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Reducing traffic externalities by multiple-cordon pricing

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Abstract The second-best congestion pricing schemes including common optimum, one cordon, and multiple cordons schemes are compared with the first-best optimum pricing scheme. A cross-subsidy effect exists in these second-best pricing models. However, the scheme with more cordons will diminish the cross-subsidy and approach an efficient and equitable outcome. The relative efficiency of a cordon pricing scheme for the case of Taipei metropolis is very high. One single cordon yields excellent performance of 93% relative efficiency. There might be some factors causing the good results: the uncongested traffic condition, the linear unit distance cost in traffic flow forming a nonlinear cost function, and the trip demands with continuous space and the same destination (the central business district) in the network.

Keywords Multiple-cordon pricing · Second-best congestion pricing · Traffic congestion externality · Social surplus

Introduction

Road congestion is a serious problem in most large cities in both developing and developed countries. To ameliorate the problem of road congestion, methods from many perspectives have been raised. Road investment by increasing road capacity is an approach from the

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supply side. However, it can cause a financial burden, especially for a local government with fiscal pressure. In addition, the induced travel demand after the road investment might cause road congestion again. From the perspective of the demand side, levying a Pigouvian tax on road users to reflect the external costs incurred by the road users is an efficient way to reduce travel demand and improve congestion during peak hours. The principle of marginal-cost pricing has been applied to peak-load problems on roads (Pigou 1920; Knight 1924). The basic concepts have been elaborated and extended by many studies (for instance, Walters 1961; Mohring and Harwitz 1962; Vickrey 1963; Strotz 1965). The models in these studies have been applied empirically to many cities worldwide.[1] However, social feasibility is another problem for the implementation of such pricing policies. Levying a tax on each trip on each road is not easily accepted by the public.[2] Some alternate policies have thus been raised for road users on some roads or in some areas depending on the limitations of policy implementation, instead of for all roads or all areas.

Cordon pricing is a second-best approach, levying a tax on trips passing the cordon line, regardless of the origin and destination of a trip. Some studies of the cordon pricing problems were intended to determine toll levels given cordon locations. May and Milne (2000) compared the effects of four kinds of road pricing schemes, including cordon pricing, by using a congested assignment network model to the city of Cambridge. Verhoef (2002a, b), proposed a mathematical method to analyze the optimal toll levels and locations of toll for a predetermined subset of paths. The demand of origin–destination (O–D) pairs can shift on the used paths following Wardrop's user equilibrium concept in a static network. Some studies have focused on methods to compute the optimal solution, and presented numerical results for given networks (Zhang and Yang 2004; Sumalee 2004). These second-best pricing schemes have been applied in real case studies to design road user charging schemes (Akiyama et al. 2004; Sumalee 2007; Sumalee et al. 2009). Recently, analysis of land consumption has also been incorporated into congestion pricing schemes (Verhoef 2005; Tikoudis et al. 2015). The consumption of land of individuals is included in utility functions with the constraint of budget for consumption and commuting cost (including congestion tax if any). Following the approach in Fujita (1989), the bid rent function is then derived after the choice of individuals and the urban configuration is formed (De Palma et al. 2011; De Lara et al. 2013).

On the other hand, continuous space models are more consistent with the transportation corridor in a city with a high-density population. Economic analysis on this type of road pricing in continuous one-dimensional space has been conducted by using a model which deals with the commuting trips with multiple origins to the same destination in a city (Mun et al. 2003). This commuting pattern fits the real case in a developed transportation corridor in cities, compared to the pattern with origin–destination pairs in many studies of road congestion pricing. The optimal cordon pricing is formulated and the cordon location and pricing rate are determined in the model. A similar model structure for cordon-congestion pricing for two-dimensional space is presented by Ho et al. (2005). This approach has been extended to the road pricing in a non-monocentric city configuration (Mun et al. 2005), as well as elevated roads and road expansion investment near a city center (Chu and Tsai 2008; Tsai and Chu 2010).

[1] These studies include Keeler and Small (1977), Dewees (1979), Gomez-Ibanez and Fauth (1980), Anderson and Mohring (1997), Nguyen (1999), May and Milne (2000), De Borger and Proost (2001), Li (2002), Niskanen and Nash (2004), Santos (2004), and Eliasson and Matsson (2006).

[2] Rietveld and Verhoef (1998) discussed the social feasibility of road pricing policies.

This study extends the cordon pricing by Mun et al. (2003) to multiple-cordon pricing in order to reflect the road users' congestion externalities much more accurately. The first-best optimum pricing is to levy a toll equal to the externality for each road user. In single-cordon pricing, some tolled road users will pay a higher toll than the externality they really incur, while the other road users will pay less than the external costs they incur or pay no toll. That is, this pricing mechanism has a cross-subsidy effect even though it can reach a high welfare level. A multiple-cordon pricing approach has three implications. First, the cross-subsidy effect in single-cordon pricing will be mitigated by multiple-cordon pricing, and generates a more equitable outcome. Second, this approach can reach a higher welfare level than can a single-cordon pricing approach. Third, the multiple-cordon pricing scheme is expected to approach the outcome of the first-best optimum pricing. That is, both the welfare level and the tolling level on the spatial distribution will approach those of the first-best pricing scheme as the number of cordons increases. The remainder of this paper is as follows. Section 2 develops a model for multiple-cordon pricing. In Sect. 3, the model is applied to the Taipei metropolitan area and its performance is compared with that of previous related studies. Section 4 provides the main findings of our study and potential avenues for future research.

The model

This study is based on the concept of road pricing from Pigou (1920), in which external marginal costs are levied on road users. Considering the feasibility of this policy, a second-best approach by using multiple-cordon pricing on a transportation corridor is used.

The model of this study is based on Mun et al. (2003). Assume a monocentric city consisting of a central business district (CBD) whose area is negligible, and the CBD is treated as a point. The residential area is around the CBD, and has uniform population density and homogenous residents who commute to the CBD by automobile. Other trips are neglected. Trip demand is elastic and depends on the costs of the trip. Let x be the distance from the CBD (km in unit), and let $q(x)$ be the travel demand, in number of trips, of a resident located at x. The inverse trip demand is given as

$$p(q(x)) = a - bq(x), \tag{1}$$

where a and b are positive parameters and $p(q(x))$ represents the marginal benefit of a trip. Since all trips are destined to the same place, the CBD, the traffic volume at x is the sum of the trips originating between x and the city boundary, B:

$$Q(x) = \int_x^B q(y)dy. \tag{2}$$

The cost for a trip from x is given by

$$C(x) = \int_0^x t(Q(y))dy, \tag{3}$$

where $t(Q(y))$ denotes the cost for driving the unit distance around y, which is increasing in traffic volume as follows:

$$t(Q(y)) = f + c \cdot Q(y), \tag{4}$$

where f denotes the cost of driving one unit distance when traffic flow is zero, and c denotes the marginal cost with respect to traffic volume.[3]

No-toll equilibrium and first-best optimum

No-toll equilibrium and first-best optimum are introduced for comparison with multiple-cordon pricing. With no toll, the equilibrium will be reached under the condition that marginal benefit equals the travel cost at each location[4]:

$$p(q(x)) = C(x), \quad \text{for all } x, \quad 0 \leq x \leq B. \tag{5}$$

Solving (5) after substituting (1) and (3) into it and differentiating both sides of the equation with respect to x twice yields the equilibrium number of trips generated from x as follows:

$$q^*(x) = \lambda_1\, exp(\alpha x) + \lambda_2\, exp(-\alpha x), \tag{6}$$

where $\alpha = \sqrt{c/b}$, and λ_1 and λ_2 are unknown constants which can be determined by boundary conditions:

$$\lambda_1 = \frac{a \cdot exp(-\alpha B) - f/\alpha}{b(exp(\alpha B) + exp(-\alpha B))}, \quad \lambda_2 = \frac{a \cdot exp(-\alpha B) + \mathrm{f}/\alpha}{b(exp(\alpha B) + exp(-\alpha B))}.$$

Total social surplus represented by total benefit minus the total costs is the goal from the perspective of the whole system. The first-best optimum is thus defined as the trip pattern which maximizes the social surplus, formulated as follows:

$$SS = \int_0^B \left[\int_0^{q(x)} p(q)dq - C(x)q(x) \right] dx. \tag{7}$$

To solve the maximization problem for (7), the optimal condition is obtained as follows:

$$p(q(x)) = C(x) + \int_0^x t'(Q(y))Q(y)dy. \tag{8}$$

The second term of the RHS in (8) represents the congestion externalities which are caused by an additional trip from x imposed on all the road users using the road between 0 and x. The first-best optimum is reached by levying a congestion tax equal to the marginal external cost of each trip incurred. In this setting, this congestion tax differs for the trips

[3] Note that the linear cost function is for every unit distance (a very small distance) of a trip. Because each point (user) on the line (road) between the city boundary and CBD generates travel demand to the same destination (the CBD), the traffic flow at x is formulated by cumulating every trip from x to B (boundary), and the cost function of a trip originating at x is thus the sum of the cost of every unit of distance from 0 to x. The traffic flow is nonlinear and increasing in the distance approaching the CBD with an increasing rate. Therefore, though the cost of driving every unit distance is linear in traffic flow, the cost function of any trip is nonlinear in distance of the trip. This setting for a continuous spatial trip pattern is suited for a developed transportation corridor which generates the trips with many different origins and one destination (the CBD).

[4] The convexity and uniqueness are satisfied and the solution can be obtained by solving the following differential equation with boundary conditions.

originating from different locations. Solving (8) after substituting (1) and (4) into it yields the optimal trip at x:

$$q^0(x) = \mu_1\, exp(\beta x) + \mu_2\, exp(-\beta x), \tag{9}$$

where $\beta = \sqrt{2b/c}$ and μ_1 and μ_2 are unknown constants which can be determined by boundary conditions:

$$\mu_1 = \frac{a \cdot exp(-\beta B) - f/\beta}{b(exp(\beta B) - exp(-\beta B))}, \quad \mu_2 = \frac{a \cdot exp(\beta B) + f/\beta}{b(exp(\beta B) - exp(-\beta B))}.$$

Multiple-cordon pricing

In this section, tolling at two cordon locations is developed. That at more than two cordon locations is discussed in the case study in the next section.

Assume that two cordon locations at x_{m1} and x_{m2} ($x_{m1} < x_{m2}$) and tolls, τ_1 and τ_2, need to be determined for two-cordon pricing. That is, the residents located between the CBD and x_{m1} will not be tolled, while those between x_{m1} and x_{m2} will be tolled by τ_1, and those outside x_{m2} will be tolled by, $\tau_1 + \tau_2$. Let $q_1(x)$, $q_2(x)$, and $q_3(x)$ represent the number of trips located in these three intervals, respectively. The traffic volume at x which is in the section inside the cordons ($0 \le x \le x_{m1}$), between the two cordons ($x_{m1} \le x \le x_{m2}$), and outside the cordons ($x_{m2} \le x \le B$) is expressed respectively as $Q_1(x)$, $Q_2(x)$, and $Q_3(x)$:

$$Q_1(x) = \int_x^{x_{m1}} q_1^{**}(y)dy + \int_{x_{m1}}^{x_{m2}} q_2^{**}(y)dy + \int_{x_{m2}}^{B} q_3^{**}(y)dy, \quad \text{for } 0 \le x \le x_{m1}, \tag{10a}$$

$$Q_2(x) = \int_x^{x_{m1}} q_2^{**}(y)dy + \int_{x_{m2}}^{B} q_3^{**}(y)dy, \quad \text{for } x_{m1} \le x \le x_{m2}, \tag{10b}$$

$$Q_3(x) = \int_x^{B} q_3^{**}(y)dy, \quad \text{for } \ x_{m2} \le x \le B. \tag{10c}$$

With two cordons, the condition for the equilibrium is that marginal benefit equals the trip cost for the trip originating from the location in each section of the road. These conditions are expressed as follows:

$$p\left(q_1^{**}(x)\right) = C_1(x), \quad \text{for } 0 \le x \le x_{m1}, \tag{11a}$$

$$p\left(q_2^{**}(x)\right) = C_2(x) + \tau_1, \quad \text{for } x_{m1} \le x \le x_{m2}, \tag{11b}$$

$$p\left(q_3^{**}(x)\right) = C_3(x) + \tau_1 + \tau_2, \quad \text{for } x_{m2} \le x \le B, \tag{11c}$$

where

$$C_1(x) = \int_0^{x} t(Q_1(y))dy, \quad \text{for } 0 \le x \le x_{m1}, \tag{12a}$$

$$C_2(x) = \int_0^{x_{m1}} t(Q_1(y))dy + \int_{x_{m1}}^{x} t(Q_2(y))dy, \quad \text{for } x_{m1} \le x \le x_{m2}, \tag{12b}$$

$$C_3(x) = \int_0^{x_{m1}} t(Q_1(y))dy + \int_{x_{m1}}^{x_{m2}} t(Q_2(y))dy + \int_{x_{m2}}^{x} t(Q_3(y))dy, \quad \text{for } x_{m2} \le x \le B. \tag{12c}$$

The equilibrium trip originating from each section can be solved as:

$$q_1^{**}(x) = \lambda_1 e^{\alpha x} + \lambda_2 e^{-\alpha x}, \quad \text{for } 0 \le x \le x_{m1}, \tag{13a}$$

$$q_2^{**}(x) = \lambda_3 e^{\alpha x} + \lambda_4 e^{-\alpha x}, \quad \text{for } x_{m1} \le x \le x_{m2}, \tag{13b}$$

$$q_3^{**}(x) = \lambda_5 e^{\alpha x} + \lambda_6 e^{-\alpha x}, \quad \text{for } x_{m2} \le x \le B, \tag{13c}$$

where $\alpha = \sqrt{c/b}$, and λ_1, λ_2, λ_3, λ_4, λ_5, and λ_6 are unknown constants which can be determined by boundary conditions (see Appendix 1).

To explore the impact of the cordon locations on trips originating from each section of the road, a comparative static analysis is conducted (see the details in Appendix 2). It is shown that $\frac{\partial q_1^{**}(x)}{\partial X_{m1}} < 0, \frac{\partial q_2^{**}(x)}{\partial x_{m1}} < 0$, and $\frac{\partial q_3^{**}(x)}{\partial X_{m1}} < 0$ when x_{m1} increases and $\frac{\partial q_1^{**}(x)}{\partial X_{m2}} < 0, \frac{\partial q_2^{**}(x)}{\partial X_{m2}} < 0$, and $\frac{\partial q_3^{**}(x)}{\partial X_{m2}} < 0$ when x_{m2} increases. This means that as the first cordon location or the second cordon location moves outward, the number of trips from each section decreases. To explore the impact of the level of toll on trips originating from each section of the road, a comparative static analysis is conducted (see the details in Appendix 2). It is shown that $\frac{\partial q_1^{**}(x)}{\partial \tau_1} > 0, \frac{\partial q_2^{**}(x)}{\partial \tau_1} < 0$, and $\frac{\partial q_3^{**}(x)}{\partial \tau_1} < 0$ when τ_1 increases. This means that as the level of toll at the first cordon increases, the trips passing this location, $q_2^{**}(x)$ and $q_3^{**}(x)$, will increase their trip costs, and thus the number of trips decreases. The decrease in trip demand passing the first cordon will thus decrease the trip cost for the trips originated from the first section of the road, and the number of trips increases. In addition, it is shown that $\frac{\partial q_1^{**}(x)}{\partial \tau_2} > 0, \frac{\partial q_2^{**}(x)}{\partial \tau_2} > 0$, and $\frac{\partial q_3^{**}(x)}{\partial \tau_2} < 0$ when τ_2 increases. Similarly, this means that as the level of toll at the second cordon increases, the number of trips from the third section decreases, and the number of trips from the other two sections of the road increases.

The optimal cordon pricing for two locations is defined as the cordon location x_{m1}, x_{m2} and tolls τ_1, τ_2 to maximize the social surplus, which is measured by the total benefits minus the total costs. The social surplus is formulated as follows:

$$\begin{aligned} SS = &\int_0^{x_{m1}} \left[\int_0^{q_1^{**}(x)} p(q)dq - C_1(x)q_1^{**}(x) \right] dx + \int_{x_{m1}}^{x_{m2}} \left[\int_0^{q_2^{**}(x)} p(q)dq - C_2(x)q_2^{**}(x) \right] dx \\ &+ \int_{x_{m2}}^{B} \left[\int_0^{q_3^{**}(x)} p(q)dq - C_3(x)q_3^{**}(x) \right] dx. \end{aligned} \tag{14}$$

The right-hand side of (14) includes three parts: the first part is the total benefits minus the total costs for the trips from the location inside the first cordon location $(0 \le x \le x_{m1})$, the second is that between the two cordon locations $(x_{m1} \le x \le x_{m2})$, and the third is that outside the second cordon location $(x_{m2} \leq x \leq B)$. The first-order conditions for this problem are $\partial SS/\partial x_{m1}$, $\partial SS/\partial x_{m2}$, $\partial SS/\partial x_{\tau 1}$, and $\partial SS/\partial x_{\tau 2}$ being nil (see Appendix 3 for the details). Solving these equations yields the cordon locations and toll for each cordon.

The numerical simulation is made in the case study for the Danshui–Beitou–Taipei city center transportation corridor in the Taipei metropolitan area. To compare the various schemes, a common optimal toll is introduced as a basic scheme. Common optimal toll is designed to maximize the social surplus by charging every user the same toll level. It thus can be treated as one cordon tolling with a fixed cordon location very close to the CBD.

Case study

The model in the previous section is applied to the Danshui–Beitou–Taipei city center transportation corridor in the Taipei metropolitan area. Danshui and Beitou are two districts in the north of Taipei city. The land along this corridor is highly developed, and the residents are distributed almost uniformly on the major road connecting Danshui, Beitou, and the Taipei city center. Most residents of these two districts commute to the Taipei city center in the morning. The commuting trip pattern in this corridor is similar to the assumption of our model. The Taipei city center in this case is broadly defined as south of the Double-River Bridge in Sulin district, which is well developed and provides many jobs. The residents commute to the city center, the CBD, in the morning peak hour (see Fig. 1).

Parameter estimation

The parameters in the model are calibrated: $B = 26.30$ km, $a = 60$ min, $b = 152.5165$ $\frac{min}{trip/hr}$, $c = 0.4860$ $\frac{min/km}{veh/hr}$, $f = 1.0$ $\frac{min}{km}$. They are explained as follows:

(1) The commuting city boundary

The farthest location the resident can commute from to the CBD in this case is the new town in Danshui, 26.30 km between this farthest location and the CBD.

(2) Demand function of travel behavior

A linear trip demand function is assumed in this model, and the parameters in the inverse demand function, a and b, need to be determined. The maximum willingness-to-pay, $a = 60$ min,[5] is estimated as the commuting time of the farthest commuter.

The slope of the inverse demand function, b, is estimated by parameter a divided by the maximum number of trips per person. In this case, the maximum number of trips per person is estimated by the trips between Danshui and the CBD and that between Beitou and the CBD.[6] However, the data we collected are the traffic flow between Danshui district and Beitou district (36,391 per day) and that between Beitou district and the CBD (77,794 per day).[7] To obtain the trips from these two districts to the CBD, two relations are set as:

[5] This implies that the average speed is about 26 km/hr, which is obtained by searching Google Maps in this corridor for the peak hours. This value needs to be multiplied by the value of time to convert into the monetary value.

[6] It is assumed that the number of trips per person is linearly negatively proportionate to the distance to the CBD.

[7] The trips between Danshui district and Beitou district are collected by the average daily traffic flow at Jin-Long Bridge on Provincial Highway No. 2 and at Gan-Zhen-Lin on Provincial Highway No. 2A. Those

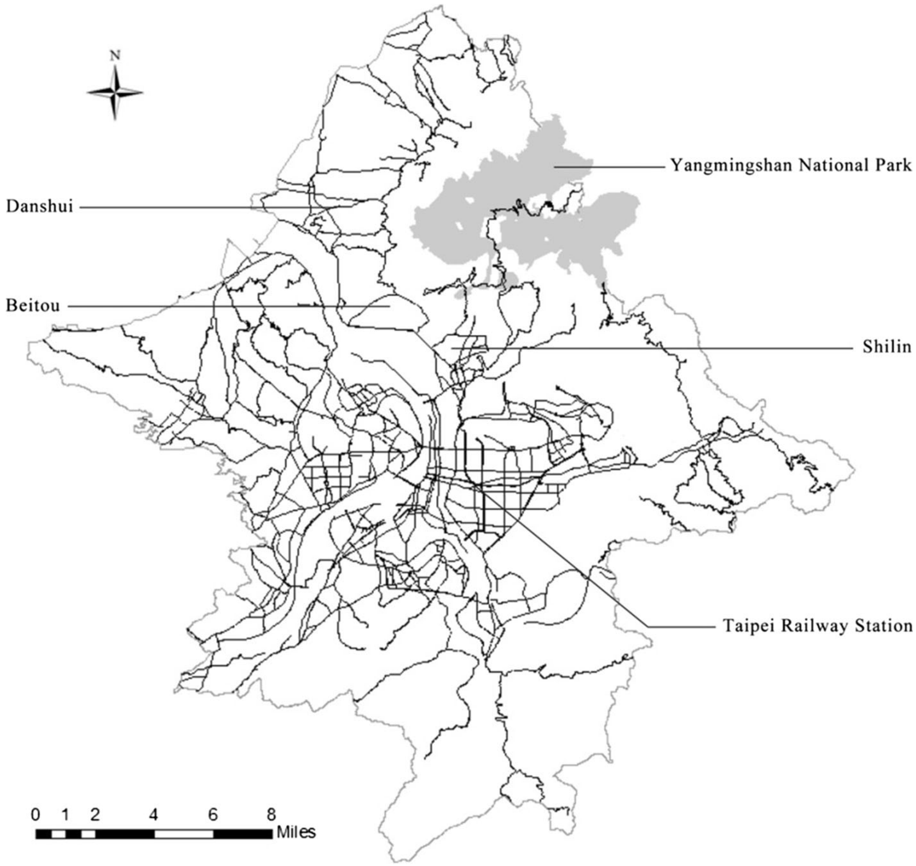

Fig. 1 Map of the Taipei metropolitan area

$$T_{12} + T_{13} = 36391,$$

$$T_{13} + T_{23} = 77794,$$

where T_{12} denotes the trips from Danshui district to Beitou district, T_{13} denotes the trips from Danshui district to the CBD, and T_{23} denotes the trips from Beitou district to the CBD. In addition, there is a relationship such that $T_{12} = 0.9\, T_{13}$.[8] We can thus obtain $T_{13} = 19{,}154$ and $T_{23} = 58{,}640$. The populations of Danshui district and Beitou district are 156,368 and 255,444, respectively. The distances from these two districts to the CBD are 12.26 and 1.35 km, respectively. The load factor is 1.62 persons/car. The number of trips per person from these two districts to the CBD is thus 0.1984 and 0.3719, respectively. The

Footnote 7 continued
between Beitou district and the CBD are based on the traffic flow at Cheng-De Road and Xi-An Street. These data are from Taipei City Traffic Engineering Office (2012) and Directorate General of Highway, Taiwan (2012).

[8] The number of trips from Danshui district to Beitou district is around 90% of that from Danshui district to the CBD. The data are from the Department of Budget, Accounting, and Statistics, Taipei City Government (2010).

maximum number of trips per person is then calculated as 0.3934. The parameter b in (1) can be estimated as 152.5165 $\frac{\text{min}}{\text{trip/hr}}$.

(3) Trip cost function

The parameters of driving cost in (4) include fixed cost and marginal cost for driving one unit of distance. The fixed cost, f, is estimated by the time cost of free flow without any congestion. The speed of free flow is assumed to be 60 km/h and thus $f = 1$ min/km.[9]

The marginal cost increases as the number of lanes of the road in the corridor decreases. There are three lanes on most of the roads in the corridor. Following the approach of Mun et al. (2003) for estimating parameter c, travel time from zone i ($i = 1, \ldots 31$) to the CBD by the model is best fitted to the actual travel time from real data. The point of the CBD in this case is Taiwan Museum in Taipei city center, and the origins of the trips consist of 12 districts in Taipei City and 19 selected districts in New Taipei City. The distance and the travel time for each trip are collected by using Google Maps in the morning rush hour. Parameter c is determined so that the sum of square errors for the 31 observations is a minimum. It follows that c = 0.4860 $\frac{\text{min/km}}{\text{veh/hr}}$.

Analysis for cordon location and tolls

This section analyzes the cordon location, level of toll, and the social surplus under schemes with no toll, the first-best optimal pricing, common optimal pricing and cordon pricing with one to four cordon locations. Toll level, trip pattern, and traffic flow are compared under these schemes. In addition, relative efficiency (RE), acceptance ratio (AR) and fairness index (FI) are also listed. The RE is to represent the improvement in social surplus compared with the first-best pricing scheme. It is defined as

$$\mathrm{RE} = \frac{SS^{**} - SS^{*}}{SS^{O} - SS^{*}} \times 100\%$$

where SS^{*}, SS^{O}, and SS^{**} denote the social surplus under the no-toll, the first-best, and the second-best scheme.

The AR is used to express how the residents may accept the pricing regime, and is defined as the percentage of the residents who are levied less than the external cost they generated.[10] The FI is used to represent the degree of fairness if fairness is defined by the closeness of toll level and external costs (Verhoef 2008). It thus denotes the fitness of the resident's toll and the external cost he (or she) generates.[11] The first-best optimal toll is exactly equal to the external cost and thus has a 100% score on the FI. It is measured as follows:

$$\mathrm{FI} = \left(1 - \frac{\text{sum of each users's difference between toll level and external costs}}{\text{total external costs}}\right) \times 100\%.$$

[9] The maximum speed on the roads in Danshui district is 70 km/h and that in Beitou district is around 50 $\sim$ 60 km/h. 60 km/h is thus assumed to be the free flow speed.

[10] This ratio may not reflect exactly the acceptance by road users. In reality, most people may resist any pricing scheme compared to no toll. However, if people pay less than the external costs they generate, it may decrease resistance against the pricing regime. In this model, the assumption of the residents being uniformly distributed on the transportation corridor makes this index easily calculated.

[11] The fairness index may be treated as one for equity impact based on horizontal dimension user groups (Maruyama and Sumalee 2007).

Table 1 Comparison of social surplus with multiple cordon pricing

	NT	FB	CO	1C	2C	3C	4C
Cordon location (km)	–	–		4.1903	2.3425	1.6274	1.2471
					8.9438	5.6732	4.1739
						11.4999	7.8804
							13.1678
Toll (min)	–	–	11.3775	13.1578	8.1003	5.8364	4.5590
					6.3454	5.0613	4.1212
						3.9990	3.5880
							2.8508
SS (min)	93.2262	103.9126	102.3647	103.2408	103.6886	103.8018	103.8467
RE (%)	–	100	85.35	93.71	97.90	98.63	99.05
FI (%)	–	100	68.49	79.79	88.68	92.13	93.97
AR (%)	–	–	65.71	70.51	64.50	61.63	59.90

SS social surplus, *RE* relative efficiency, *FI* fairness index, *AR* acceptance ratio, *NT* no toll, *FB* first-best optimum, *CO* common optimum, *1C* one cordon, *2C* two cordons, *3C* three cordons, *4C* four cordons

Table 1 shows the social surplus is 93.2262 min[12] with no toll and 103.9126 min with the first-best optimal congestion pricing, which can improve 11.46% by social surplus compared with no toll scheme.[13] With common optimal tolling, the social surplus is 102.3647 min and the RE is 85.35%. With cordon pricing, the social surplus is 103.2408 min for one cordon, and this increases and approaches that with the first-best optimal congestion pricing for more cordons.[14] The RE also increases from 93.71 to 99.05% as the number of cordons rises from one to four.

The one cordon location is at 4.1903 km from the CBD. The two cordon locations are at 2.3425 and 8.9438 km from the CBD. With more cordons, the density of cordon locations is higher in the area close to the CBD because the traffic volume is greater there.

The toll is 11.3775 min. for every resident for common optimum, and 13.1578 min. for each resident outside the cordon location for one-cordon pricing. However, more cordons will lower the toll in each cordon location. For four cordons, the toll is 4.5590, 4.1212, 3.5880, and 2.8508 min. for the four cordon locations. From the perspective of equity, more cordons provide a higher level of equity. For one cordon, the difference between the residents inside and outside of the cordon line is 13.1578 min. There exists a certain cross-subsidy effect in the model. This means that some residents pay more than

[12] The unit of social net benefit and level of toll is minutes instead of a monetary unit. It can be multiplied by the value of time to yield the monetary unit.

[13] The value for the analysis is under normalization. For the Taipei metropolis, the number of residents is 4,642,879 in 2012 (assuming half of residents in New Taipei city commute to the CBD in Taipei). Nearly 60% of commuting trips are by private vehicles. The normalization multiplier is thus equal to 4,642,879 (60%)/26.3 = 105,921. Time value is NT$ 4.85 per minute. The social surplus is thus NT$ 47,891,870 with no toll.

[14] In this case, even one-cordon pricing provides good performance. Are the linear demand function and linear unit cost function responsible for this result? If the demand function follows the law of demand (an increase in price will decrease the quantity of demand), the performance with cordon pricing will not change too much. In addition, though the cost of driving every unit distance is linear in traffic flow, the cost function of any trip is nonlinear in distance of the trip (see footnote 3). Thus, the different function form of demand and cost will not make a large change in the outcome. The continuous spatial distribution of a single transportation corridor may be largely responsible for this result.

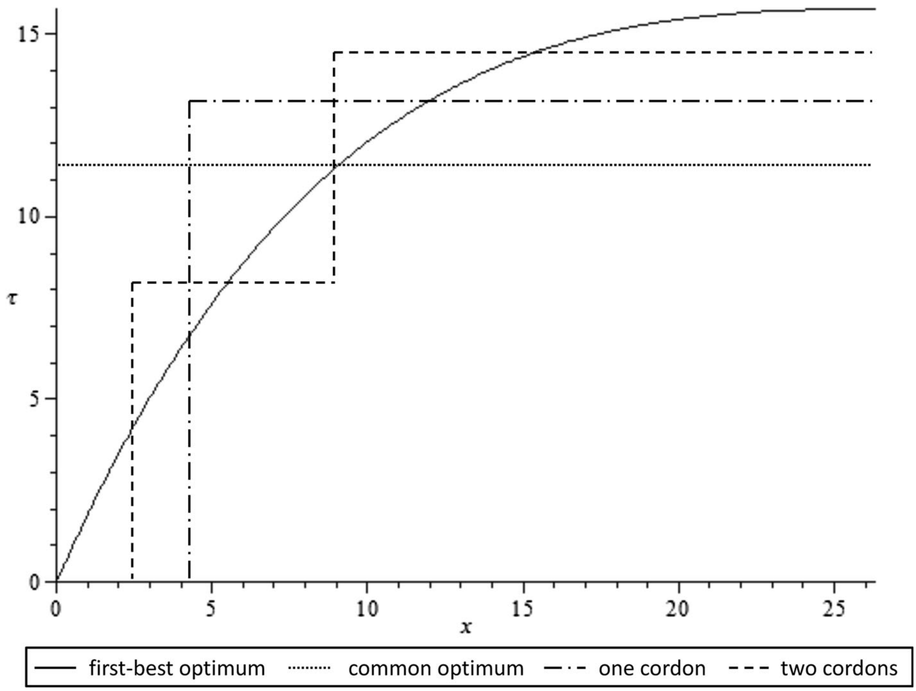

Fig. 2 Toll levels of first-best optimum, common optimum, and cordon(s) pricing

the external costs they generate, while the other road users pay less than the external costs they incur or pay no toll. For one cordon, the residents located between 4.19 and 11.95 km pay more than the external costs they generate, while the residents between 11.95 and 26.3 km (city boundary) pay less than the external costs they generate, and the residents between the CBD and 4.19 km pay no toll (see Fig. 2). Thus, this equity problem exists in the cordon pricing scheme. However, the scheme with more cordons provides less difference between the residents who pay more and those who pay less than the external costs (see Figs. 3, 4). Specifically, in the scheme with more cordons, the residents pay the toll close to the external costs they generate. This can be shown by the percentage of FI. It is 68.49% for common optimum, 79.79% for one cordon, and 93.97% for four cordons. However, the residents who are levied higher than the external costs they generate may resist the tolling. For instance, the residents located between the CBD and 9.0188 km (34.29% of total road users) may resist the common optimum pricing, while those located between 4.1903 and 11.9466 km (29.41% of total road users) may resist the one-cordon pricing. The AR decreases as more cordons are used.[15] However, this ratio may not reflect the real acceptance exactly, especially for the scheme of a large number of cordons. Specifically, the acceptance may greatly increase for the cordon pricing with many cordons.

[15] The AR may be underestimated for the tolling schemes of more cordons. For a reasonable guess, AR will approach only 50% as the tolling is for infinite cordons. However, the toll level is very close to the external cost even though half of the users are charged a little bit higher than the external costs.

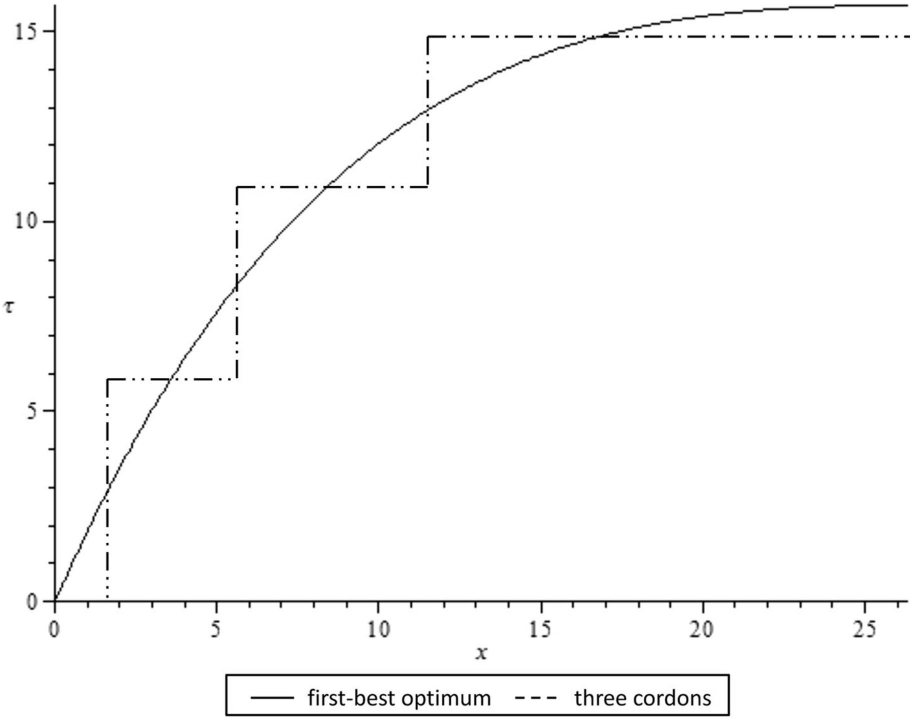

Fig. 3 Toll levels of first-best optimum and three-cordon pricing

The number of trips for one resident originating from location x with no toll, first-best optimum, common optimum and one cordon are shown in Figs. 5, 6, 7 and 8.[16] In Fig. 5, the curve for trip number is discontinuous at the cordon location. The trips from the area outside the cordon location will decrease, and those from the area inside the cordon location will increase when the trips are compared with the scheme with no toll. Compared with the first-best optimum congestion pricing, the number of trips from the area inside the cordon location is larger than that of the first-best optimum. However, for the area outside the cordon location, the number of trips from the area near the cordon location is smaller than that of the first-best optimum because the residents pay higher travel costs than the external costs they generate. Alternately, the number of trips from the area near the suburb is larger than that of the first-best optimum because the road users pay a lower travel cost than the external cost they generate. For the scheme with two cordons, the trip curve consists of two discontinuous gaps at the two cordon locations. The pattern of the curve is similar to that of the scheme with one cordon. For the schemes with more than two cordons, the trip curve is much closer to that of the first-best optimum (see the scheme with four cordons in Fig. 8).

The traffic flows at location x with no toll, first-best optimum, and cordon pricing are shown in Figs. 9 and 10.[17] The scheme of cordon pricing decreases the traffic flow by a

[16] Note that the normalization multiplier 105,291 needs to be multiplied by the number of trips for one resident on these figures to obtain the total trips for the corridor.

[17] The traffic flow Q(x) in the figure has to multiply the normalization multiplier 105,291 (see footnote 12). Specifically, Q(5) = 2.5 means 263,228 vehicles/day passing the point 5 km from the CBD and commuting to the CBD.

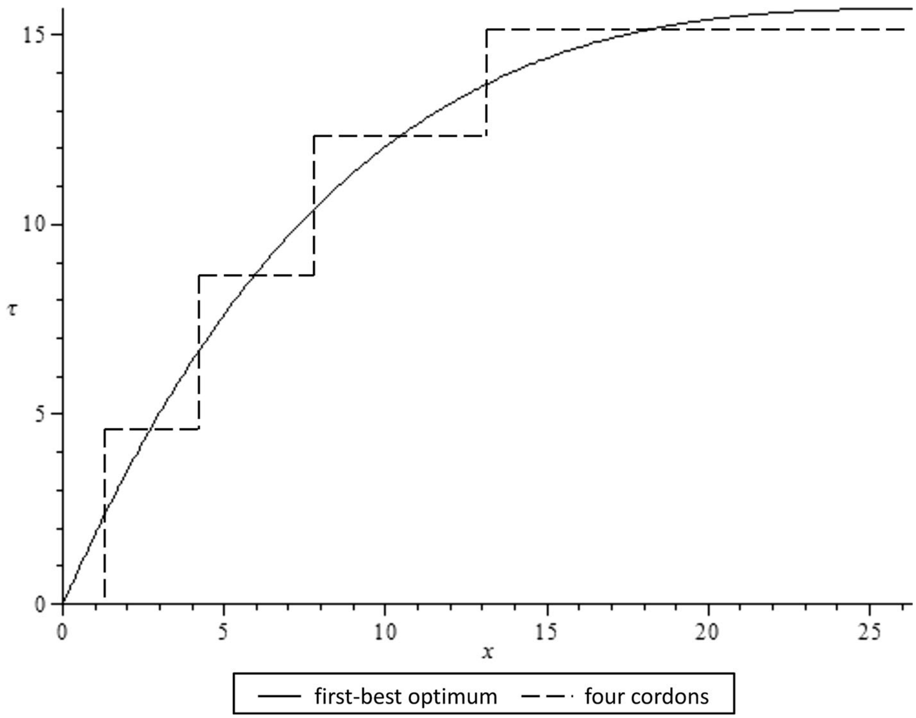

Fig. 4 Toll levels of first-best optimum and four-cordon pricing

significant amount, and results in traffic flow close to that of the first-best optimum, especially in the scheme with four cordons.

Sensitivity analysis

This section explores the impact on social surplus, cordon location, tolling level, and RE when the values of the parameters in Sect. 3.1 increase 10% (see Table 2).

(1) Maximum willingness-to-pay, parameter a
When the value of parameter a increases, it means the willingness-to-pay increases, and this will cause the marginal benefit curve to shift upward. It thus increases both the equilibrium demand for trips and the social surplus in any scheme. In addition, any cordon location will be set farther from the CBD, and the level of toll at any cordon location increases to reflect the increase in trips.

(2) Slope of the inverse demand function, parameter b
As the value of parameter b increases, the social surplus in any scheme decreases. In addition, any cordon location will be set farther from the CBD, but tolling level at any cordon location decreases. The reason is that the marginal benefit decreases, thus decreasing the trip demand and traffic flow. This contributes to lower external cost, and thus decreases the level of toll for cordon pricing.

(3) City boundary, parameter B
The traffic is cumulated by the trips from continuous points on a corridor to the CBD. The city boundary is the end point of the corridor, and thus an increase in

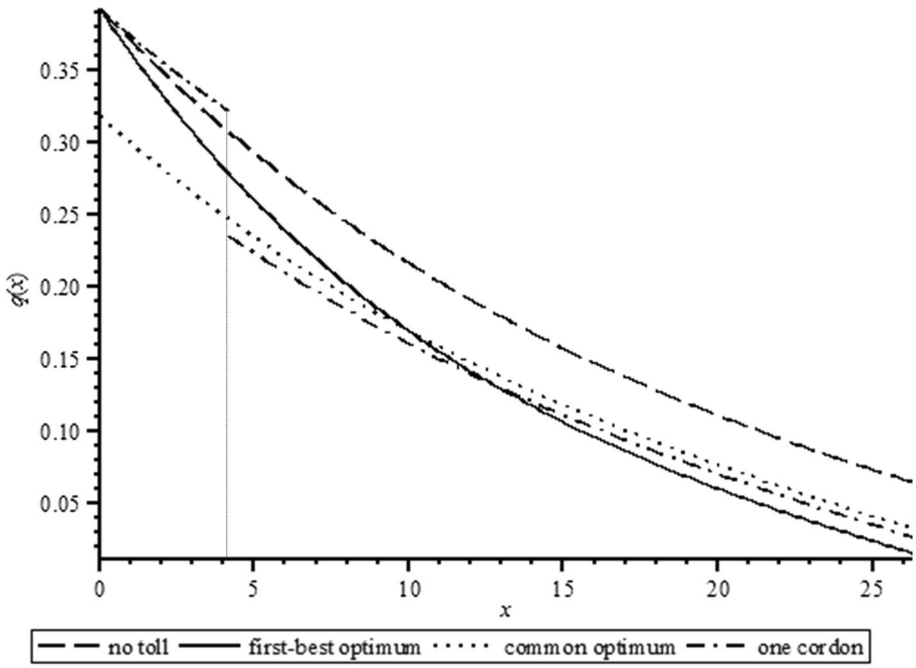

Fig. 5 Trips of a resident with no toll, first-best optimum, common optimum and one-cordon pricing

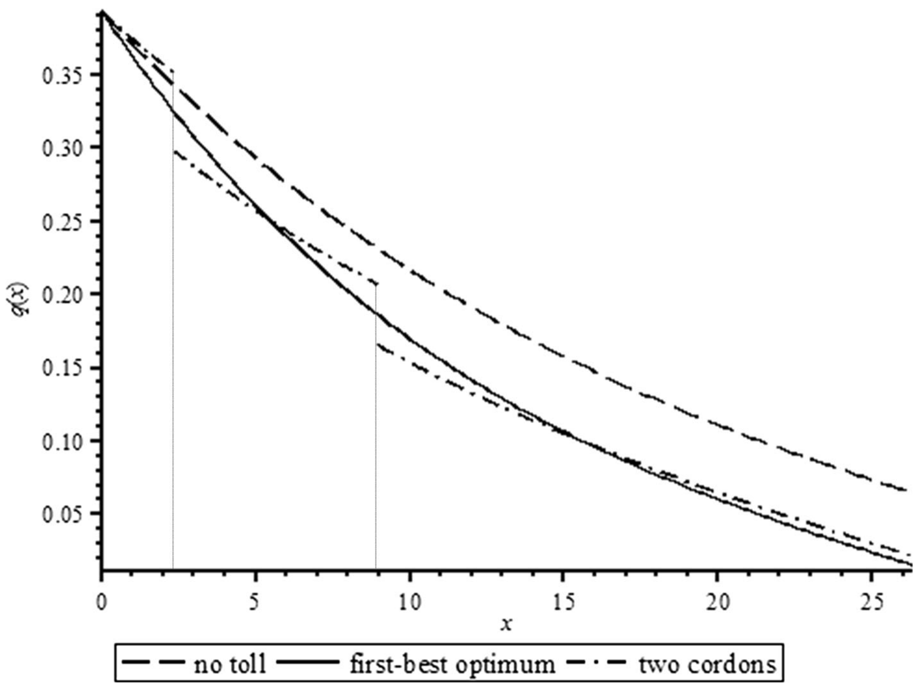

Fig. 6 Trips of a resident with no toll, first-best optimum and two-cordon pricing

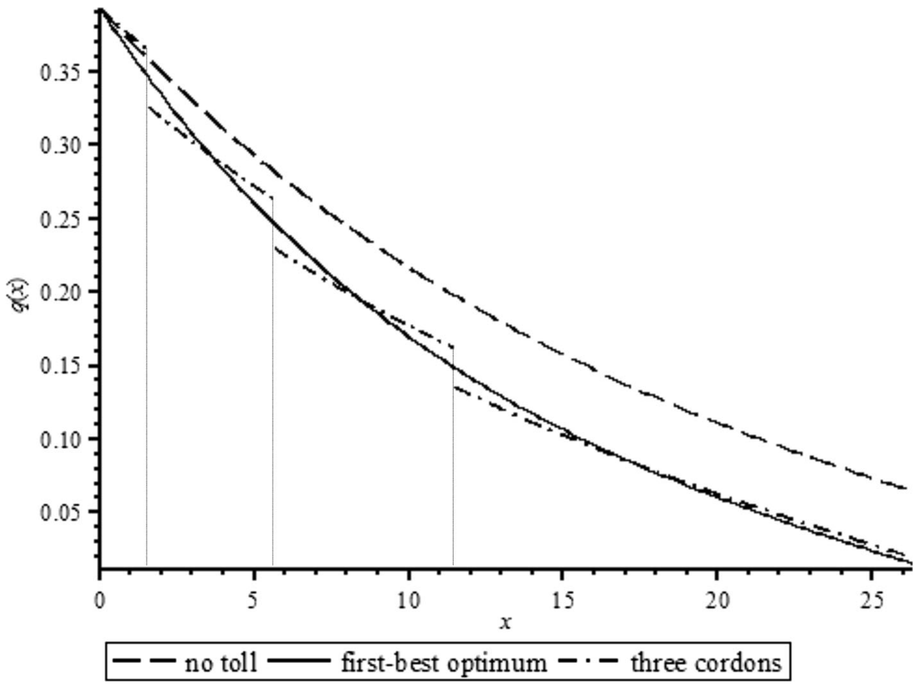

Fig. 7 Trips of a resident with no toll, first-best optimum and three-cordon pricing

B will incur more trips to the CBD. This increase in trip demand will cause an effect similar to an increase in willingness-to-pay. That is, the social surplus in any scheme increases. In addition, any cordon location will be set farther from the CBD, and toll at any cordon location increases.

(4) Marginal cost, parameter c

As marginal cost increases, trip demand decreases, while the external cost increases. The former causes a decrease in the social surplus in any scheme, and thus any cordon location will be set near the CBD. The latter causes an increase in the level of toll at any cordon location.

(5) Fixed cost, parameter f

Parameter f represents the fixed cost with respect to traffic volume. An increase in fixed cost will increase the total costs while keeping the marginal cost unchanged. The trip demand thus decrease, inducing a decrease in the social surplus in any scheme, and any cordon location will be set near the CBD. The decrease in trip demand also induces an indirect effect of reducing the external costs. This thus causes a decrease in the level of toll at any cordon location.

Comparison with related studies

The RE of cordon pricing for Taipei Metropolis is very high. It is 93.71% for one-cordon pricing, which is very close to that (93.98%) in Mun et al. (2003). This RE increases to 99.05% for four-cordon pricing. This means the cordon pricing, especially the multiple-cordon scheme, is a good substitute for the first-best optimum. However, the previous

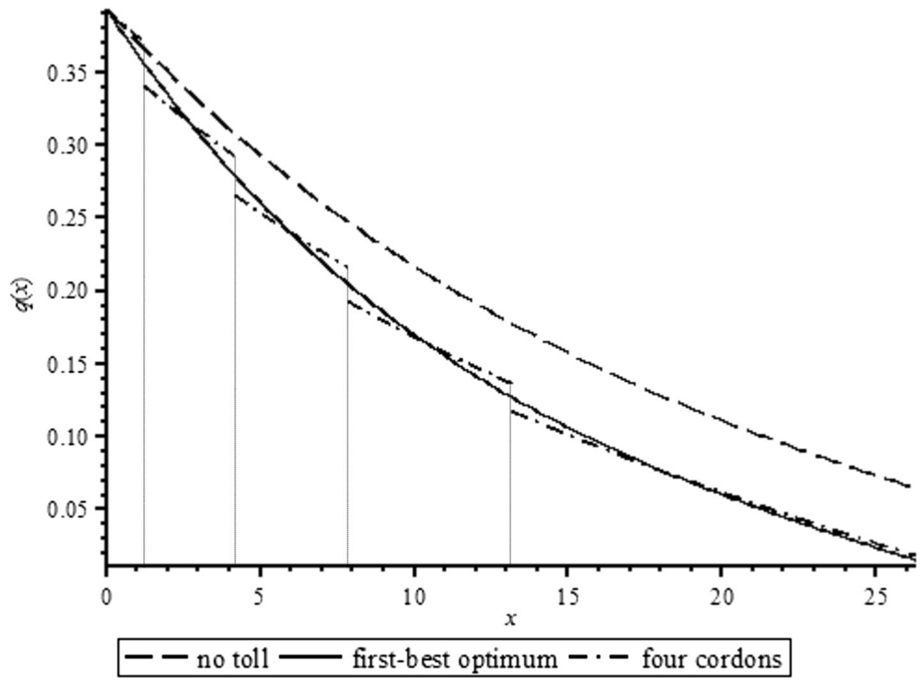

Fig. 8 Trips of a resident with no toll, first-best optimum and four-cordon pricing

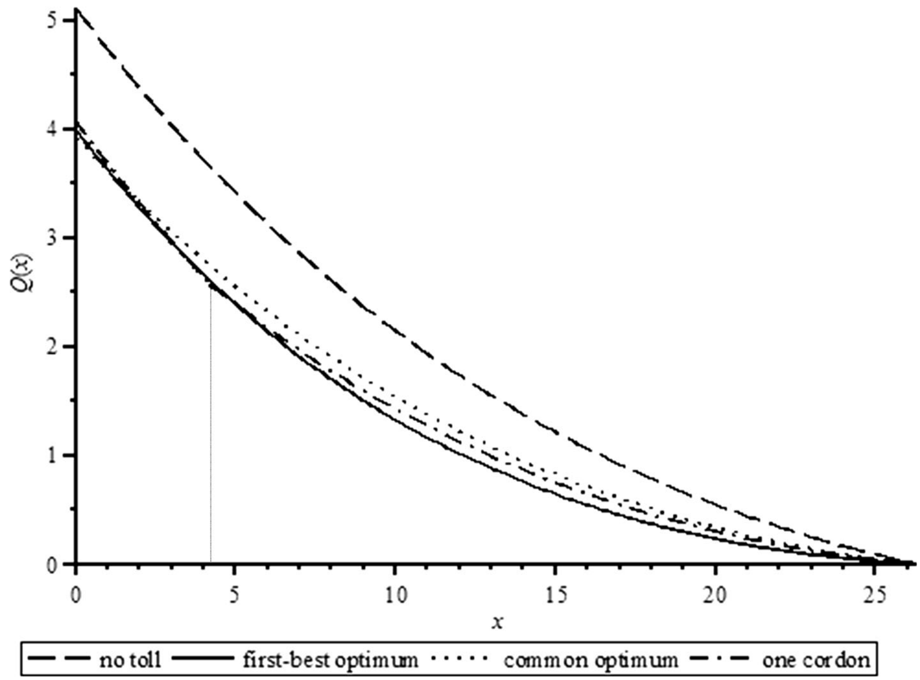

Fig. 9 Traffic flow with no toll, first-best optimum and one-cordon pricing

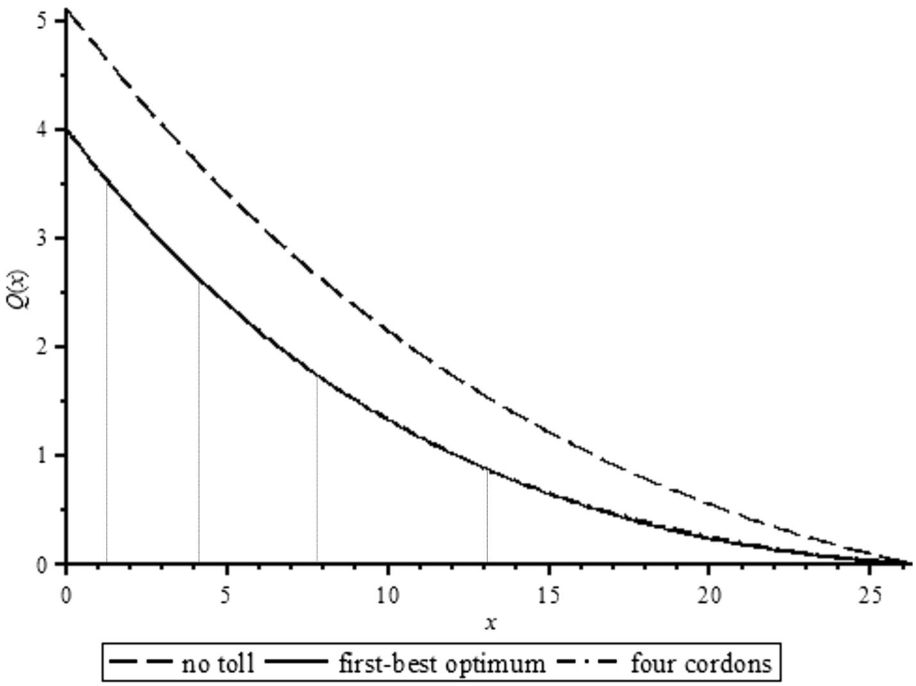

Fig. 10 Traffic flow with no toll, first-best optimum and four-cordon pricing

studies on cordon pricing yield quite different values of RE, which may be due to differences in spatial continuity, network setting, and urban configuration.

The trip demand function in our model (from Mun et al. 2003) is for multiple origins to the same destination (the CBD) on one-dimensional continuous space. The city is densely populated, with a radial transportation road system from the CBD, which is at the city center. The continuous space is more consistent with the real distribution for a metropolis. Cordon pricing can set the exact location for tolling to pursue the maximal surplus and reach a high RE due to the continuous space setting. For a two-dimensional continuous space, the four-cordon pricing reaches 99.54% RE for the situation in which all commuters travel to the CBD, which is not at the center of the space (Ho et al. 2005).[18]

Most studies introducing a network for cordon pricing have abandoned two settings of Mun et al. (2003). One is that the demand is based on continuous space. The other is that all commuters travel to the same destination (the CBD). The first is due to simplification of the trip demand pattern, which changed the trip distribution on continuous space to trips on O–D pairs between zonal centroids. The continuous space setting for trip demand cannot thus be sustained. The trip costs from different origins on the same link cannot be reflected in the demand for trips in an O–D pair setting. The second is because the trips of O–D pairs mean multiple destinations instead of only one destination (the CBD). Some road users can thus make a route choice to avoid the tolling by cordon pricing and decrease the RE (Zhang and Yang 2004). However, these road users may not easily avoid the tolling by route choice if these trips have the same destination (the CBD). The change for these two

[18] We found the relative efficiency from their net benefit. This high relative efficiency is obtained even though they set an upper bound constraint for the toll level of an initial-cordon scenario.

Table 2 Comparison of social surplus with 10% increase in each parameter

		NT	FB	CO	1C	2C	3C		4C	
a	CL	–	–	–	4.2619	2.3870	1.6596	5.7853	1.2723	4.2587
						9.1075	11.7134		8.0384	13.4117
	Toll	–	–	12.9237	14.9839	9.2392	6.6620	5.7790	5.2063	4.7064
						7.2487	4.5775		4.1003	3.2087
	SS	115.0257	129.3537	127.3167	128.4678	129.0596	129.2098		129.2695	
	RE		100.00%	85.78%	93.82%	97.95%	99.00%		99.41%	
b	CL	–	–	–	4.2847	2.3975	1.6663	5.7965	1.2772	4.2685
						9.1190	11.7101		8.0438	13.3970
	Toll	–	–	11.9028	12.7352	7.8329	5.6426	4.9033	4.4077	3.9898
						6.1600	3.8901		3.4809	2.7776
	SS	88.2830	97.5669	96.2212	96.9834	97.3722	97.4706		97.5096	
	RE	–	100.00%	85.51%	93.71%	97.90%	98.96%		99.38%	
B	CL	–	–	–	4.2964	2.3906	1.6569	5.7945	1.2679	4.2520
						9.1881	11.8200		8.0514	13.5390
	Toll	–	–	11.6565	13.4182	8.2519	5.9382	5.1327	4.6341	4.1809
						6.4138	4.0188		3.6267	2.8526
	SS	92.2160	103.9248	102.2701	103.2106	103.2436	103.8086		103.8559	
	RE	–	100.00%	85.87%	93.90%	94.18%	99.01%		99.41%	
c	CL	–	–	–	4.0917	2.2852	1.5868	5.5429	1.2156	4.0742
						8.7569	11.2724		7.7060	12.9169
	Toll	–	–	11.7387	13.5643	8.3577	6.0225	5.2126	4.7043	4.2470
						6.5223	4.1021		3.6902	2.9198
	SS	89.4303	100.5434	98.9346	99.8447	100.3106	100.4283		100.4750	
	RE	–	100.00%	85.52%	93.71%	97.91%	98.96%		99.38%	

Table 2 continued

		NT	FB	CO	1C	2C	3C		4C	
f	CL	–	–	–	4.1082	2.2914	1.5903	5.5428	1.2179	4.0752
						8.7522	11.2458		7.6946	12.8732
	Toll	–	–	10.9691	12.6486	7.7710	5.5935	4.8051	4.3667	3.9477
						6.0794	3.8231		3.4353	2.7209
	SS	90.8607	100.7577	99.3549	100.1522	100.5568	100.6586		100.6988	
	RE	–	100.00%	85.83%	93.88%	97.97%	99.00%		99.40%	

CL cordon location, *SS* social surplus, *RE* relative efficiency, *NT* no toll, *FB* first-best optimum, *CO* common optimum, *1C* one cordon, *2C* two cordons, *3C* three cordons, *4C* four cordons settings will reduce the power of the tolling of cordon pricing. For example, Akiyama et al. (2004) use the trip demand pattern by O–D pairs to analyze the cordon pricing for the Osaka metropolitan area. There are 630 links, 241 nodes, and 36 zones for the analysis in this area. The RE is 65.5% for one-cordon pricing and 76.8% for three-cordon pricing. Zhang and Yang (2004) analyze the cordon with a network setting for Shanghai metropolis, the largest city in China. There are 690 links, 187 nodes, and 30 zones. The RE fell to 28.30% for one-cordon pricing and 41.51% for two-cordon pricing. These two studies have very different RE because these two metropolitan areas have a large difference in population numbers, with similar settings for a network.[19] Abandoning continuous space in Shanghai metropolis thus caused a larger impact on the efficiency of cordon pricing.

The urban configuration has been incorporated into congestion pricing recently. Households distribute their incomes to consume composite goods, housing (or land) after paying the commuting costs to travel to the CBD in a monocentric city. The bid-rent function can be derived to determine the city boundary in a closed city model. Various congestion pricing regimes can thus influence the commuting costs and thus the consumption of land and bid-rent, and can shrink the city boundary. This kind of model keeps the two settings in Mun et al. (2003) which were abandoned in the network models above. The RE is higher than that from the models with a network. Verhoef (2005) found 88% RE for cordon pricing by considering leisure time as a variable to reflect the effective work time (and the commuting time), and Tikoudis et al. (2015) found 84.3% RE by extending the model of Verhoef (2005) to consider tax revenue recycling to finance labor tax cuts. De Palma et al. (2011) found 63 and 73% RE for one-cordon and two-cordon pricing regimes by considering the transport cost depending on road occupancy, which is defined by the ratio of the number of households located further away from the center to the amount of land devoted to transport use.

Conclusions

The second-best congestion pricing schemes including common optimum, one cordon, and multiple cordons schemes are compared with the first-best optimum pricing scheme.

There exists a certain cross-subsidy effect in these second-best pricing models. Specifically, some tolled road users will pay a higher toll than the externality they really incur, while the other road users will pay less than the external costs they incur or pay no toll. This effect causes the former to make inefficiently too few trips and the latter inefficiently too many trips. However, the scheme with more cordons will diminish the cross-subsidy and approach an efficient and equitable outcome. That is, the toll and the social surplus under the first-best optimum pricing scheme can be closely approached as the number of cordons increases in cordon pricing in the case study for the Taipei metropolis.

The relative efficiency (RE) of the cordon pricing schemes, which represents an improvement in social surplus, compared with the first-best optimum pricing scheme for the case of Taipei metropolis is very high. Moreover, the single cordon yields excellent performance of 93% RE. This is consistent with the previous studies which have similar model settings. There might be some factors causing this performance. First, the traffic condition is very uncongested in this case. Second, the linear unit distance cost in traffic flow forming a nonlinear cost function provides a good estimate. Third, the trip demands

[19] Shanghai metropolis had population of 34,000,000 residents, while Osaka metropolis had 19,342,000 in 2010.

with continuous space and the same destination (the CBD) in the network fits the trip pattern in Taipei metropolis well.

There are some major assumptions employed for the cordon models in this study.

The first is the same destination (the CBD) for all trips mentioned above. For a city with a more dispersed trip pattern, it will be difficult for the cordon pricing to levy on the residents the exact external costs they generate. A survey of actual trip patterns may provide real data to help set the locations of cordons. The second is the uniform population density on space. This assumption may not hold for a large city in which the land rent is decreasing in the distance from the CBD. That is, population density will be higher in the area closer to the CBD. In this case, the external costs will be higher for the residents near the CBD compared with the case of uniform population density. It is expected that the toll level for the residents near the CBD will increase and the locations of cordons will be set near to the CBD. Third, the trips by private vehicle are assumed to use automobiles only. In an Asian city, many residents may use motorcycles as a commuting mode. However, the current model can be modified to include these two types of trips. Fourth, a linear unit distance cost function in traffic flow is used for the model. The assumption means that the cost function for a resident will be nonlinear in distance due to the cumulative traffic flow based on the continuous space setting. Thus, it doesn't seem to be a bad method of estimation for the trip costs under the continuous space setting.

There are at least three avenues for future research. First, this model can be extended to a city with multiple CBD rather than a monocentric city structure. Second, multiple transportation modes including automobiles, motorcycles and buses can be incorporated into the model to analyze the case of mixed traffic flow. Third, the approach of dynamic traffic flow can be employed for this model to internalize the congestion externalities.

Acknowledgements The authors wish to thank the Editor and the anonymous referees for very helpful comments in the paper revision.

Appendix 1

The following boundary conditions are used for determining the values of the six unknowns above:

(1) The condition with the CBD ($x = 0$)
When the location is the CBD, the following relations hold:

$$p\left(q_1^{**}(0)\right) = C_1(0) = 0, \tag{15a}$$

$$p' \frac{dq_1^{**}(0)}{dx} - t(Q_1(0)) = 0. \tag{15b}$$

(2) The condition with the first cordon location ($x = x_{m1}$)
When the location is the first cordon location, combining (11a) and (11b) with the trip cost condition,$C_1(x_{m1}) = C_2(x_{m1})$, yields:

$$p\left(q_1^{**}(x_{m1})\right) = p\left(q_2^{**}(x_{m1})\right) - \tau_1. \tag{16}$$

(3) The condition with the second cordon location ($x = x_{m2}$)
When the location is the second cordon location, combining (11b) and (11c) with the trip cost condition, $C_2(x_{m2}) = C_3(x_{m2})$, yields:

$$p\left(q_2^{**}(x_{m2})\right) = p\left(q_3^{**}(x_{m2})\right) - \tau_2. \tag{17}$$

(4) The condition with city boundary ($x = B$)
When the location is at the city boundary, the condition that marginal benefit equals the trip cost and the derivative for this condition must hold:

$$p\left(q_3^{**}(B)\right) = C_3(B) + \tau_1 + \tau_2, \tag{18a}$$

$$p' \frac{dq_3^{**}(B)}{dx} - t(Q_3(B)) = 0. \tag{18b}$$

From the above conditions, (15a–18b), the six unknown constants are determined as follows:

$$\lambda_1 = \frac{-2f/\alpha + 2ae^{-\alpha B} + \tau_1\left(e^{\alpha(B-x_{m1})} - e^{-\alpha(B-x_{m1})}\right) + \tau_2\left(e^{\alpha(B-x_{m2})} - e^{-\alpha(B-x_{m2})}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)},$$

$$\lambda_2 = \frac{2f/\alpha + 2ae^{\alpha B} - \tau_1\left(e^{\alpha(B-x_{m1})} - e^{-\alpha(B-x_{m1})}\right) - \tau_2\left(e^{\alpha(B-x_{m2})} - e^{-\alpha(B-x_{m2})}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)},$$

$$\lambda_3 = \frac{-2f/\alpha + 2ae^{-\alpha B} - \tau_1\left(e^{-\alpha(B-x_{m1})} + e^{-\alpha(B+x_{m1})}\right) + \tau_2\left(e^{\alpha(B-x_{m2})} - e^{-\alpha(B-x_{m2})}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)},$$

$$\lambda_4 = \frac{2f/\alpha + 2ae^{-\alpha B} - \tau_1\left(e^{\alpha(B-x_{m1})} + e^{\alpha(B+x_{m1})}\right) - \tau_2\left(e^{\alpha(B-x_{m2})} - e^{-\alpha(B-x_{m2})}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)},$$

$$\lambda_5 = \frac{-2f/\alpha + 2ae^{-\alpha B} - \tau_1\left(e^{-\alpha(B-x_{m1})} + e^{-\alpha(B+x_{m1})}\right) - \tau_2\left(e^{-\alpha(B-x_{m2})} - e^{-\alpha(B+x_{m2})}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)},$$

$$\lambda_6 = \frac{2f/\alpha + 2ae^{-\alpha B} - \tau_1\left(e^{\alpha(B-x_{m1})} + e^{\alpha(B+x_{m1})}\right) - \tau_2\left(e^{\alpha(B-x_{m2})} - e^{\alpha(B+x_{m2})}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)}.$$

Appendix 2

$$\frac{\partial q_1^{**}(x)}{\partial x_{m1}} = \frac{-\tau_1\alpha\left(e^{-\alpha(-x+B-x_{m1})} - e^{-\alpha(x+B-x_{m1})} + e^{\alpha(x+B-x_{m1})} - e^{\alpha(-x+B-x_{m1})}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)} < 0, \quad \textit{for } 0 \le x \le x_{m1}, \tag{19a}$$

$$\frac{\partial q_2^{**}(x)}{\partial x_{m1}} = \frac{-\tau_1\alpha\left(e^{-\alpha(-x+B-x_{m1})} - e^{-\alpha(-x+B+x_{m1})} + e^{\alpha(-x+B+x_{m1})} - e^{\alpha(-x+B-x_{m1})}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)} < 0, \quad \textit{for } x_{m1} \le x \le x_{m2}, \tag{19b}$$

$$\frac{\partial q_3^{**}(x)}{\partial x_{m1}} = \frac{-\tau_1\alpha\left(e^{-\alpha(-x+B-x_{m1})} - e^{-\alpha(-x+B+x_{m1})} + e^{\alpha(-x+B+x_{m1})} - e^{\alpha(-x+B-x_{m1})}\right)}{2b\left(e^{\alpha B} + e^{-\alpha B}\right)} < 0, \quad \textit{for } x_{m2} \le x \le B, \tag{19c}$$

$$\frac{\partial q_1^{**}(x)}{\partial x_{m2}} = \frac{-\tau_2\alpha\left(e^{-\alpha(-x+B-x_{m2})} - e^{-\alpha(x+B-x_{m2})} + e^{\alpha(x+B-x_{m2})} - e^{\alpha(-x+B-x_{m2})}\right)}{2b(e^{\alpha B} + e^{-\alpha B})} < 0, \quad \textit{for } 0 \le x \le x_{m1}, \tag{20a}$$

$$\frac{\partial q_2^{**}(x)}{\partial x_{m2}} = \frac{-\tau_2\alpha\left(e^{-\alpha(-x+B-x_{m2})} - e^{-\alpha(x+B-x_{m2})} + e^{\alpha(x+B-x_{m2})} - e^{\alpha(-x+B-x_{m2})}\right)}{2b(e^{\alpha B} + e^{-\alpha B})} < 0, \quad \textit{for } x_{m1} \le x \le x_{m2}, \tag{20b}$$

$$\frac{\partial q_3^{**}(x)}{\partial x_{m2}} = \frac{-\tau_2\alpha\left(e^{-\alpha(-x+B-x_{m2})} - e^{-\alpha(-x+B+x_{m2})} + e^{\alpha(-x+B+x_{m2})} - e^{\alpha(-x+B-x_{m2})}\right)}{2b(e^{\alpha B} + e^{-\alpha B})} < 0, \quad \textit{for } x_{m2} \le x \le B \tag{20c}$$

$$\frac{\partial q_1^{**}(x)}{\partial \tau_1} = \frac{\left(e^{-\alpha(x+B-x_{m1})} - e^{-\alpha(-x+B-x_{m1})} + e^{\alpha(x+B-x_{m1})} - e^{\alpha(-x+B-x_{m1})}\right)}{2b(e^{\alpha B} + e^{-\alpha B})} > 0, \quad \textit{for } 0 \le x \le x_{m1}, \tag{21a}$$

$$\frac{\partial q_2^{**}(x)}{\partial \tau_1} = \frac{-\left(e^{-\alpha(-x+B-x_{m1})} + e^{-\alpha(-x+B+x_{m1})} + e^{\alpha(-x+B+x_{m1})} + e^{\alpha(-x+B-x_{m1})}\right)}{2b(e^{\alpha B} + e^{-\alpha B})} < 0, \quad \textit{for } x_{m1} \le x \le x_{m2}, \tag{21b}$$

$$\frac{\partial q_3^{**}(x)}{\partial \tau_1} = \frac{-\left(e^{-\alpha(-x+B-x_{m1})} + e^{-\alpha(-x+B+x_{m1})} + e^{\alpha(-x+B+x_{m1})} + e^{\alpha(-x+B-x_{m1})}\right)}{2b(e^{\alpha B} + e^{-\alpha B})} < 0, \quad \textit{for } x_{m2} \le x \le B, \tag{21c}$$

$$\frac{\partial q_1^{**}(x)}{\partial \tau_2} = \frac{\left(e^{-\alpha(B+x-x_{m2})} - e^{-\alpha(B-x-x_{m2})} + e^{\alpha(B+x-x_{m2})} - e^{\alpha(B-x-x_{m2})}\right)}{2b(e^{\alpha B} + e^{-\alpha B})} > 0, \quad \textit{for } 0 \le x \le x_{m1}, \tag{22a}$$

$$\frac{\partial q_2^{**}(x)}{\partial \tau_2} = \frac{\left(e^{-\alpha(B+x-x_{m2})} - e^{-\alpha(B-x-x_{m2})} + e^{\alpha(B+x-x_{m2})} - e^{\alpha(B-x-x_{m2})}\right)}{2b(e^{\alpha B} + e^{-\alpha B})} > 0, \quad \textit{for } x_{m1} \le x \le x_{m2}, \tag{22b}$$

$$\frac{\partial q_3^{**}(x)}{\partial \tau_2} = \frac{-\left(e^{-\alpha(-x+B-x_{m2})} + e^{-\alpha(-x+B+x_{m2})} + e^{\alpha(-x+B+x_{m2})} + e^{\alpha(-x+B-x_{m2})}\right)}{2b(e^{\alpha B} + e^{-\alpha B})} < 0, \quad \textit{for } x_{m2} \le x \le B. \tag{22c}$$

Appendix 3

Substituting trip in (13a–13c) into (14), the first-order conditions are:

$$\begin{aligned}
&\int_0^{q_1^{**}(x_{m1})} p(q)dq - C_1(x_{m1})q_1^{**}(x_{m1}) - \left[\int_0^{q_2^{**}(x_{m1})} p(q)dq - C_2(x_{m1})q_2^{**}(x_{m1})\right] \\
&\quad + \int_0^{x_{m1}} \left[p\left(q_1^{**}(x)\right) - C_1(x) - E(x)\right] \frac{\partial q_1^{**}(x)}{\partial x_{m1}} dx \\
&\quad + \int_{x_{m1}}^{x_{m2}} \left[p\left(q_2^{**}(x)\right) - C_2(x) - E(x)\right] \frac{\partial q_2^{**}(x)}{\partial x_{m1}} dx + \int_{x_{m2}}^{B} \left[p\left(q_3^{**}(x)\right)\right. \\
&\quad \left. - C_3(x) - E(x)\right] \frac{\partial q_3^{**}(x)}{\partial x_{m1}} dx = 0,
\end{aligned} \tag{23a}$$

$$\begin{aligned}
&\int_0^{q_2^{**}(x_{m2})} p(q)dq - C_2(x_{m2})q_2^{**}(x_{m2}) - \left[\int_0^{q_3^{**}(x_{m2})} p(q)dq - C_3(x_{m2})q_3^{**}(x_{m2})\right] \\
&\quad + \int_0^{x_{m1}} \left[p\left(q_1^{**}(x)\right) - C_1(x) - E(x)\right] \frac{\partial q_1^{**}(x)}{\partial x_{m2}} dx \\
&\quad + \int_{x_{m1}}^{x_{m2}} \left[p\left(q_2^{**}(x)\right) - C_2(x) - E(x)\right] \frac{\partial q_2^{**}(x)}{\partial x_{m2}} dx \\
&\quad + \int_{x_{m2}}^{B} \left[p\left(q_3^{**}(x)\right) - C_3(x) - E(x)\right] \frac{\partial q_3^{**}(x)}{\partial x_{m2}} dx = 0,
\end{aligned} \tag{23b}$$

$$\begin{aligned}
&\int_0^{x_{m1}} \left[p\left(q_1^{**}(x)\right) - C_1(x) - E(x)\right] \frac{\partial q_1^{**}(x)}{\partial \tau_1} dx + \int_{x_{m1}}^{x_{m2}} \left[p\left(q_2^{**}(x)\right) - C_2(x) - E(x)\right] \frac{\partial q_2^{**}(x)}{\partial \tau_1} dx \\
&\quad + \int_{x_{m2}}^{B} \left[p\left(q_3^{**}(x)\right) - C_3(x) - E(x)\right] \frac{\partial q_3^{**}(x)}{\partial \tau_1} dx = 0,
\end{aligned} \tag{23c}$$

$$\begin{aligned}
&\int_0^{x_{m1}} \left[p\left(q_1^{**}(x)\right) - C_1(x) - E(x)\right] \frac{\partial q_1^{**}(x)}{\partial \tau_2} dx + \int_{x_{m1}}^{x_{m2}} \left[p\left(q_2^{**}(x)\right) - C_2(x) - E(x)\right] \frac{\partial q_2^{**}(x)}{\partial \tau_2} dx \\
&\quad + \int_{x_{m2}}^{B} \left[p\left(q_3^{**}(x)\right) - C_3(x) - E(x)\right] \frac{\partial q_3^{**}(x)}{\partial \tau_2} dx = 0,
\end{aligned} \tag{23d}$$

$$E(x) = \begin{cases} \int_0^x t'(Q_1(y))Q_1(y)dy, \quad 0 \le x \le x_{m1} \\ \int_0^{x_{m1}} t'(Q_1(y))Q_1(y)dy + \int_{x_{m1}}^{x} t'(Q_2(y))Q_2(y)dy, \quad x_{m1} \le x \le x_{m2} \\ \int_0^{x_{m1}} t'(Q_1(y))Q_1(y)dy + \int_{x_{m1}}^{x_{m2}} t'(Q_2(y))Q_2(y)dy + \int_{x_{m2}}^{B} t'(Q_3(y))Q_3(y)dy, \quad x_{m2} \le x \le B. \end{cases} \tag{24}$$

$E(x)$ represents the externality incurred from an additional trip from x on all drivers using the road between 0 and x. The first two parts of (23a) represent the direct effect on total surplus caused by the outward move of the first cordon location. The other three parts of (23a) represent the indirect effect on total surplus via the change in trips on the three sections of the road by an outward move for the first cordon location. Similarly, (23b) represents the direct effect and indirect total surplus caused by an outward move for the second cordon location. In addition, (23c) and (23d) represent the indirect effect on total surplus via the change in the trips on the three sections of the road by a unit increase of toll at the first cordon location and at the second cordon location, respectively.

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