquad Z_2 = R_2 + j\omega L_2; \qquad Z_M = j\omega M$$

$$\dot{I} - \dot{I}_1 - \dot{I}_2 = 0$$

$$\dot{I}_1 Z_1 + \dot{I}_2 . Z_M = \dot{U}$$

$$\dot{I}_1 Z_M + \dot{I}_2 . Z_2 = \dot{U}$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{U} = \dot{I}_1 (R_1 + j\omega L_1) + \dot{I}_2 . j\omega M$$

$$\dot{U} = \dot{I}_2 (R_2 + j\omega L_2) + \dot{I}_1 . j\omega M_{12}$$

$$\dot{I} - \dot{I}_1 - \dot{I}_2 = 0$$

$$\dot{I}_1 Z_1 + \dot{I}_2 . Z_M = \dot{U}$$

$$\dot{I}_1 Z_M + \dot{I}_2 . Z_2 = \dot{U}$$

$$\dot{I}_1 = \frac{\begin{vmatrix} \dot{U} & Z_M \\ \dot{U} & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{\dot{U}(Z_2 - Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

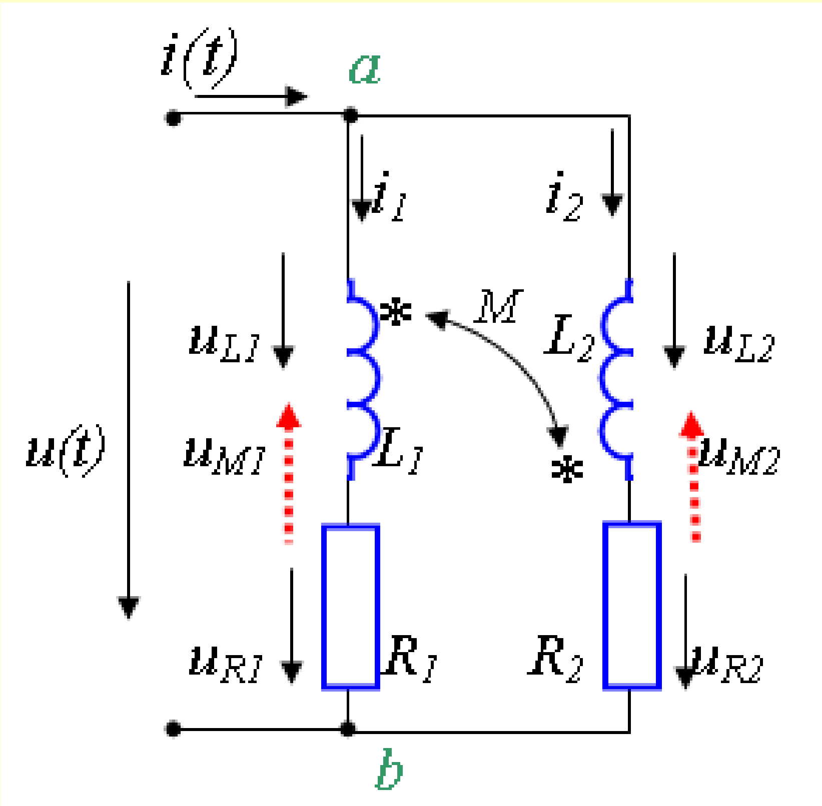
$$\dot{I}_2 = \frac{\begin{vmatrix} Z_1 & \dot{U} \\ Z_M & \dot{U} \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{\dot{U}(Z_1 - Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

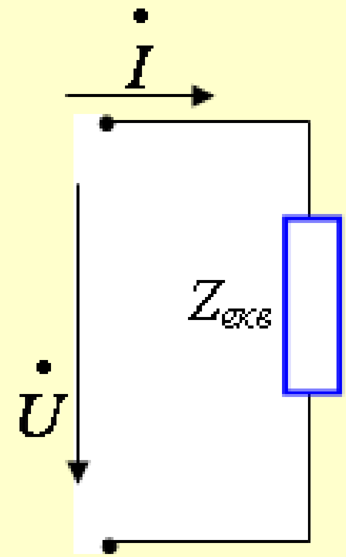
$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{I} = \frac{\dot{U}(Z_1 + Z_2 - 2Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

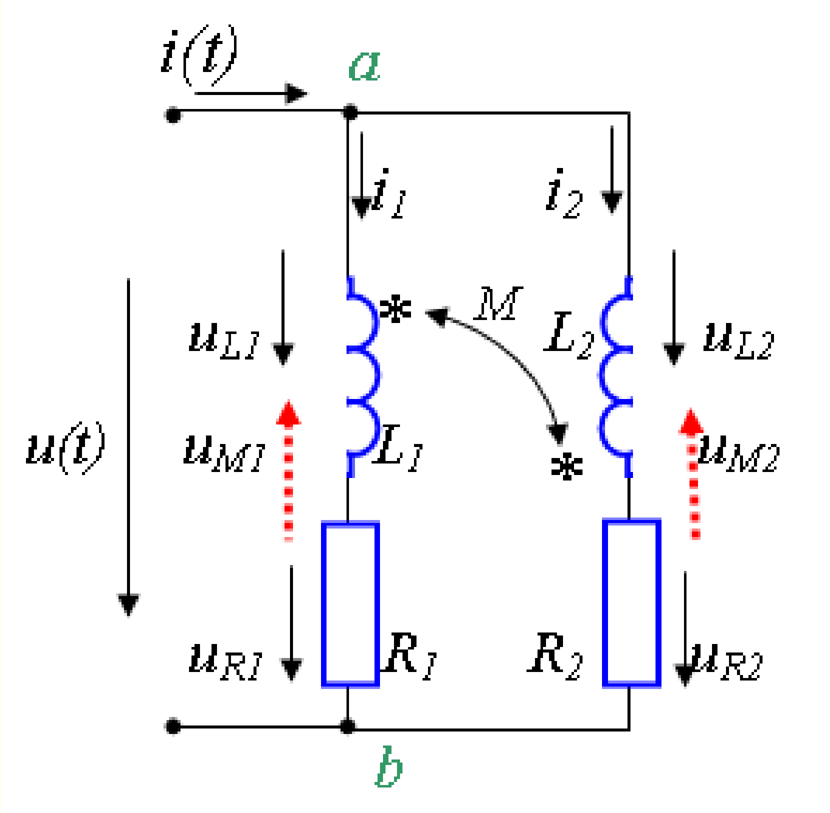
$$\boxed{Z_{екв} = \frac{\dot{U}}{\dot{I}} = \frac{Z_1 Z_2 - Z_M{}^2}{Z_1 + Z_2 - 2Z_M}}$$

Разноименни изводи в общата точка





Разноименни изводи в общата точка



$$i(t) = i_1(t) + i_{2_1}(t)$$

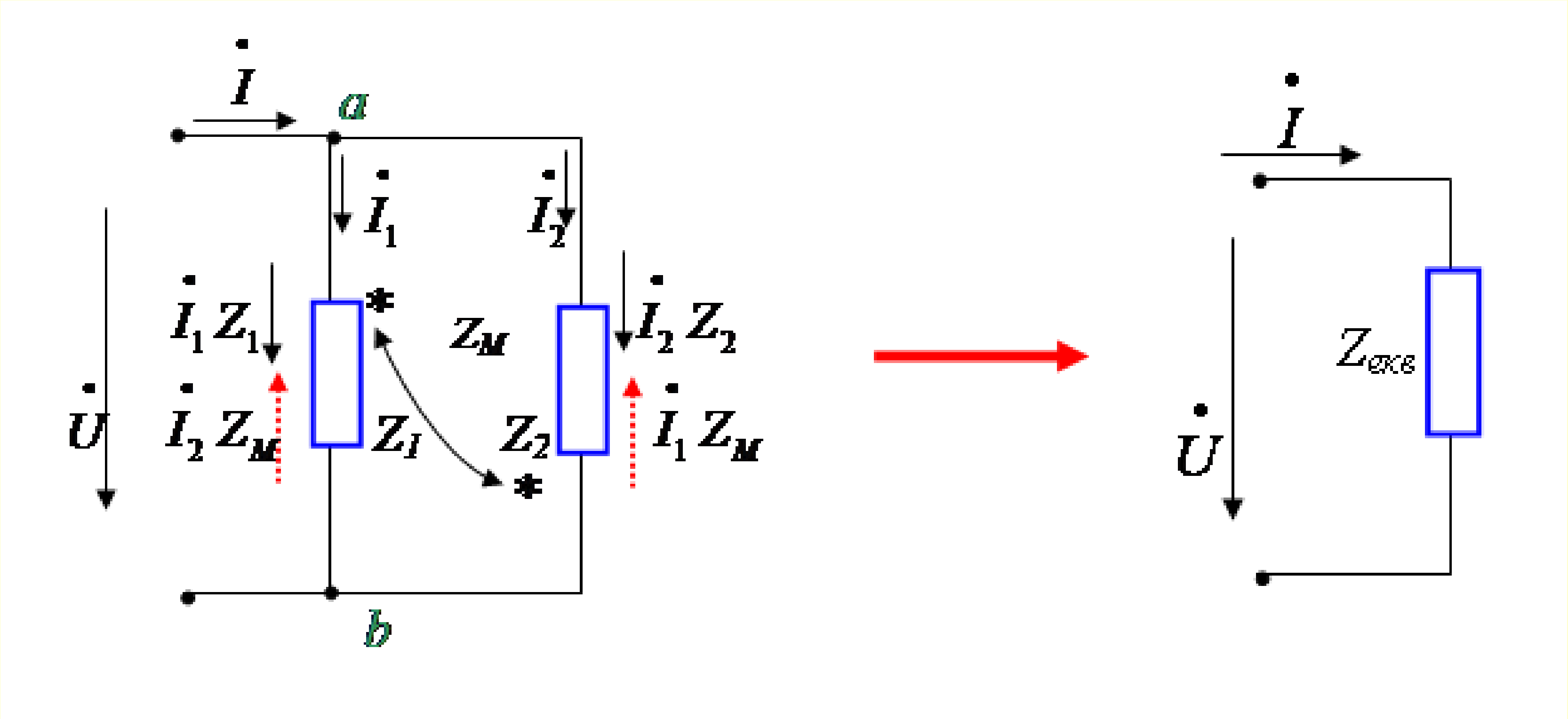
$$u(t) = u_{R_1}(t) + u_{L_1}(t) - u_{M_1}(t)$$

$$u(t) = u_{R_2}(t) + u_{L_2}(t) - u_{M_2}(t)$$

$$i(t) = i_1(t) + i_{2_1}(t)$$

$$u(t) = R_1 i_1(t) + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$u(t) = R_2 i_2(t) + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$



$$\dot{I} - \dot{I}_1 - \dot{I}_2 = 0$$

$$\dot{I}_1 Z_1 - \dot{I}_2 . Z_M = \dot{U}$$

$$-\dot{I}_1 Z_M + \dot{I}_2 . Z_2 = \dot{U}$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{U} = \dot{I}_1 (R_1 + j\omega L_1) - \dot{I}_2 . j\omega M$$

$$\dot{U} = \dot{I}_2 (R_2 + j\omega L_2) - \dot{I}_1 . j\omega M$$

$$\dot{I} - \dot{I}_1 - \dot{I}_2 = 0$$

$$\dot{I}_1 Z_1 - \dot{I}_2 .Z_M = \dot{U}$$

$$-\dot{I}_1 Z_M + \dot{I}_2 .Z_2 = \dot{U}$$

$$\dot{I}_1 = \frac{\begin{vmatrix} \dot{U} & -Z_M \\ \dot{U} & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{\dot{U}(Z_2 + Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

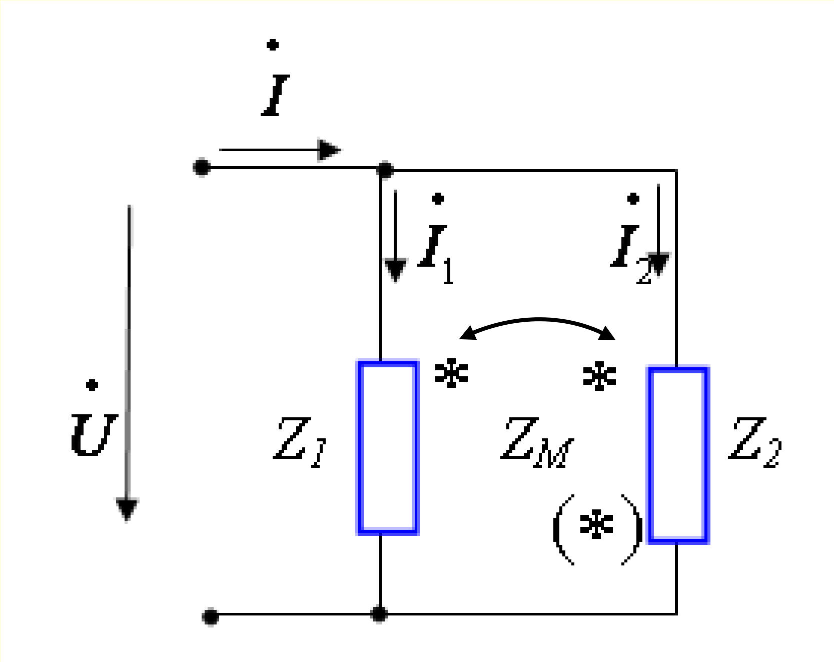
$$\dot{I}_2 = \frac{\begin{vmatrix} Z_1 & \dot{U} \\ -Z_M & \dot{U} \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{\dot{U}(Z_1 + Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{I} = \frac{\dot{U}(Z_1 + Z_2 + 2Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

$$\boxed{Z_{екв} = \frac{\dot{U}}{\dot{I}} = \frac{Z_1 Z_2 - Z_M{}^2}{Z_1 + Z_2 + 2Z_M}}$$

Извод



$$Z_{екв} = \frac{Z_1 Z_2 - Z_M{}^2}{Z_1 + Z_2 \mp 2Z_M}$$

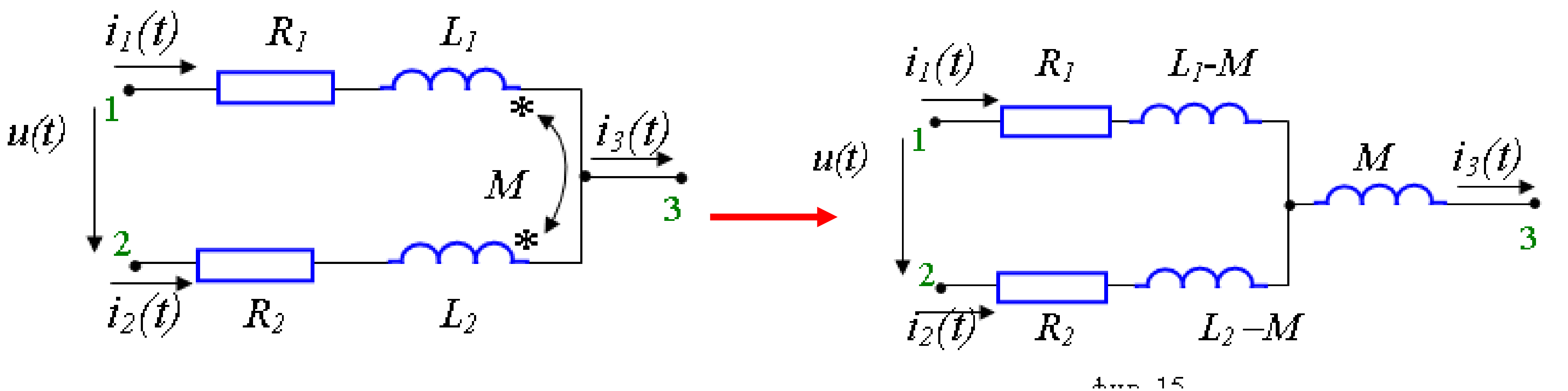
Знакът пред 2ZM се определя от начина на свързване:

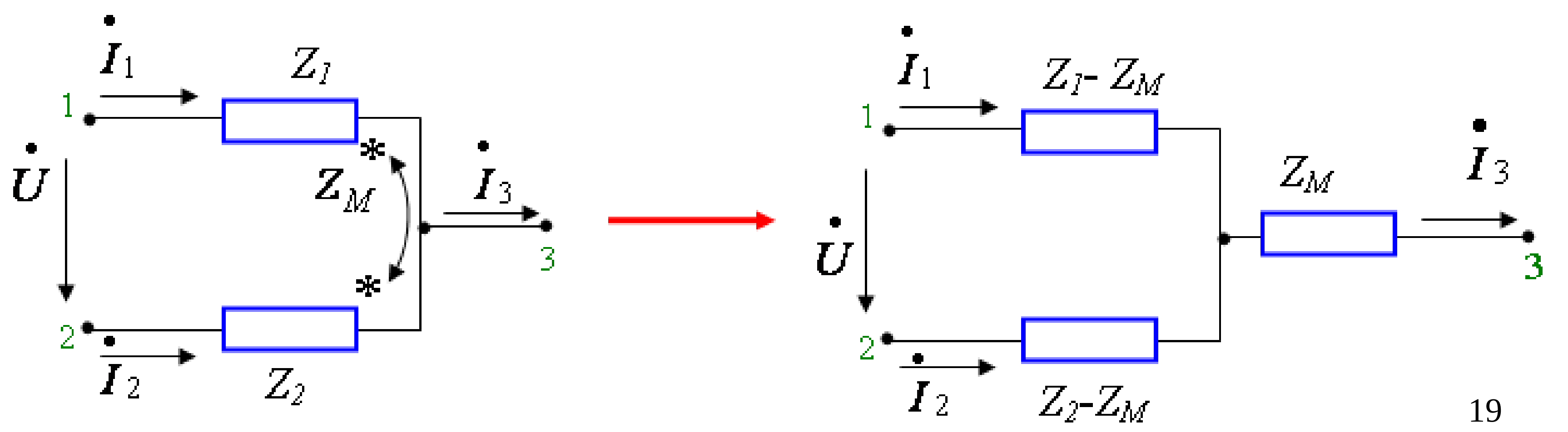
“ - " при едноименни изводи в общата точка

“+" при разноименни изводи в общата точка.

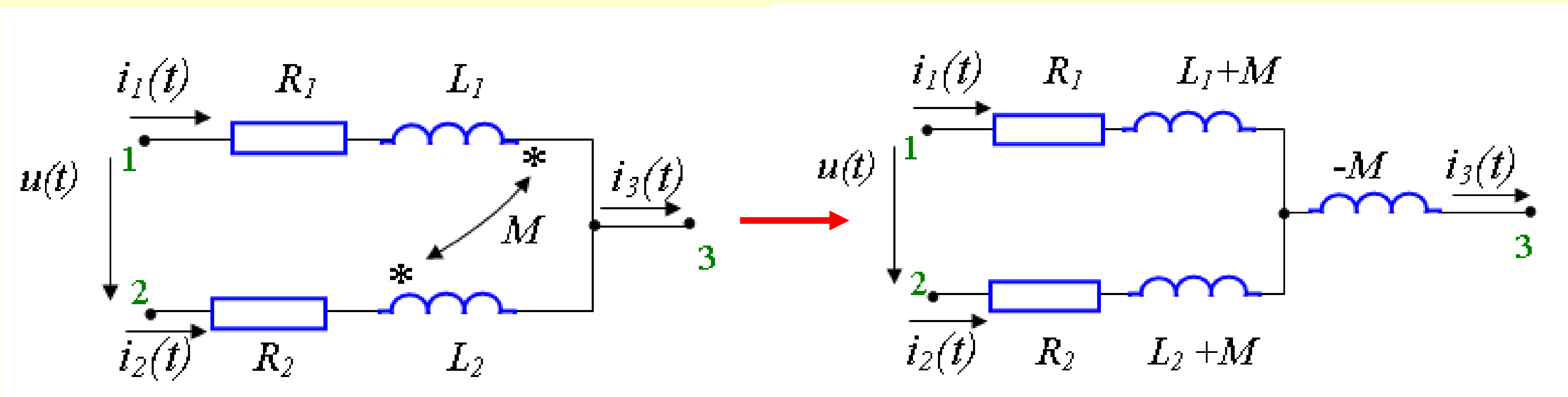
Преобразуване на триполюсно съединение с индуктивна връзка

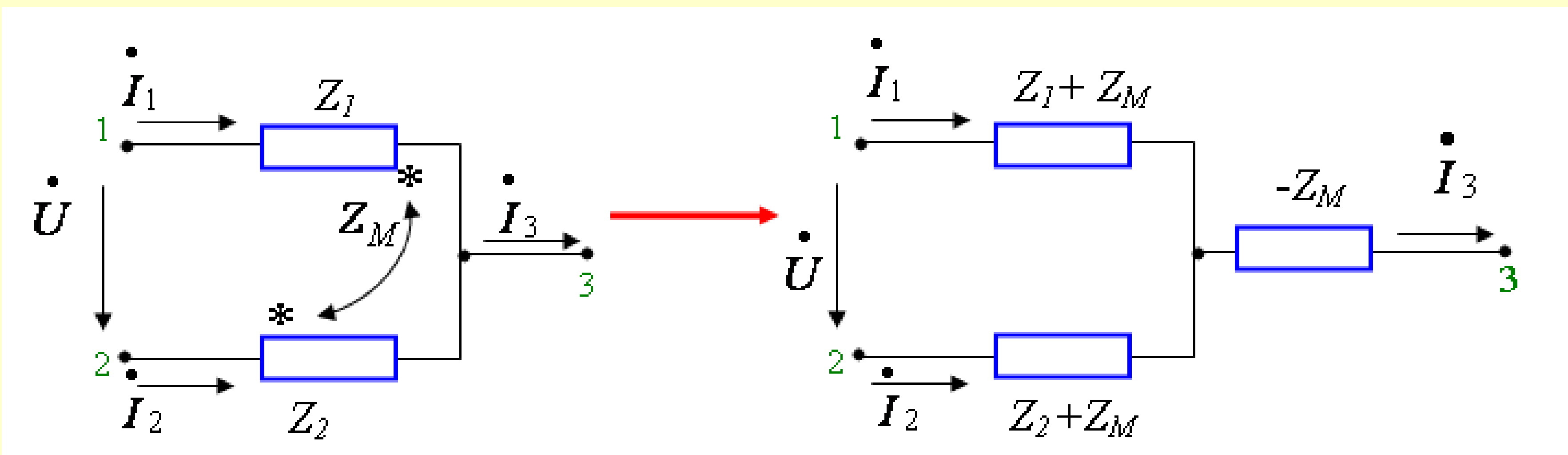
Едноименните изводи са в обща точка





<u>Разноименните</u> изводи са в обща точка





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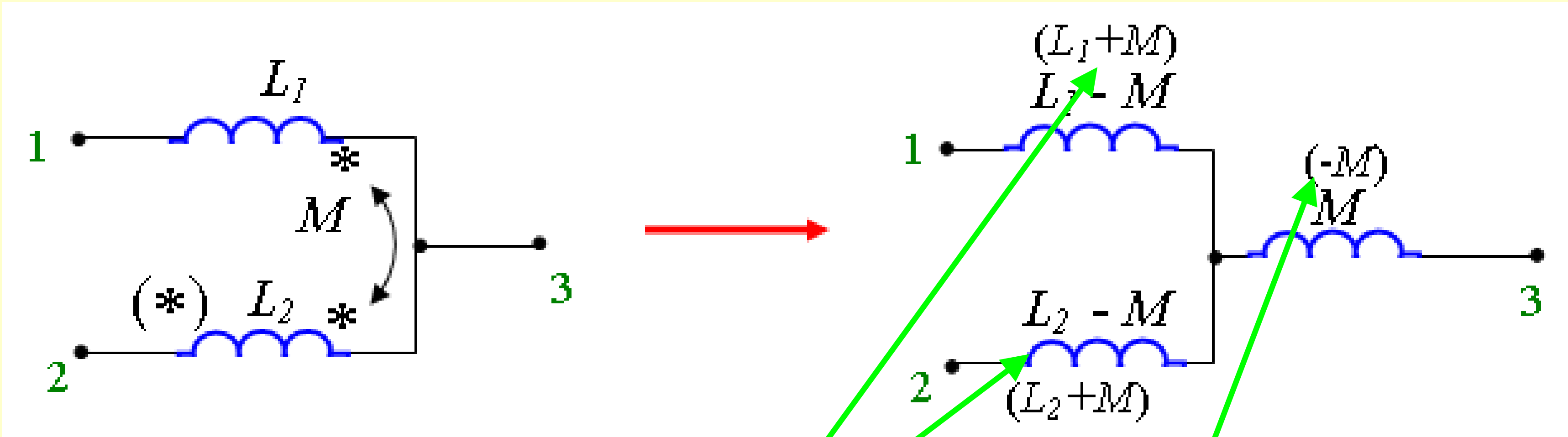
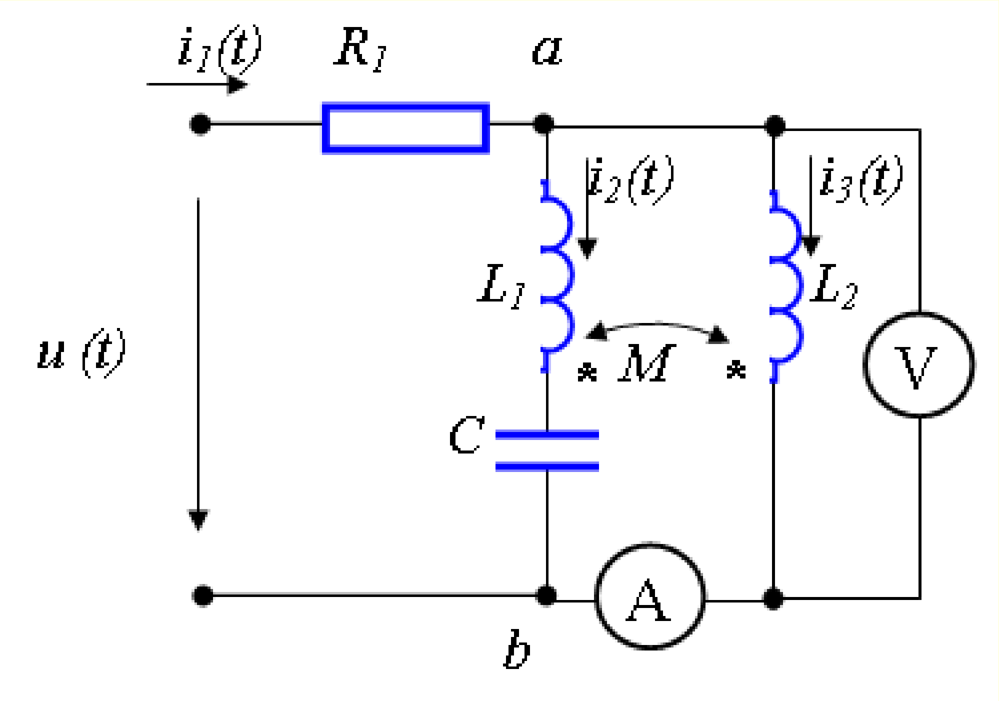


Знакът пред *M* се определя от начина на свързване:

- изразът без скоби е при **едноименни изводи в общата точка**
- а изразът в скобите е при **разноименни изводи в общата точка**.

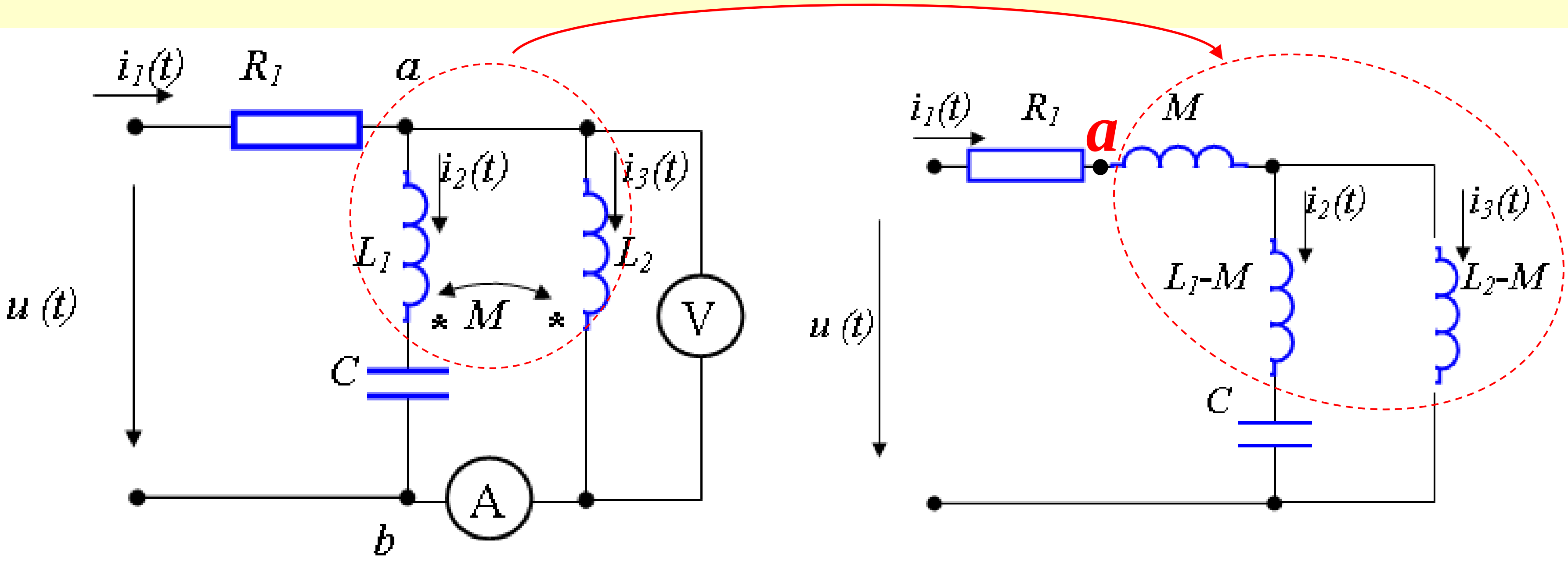
Посоките на токовете нямат значение за преобразуването.

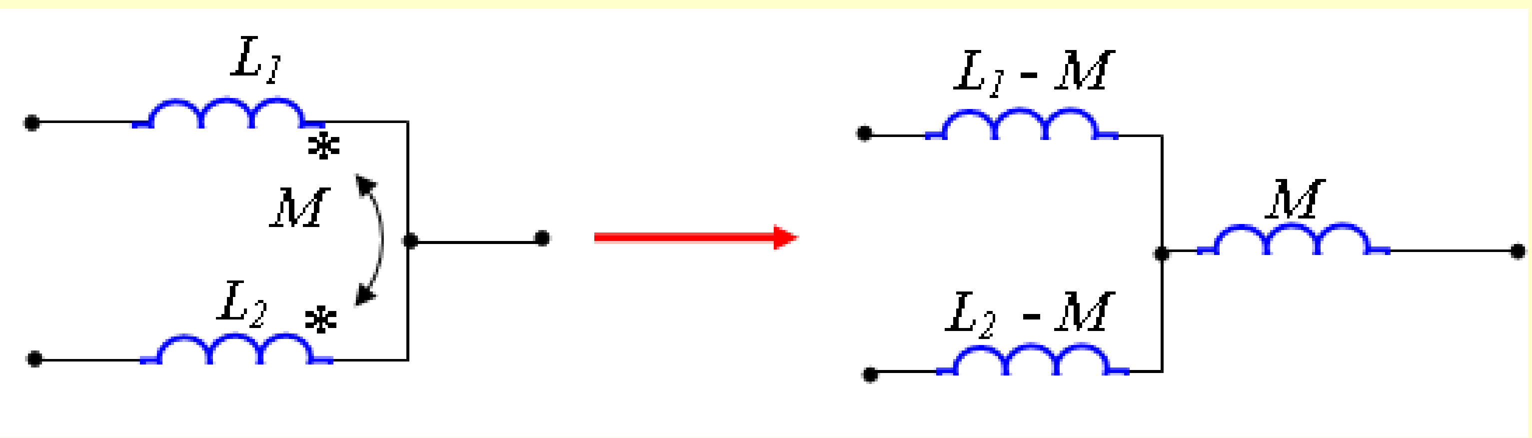
Пример за анализ на верига с индуктивни връзки посредством **отстраняване на индуктивната връзка:**

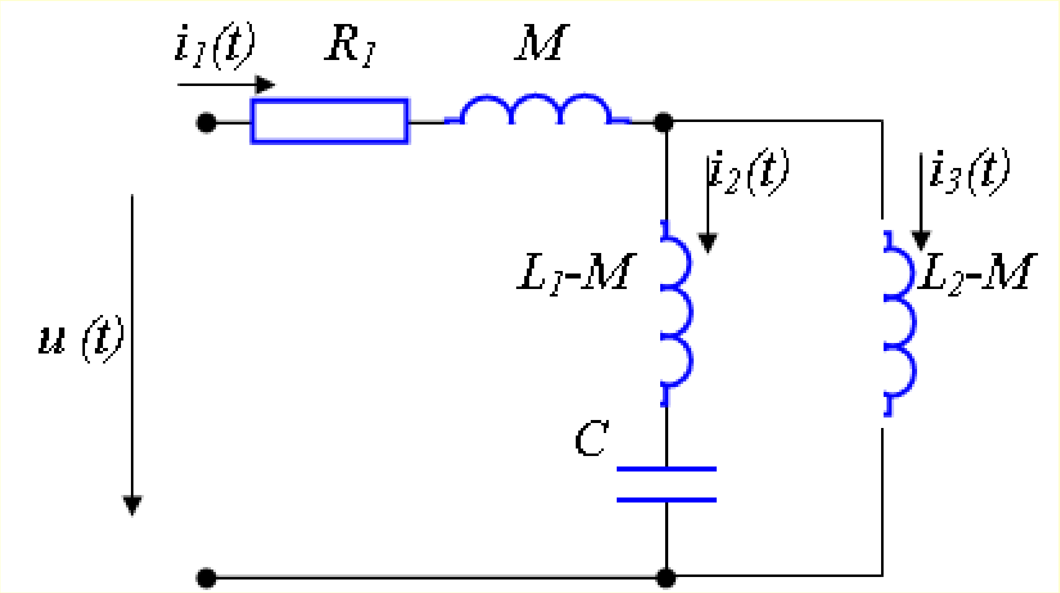


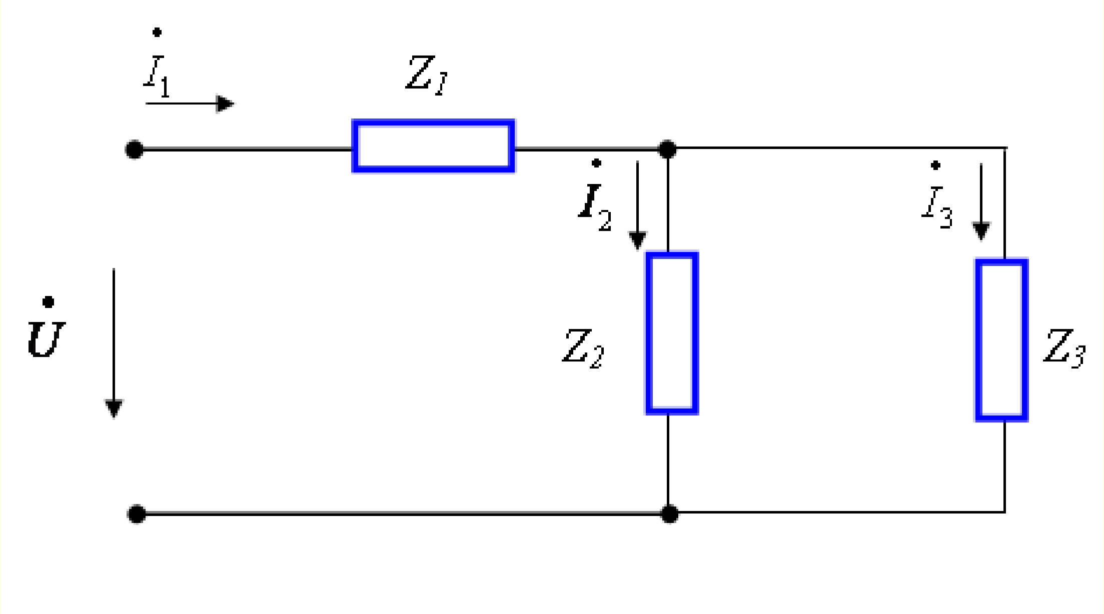
$u(t) = 141\sin(\omega t+90)V$

$f=160Hz,$
$L_1=40\ mH,$
$L_2=30\ mH,$
$M=10\ mH,$
$C=100\ \mu F$
$R_1 = 10\Omega,$


i1(t)
R1
a
i2(t)
i3(t)
L1
L2
M
C
u (t)
V
A
b
M
L1-M
L2-M


L1
M
L2
L1 - M
L2 - M
M

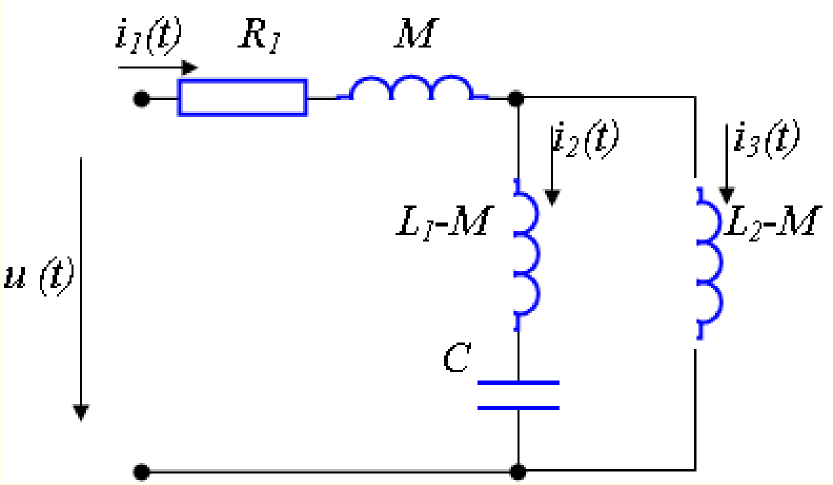


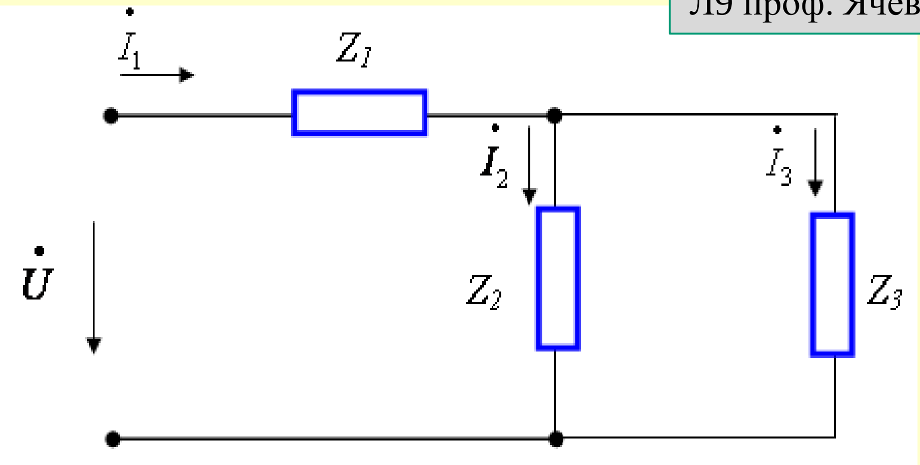


$$Z_1 = R_1 + j\omega M$$

$$Z_2 = j\omega(L_1 - M) - j\frac{1}{\omega C}$$

$$Z_3 = j\omega(L_2 - M)$$





$u(t) = 141\sin(\omega t+90)V$

$f=160Hz,$
$L_1=40\ mH,$
$L_2=30\ mH,$
$M=10\ mH,$
$C=100\ \mu F$
$R_1 = 10\Omega,$

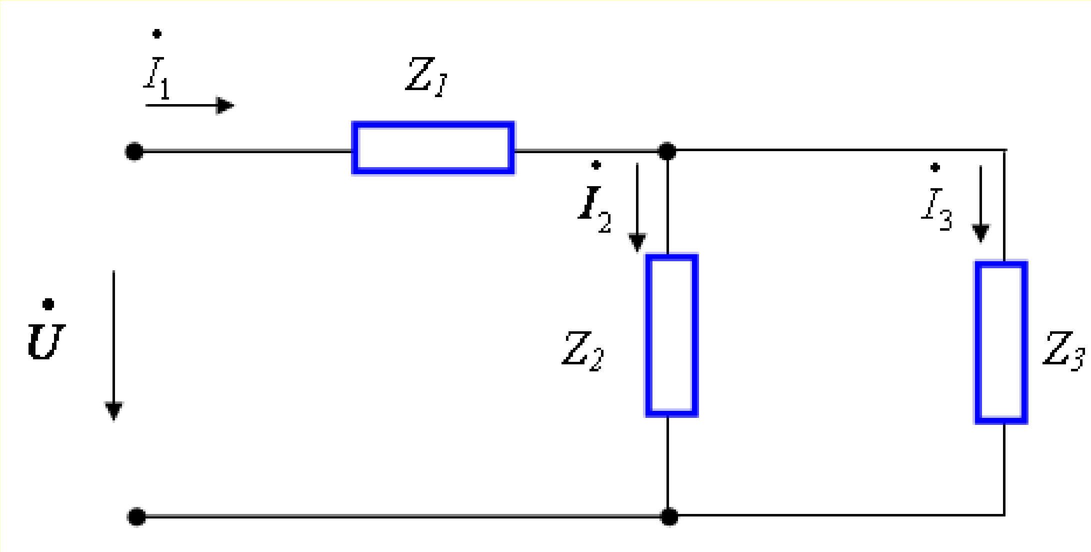
$$\dot{U} = Ue^{j\psi_u} = \frac{u_m}{\sqrt{2}}e^{j\psi_u} = \frac{141}{\sqrt{2}}e^{j90} =$$
$$100.[\cos(90) + j\sin(90)] = 100.(0 + j) = j100V$$

$$\omega = 2\pi f = 2\pi.160 \approx 1000 = 10^3 rad/s$$

$$Z_1 = R_1 + j\omega M = (10 + j.10^3 10.10^{-3}) = (10 + j10)\ \Omega$$

$$Z_2 = j\omega(L_1 - M) - j\frac{1}{\omega C} = j.10^3.(40-10).10^{-3} - j\frac{1}{10^3.100.10^{-6}} = j30 - j10 = j20\ \Omega$$

$$Z_3 = j\omega(L_2 - M) = j.10^3(30-10).10^{-3} = j20\ \Omega$$



$\dot{U} = j100V$

$Z_1 = (10 + j10)\ \Omega$

$Z_2 = j20\ \Omega$

$Z_3 = j20\ \Omega$

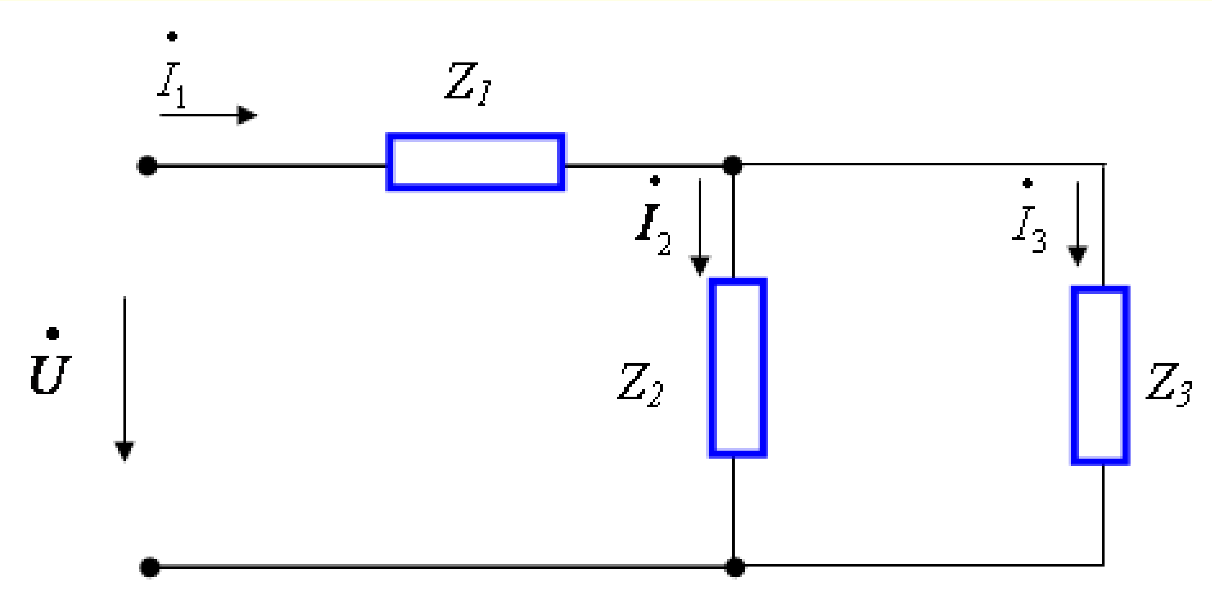
$$Z_{ekв} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$\Rightarrow \quad Z_{ekв} = 10 + j10 + \frac{\cancel{j}2\cancel{0}.j20}{\cancel{j}4\cancel{0}} = 10 + j10 + j10 = (10 + j20)\Omega$$

$$\dot{I}_1 = \frac{\dot{U}}{Z_{ek}} = \frac{j100}{10 + j20} = \frac{j10}{1 + j2} = \frac{j10(1 - j2)}{(1 + j2)(1 - j2)} =$$

$$\frac{j10(1 - j2)}{5} = \frac{20 + j10}{5} = (4 + 2j)A$$

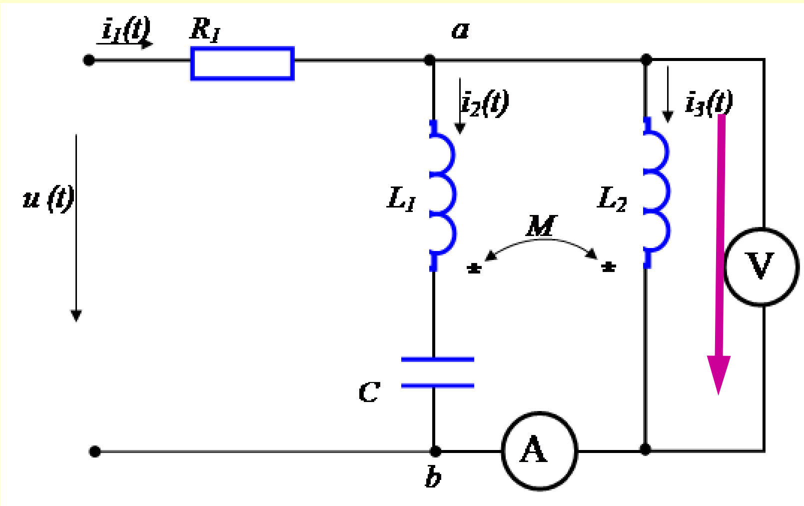
5. Определяме токовете $\dot{I}_2$ и $\dot{I}_3$ в двата паралелни клона

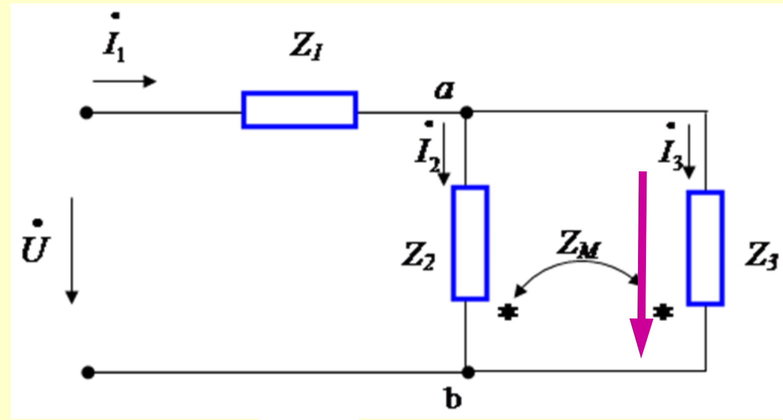


$$\dot{I}_2 = \dot{I}_1 \frac{Z_3}{Z_2 + Z_3} = (4 + 2j)\frac{j20}{j40} = (4 + 2j)\frac{1}{2} = (2 + j)A$$

$$\dot{I}_3 = \dot{I}_1 - \dot{I}_2 = 4 + 2j - 2 - j = (2 + j)A$$

6. Определяме напрежението на волтметъра





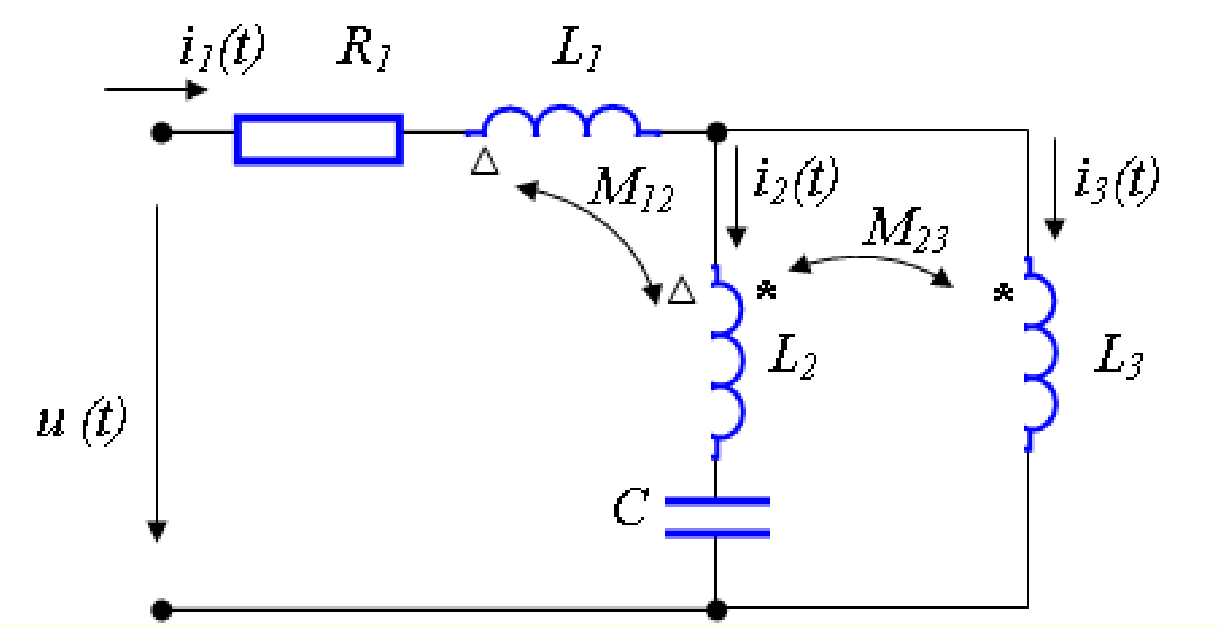
$$U_V = U_{ab}$$

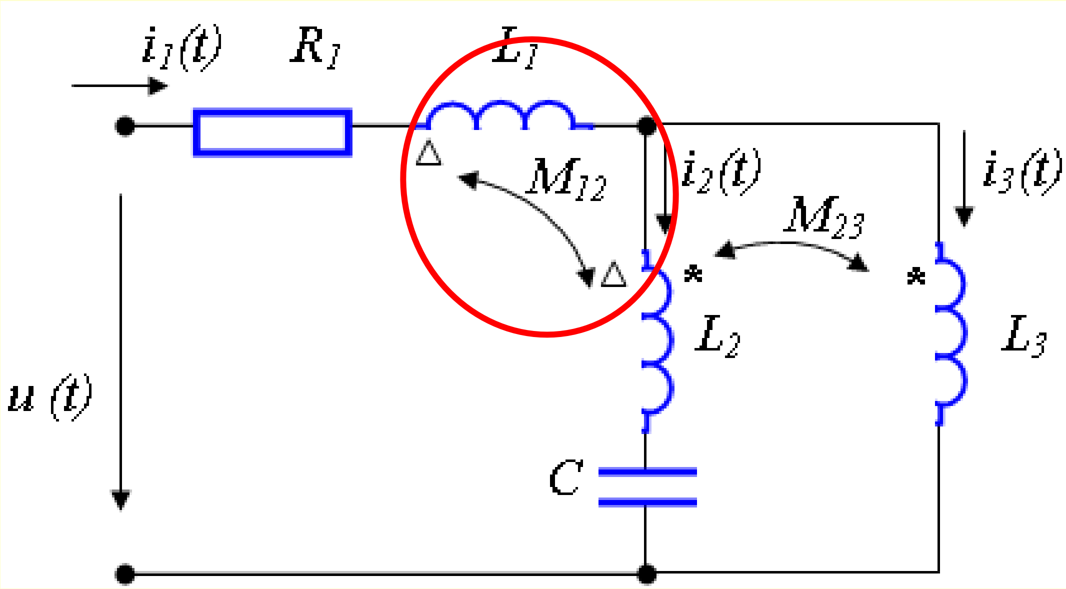
$$\dot{U}_{ab} = j\omega L_2.\dot{I}_3 + j\omega M.\dot{I}_2 = (2+j)j30 + (2+j)j10 = (-40+j80)V$$

$$\Rightarrow U_V = U_{ab} = \sqrt{(-40)^2 + 80^2} = 40\sqrt{5} = 89{,}44V$$

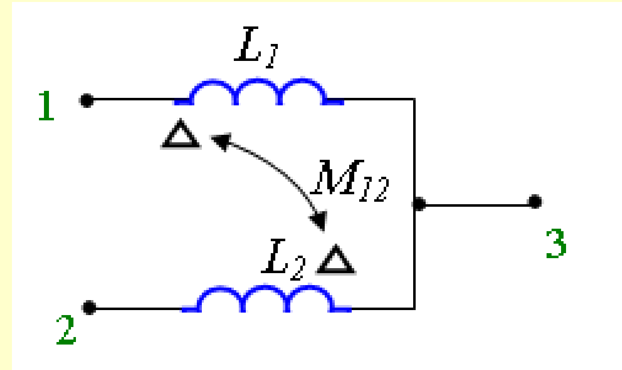
Пример 2: за **отстраняване на индуктивни връзки:**

Да се преобразува веригата до еквивалентна по отношение на токовете посредством отстраняване на индуктивните възки.

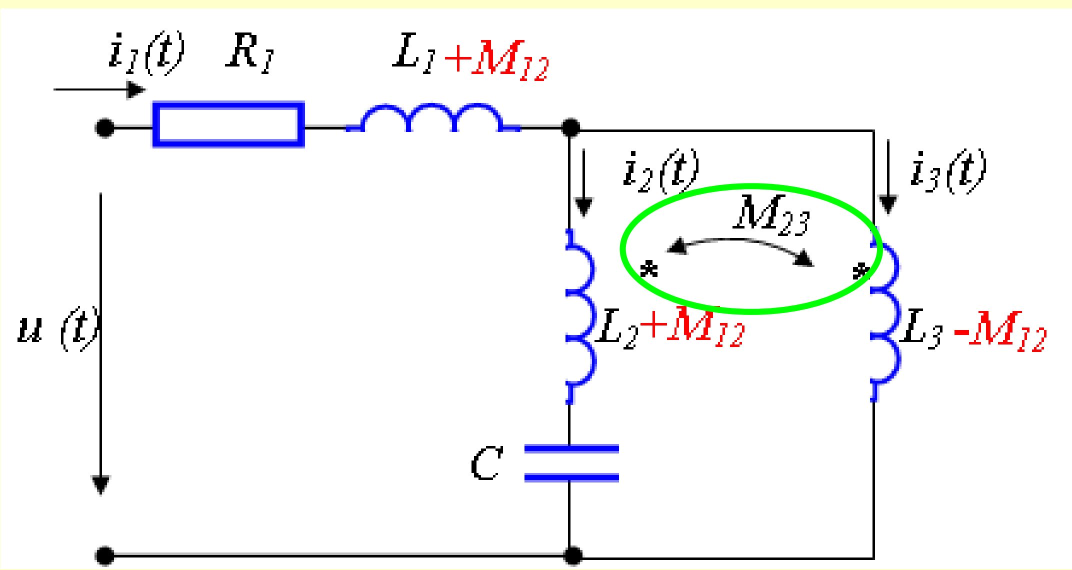




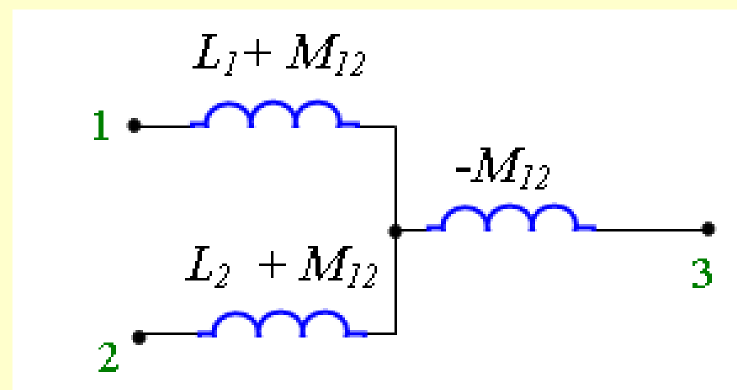
$i_1(t)$
R_1
L_1
M_{12}
$i_2(t)$
$i_3(t)$
M_{23}
*
*
L_2
L_3
$u\ (t)$
C



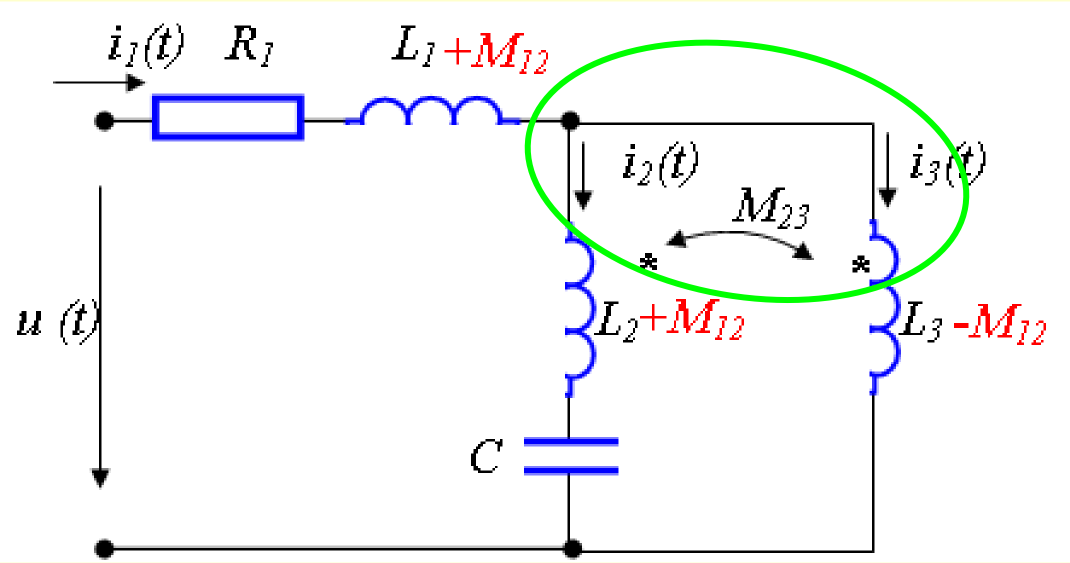
L_1
1
M_{12}
3
L_2
2



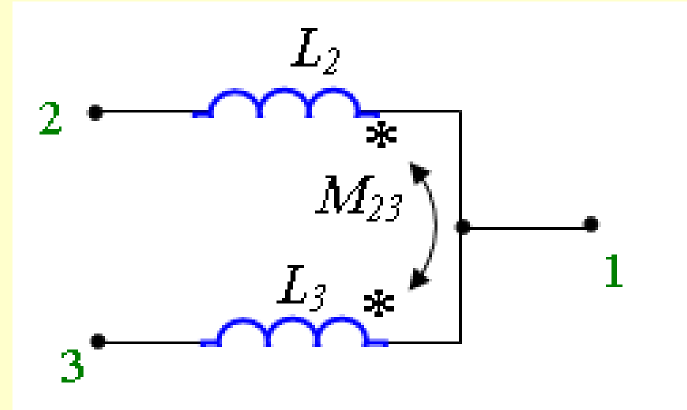
$i_1(t)$
R_1
L_1+M_{12}
$i_2(t)$
$i_3(t)$
M_{23}
*
*
$u\ (t)$
L_2+M_{12}
L_3-M_{12}
C



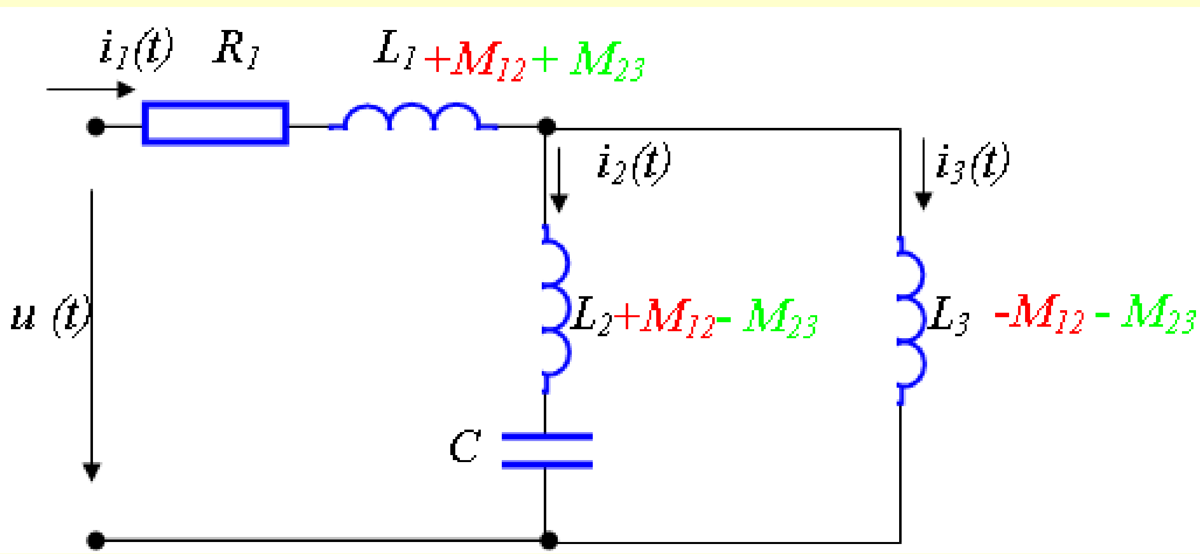
L_1+M_{12}
1
$-M_{12}$
L_2+M_{12}
3
2



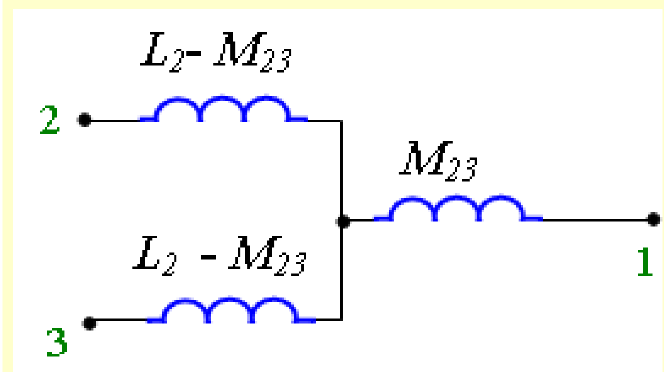
$i_1(t)$
R_1
L_1+M_{12}
$i_2(t)$
$i_3(t)$
M_{23}
*
*
$u\ (t)$
L_2+M_{12}
$L_3\ -M_{12}$
C



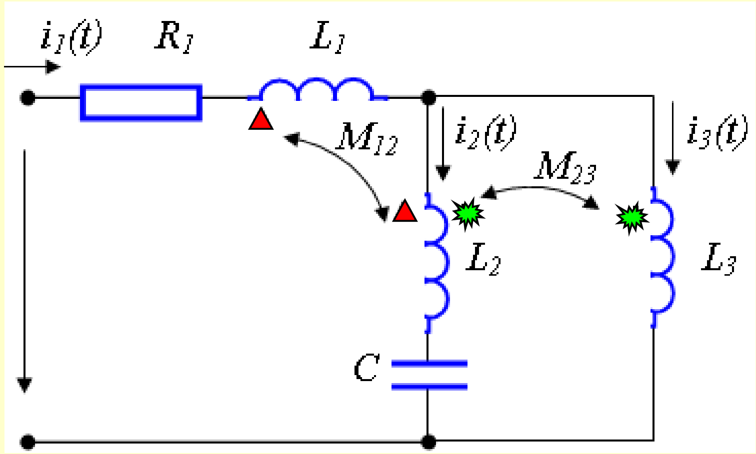
L_2
2
*
M_{23}
1
L_3
*
3

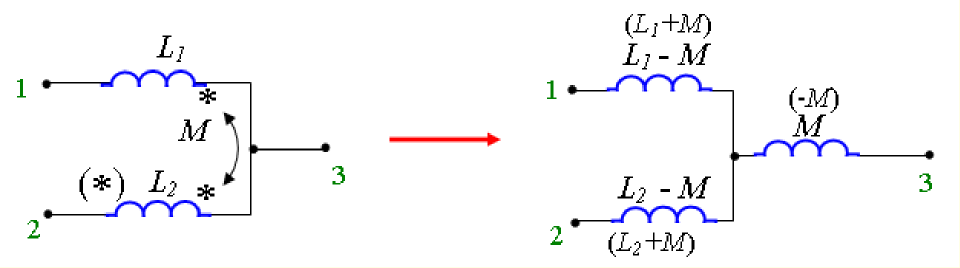


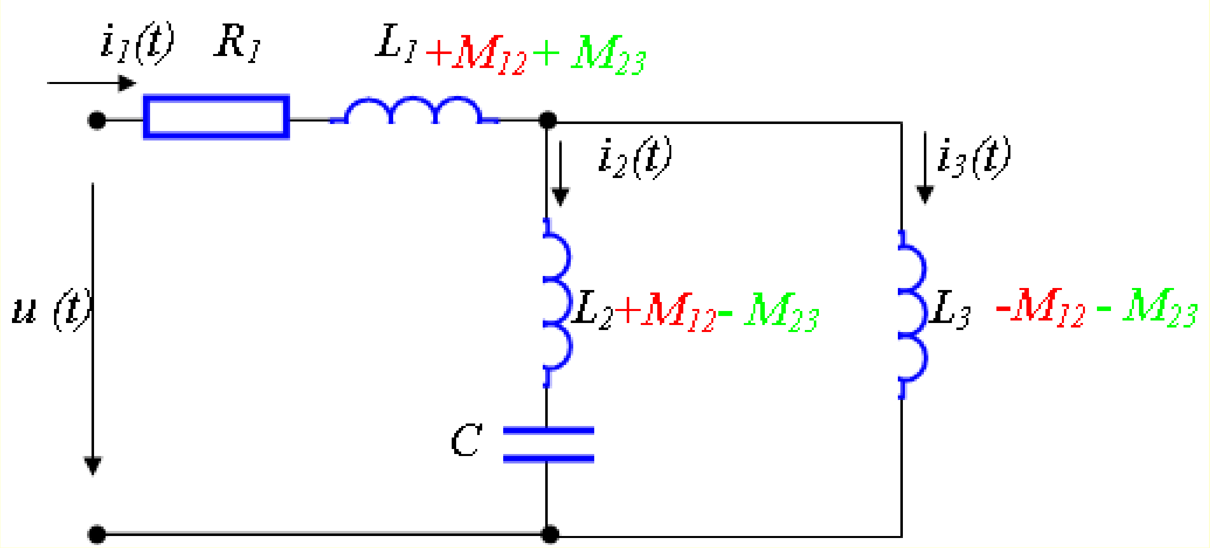
$i_1(t)$
R_1
$L_1+M_{12}+M_{23}$
$i_2(t)$
$i_3(t)$
$u\ (t)$
$L_2+M_{12}-M_{23}$
$L_3\ -M_{12}-M_{23}$
C



L_2-M_{23}
2
M_{23}
$L_2\ -M_{23}$
1
3

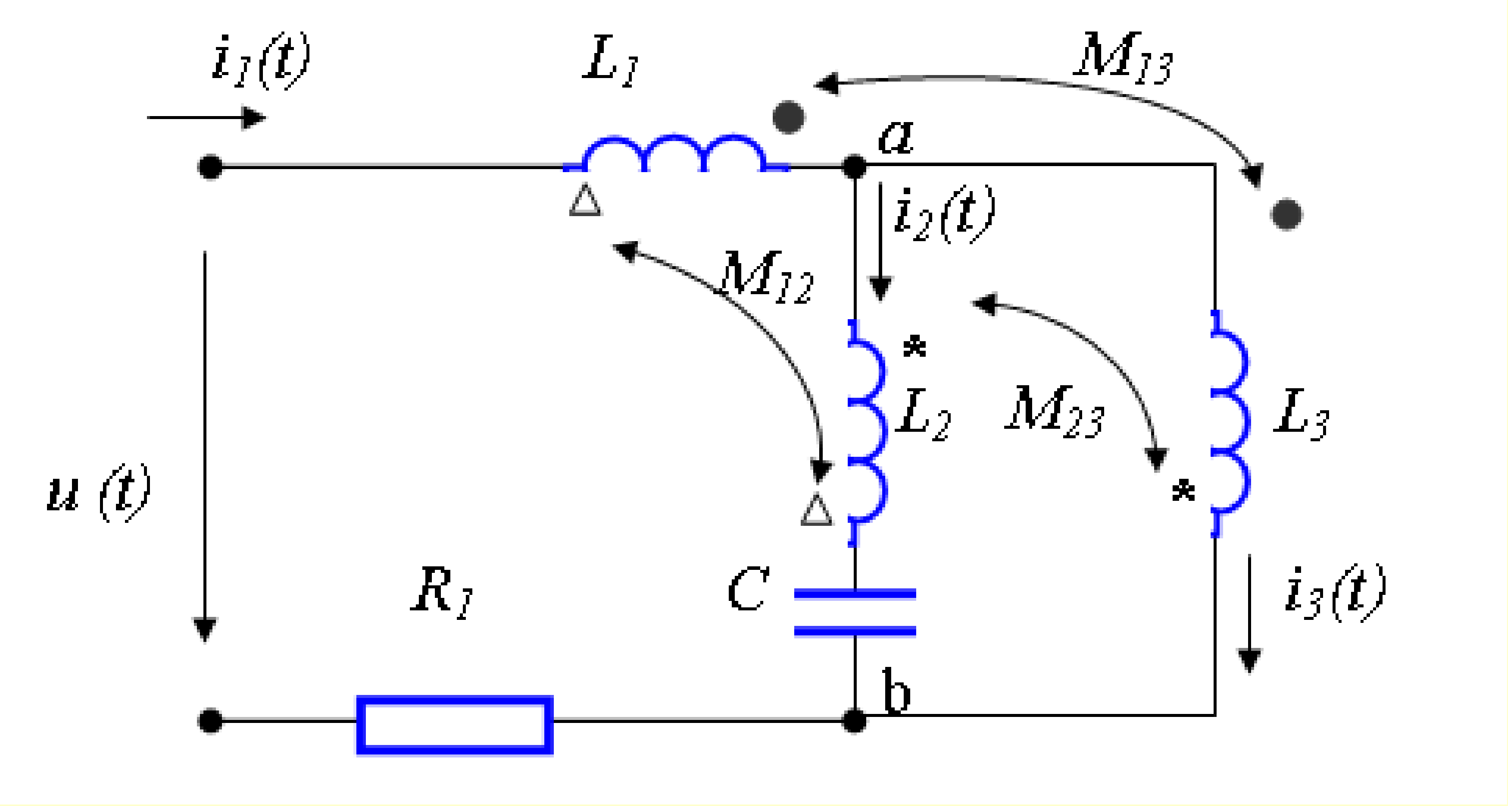

$i_1(t)$
R_1
L_1
M_{12}
$i_2(t)$
M_{23}
$i_3(t)$
L_2
L_3
C

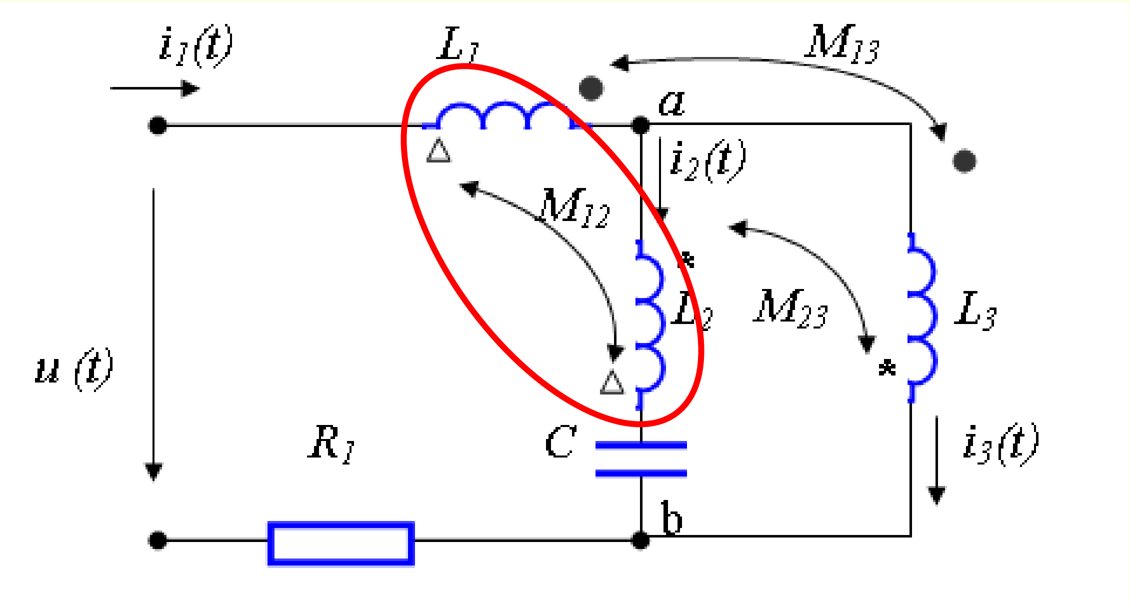

L_1
1
*
M
3
(*) L_2 *
2
(L_1+M)
$L_1 - M$
1
(-M)
M
3
$L_2 - M$
2
(L_2+M)


$i_1(t)$
R_1
$L_1 + M_{12} + M_{23}$
$i_2(t)$
$i_3(t)$
$u(t)$
$L_2 + M_{12} - M_{23}$
$L_3 \; -M_{12} - M_{23}$
C

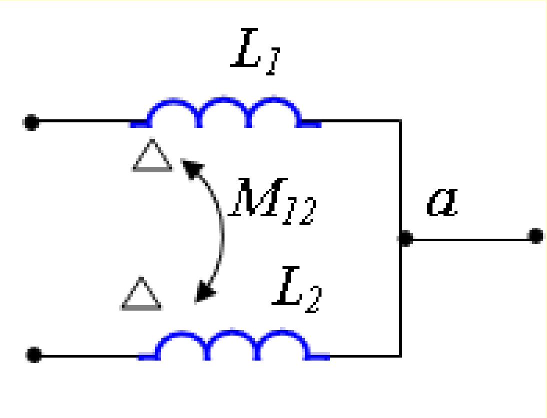
Пример 3 : **Отстраняване на индуктивни връзки:**

Да се преобразува веригата до еквивалентна по отношение на токовете посредством отстраняване на индуктивните възки.

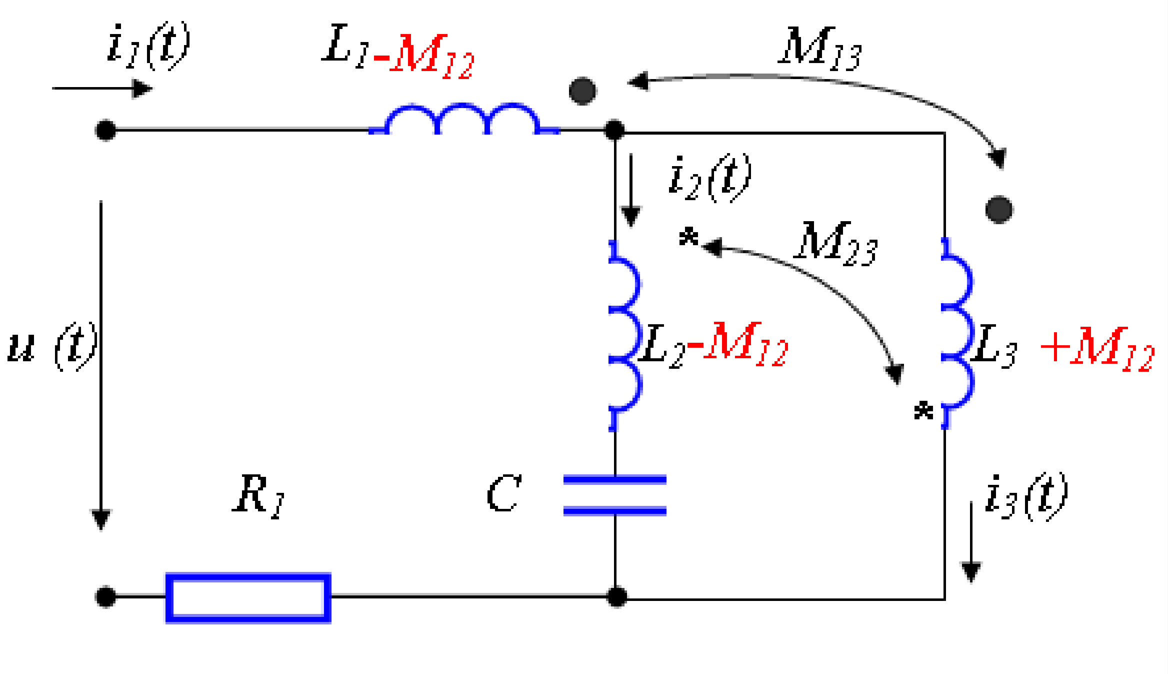




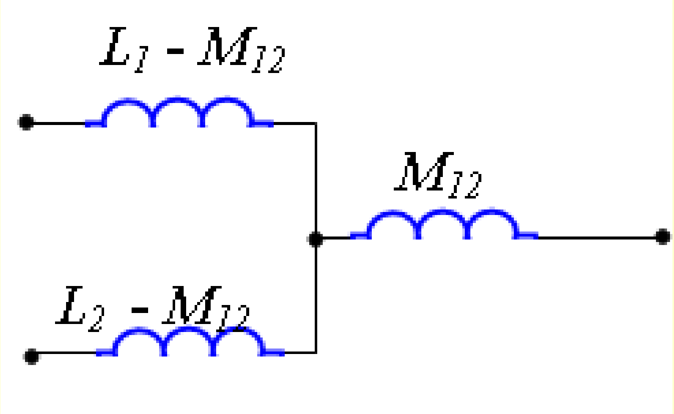
i1(t)
L1
M13
a
i2(t)
M12
L2
M23
L3
u (t)
R1
C
i3(t)
b



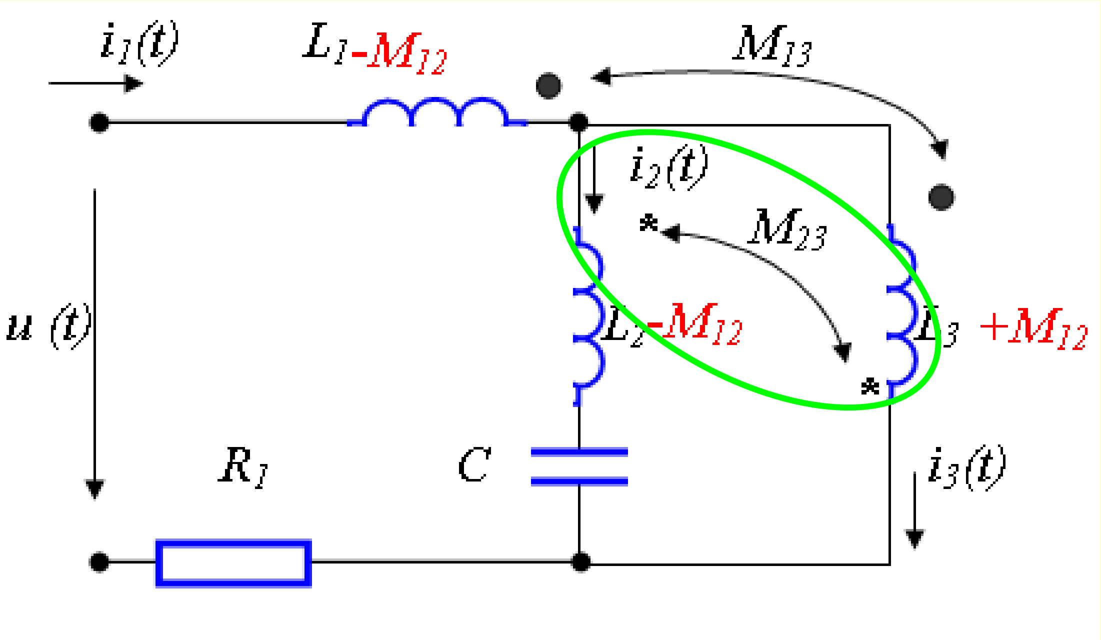
L1
M12
a
L2



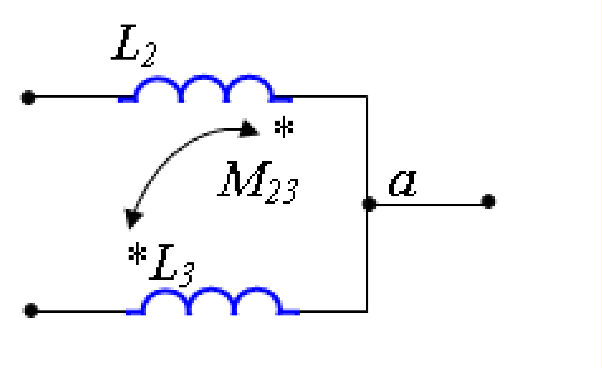
i1(t)
L1-M12
M13
i2(t)
M23
u (t)
L2-M12
L3 +M12
R1
C
i3(t)



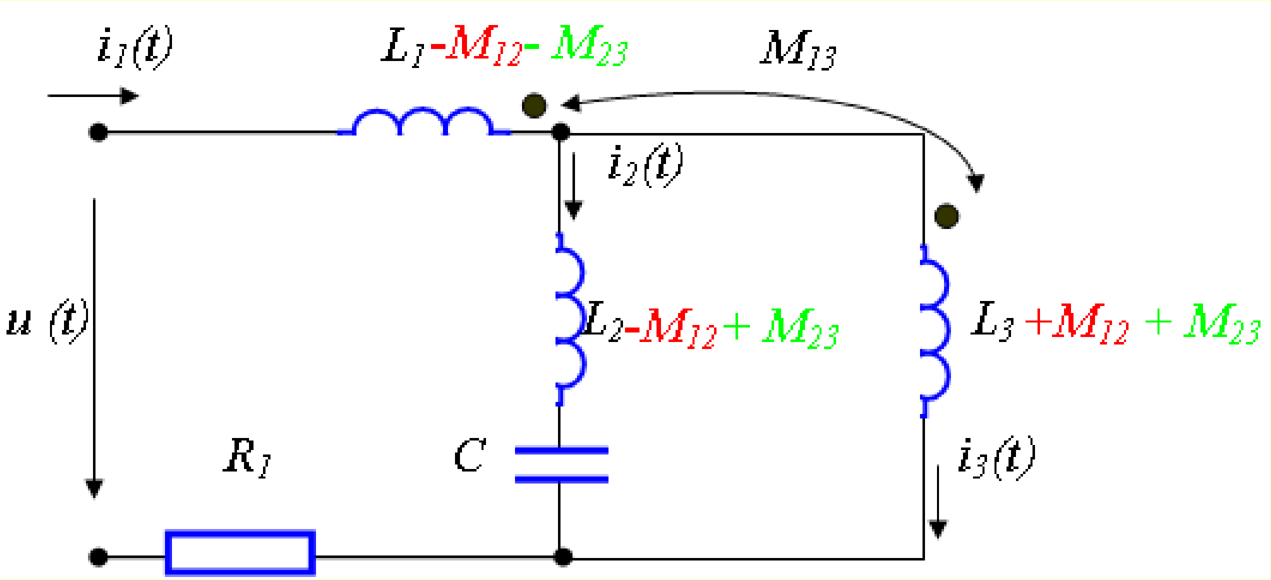
L1 - M12
M12
L2 - M12



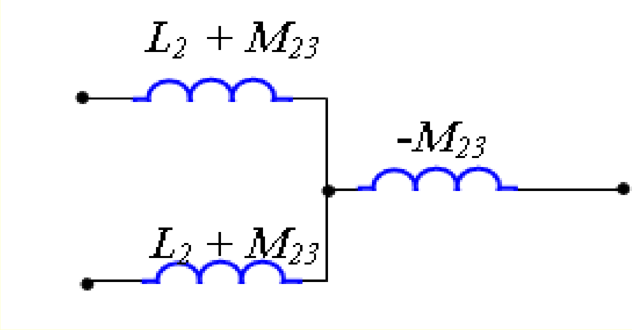
$i_1(t)$
L_1-M_{12}
M_{13}
$i_2(t)$
M_{23}
$u\ (t)$
L_2-M_{12}
L_3 +M_{12}
R_1
C
$i_3(t)$



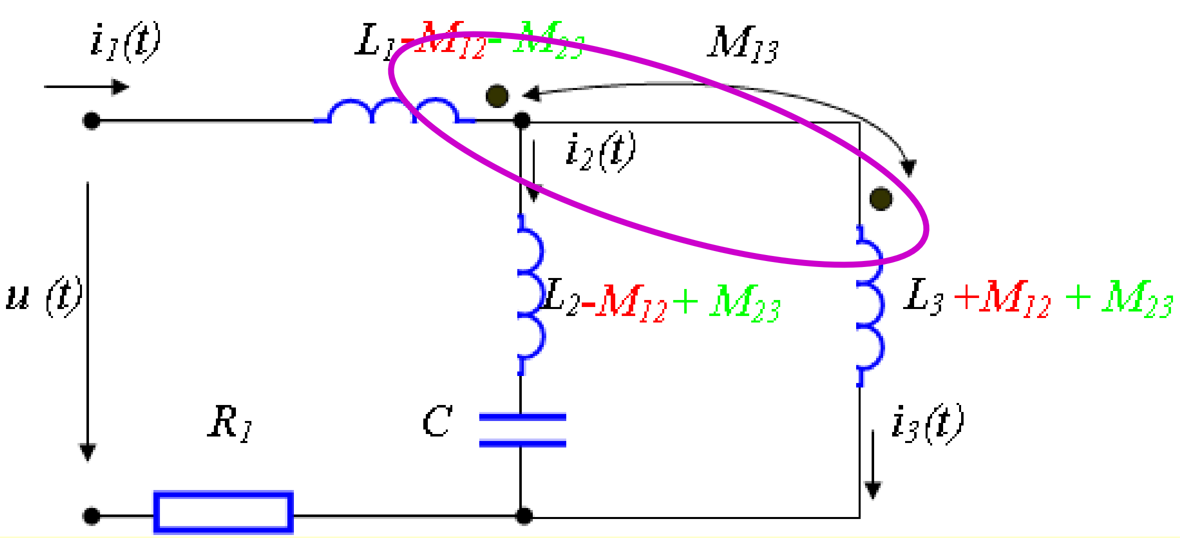
L_2
M_{23}
a
L_3

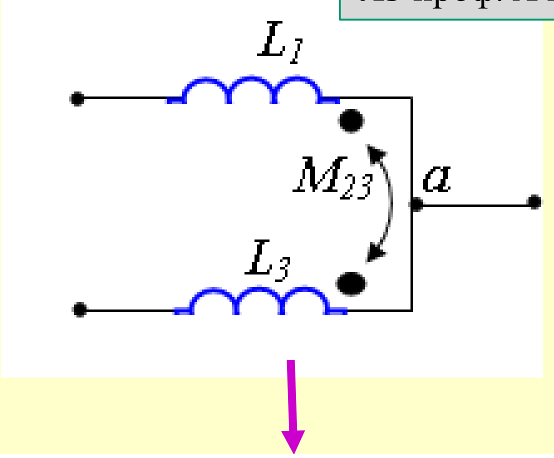


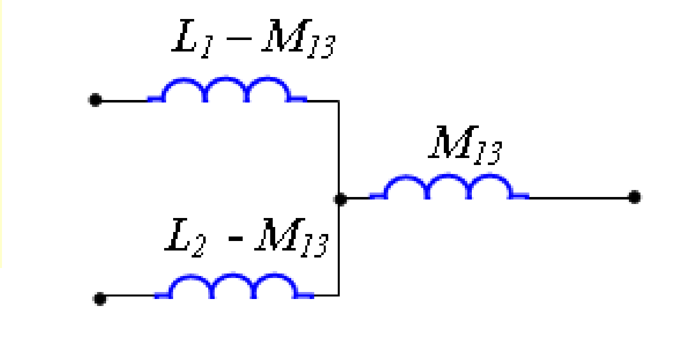
$i_1(t)$
L_1-M_{12}- M_{23}
M_{13}
$i_2(t)$
$u\ (t)$
L_2-M_{12}+ M_{23}
L_3 +M_{12} + M_{23}
R_1
C
$i_3(t)$

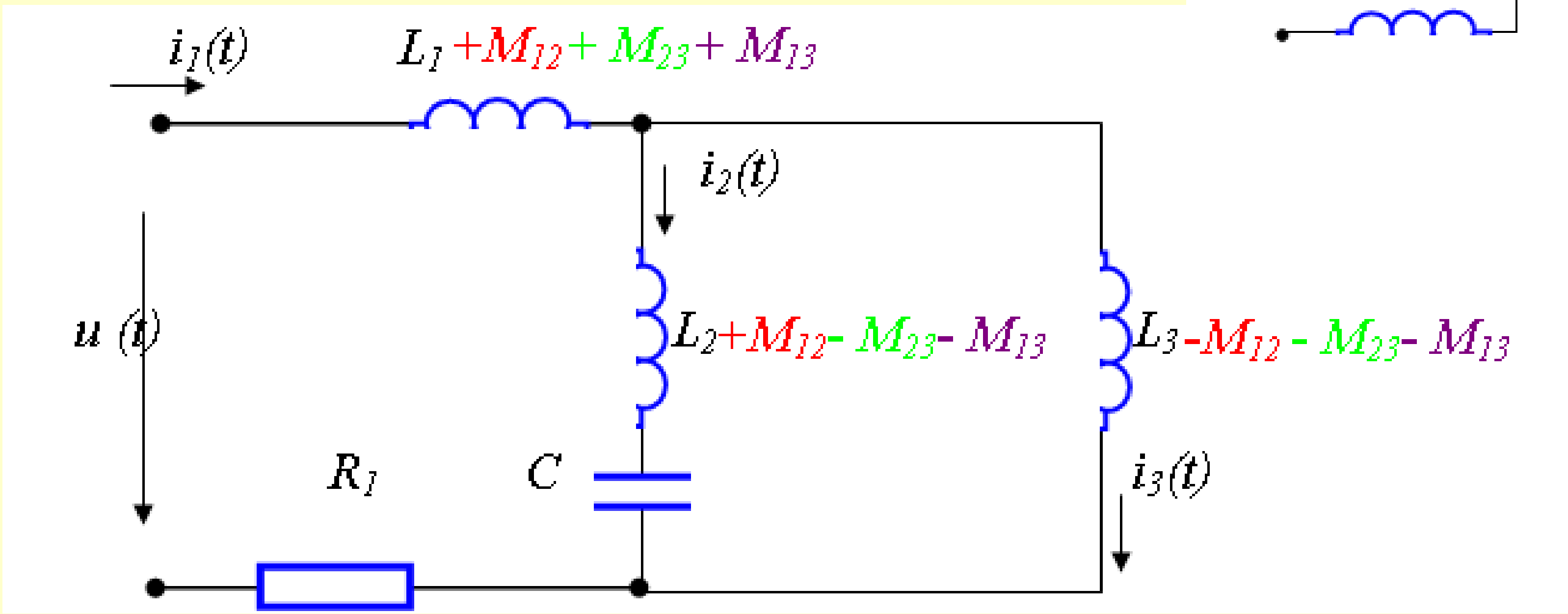


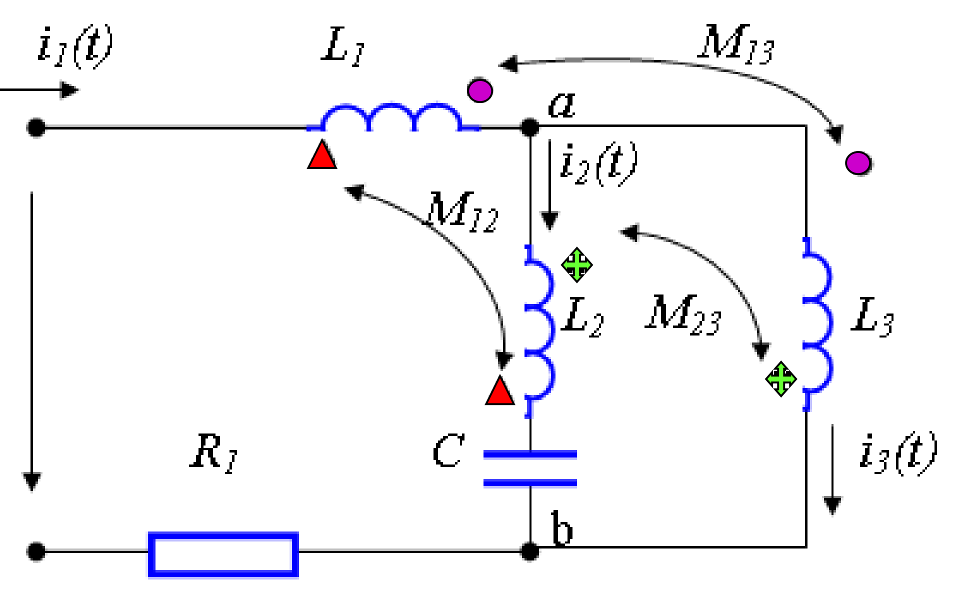
L_2 + M_{23}
-M_{23}
L_2 + M_{23}

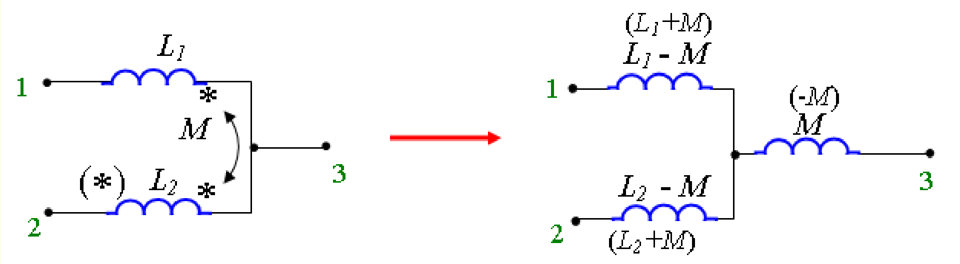

i1(t)
L1-M12- M23
M13
i2(t)
u (t)
L2-M12+ M23
L3 +M12 + M23
R1
C
i3(t)

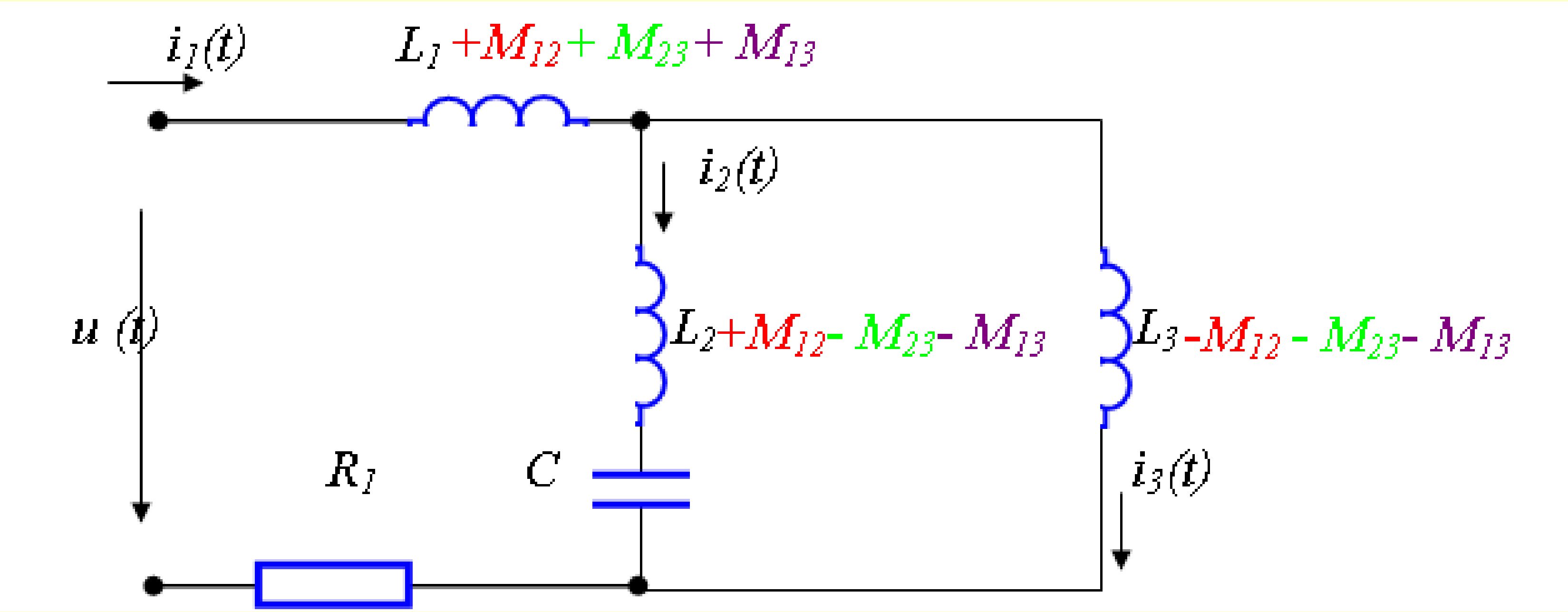

L1
M23
a
L3


L1 – M13
M13
L2 - M13

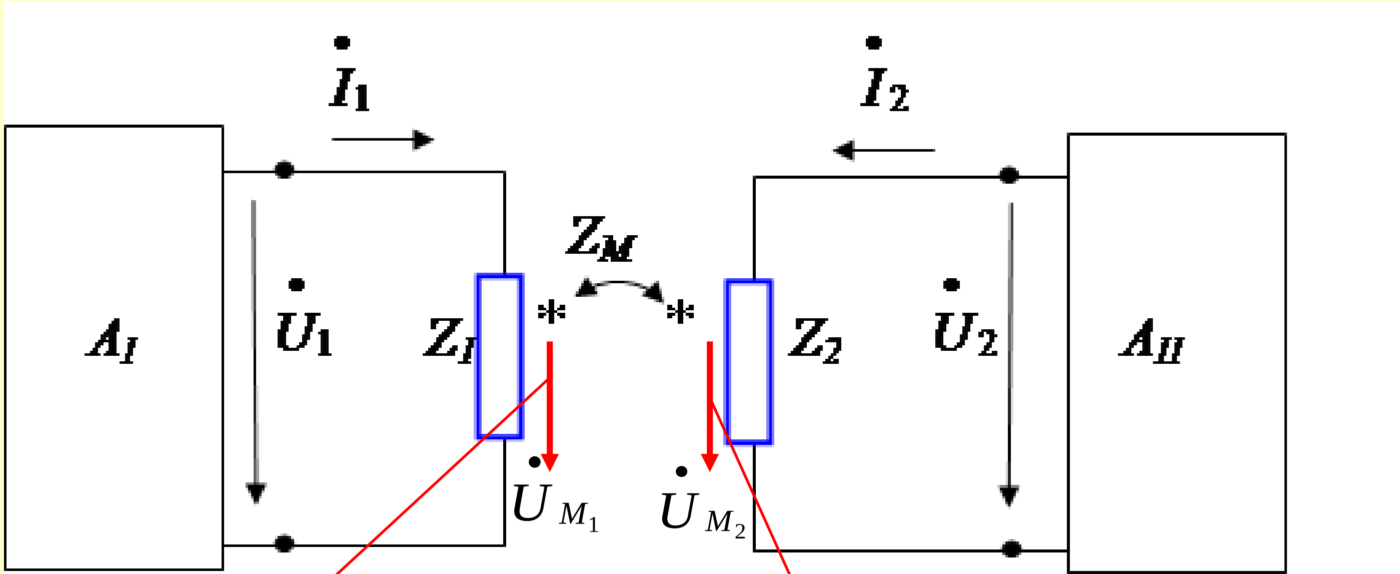

i1(t)
L1 +M12+ M23+ M13
i2(t)
u (t)
L2+M12- M23- M13
L3 -M12 - M23- M13
R1
C
i3(t)


i1(t)
L1
M13
a
i2(t)
M12
L2
M23
L3
R1
C
b
i3(t)


L1
1
*
M
3
(*) L2
*
2
(L1+M)
L1 - M
1
(-M)
M
3
L2 - M
2
(L2+M)


i1(t)
L1 +M12+ M23+ M13
i2(t)
u (t)
L2+M12- M23- M13
L3 -M12 - M23- M13
R1
C
i3(t)

Предаване на мощност по индуктивен път



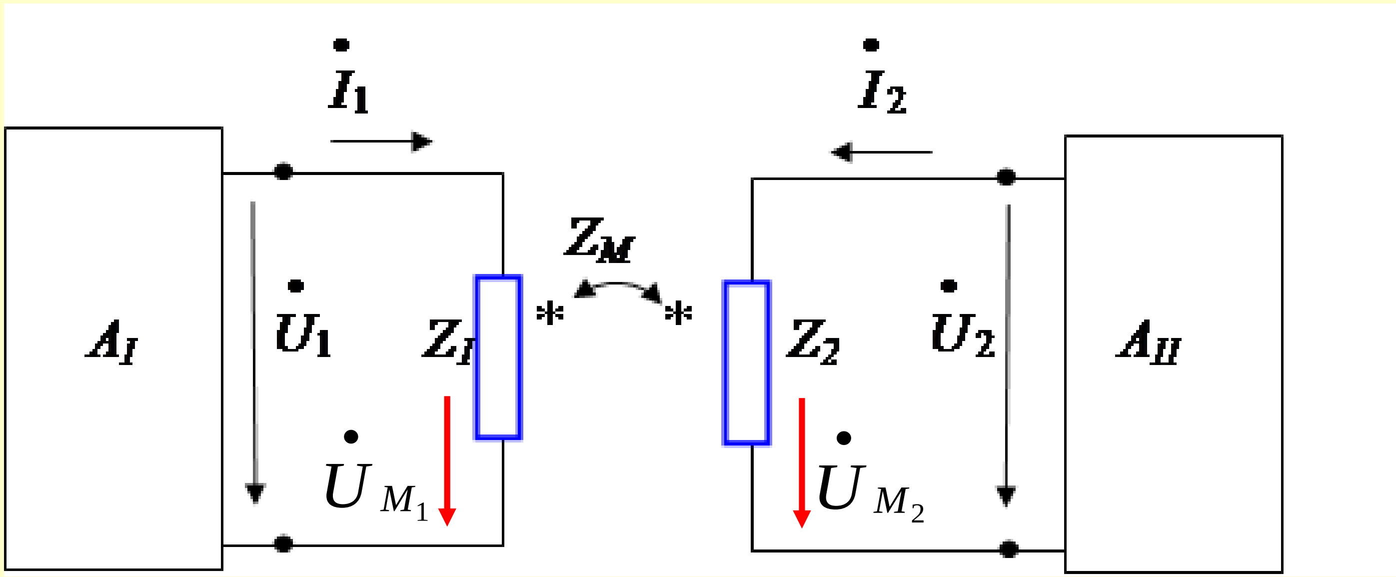
$$\dot I_1 = I_1 e^{j\psi_1}$$

$$\dot U_{M_1} = \dot I_2 Z_M = \dot I_2\, j\omega M$$

$$\dot I_2 = I_1 e^{j\psi_2}$$

$$\dot U_{M_2} = \dot I_1 Z_M = \dot I_1\, j\omega M$$

Предаване на мощност по индуктивен път



$$\dot{I}_1 = I_1 e^{j\psi_1} \rightarrow \boxed{\overset{*}{I}_1 = I_1 e^{-j\psi_1}}$$

$$\dot{U}_{M_1} = \dot{I}_2 Z_M = \dot{I}_2\, j\omega M$$

$$\dot{I}_2 = I_1 e^{j\psi_2} \rightarrow \boxed{\overset{*}{I}_2 = I_1 e^{-j\psi_2}}$$

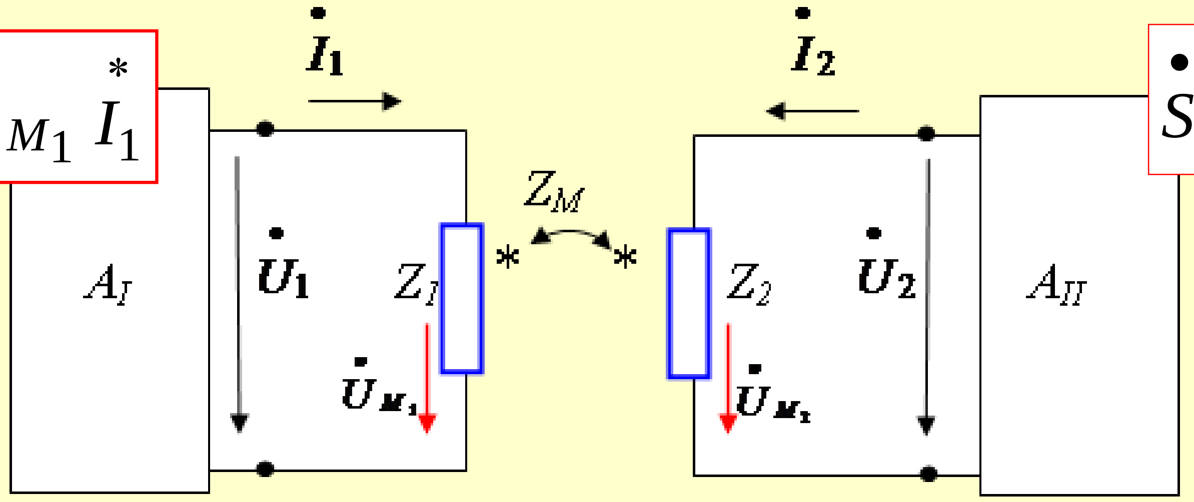
$$\dot{U}_{M_2} = \dot{I}_1 Z_M = \dot{I}_1\, j\omega M$$

$$\boxed{\dot{S}_{M_1} = \dot{U}_{M_1} \overset{*}{I}_1}$$

$$\boxed{\dot{S}_{M_2} = \dot{U}_{M_2} \overset{*}{I}_2}$$

Предаване на мощност по индуктивен път

$$\dot{S}_{M_1} = \dot{U}_{M_1}\overset{*}{I}_1$$



$$\dot{S}_{M_2} = \dot{U}_{M_2}\overset{*}{I}_2$$

$$\dot{S}_{M_1} = P_{M_1} + jQ_{M_1}$$

$$P_{M_1} = \mathrm{Re}[\dot{S}_{M_1}]$$

$$Q_{M_1} = \mathrm{Im}[\dot{S}_{M_1}]$$

$$\dot{S}_{M_2} = P_{M_2} + jQ_{M_2}$$

$$P_{M_2} = \mathrm{Re}[\dot{S}_{M_2}]$$

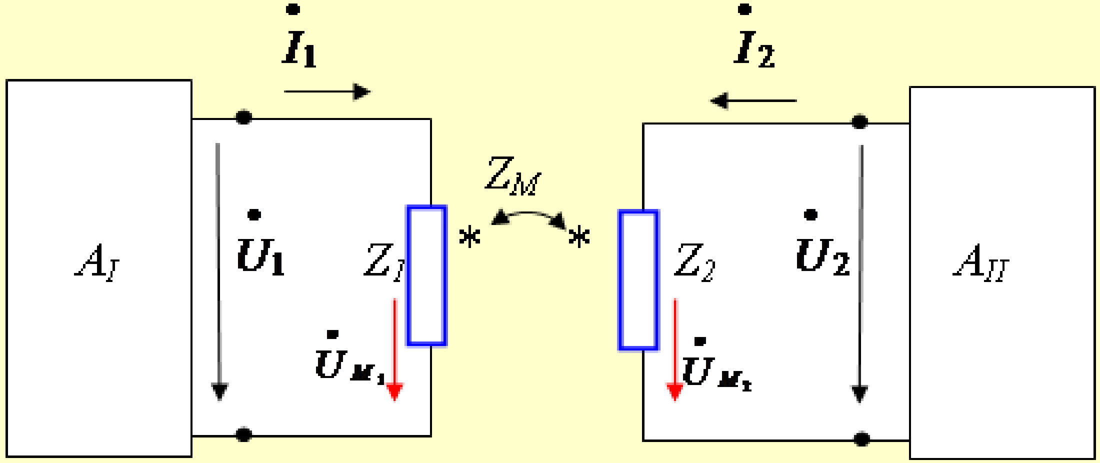
$$Q_{M_2} = \mathrm{Im}[\dot{S}_{M_2}]$$

$$P_{M_1} = -P_{M_2}$$

$$Q_{M_1} = Q_{M_2}$$

Предаване на активна мощност по индуктивен път

$P_{M1} = \mathrm{Re}[\dot{S}_{M_1}]$



$P_{M_2} = \mathrm{Re}[\dot{S}_{M_2}]$

*При **еднакво** ориентирани спрямо едноименните изводи токове*

Ако $P_{M1} = \mathrm{Re}[\dot{S}_{M_1}] \rangle\, 0$ то клон 1 прехвърля енергия към клон 2,

Ако $P_{M1} = \mathrm{Re}[\dot{S}_{M_1}] \langle\, 0$ то клон 1 приема енергия от клон 2.

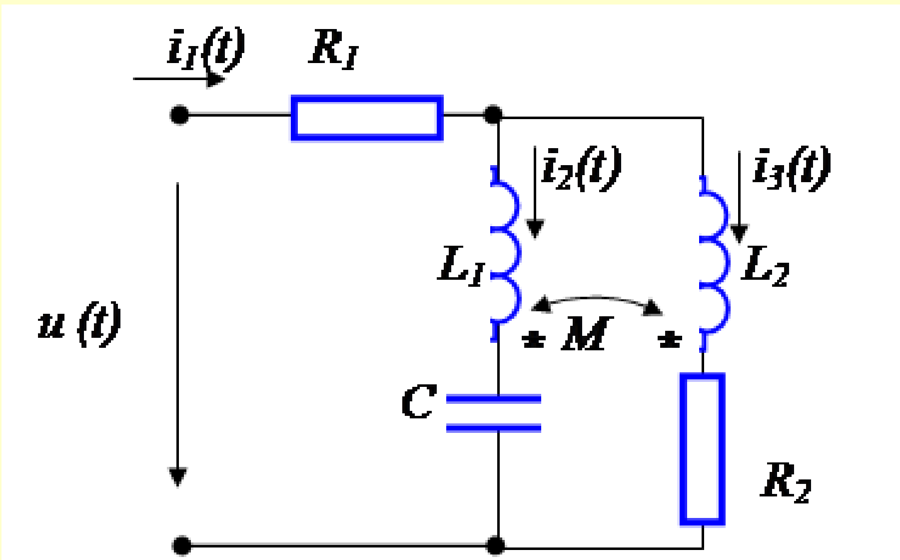
*При **различно** ориентирани спрямо едноименните изводи токове*

Ако $P_{M1} = \mathrm{Re}[\dot{S}_{M_1}] \rangle\, 0$ то клон 1 приема енергия от клон 2,

Ако $P_{M1} = \mathrm{Re}[\dot{S}_{M_1}] \langle\, 0$ то клон 1 прехвърля енергия към клон 2.

Пример:
Определяне на мощност, предавана по индуктивен път

Да се определи мощността, предавана по индуктивен път за веригата:



$u(t) = 200\sin(\omega t - 45) V$

$f = 160 Hz$,

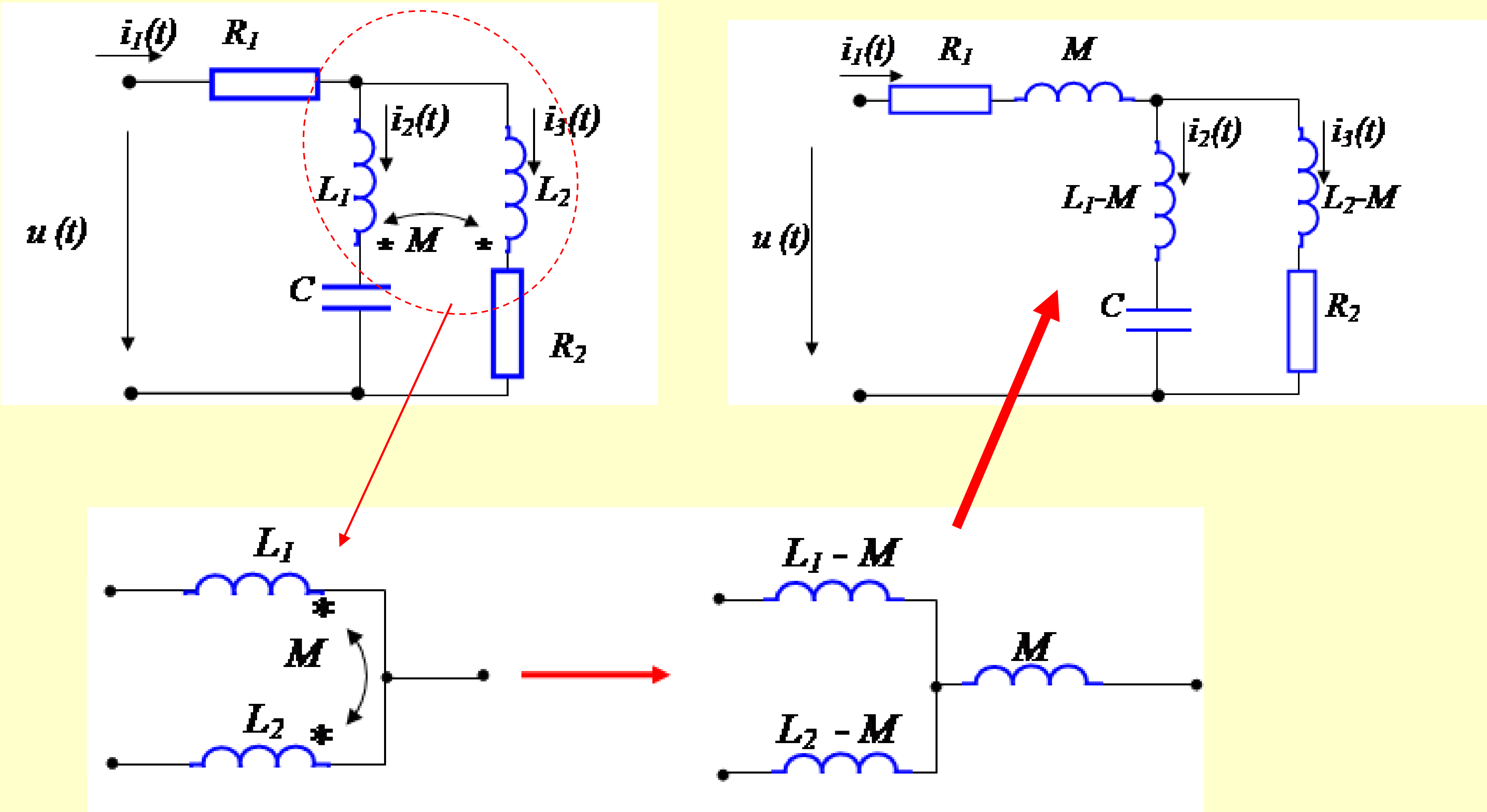
$L_2 = 20\ mH$,

$L_1 = M = 10\ mH$,

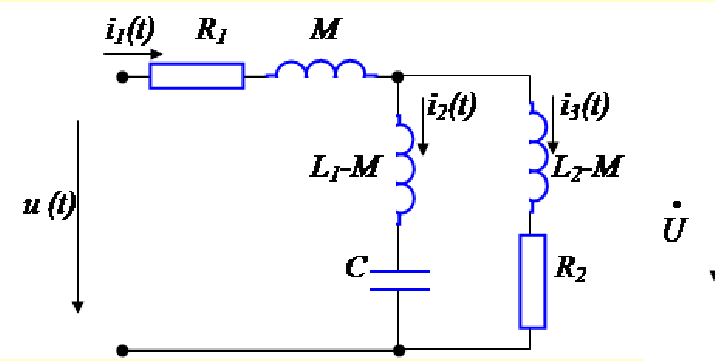
$C = 50\ \mu F$

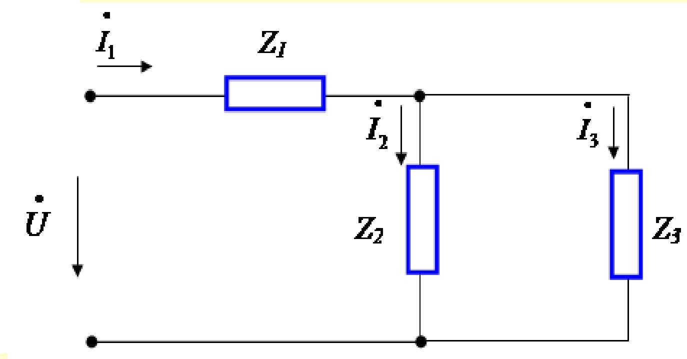
$R_1 = R_2 = 10 \Omega$

Определяне на мощност, предавана по индуктивен път



Определяне на мощност, предавана по индуктивен път





$u(t) = 200\sin(\omega t - 45)V$

$f=160Hz$,

$L_2=20\ mH$,

$L_1= M=10\ mH$,

$C=50\ \mu F$

$R_1 = R_2=10\Omega$

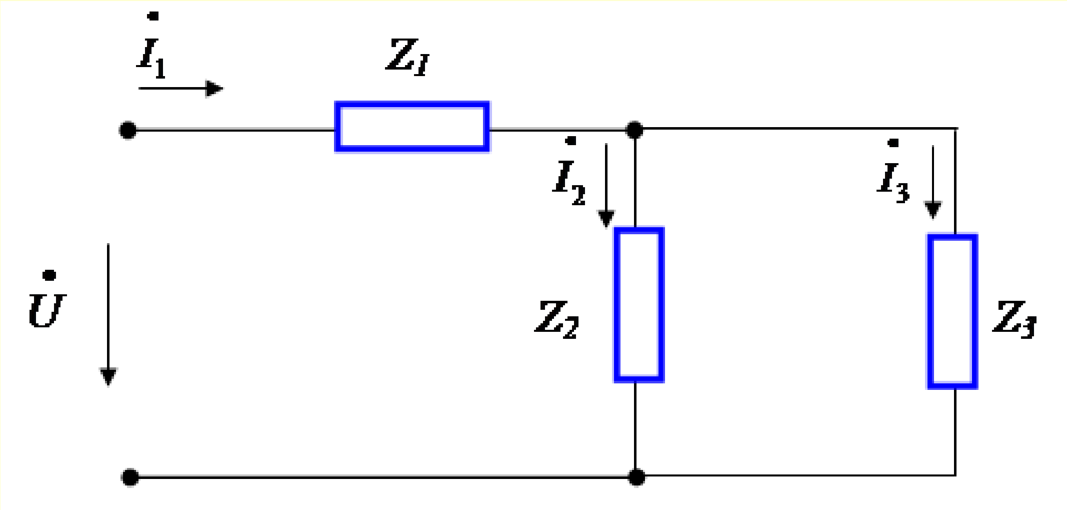
$$\omega = 2\pi f = 2\pi.160 \approx 1000 = 10^3\, rad/s$$

$$Z_1 = R_1 + j\omega M = (10 + j.10^3 10.10^{-3}) = (10 + j10)\ \Omega$$

$$Z_2 = j\omega(L_1 - M) - j\frac{1}{\omega C} = j.10^3.(10-10).10^{-3} - j\frac{1}{10^3.50.10^{-6}} = 0 - j20 = -j20\ \Omega$$

$$Z_3 = R_2 + j\omega(L_2 - M) = 10 + j.10^3(20-10).10^{-3} = 10 + j10\ \Omega$$

Определяне на мощност, предавана по индуктивен път

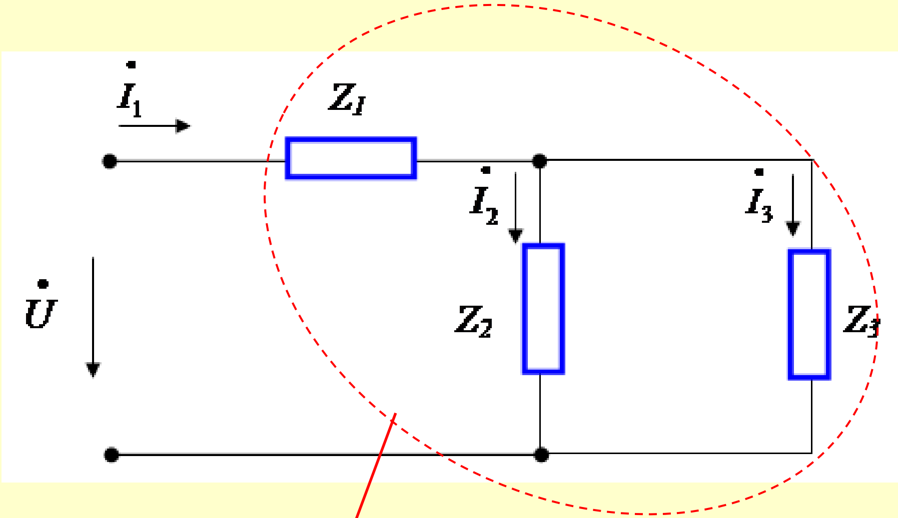


$u(t) = 200\sin(\omega t+45)V$
$f=160Hz,$
$L_2=20\ mH,$
$L_1= M=10\ mH,$
$C=50\ \mu F$
$R_1 = R_2=10\Omega$

$$\dot{U} = Ue^{j\psi_u} = \frac{u_m}{\sqrt{2}}e^{j\psi_u} = \frac{200}{\sqrt{2}}e^{j45} =$$

$$\frac{200}{\sqrt{2}}.(\cos 45 + j\sin 45) = \frac{200}{\sqrt{2}}(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}) = 100.(1+j) = (100+j100)V$$

Определяне на мощност, предавана по индуктивен път

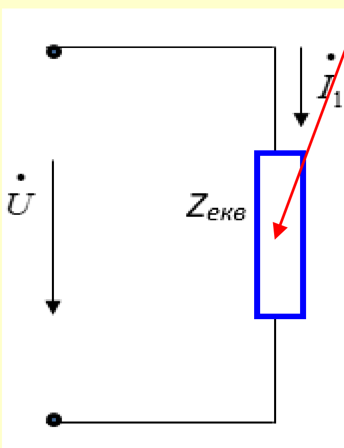


$$\dot{U} = (100 + j100)V$$

$$Z_1 = (10 + j10)\,\Omega$$

$$Z_2 - j20\,\Omega$$

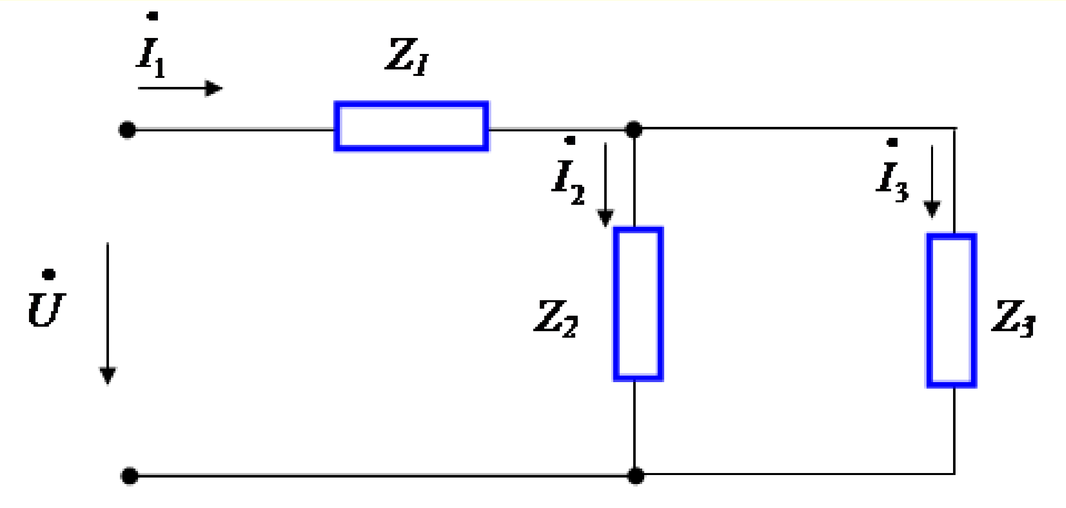
$$Z_3 = 10 + j10\,\Omega$$



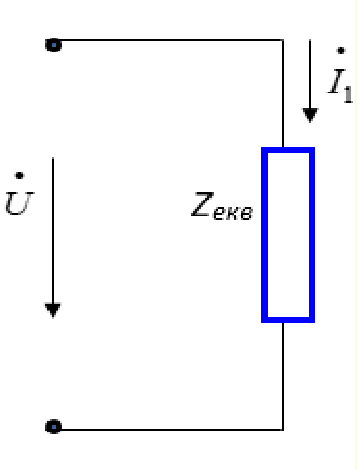
$$Z_{екв} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$\Rightarrow \quad Z_{екв} = 10 + j10 + \frac{-j20.(10 + j10)}{10 - j10} = 10 + j10 + \frac{20(-j).(1 + j1)}{1 - j1} =$$

$$= 10 + j10 + \frac{20.(1 - j1)}{1 - j1} = 10 + j10 + 20 = (30 + j10)\Omega$$



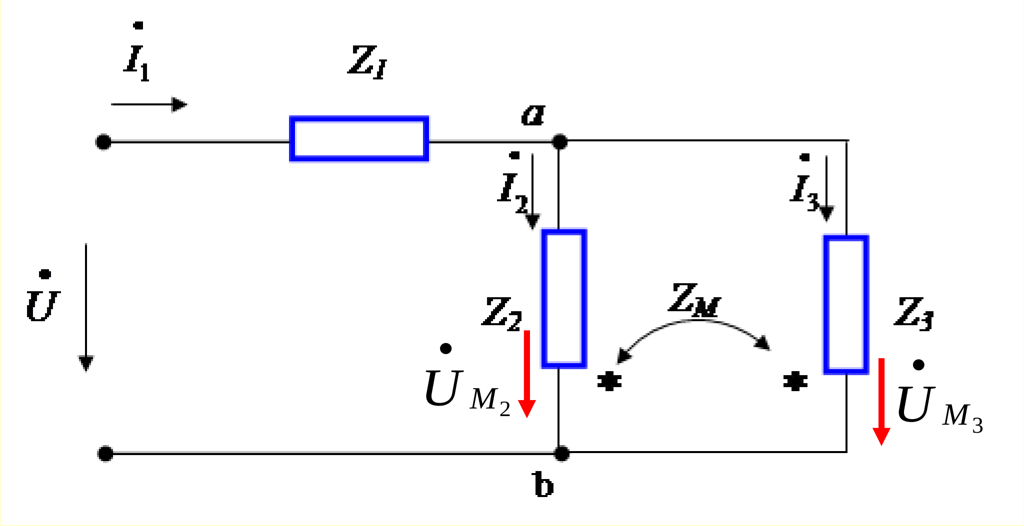
$$\dot{I}_1 = \frac{\dot{U}}{Z_{ek}} = \frac{100 + j100}{30 + j10} = \frac{10 + j10}{3 + j} = \frac{10(1 + j)(3 - j)}{(3 + j)(3 - j)} = \frac{10(3 + 3j - j + 1)}{10} = (4 + 2j)A$$



$$\dot{I}_3 = \dot{I}_1 \frac{Z_2}{Z_2 + Z_3} = (4 + 2j)\frac{-j20}{10 - j10} = \frac{-j2(4 + 2j)}{1 - j} = \frac{2(2 - 4j)}{1 - j} = \frac{2(2 - 4j)(1 + j)}{(1 - j)(1 + j)} = \frac{2(2 - 4j)(1 + j)}{2} = (6 - 2j)A$$

$$\dot{I}_2 = \dot{I}_1 - \dot{I}_3 = 4 + 2j - 6 + 2j = (-2 + 4j)A$$

Определяне на мощност, предавана по индуктивен път



$$P_{M2} = 200W \rangle\ 0$$
$$P_{M3} = -200W \langle\ 0$$

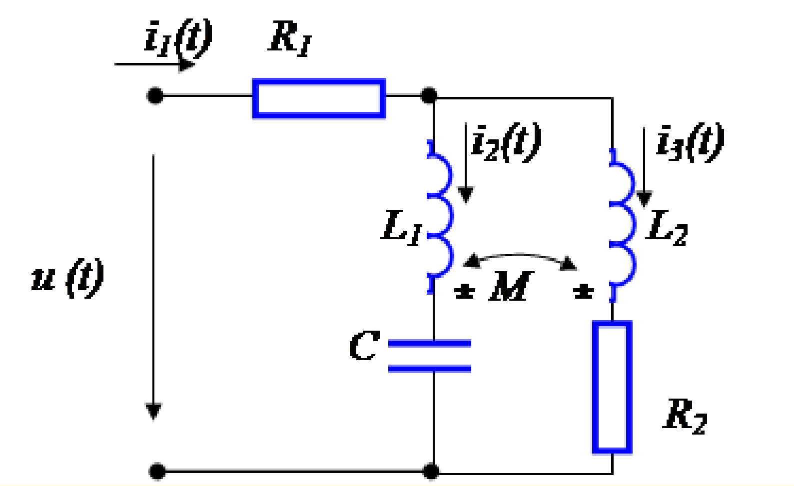
$$\dot{I}_1 = (4+2j)A \qquad \dot{I}_2 = (-2+4j)A \qquad \dot{I}_3 = (6-2j)A$$

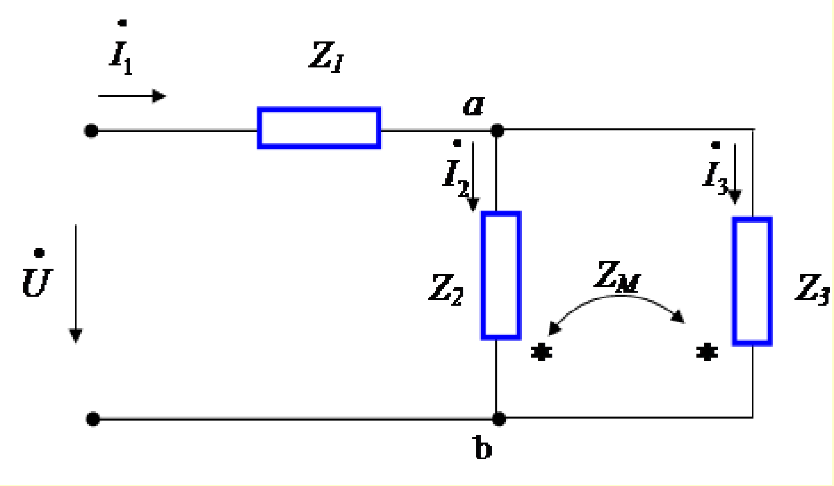
$$P_{M2} = \mathrm{Re}[\dot{U}_{M_2}\overset{*}{I}_2] = \mathrm{Re}[\dot{I}_3\, j\omega M.\overset{*}{I}_2] = \mathrm{Re}[(6-j2)\,j10.(-2-j4)] =$$
$$= \mathrm{Re}[(20+j60)(-2-j4)] = 20.(-2)+4.60 = -40+240 = 200W$$

$$P_{M3} = \mathrm{Re}[\dot{U}_{M_3}\overset{*}{I}_3] = \mathrm{Re}[\dot{I}_2\, j\omega M.\overset{*}{I}_3] = \mathrm{Re}[(-2+j4)\,j10.(6+j2)] =$$
$$= \mathrm{Re}[(-40-j20)(6+j2)] = -40.6+2.20 = -240+40 = -200W$$

Следователно енергията се предава от клон «2» към клон «3»

Баланс на мощностите във веригата





$$\dot{I}_1 = (4+2j)A \qquad \dot{I}_2 = (-2+4j)A \qquad \dot{I}_3 = (6-2j)A$$

Мощността на ***източника*** е:

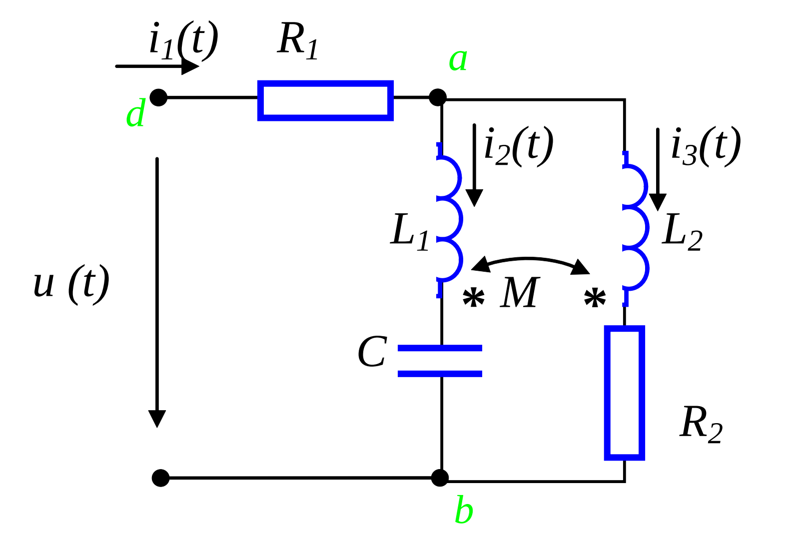
$$P_{вх} = \mathrm{Re}[\dot{U}.\overset{*}{I}_1] = \mathrm{Re}[(100+j100)(4-j2)] =$$
$$= 20.(-2) + 4.60 = 100.4 + 100.2 = 600W$$

Мощността изразходвана в ***резисторите*** е:

$$P_{R1} = I_1^2.R1 = (4^2+2^2).10 = 20.10 = 200W$$
$$P_{R2} = I_3^2.R2 = (6^2+2^2).10 = 40.10 = 400W$$

$$\boxed{\Rightarrow \quad P_{вх} = P_{кон} = 600W}$$

Баланс на разпределението на активната мощност в отделните клонове:



$$\dot{I}_1 = (4+2j)A$$
$$\dot{I}_2 = (-2+4j)A$$
$$\dot{I}_3 = (6-2j)A$$

$$P_{R1} = I_1^2.R1 = 200W$$
$$P_{R2} = I_3^2.R2 = 400W$$

Мощности в ***отделните клонове*** :

$$P_{кл1} = \mathrm{Re}[\dot{U}_{da}.\overset{*}{I}_1] = 200W \rightarrow P_{R1} = I_1^2.R1 = 200W$$

$$P_{кл2} = \mathrm{Re}[\dot{U}_{ab}.\overset{*}{I}_2] = 200W \rightarrow P_{кл2} = P_{M2}$$

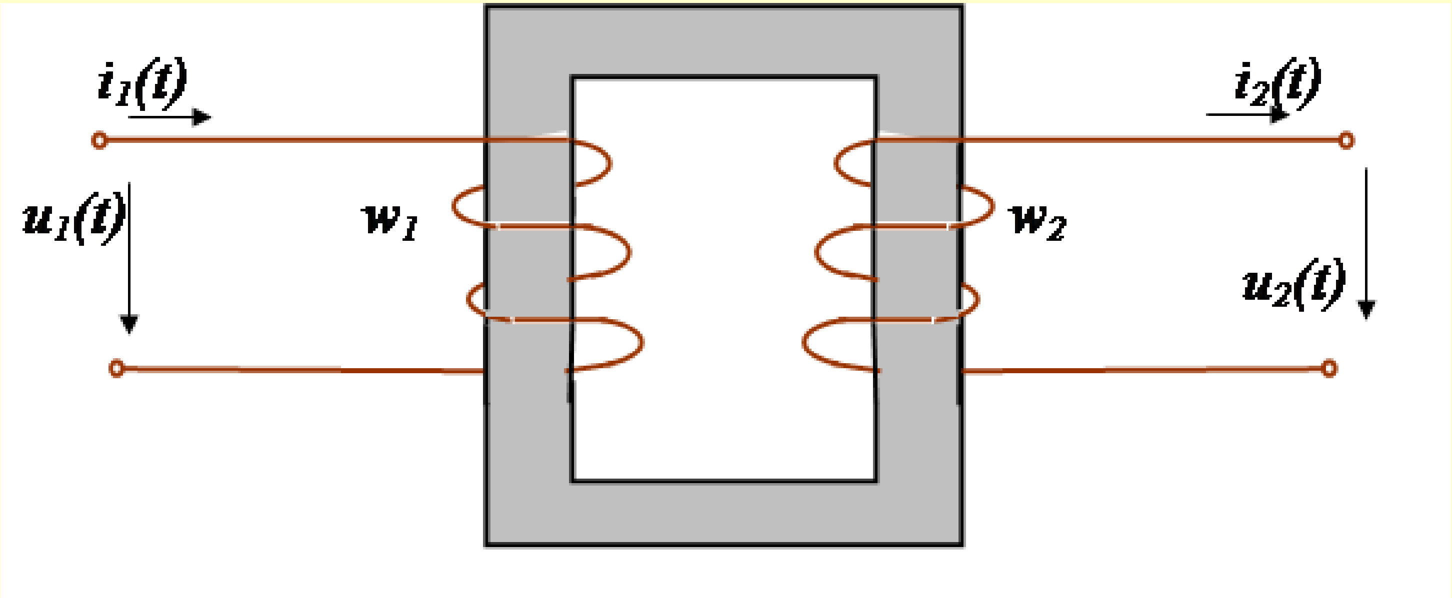
$$P_{кл3} = \mathrm{Re}[\dot{U}_{ab}.\overset{*}{I}_3] = 200W$$

$$P_{R2} = P_{M2} + P_{кл3} = 200+200 = 400W$$

$$\dot{U}_{da} = \dot{I}_1 R1 = (4+j2).10 = (40+j20)V$$

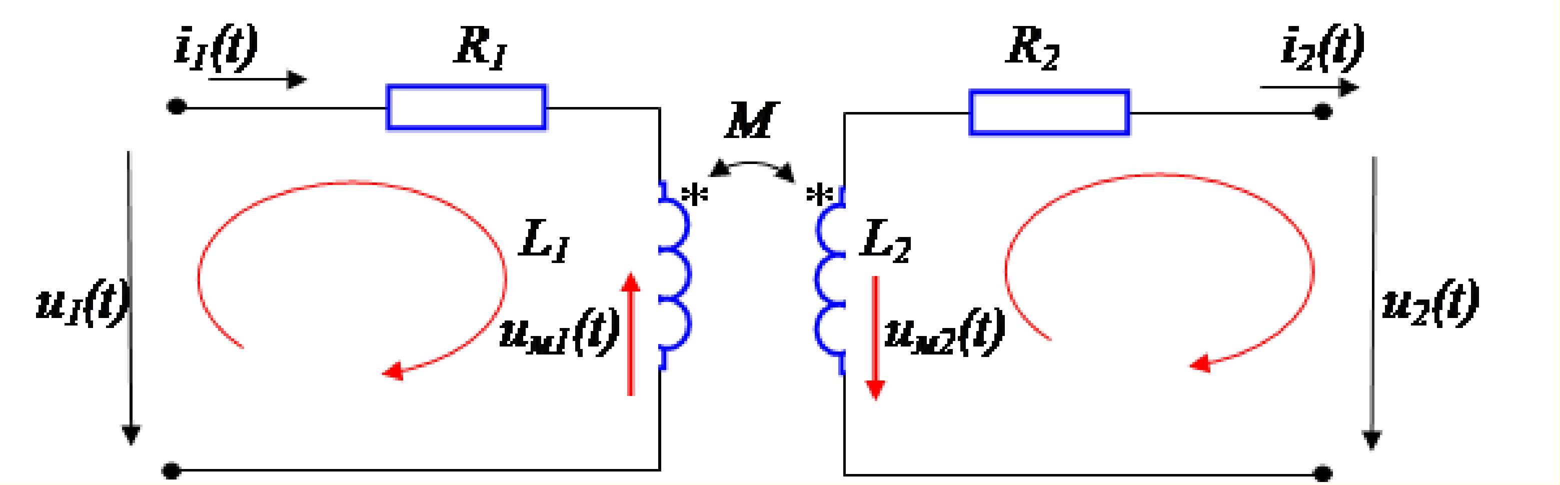
$$\dot{U}_{ab} = \dot{U} - \dot{I}_1 R1 = 100+j100-(4+j2).10 = (60+j80)V$$

<u>**Трансформаторно съединение.**</u>



- Трансформаторното съединение е съединение на две намотки с общо ядро
- Връзката между тях се осъществява посредством <u>**променливо**</u> магнитно поле.
- Енергията се предава от първичната към вторичната намотка на базата на взаимна индукция.

Уравнения на линеен трансформатор



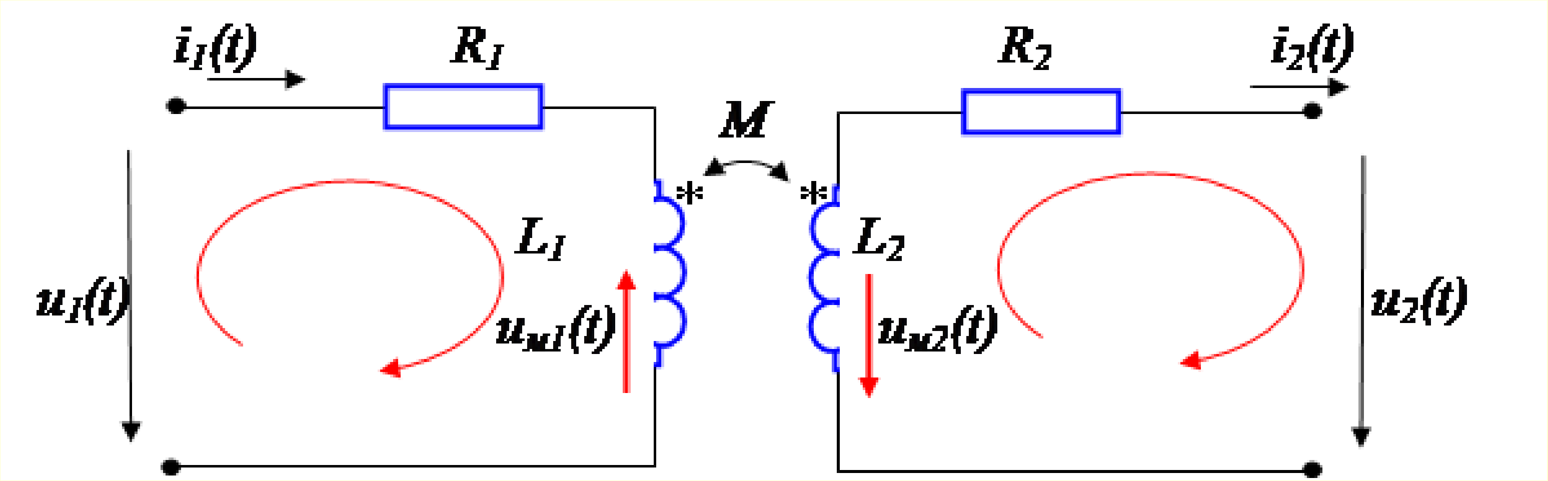
$$u_1(t) = u_{R_1}(t) + u_{L_1}(t) - u_{M_1}(t)$$

$$-u_2(t) = u_{R_2}(t) + u_{L_2}(t) - u_{M_2}(t)$$

$$u_{R_1}(t) = R_1 i_1(t); \quad u_{L_1}(t) = L_1 \frac{di_1}{dt}; \quad u_{M_1}(t) = M \frac{di_2}{dt}$$

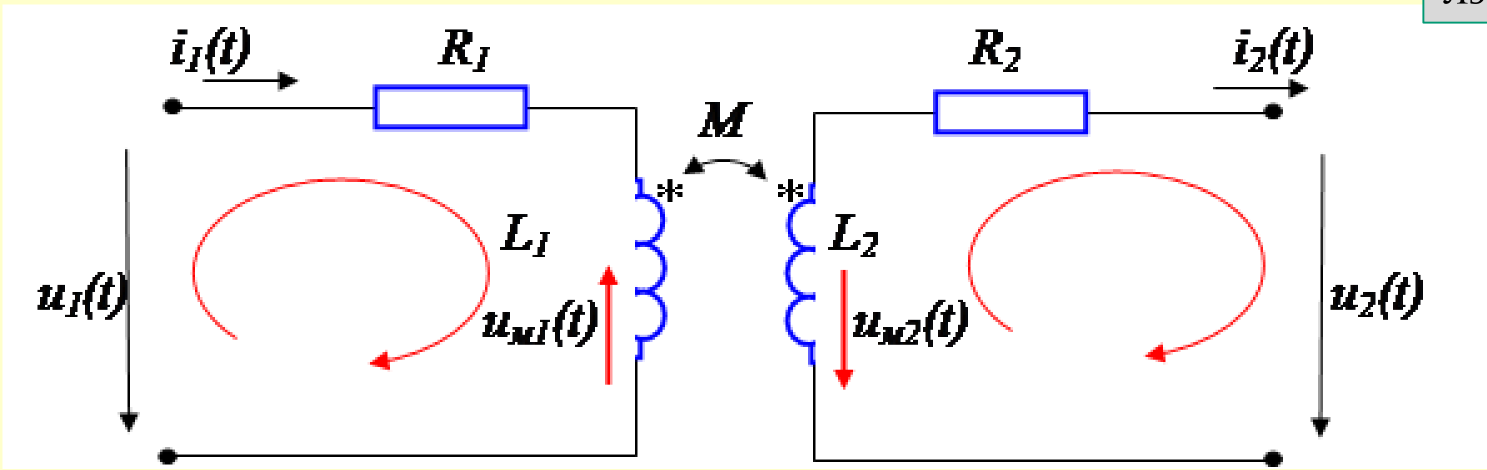
$$u_{R_2}(t) = R_2 i_2(t); \quad u_{L_2}(t) = L_2 \frac{di_2}{dt}; \quad u_{M_2}(t) = M \frac{di_1}{dt}$$

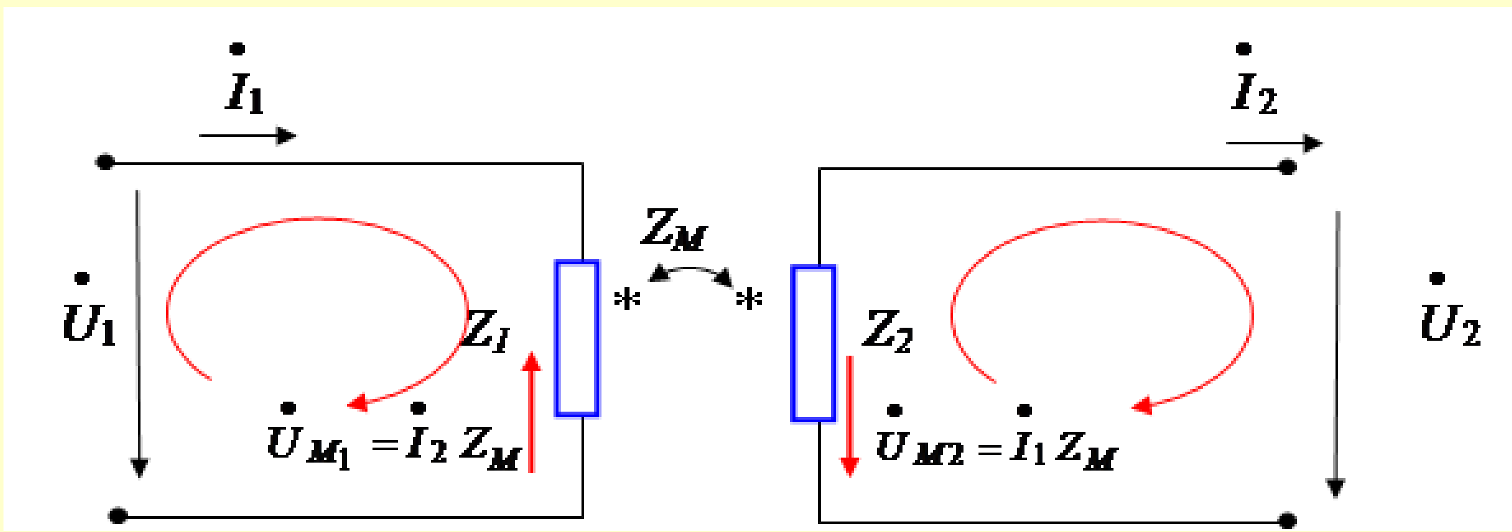
Уравнения на линеен трансформатор



$$u_1(t) = R_1 i_1(t) + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$-u_2(t) = R_2 i_2(t) + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$





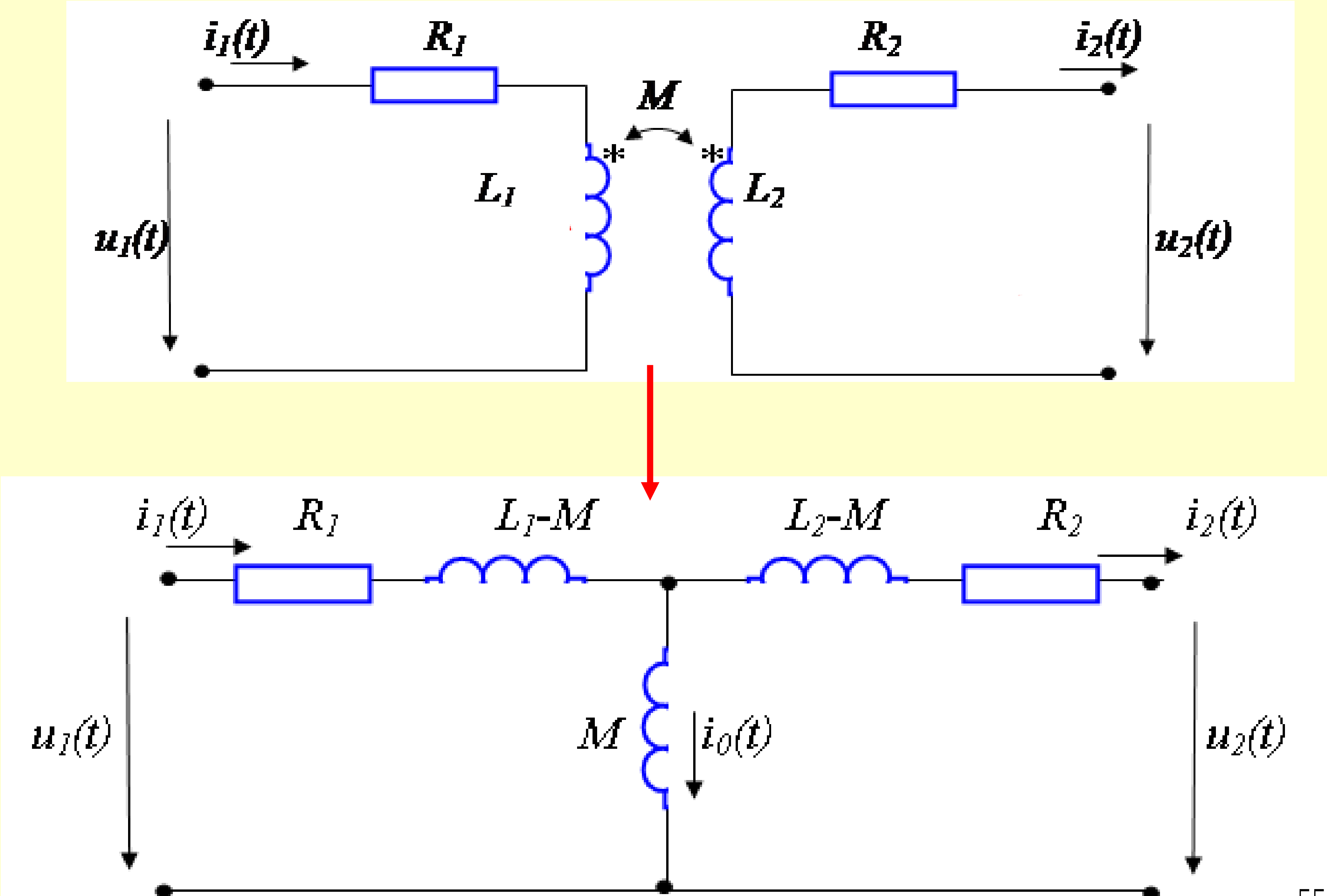
$$\dot{U}_1 = \dot{I}_1 Z_1 - \dot{I}_2 Z_M$$

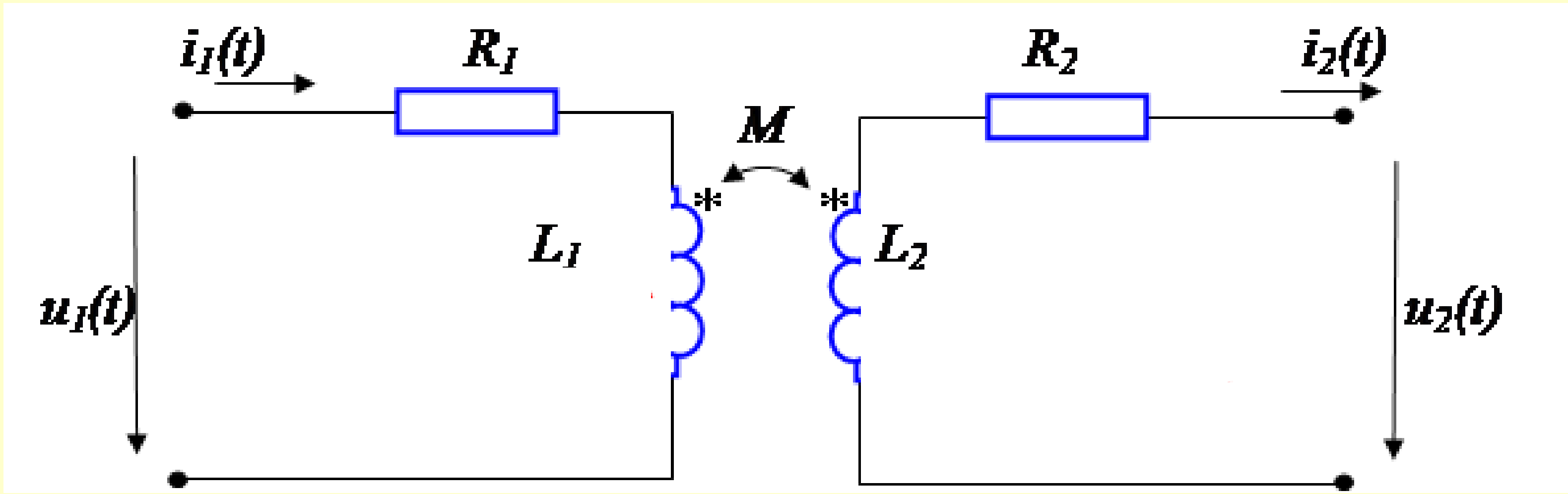
$$-\dot{U}_2 = \dot{I}_2 Z_2 - \dot{I}_1 Z_M$$

$$\dot{U}_1 = \dot{I}_1 (R_1 + j\omega L_1) - \dot{I}_2 \, j\omega M$$

$$-\dot{U}_2 = \dot{I}_2 (R_2 + j\omega L_2) - \dot{I}_1 \, j\omega M$$

Еквивалентна схема на линеен трансформатор

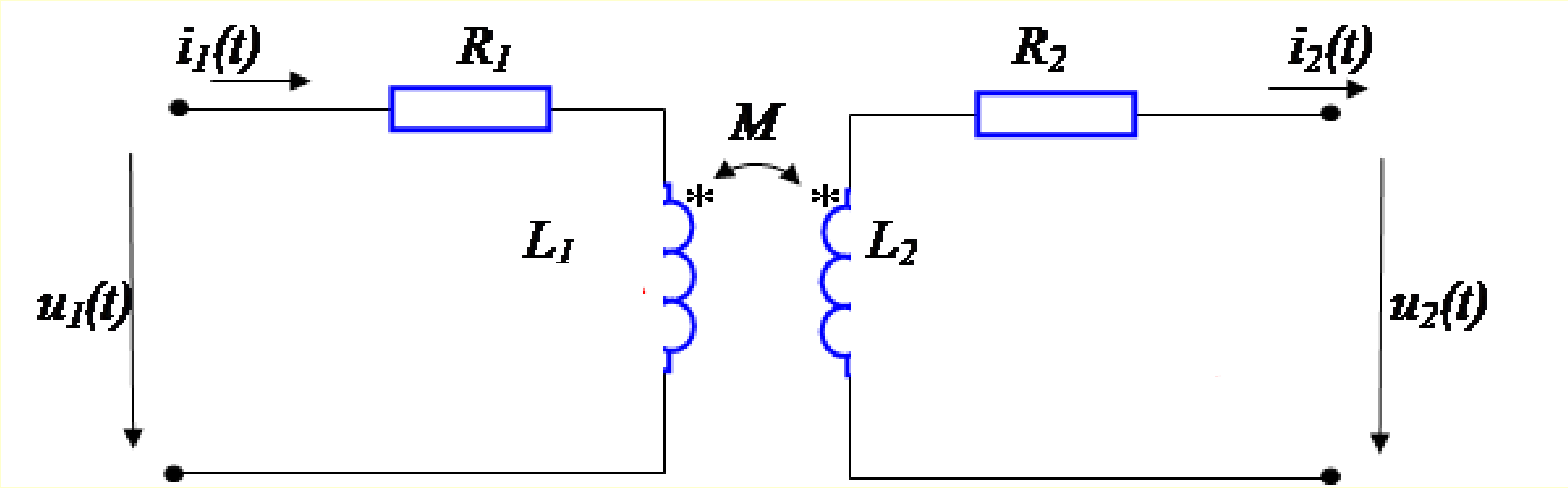




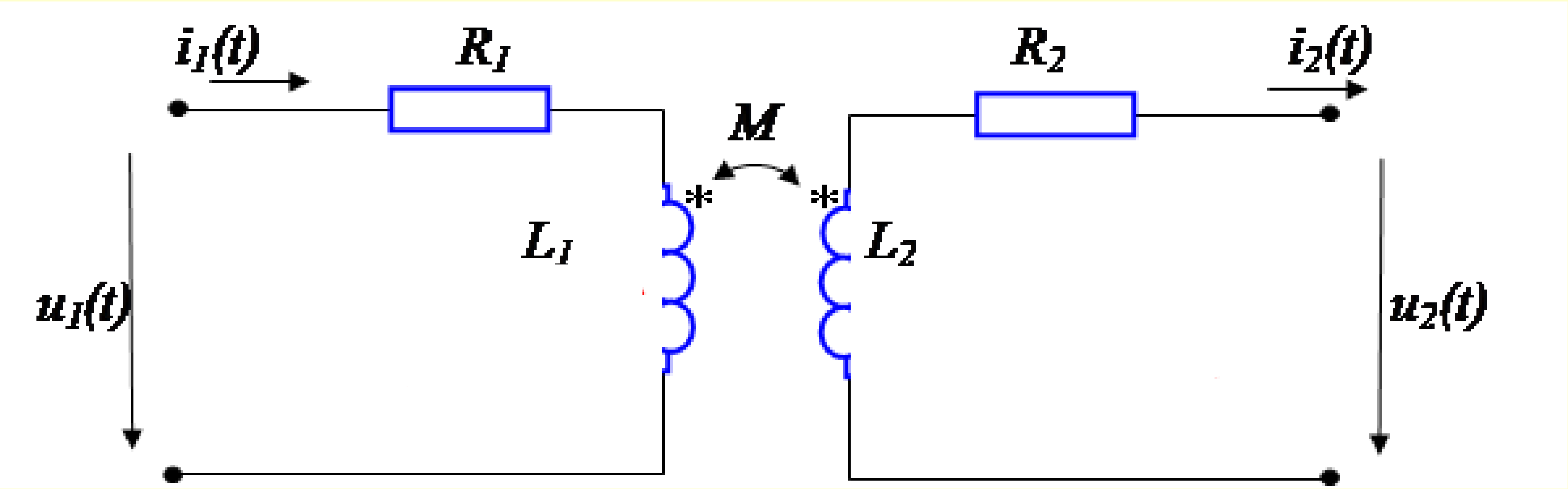
$$\begin{cases} u_1(t) = R_1 i_1(t) + L_1 \dfrac{di_1}{dt} - M\dfrac{di_2}{dt} \\ -u_2(t) = R_2 i_2(t) + L_2 \dfrac{di_2}{dt} - M\dfrac{di_1}{dt} \end{cases}$$

$$+\left(M\frac{di_1}{dt} - M\frac{di_1}{dt}\right)$$

$$+\left(M\frac{di_2}{dt} - M\frac{di_2}{dt}\right)$$



$$
\begin{cases}
u_1(t) = R_1 i_1(t) + L_1 \dfrac{di_1}{dt} - M\dfrac{di_2}{dt} + (M\dfrac{di_1}{dt} - M\dfrac{di_1}{dt}) \\
-u(t) = R_2 i_2(t) + L_2 \dfrac{di_2}{dt} - M\dfrac{di_1}{dt} + (M\dfrac{di_2}{dt} - M\dfrac{di_2}{dt})
\end{cases}
$$



$$u_1(t) = R_1 i_1(t) + (L_1 - M)\frac{di_1}{dt} + M(\frac{di_1}{dt} - \frac{di_2}{dt})$$

$$-u(t) = R_2 i_2(t) + (L_2 - M)\frac{di_2}{dt} - M(\frac{di_1}{dt} - \frac{di_2}{dt})$$

$$i_0 = i_1 - i_2$$

$$\frac{di_0}{dt} = \frac{di_1}{dt} - \frac{di_2}{dt}$$

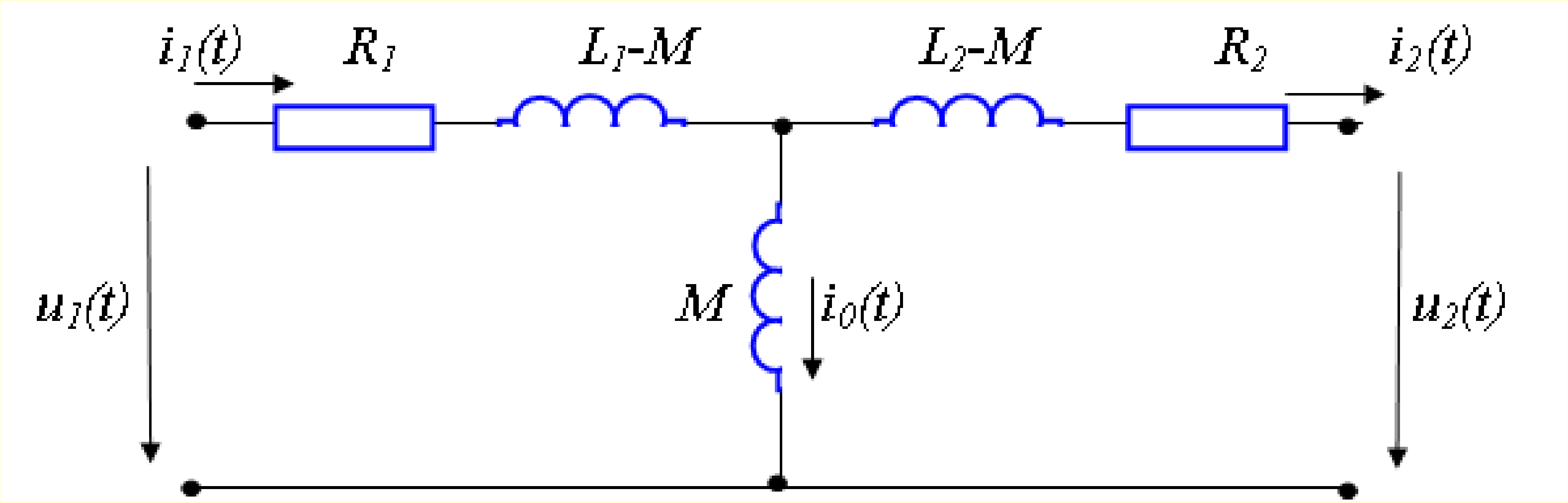
$$u_1(t) = R_1 i_1(t) + (L_1 - M)\frac{di_1}{dt} + M\frac{di_0}{dt}$$

$$-u(t) = R_2 i_2(t) + (L_2 - M)\frac{di_2}{dt} - M\frac{di_0}{dt}$$

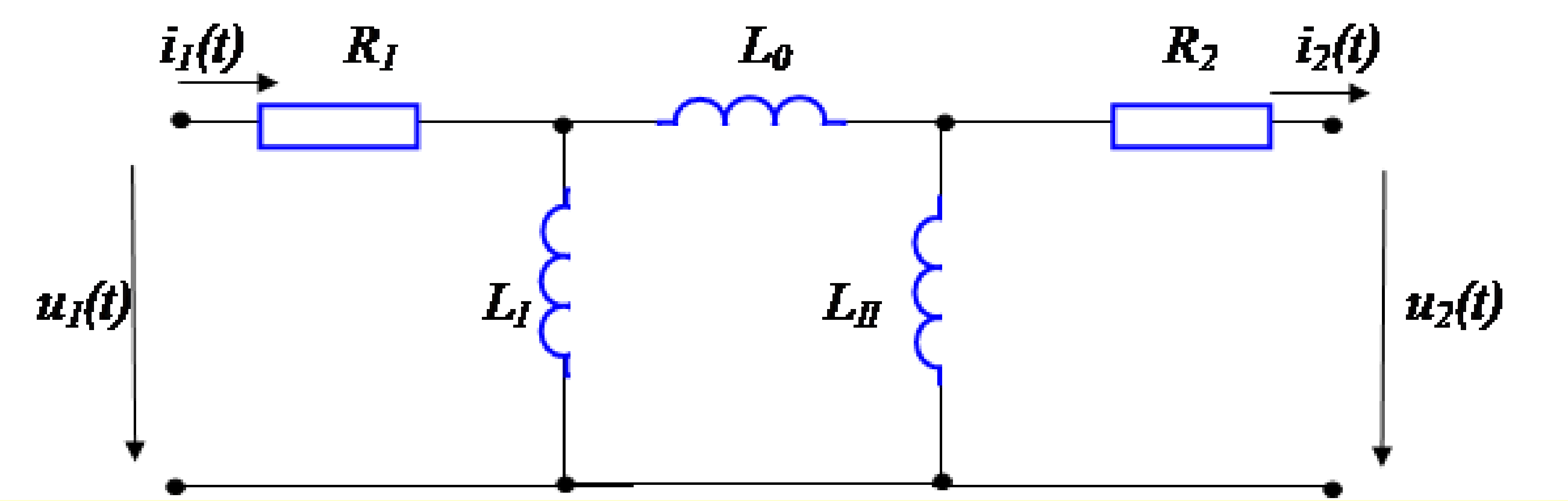
Т-образна еквивалентна схема на линеен трансформатор

$$u_1(t) = R_1 i_1(t) + (L_1 - M)\frac{di_1}{dt} + M\frac{di_0}{dt}$$

$$-u(t) = R_2 i_2(t) + (L_2 - M)\frac{di_2}{dt} - M\frac{di_0}{dt}$$

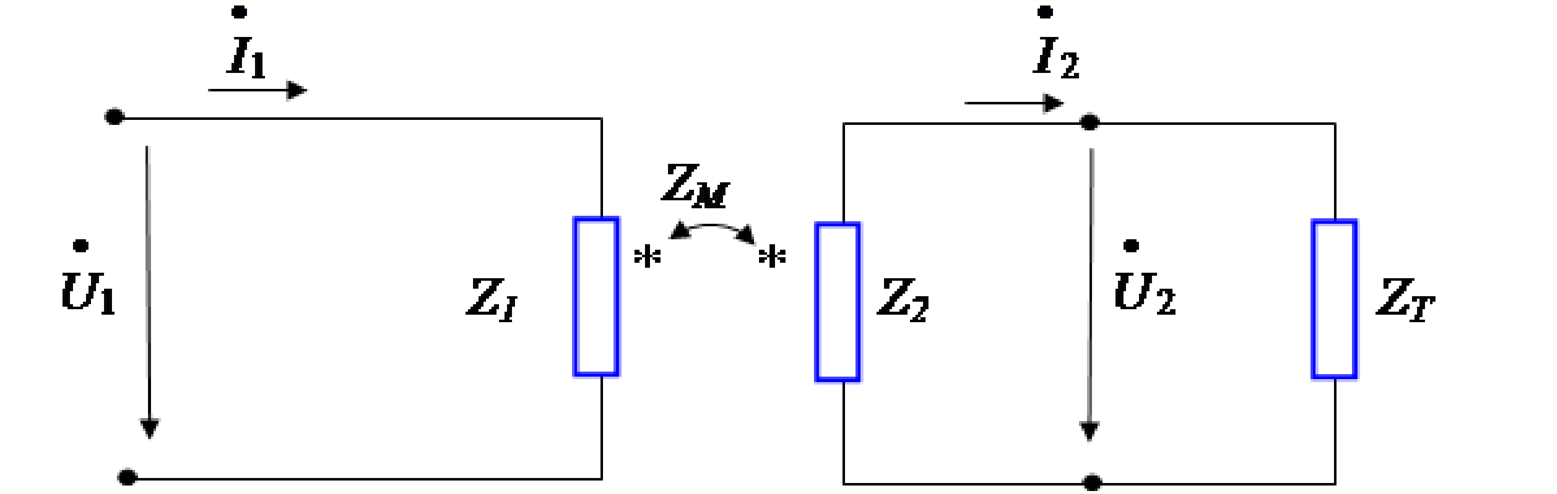


П-образна еквивалентна схема на линеен трансформатор



$$L_0 = \frac{L_1 L_2 - M^2}{M}; \quad L_I = \frac{L_1 L_2 - M^2}{L_2 - M}; \quad L_{II} = \frac{L_1 L_2 - M^2}{L_1 - M}$$

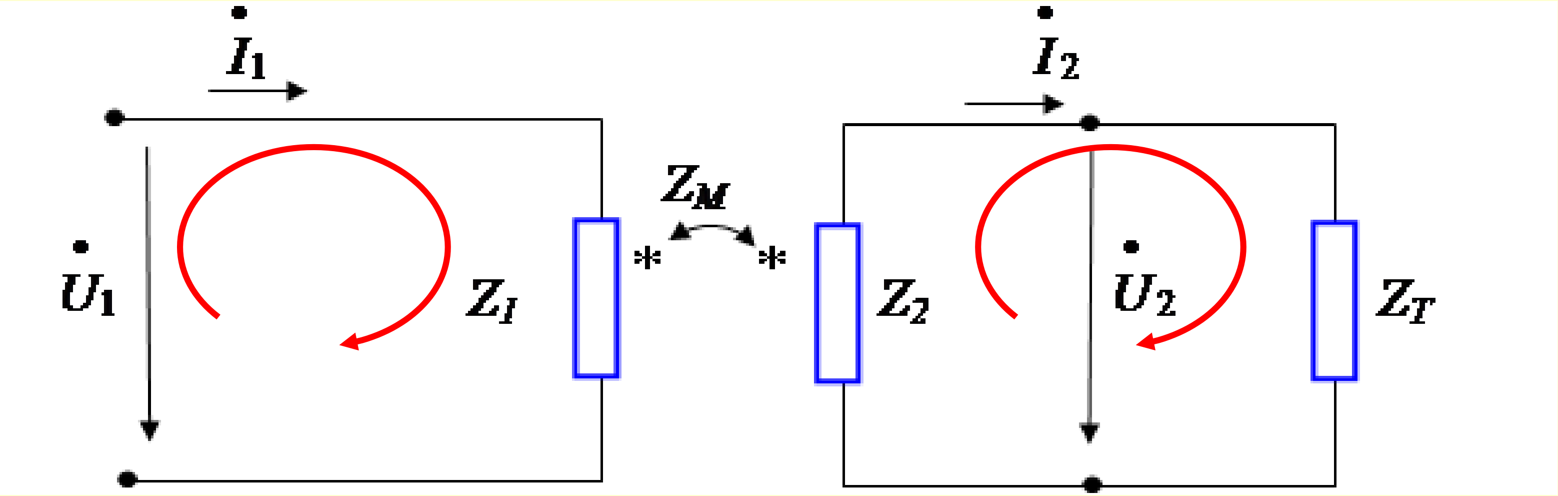
Векторна диаграма на натоварен трансформатор



$$Z_1 = R_1 + j\omega L_1;$$
$$Z_2 = R_2 + j\omega L_2;$$
$$Z_M = j\omega M$$

$$Z_T = R_T + j\omega L_T$$

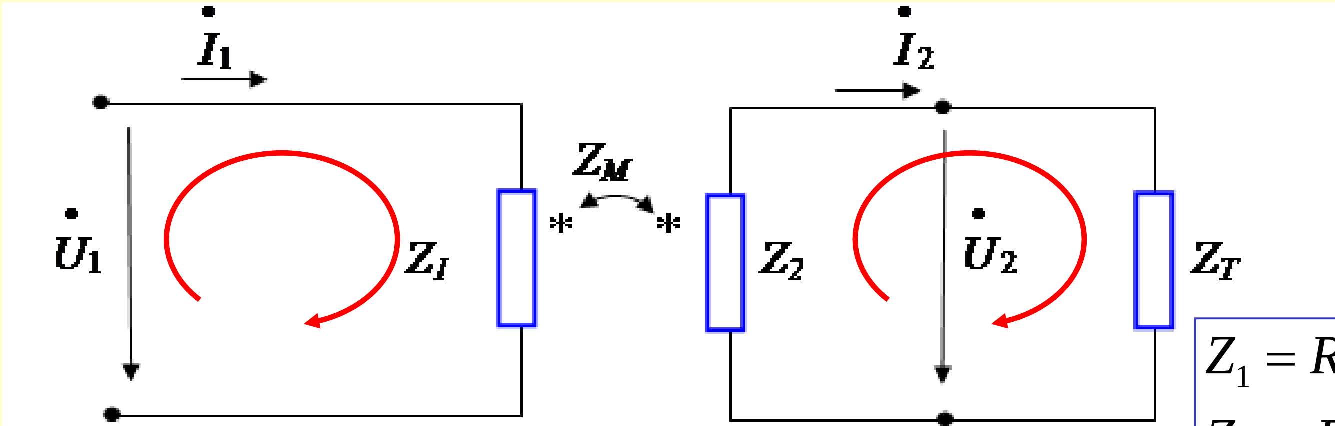
Векторна диаграма на натоварен трансформатор



$$\dot{U}_1 = \dot{I}_1 Z_1 - \dot{I}_2 Z_M$$

$$\dot{0} = \dot{I}_2 Z_2 + \dot{I}_2 Z_T - \dot{I}_1 Z_M \quad \Rightarrow \dot{I}_1 Z_M = \dot{I}_2 Z_2 + \dot{I}_2 Z_T$$

Векторна диаграма на натоварен трансформатор



$$\dot{U}_1 = \dot{I}_1 Z_1 - \dot{I}_2 Z_M$$

$$\dot{0} = \dot{I}_2 Z_2 + \dot{I}_2 Z_T - \dot{I}_1 Z_M$$

$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = R_2 + j\omega L_2$$

$$Z_M = j\omega M$$

$$Z_T = R_T + j\omega L_T$$

$$\dot{U}_1 = \dot{I}_1 R_1 + \dot{I}_1 j\omega L_1 - \dot{I}_2 j\omega M$$

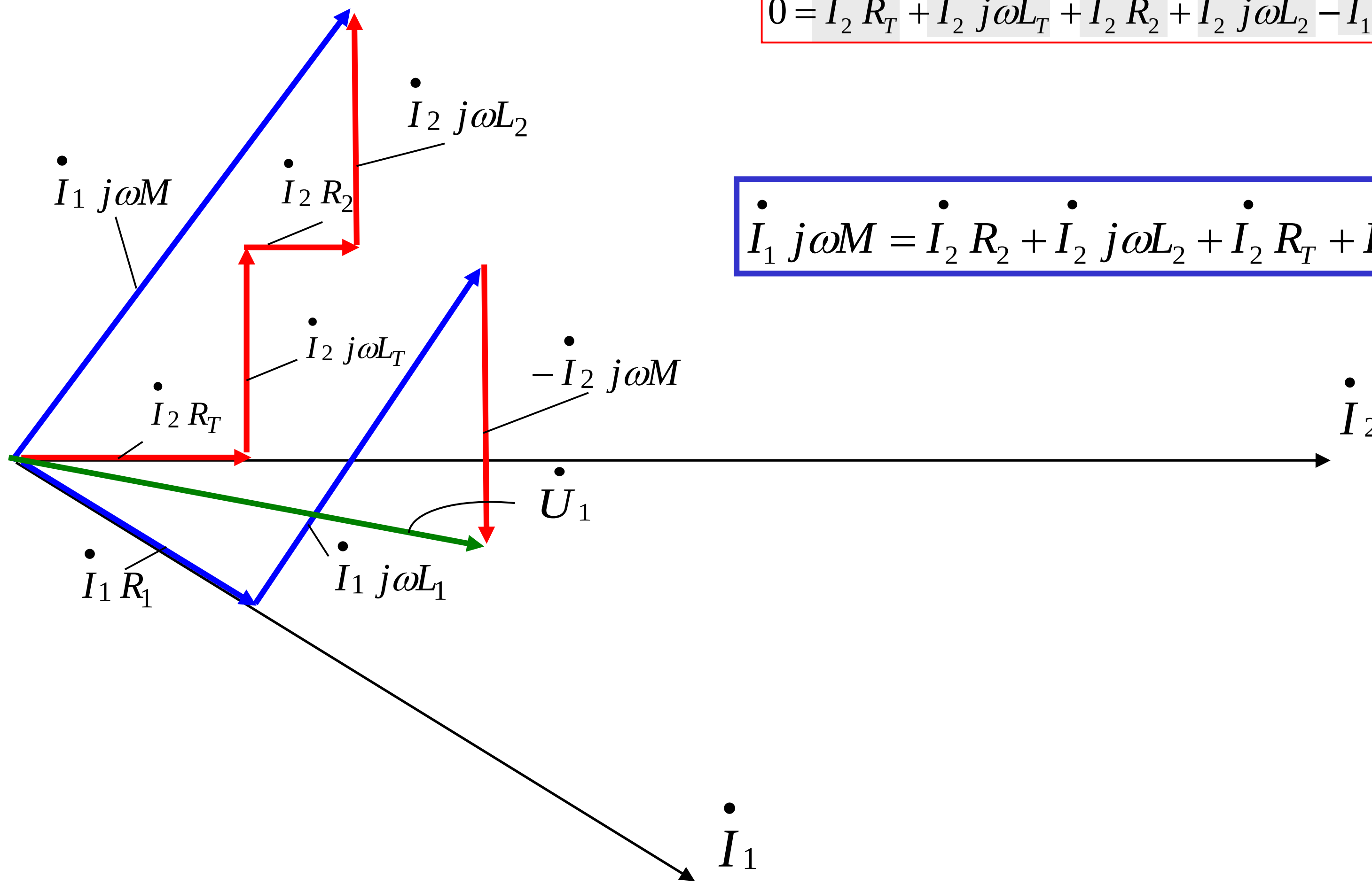
$$0 = \dot{I}_2 R_T + \dot{I}_2 j\omega L_T + \dot{I}_2 R_2 + \dot{I}_2 j\omega L_2 - \dot{I}_1 j\omega M$$

Векторна диаграма на натоварен трансформатор

$$\dot{U}_1 = \dot{I}_1 R_1 + \dot{I}_1 j\omega L_1 - \dot{I}_2 j\omega M$$

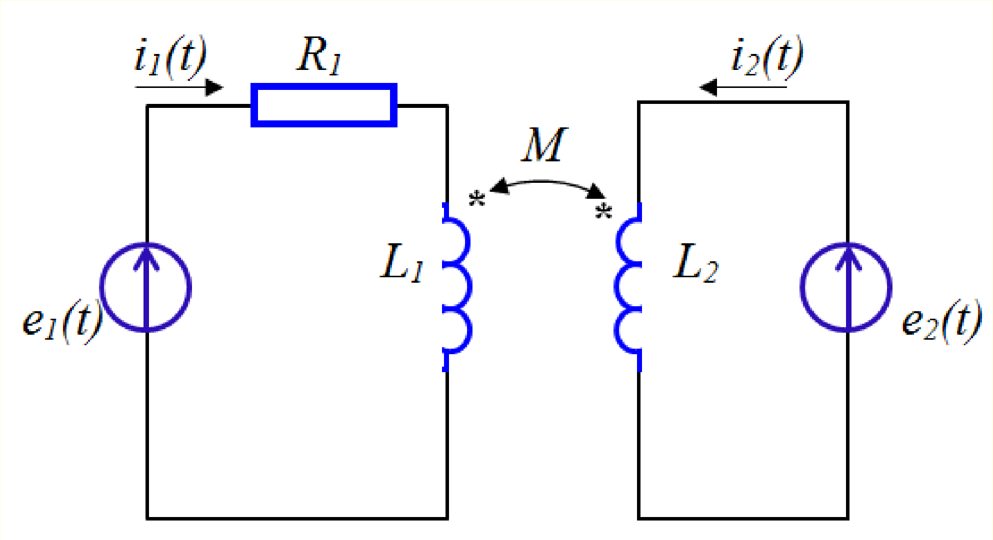
$$0 = \dot{I}_2 R_T + \dot{I}_2 j\omega L_T + \dot{I}_2 R_2 + \dot{I}_2 j\omega L_2 - \dot{I}_1 j\omega M$$

$$\boxed{\dot{I}_1 j\omega M = \dot{I}_2 R_2 + \dot{I}_2 j\omega L_2 + \dot{I}_2 R_T + \dot{I}_2 j\omega L_T}$$



Пример : Трансформаторно съединение

За веригата показана на фигурата е известно:



$$e_1(t) = 500\sqrt{2}\sin\omega t\ V$$

$$e_2(t) = 100\sqrt{2}\sin(\omega t + 90)\ V$$

$$R_1 = 100\Omega$$

$$\omega L_1 = 500\,\Omega$$

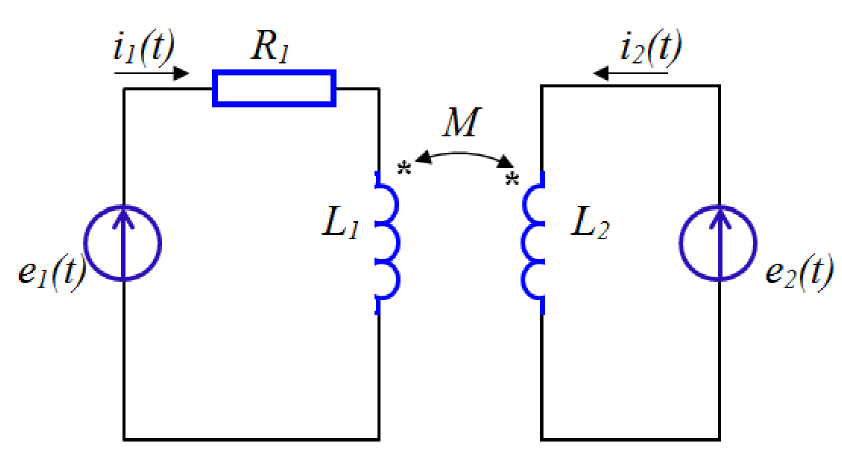
$$\omega L_2 = 50\,\Omega$$

$$\omega M = 100\,\Omega$$

a) Да се определят токовете $i_1(t)$ и $i_2(t)$;

b) Да се направи баланс на активната мощност.

Решение

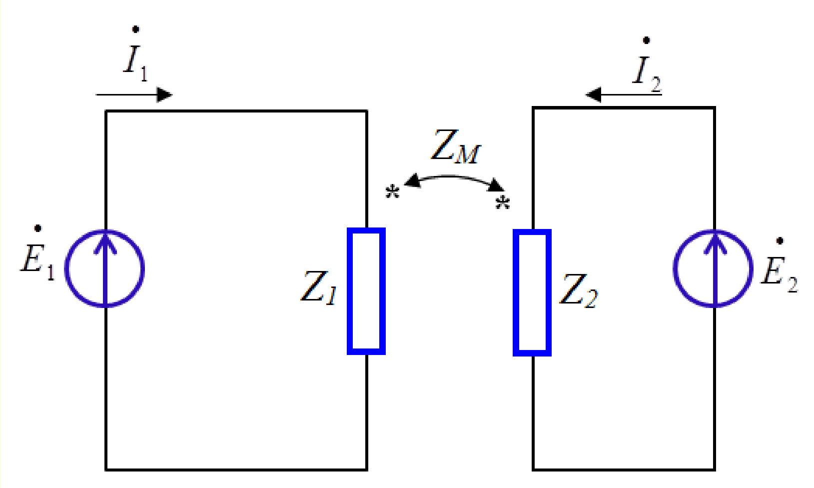


$$e_1(t) = 500\sqrt{2}\sin\omega t \; V$$

$$\dot{E}_1 = E_1 e^{j\psi_{e_1}} = \frac{e_{m1}}{\sqrt{2}} e^{j\psi_{e_1}} = \frac{500\sqrt{2}}{\sqrt{2}} e^{j0}$$

$$= 500.(\cos 0 + j\sin 0) =$$

$$= 500.(1 + j0) = 500V$$



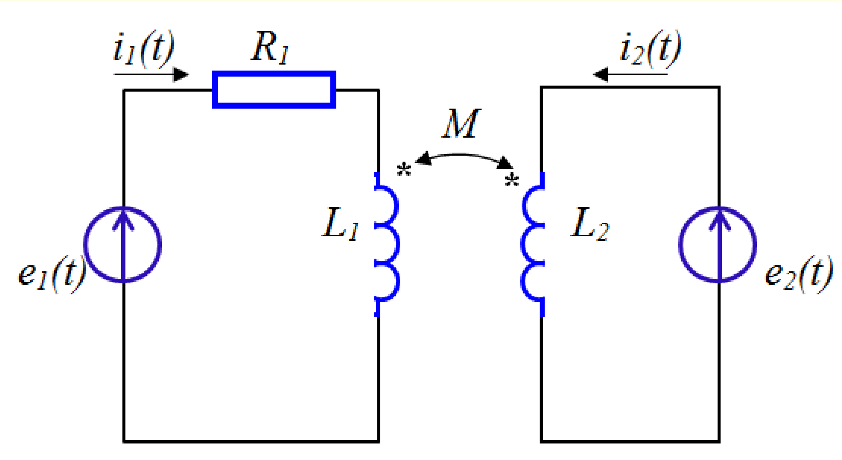
$$e_2(t) = 100\sqrt{2}\sin(\omega t + 90^\circ) \; V$$

$$\dot{E}_2 = E_2 e^{j\psi_{e_2}} = \frac{e_{m2}}{\sqrt{2}} e^{j\psi_{e2}} = \frac{100\sqrt{2}}{\sqrt{2}} e^{j90}$$

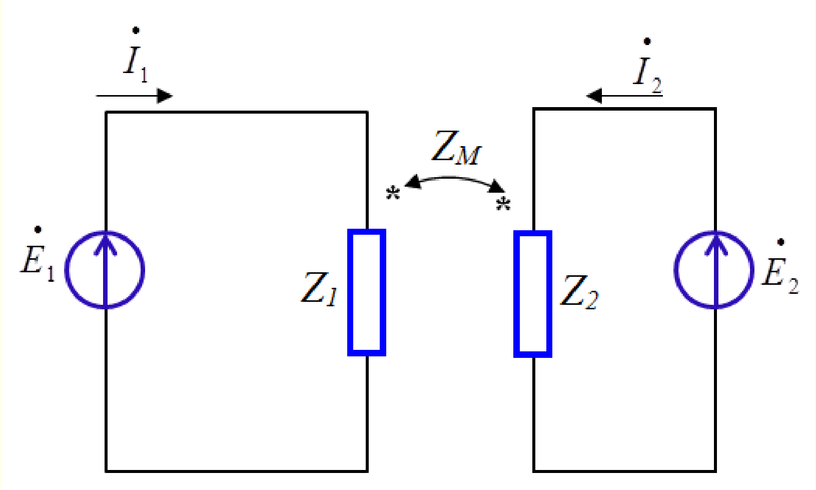
$$= 100.(\cos 90 + j\sin 90) =$$

$$= 100.(0 + j1) = j100V$$

Решение

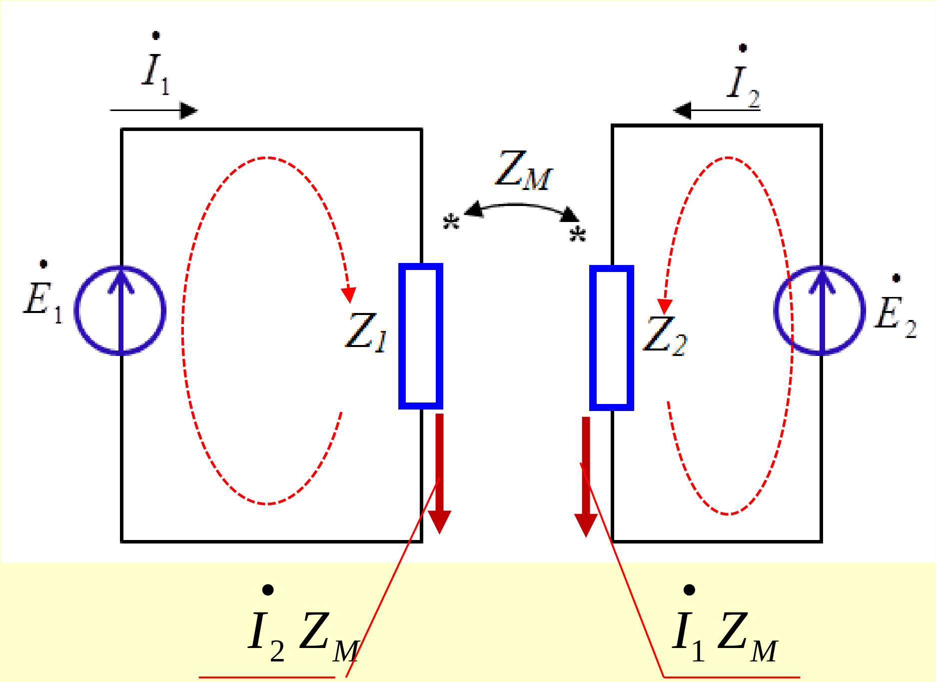


$$R_1 = 100\,\Omega$$
$$\omega L_1 = 500\,\Omega$$
$$\omega L_2 = 50\,\Omega$$
$$\omega M = 100\,\Omega$$



$$Z_1 = R_1 + j\omega L_1 = (100 + j500)\,\Omega$$
$$Z_2 = j\omega L_2 = j50\,\Omega$$
$$Z_M = j\omega M = j100\,\Omega$$

Определяне на токовете



$$\dot{E}_1 = 500V$$
$$\dot{E}_2 = j100V$$

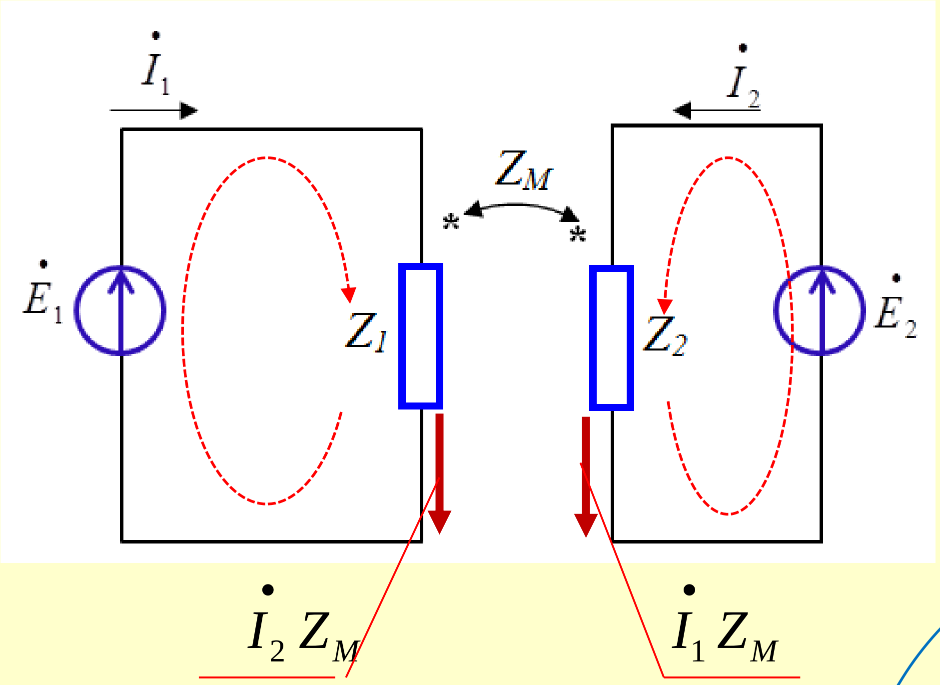
$$Z_1 = (100 + j500)\,\Omega$$
$$Z_2 = j50\,\Omega$$
$$Z_M = j100\,\Omega$$

$$\dot{I}_1 Z_1 + \dot{I}_2 Z_M = \dot{E}_1$$
$$\dot{I}_2 Z_2 + \dot{I}_1 Z_M = \dot{E}_2$$

$\rightarrow$

$$\dot{I}_1(100 + j500) + \dot{I}_2\, j100 = 500$$
$$\dot{I}_2\, j50 + \dot{I}_1\, j100 = j100$$

Определяне на токовете



$$\dot{I}_1(100+j500)+\dot{I}_2\ j100=500$$

$$\dot{I}_2\ j50+\dot{I}_1\ j100=j100$$

$$\dot{I}_1(1+j5)+\dot{I}_2.j=5$$

$$\dot{I}_2.j+\dot{I}_1.j2=j2$$

$$\dot{I}_1(1+j5-j2)=5-j2 \quad \Rightarrow \dot{I}_1(1+j3)=5-j2$$

$$\Rightarrow \dot{I}_1=\frac{5-j2}{(1+j3)}=\frac{(5-j2)}{(1+j3)}.\frac{(1-j3)}{(1-j3)}=$$

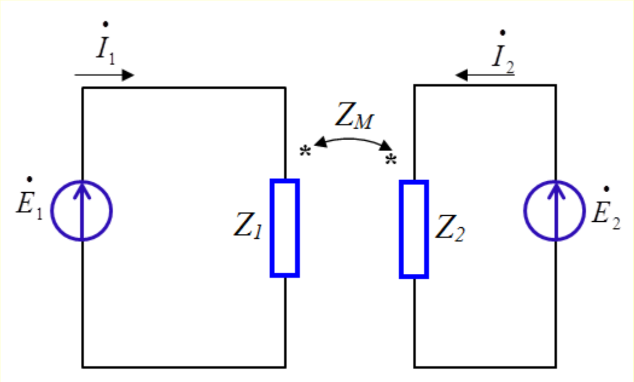
$$=\frac{(5-j2-j15-6)}{1^2+3^2}=\frac{(-1-j17)}{10}=(-0{,}1-j1{,}7)A$$

$$\boxed{\dot{I}_2.j+\dot{I}_1.j2=j2}$$

$$\Rightarrow \dot{I}_2=-2\dot{I}_1+2=$$

$$-2(-0{,}1-j1{,}7)+2=(2{,}2+j3{,}4)A$$

Определяне на моментните стойности на токовете $i_1(t)$ и $i_2(t)$

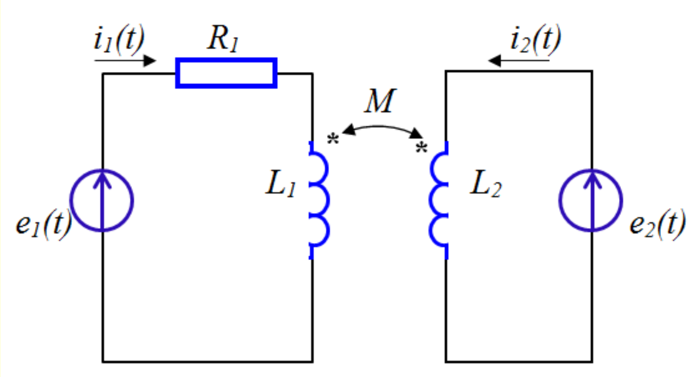


$$\dot{I}_1 = (-0{,}1 - j1{,}7)A$$

$$\dot{I}_2 = (2{,}2 + j3{,}4)A$$

$$\dot{I}_1 = \sqrt{(-0{,}1)^2 + (-1{,}7)^2}\, e^{jarctg\frac{-1{,}7}{-0{,}1}} = 1{,}703e^{-j93}A$$

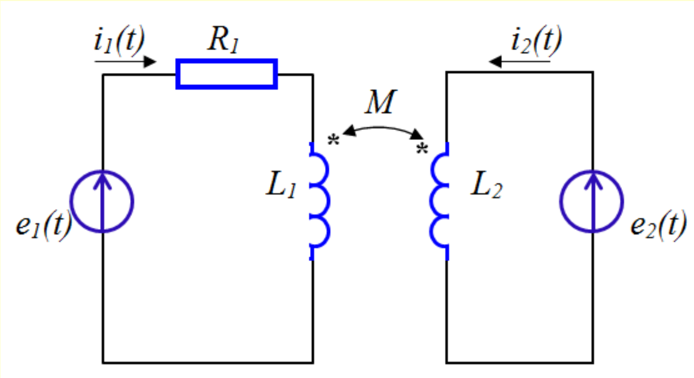
$$\dot{I}_2 = \sqrt{2{,}2^2 + 3{,}4^2}\, e^{jarctg\frac{3{,}4}{2{,}2}} = 4{,}05e^{j57}A$$

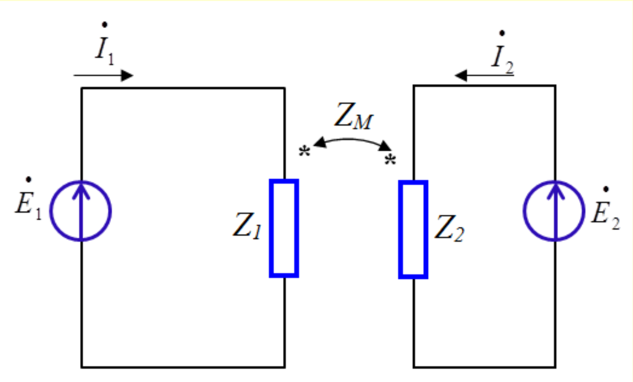


$$i_1(t) = I_1\sqrt{2}\sin(\omega t + \psi_{i1}) = 1{,}703\sqrt{2}\sin(\omega t - 93^\circ) = 2{,}41\sin(\omega t - 93^\circ)A$$

$$i_2(t) = I_2\sqrt{2}\sin(\omega t + \psi_{i2}) = 4{,}05\sqrt{2}\sin(\omega t + 57^\circ) = 5{,}73\sin(\omega t + 57^\circ)A$$

Баланс на активната мощност





$R_1 = 100\,\Omega$

$\dot{E}_1 = 500\,V$

$\dot{E}_2 = j100\,V$

$\dot{I}_1 = (-0{,}1 - j1{,}7)\,A$

$\dot{I}_2 = (2{,}2 + j3{,}4)\,A$

Мощност на ***източниците*** :

$$P_{e1} = \mathrm{Re}[\dot{E}_1 . \overset{*}{I}_1] = \mathrm{Re}[500(-0{,}1 + j1{,}7)] = -50\,W$$

→ *Работи като активен консуматор*

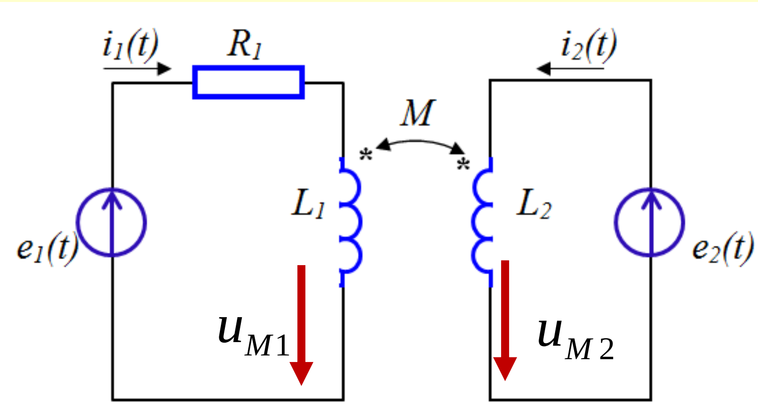
$$P_{e2} = \mathrm{Re}[\dot{E}_2 . \overset{*}{I}_2] = \mathrm{Re}[j100(2{,}2 - j3{,}4)] =$$
$$= j100.(-j3{,}4) = 340\,W$$

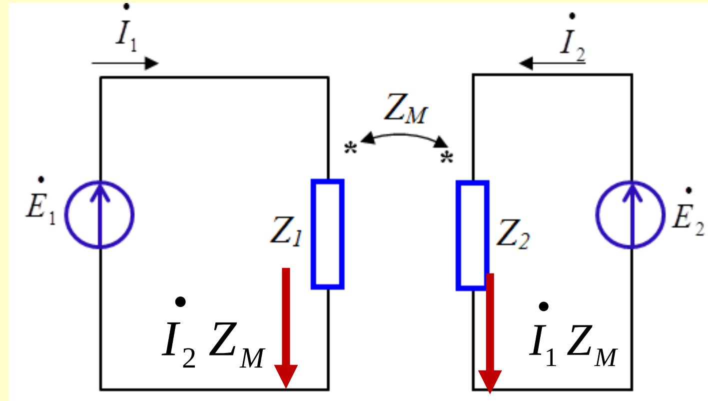
Мощност изразходвана в ***резистора***:

$$P_{R1} = I_1^2 . R1 = [(-0{,}1)^2 + (-1{,}7)^2].100$$
$$= 2{,}9.100 = 290W$$

$$\Rightarrow \quad P_{e2} = |P_{e1}| + P_{R1} = 50 + 290 = 340W$$

*Мощност предавана по **индуктивен** път:*





$\dot{I}_1 = (-0,1 - j1,7)A$

$\dot{I}_2 = (2,2 + j3,4)A$

$Z_M = j100\,\Omega$

$$P_{M1} = \mathrm{Re}[\dot{U}_{M1} . \overset{*}{I}_1] = \mathrm{Re}[\dot{I}_2 Z_M . \overset{*}{I}_1] = \mathrm{Re}[(2,2 + j3,4)\, j100(-0,1 + j1,7)] =$$

$$\mathrm{Re}[(j220 - 340)(-0,1 + j1,7)] = j220 . j1,7 + (-340).(-0,1) = -374 + 34 = -340\,W$$

$$P_{M2} = \mathrm{Re}[\dot{U}_{M2} . \overset{*}{I}_2] = \mathrm{Re}[\dot{I}_1 Z_M . \overset{*}{I}_2] = \mathrm{Re}[(-0,1 - j1,7)\, j100(2,2 - j3,4)] =$$

$$\mathrm{Re}[(-j10 + 170)(2,2 - j3,4)] = (-j10).(-j3,4) + 170.2,2 = -34 + 374 = 340\,W$$

$$P_{M1} = -340\,W \langle\ 0$$

$$P_{M2} = \ 340 W \rangle\ 0$$

***Енергията се предава от** <u>«2» към «1»</u>*

$$\Rightarrow \boxed{P_{e2} = P_{M2}} = |P_{e1}| + P_{R1} = 50 + 290 = 340W$$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