Linear Algebra Preparation for Emergent Neural Network Architectures (LAPENNA)

Unit 6

Convolutional neural Network

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Julian Halloy, Stephen Qiu, Eric Zhao

March 10, 2022

University of Tennessee, Knoxville

Acknowledgements:

- Support from NSF, UTK, JICS, ICL, NICS
- LAPENNA, www.jics.utk.edu/lapenna, NSF award #202409
- www.icl.utk.edu, cfdlab.utk.edu, www.xsede.org, www.jics.utk.edu/recsem-reu,
- MagmaDNN is a project grown out from the RECSEM REU Summer program supported under NSF award #1659502
- Source code: www.bitbucket.org/icl/magmadnn
- www.bitbucket.org/cfdl/opendnnwheel

INNOVATIVE COMPUTING LABORATORY

Overview of LAPENNA

The major goal of The Linear Algebra Preparation for Emergent Neural Network Architectures (LAPENNA) program is to provide essential knowledge to advance literacy in AI to sustain the growth and development of the workforce in the cyberinfrastructure (CI) ecosystem. This program aims to prepare college faculty, researchers, and industrial practitioners to design, enable and direct their own course curricula, collaborative projects, and training programs for in-house data-driven sciences programs. The LAPENNA program focuses on delivering computational techniques, numerical algorithms and libraries, and implementation of AI software on emergent CPU and GPU platforms.

Ecosystem Linear Algebra Preparation for Emergent Neural Network Architectures (LAPENNA)

Modeling, Numerical Linear Algebra, Data Analytics, Machine Learning, DNN, GPU, HPC

Session 1	Session 2	Session 3	Session 4
7/2020 - 12/2020	1/2021- 6/2021	7/2021 - 1/2022	1/2022 - 6/2022
16 participants	16 participants	16 participants	16 participants
Faculty/Students	Faculty/Students	Faculty/Students	Faculty/Students
10 webinars	10 webinars	10 webinars	10 webinars
4 Q & A	4 Q & A	4 Q & A	4 Q & A

Colleges courses, continuous integration, online courses, projects, software support

Web-based resources, tutorials, webinars, training, outreach

Teams and Participants of LAPENNA

- ✓ **PIs : Kwai Wong (JICS), Stan Tomov (ICL), University of Tennessee, Knoxville**
  - **Stephen Qiu, Julian Halloy, Eric Zhao (Students)**

- ✓ **Team : Clemson University**
- ✓ **Teams : University of Arkansas**
- ✓ **Team : University of Houston, Clear Lake**
- ✓ **Team : Miami University, Ohio**
- ✓ **Team : Boston University**
- ✓ **Team : West Virginia University**
- ✓ **Team : Louisiana State University, Alexandria**
- ✓ **Teams : Jackson Laboratory**
- ✓ **Team : Georgia State University**
- ✓ **Teams : University of Tennessee, Knoxville**
- ✓ **Teams : Morehouse College, Atlanta**
- ✓ **Team : North Carolina A & T University**
- ✓ **Team : Clark Atlanta University, Atlanta**
- ✓ **Team : Alabama A & M University**
- ✓ **Team : Slippery Rock University**
- ✓ **Team : University of Maryland, Baltimore County**

Meeting Time

- ✓ **Webinar Meeting time. Thursday 8:00 – 10:00 pm ET,**
- ✓ **Tentative schedule, www.jics.utk.edu/lapenna --> Spring 2022**

Topic: LAPENNA Spring 2022 Webinar
Time: Feb 3, 2022 08:00 PM Eastern Time (US and Canada)
Every week on Thu, 12 occurrence(s)
Feb 3, 2022 07:30 PM
Feb 10, 2022 07:30 PM
Feb 17, 2022 07:30 PM
Feb 24, 2022 07:30 PM
Mar 3, 2022 07:30 PM
Mar 10, 2022 07:30 PM
Mar 17, 2022 07:30 PM
Mar 24, 2022 07:30 PM
Mar 31, 2022 07:30 PM
Apr 7, 2022 07:30 PM
Apr 14, 2022 07:30 PM
Apr 21, 2022 07:30 PM

Join from PC, Mac, Linux, iOS or Android: https://tennessee.zoom.us/j/94140469394
Password: 708069

Schedule of LAPENNA Spring 2022
Thursday 8:00pm -10:00pm Eastern Time

The goal of The Linear Algebra Preparation for Emergent Neural Network Architectures (LAPENNA) program is to provide essential knowledge to advance literacy in AI to sustain the growth and development of the workforce in the cyberinfrastructure (CI) ecosystem for data-driven applications.

Month	Week	Date	Topics
February	Week 01	3	Logistics, High Performance Computing
	Week 02	10	Computational Ecosystem, Linear Algebra
	Week 03	17	Introduction to DNN, Forward Path (MLP)
	Week 04	24	Backward Path (MLP), Math, Example
March	Week 05	3	Backpro (MLP), CNN Computation
	Week 06	10	CNN Backpropagation, Example
	Week 07	17	CNN Network, Linear Algebra
	Week 08	24	Segmentation, Unet,
	Week 09	31	Object Detection, RC vehicle
April	Week 10	7	RNN, LSTEM, Transformers
	Week 11	14	DNN Computing on GPU
	Week 12	21	Overview, Closing
June or July	Workshop	To be arranged	4 Days In Person at UTK

Topics of Unit 6 - CNN

- ✓ **UNIT 6 : Convolutional Neural Network (CNN)**
  - **Overview Forward and Backpropagation of MLP**
  - **CNN computation**
  - **CNN forward path calculation**
  - **MNIST TensorFlow code**
  - **CIFAT 10 example**
  - **AlexNet, number of parameters**
  - **CNN Backward Calculations**
  - **Homework exercises**

Supervised learning is a type of machine learning which automate decision-making processes by generalizing from known examples. That is, the users provides the algorithm with pairs of inputs and desired outputs. The algorithm finds a way to produce the desired output given an input. In other words, we construct a model for the problem. The model is governed by some unknown parameters which we will learn from the data.

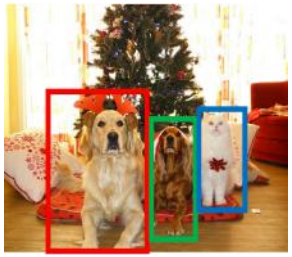
Supervised Learning Data:
(x, y) x is data, y is label

Goal:
Learn a function to map x -> y

Examples: Classification, regression, object detection, semantic segmentation, image captioning, etc.

CAT

Classification

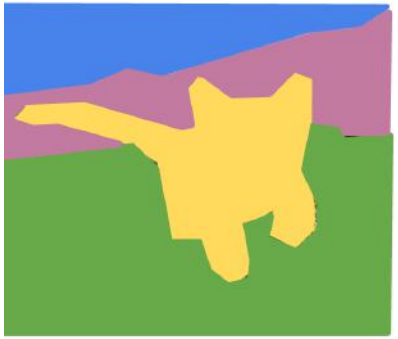


DOG, DOG, CAT

Object Detection

A cat sitting on a suitcase on the floor

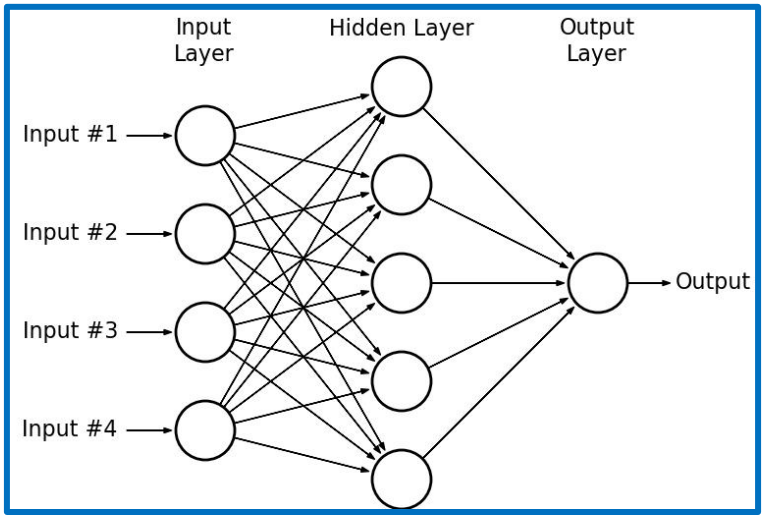
Image captioning

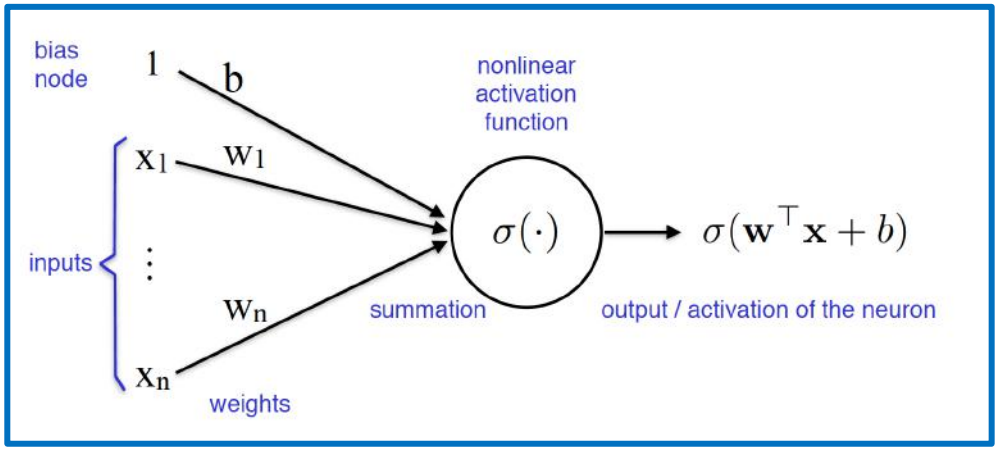


GRASS, CAT, TREE, SKY

Semantic Segmentation

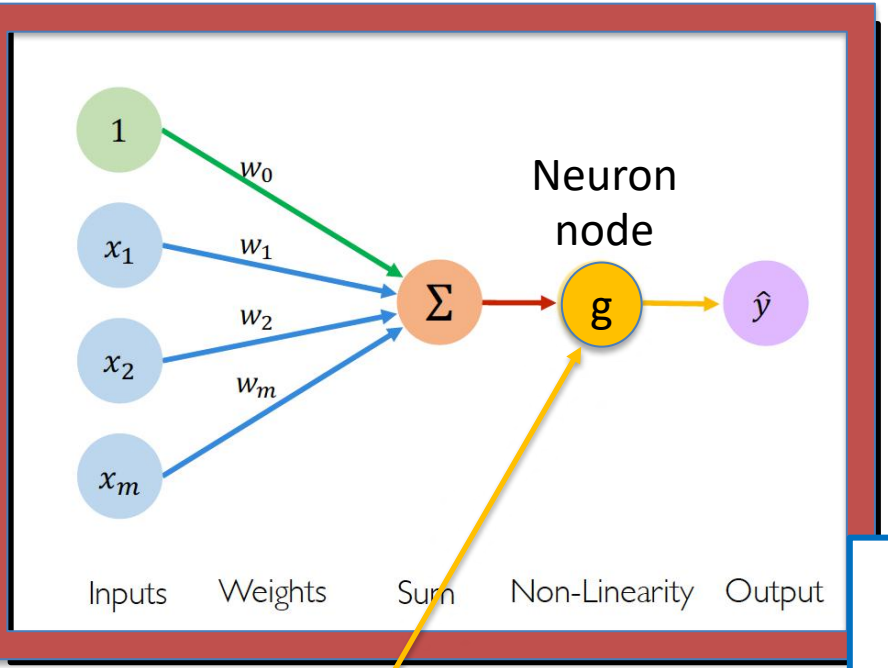
NN Modeling





- ✓ A node in the neural network is a mathematical function or activation function which maps input to output values.
- ✓ Inputs represent a set of vectors containing weights (w) and bias (b). They are the sets of parameters to be determined.
- ✓ Many nodes form a **neural layer**, **links** connect layers together, defining a NN model.
- ✓ Activation function (f or σ), is generally a nonlinear data operator which facilitates identification of complex features.

DNN MLP Forward Steps



$$\hat{y} = g(w_0 + \boldsymbol{X}^T \boldsymbol{W})$$

$$\text{where: } \boldsymbol{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \text{ and } \boldsymbol{W} = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix}$$

Output

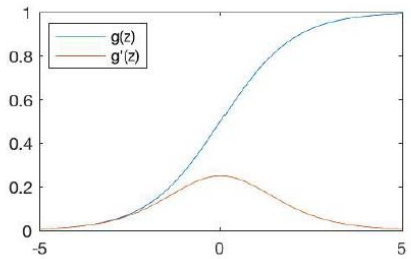
Linear combination of inputs

$$\hat{y} = g\left(w_0 + \sum_{i=1}^{m} x_i \, w_i\right)$$

Non-linear activation function

Bias

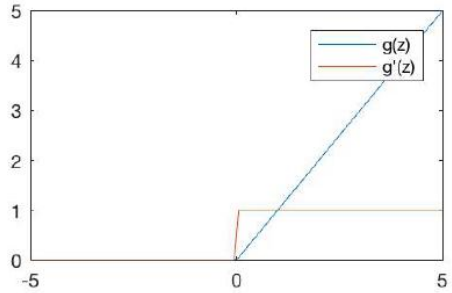
Sigmoid Function



$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

tf.nn.sigmoid(z)



$$g(z) = \max(0, z)$$

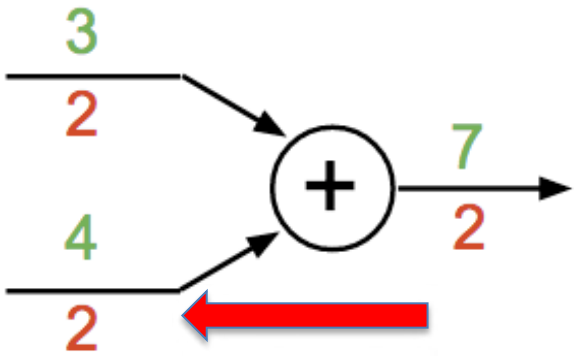
$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$

tf.nn.relu(z)

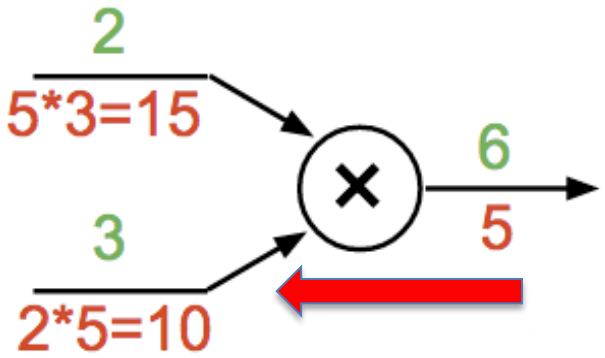
Forward and Backpropagation Operators

Forward path : backward path

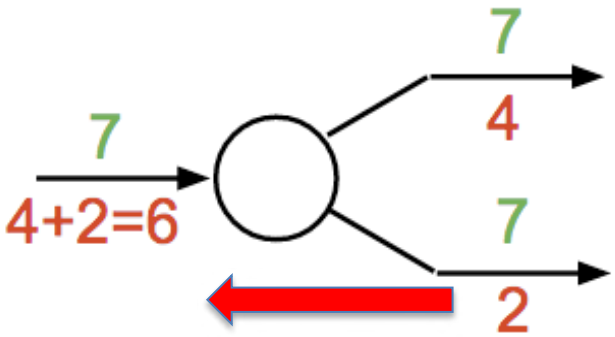
add gate: gradient distributor



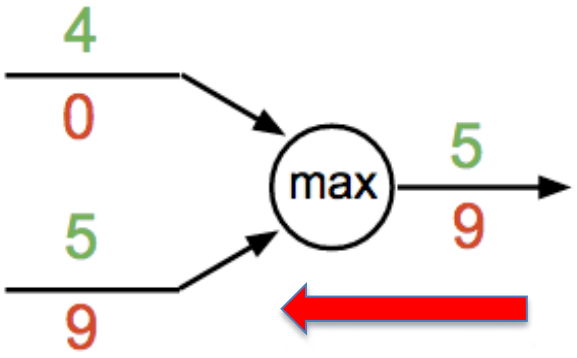
mul gate: "swap multiplier"



copy gate: gradient adder



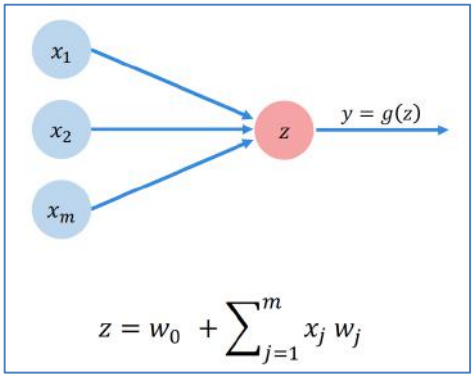
max gate: gradient router



http://cs231n.stanford.edu/slides/2020/lecture_4.pdf

Forward and backward computation operators

Z = (x1*w1 + x2*w2) + b = (-1.0*-2.0 + -2.0*3.0) + -3.0 = 4.0 – 3.0 = 1.0



Sigmoid function

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Y = σ (z) = 1 / (1 + e $^{-Z}$)
= 1 / (1 + e $^{-1}$) = 0.731

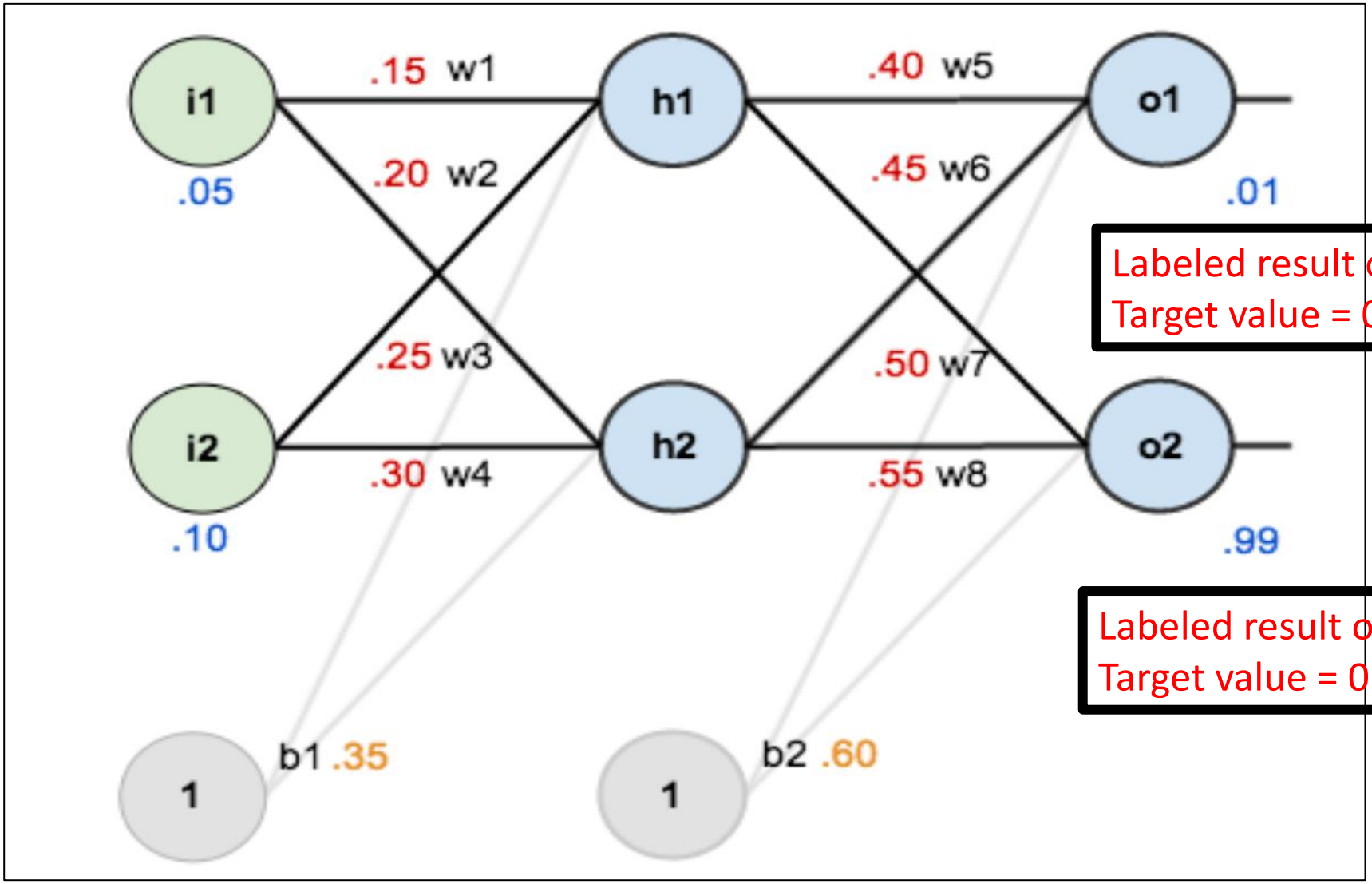
Z = (x1*w1 + x2*w2) + b

multiple operator :
downstream gradient =
upstream gradient * input value

add operator :
Downstream gradient =
upstream gradient

function operator :
downstream gradient =
Upstream gradient * local function gradient

DNN Example


i1
i2
h1
h2
o1
o2
1
1
.15 w1
.20 w2
.25 w3
.30 w4
.40 w5
.45 w6
.50 w7
.55 w8
b1 .35
b2 .60
.05
.10
.01
.99
Labeled result of out1
Target value = 0.01
Labeled result of out2
Target value = 0.99

```
# e constant
e = 2.7182818284
# initial values
i1 = 0.05
i2 = 0.10
# initial weights
w1 = 0.15
w2 = 0.20
w3 = 0.25
w4 = 0.30
w5 = 0.40
w6 = 0.45
w7 = 0.50
w8 = 0.55
# bias
b1 = 0.35
b2 = 0.60
# targets
To1 = 0.01
To2 = 0.99
```

```
# forward propagation
h1 = 1/(1+e**(-(w1*i1 + w2*i2+b1)))
print("h1: " + str(h1))

h2 = 1/(1+e**(-(w3*i1 + w4*i2+b1)))
print("h2: " + str(h2))

o1 = 1/(1+e**(-(w5*h1 + w6*h2+b2)))
print("o1: " + str(o1))

o2 = 1/(1+e**(-(w7*h1 + w8*h2+b2)))
print("o2: " + str(o2))
```

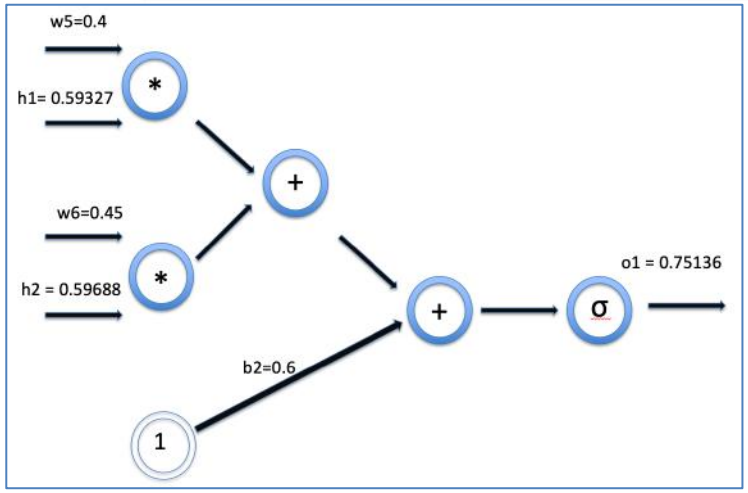
```
h1: 0.5932699921052087 h2: 0.5968843782577157
o1: 0.7513650695475076 o2: 0.772928465316421
```

```
# Error
Eo1 = 0.5*(To1-o1)**2
print("Error o1: " + str(Eo1))
Eo2 = 0.5*(To2-o2)**2
print("Error o2: " + str(Eo2))

E = Eo1 + Eo2
print("Total Error: " + str(E))
```

```
Error o1: 0.2748110831725904
Error o2: 0.023560025584942117
Total Error: 0.29837110875753253
```

$$E_{o1} = \frac{1}{2}(target_{o1} - out_{o1})^2 = \frac{1}{2}(0.01 - 0.75136507)^2 = 0.274811083$$

target=0.01 | o1=0.75136 | Eo1=0.27481



Forward Path to Objective Function

target1 = 0.01

o1 = 0.75136

Cost f

Eo1 = 0.27481

+

ET = 0.298371

grad ET = 1.0

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$

Etotal = Eo1 + Eo2 = 0.298371109

Eo2 = 0.02356

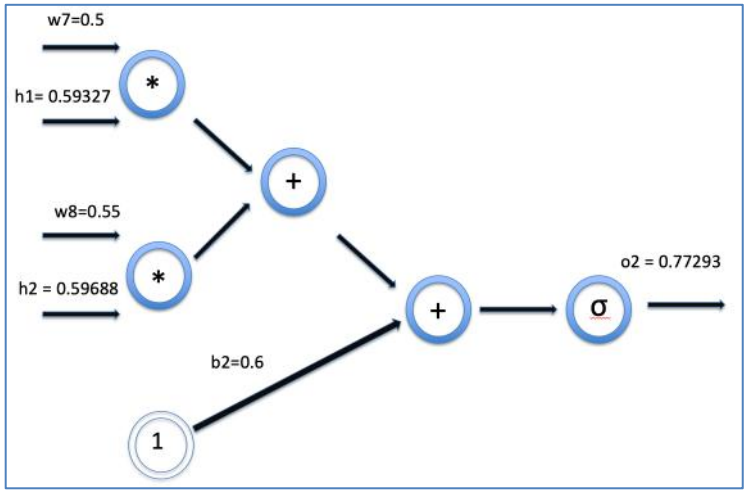
o2 = 0.77293

Cost f

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \tilde{y}_i)^2$$

target2 = 0.99

target=0.99 | o2=0.77293 | Eo2=0.02356



```
# Backpropagation
n = 0.5

grad_o1 = o1 - To1
grad_o2 = o2 - To2

print("grad_o1: "+str(grad_o1))
print("grad_o2: "+str(grad_o2))

grad_d1 = (grad_o1)*o1*(1-o1)
grad_d2 = (grad_o2)*o2*(1-o2)

print("grad_d1: "+str(grad_d1))
print("grad_d2: "+str(grad_d2))

grad_w5 = grad_d1*h1
print("grad_w5: "+str(grad_w5))

w5_final = w5 - n*grad_w5

grad_w6 = grad_d1*h2
print("grad_w6: "+str(grad_w6))
w6_final = w6 - n*grad_w6

grad_w7 = grad_d2*h1
print("grad_w7: "+str(grad_w7))
w7_final = w7 - n*grad_w7

grad_w8 = grad_d2*h2
print("grad_w8: "+str(grad_w8))
w8_final = w8 - n*grad_w8
```

```
grad_h1 = w5*grad_d1 + w7*grad_d2
print("grad_h1: "+str(grad_h1))
grad_d11 = grad_h1*h1*(1-h1)
print("grad_d11: "+str(grad_d11))

grad_w1 = grad_d11*i1
print("grad_w1: "+str(grad_w1))
w1_final = w1 - n*grad_w1

grad_w2 = grad_d11*i1
print("grad_w2: "+str(grad_w2))
w2_final = w2 - n*grad_w2

grad_h2 = w6*grad_d1 + w8*grad_d2
print("grad_h2: "+str(grad_h2))
grad_d22 = grad_h2*h2*(1-h2)
print("grad_d22: "+str(grad_d22))

grad_w3 = grad_d22*i1
print("grad_w3: "+str(grad_w3))
w3_final = w3 - n*grad_w3

grad_w4 = grad_d22*i2
print("grad_w4: "+str(grad_w4))
w4_final = w4 - n*grad_w4
```

```
print("w1+: "+str(w1_final))
print("w2+: "+str(w2_final))
print("w3+: "+str(w3_final))
print("w4+: "+str(w4_final))
print("w5+: "+str(w5_final))
print("w6+: "+str(w6_final))
print("w7+: "+str(w7_final))
print("w8+: "+str(w8_final))
```

w1+: 0.1497807161327648
w2+: 0.1997807161327648 2
w3+: 0.2497511436323716 4
w4+: 0.2995022872647432 6
w5+: 0.3589164797177565 3
w6+: 0.4086661860761087
w7+: 0.5113012702391395
w8+: 0.5613701211083925

grad_o1: 0.7413650695475076
grad_o2: -0.2170715346835789 8
grad_d1: 0.1384985616294507 6
grad_d2: -0.0380982365180384 4
grad_w5: 0.0821670405644870 1
grad_w6: 0.0826676278477826 3
grad_w7: -0.0226025404782790 4
grad_w8: -0.0227402422167847 7
grad_h1: 0.0363530639276108 6
grad_d11: 0.0087713546894077 9
grad_w1: 0.0004385677344703895
grad_w2: 0.0004385677344703895
grad_h2: 0.0413703226483317 1
grad_d22: 0.0099542547051342 71
grad_w3: 0.0004977127352567136
grad_w4: 0.0009954254705134271

MNIST Example (28x28 pixels)

Handwritten digits in the MNIST dataset are 28x28 pixel grayscale images. The simplest approach for classifying them is to use the 28x28=784 pixels as inputs for a 1-layer neural network.

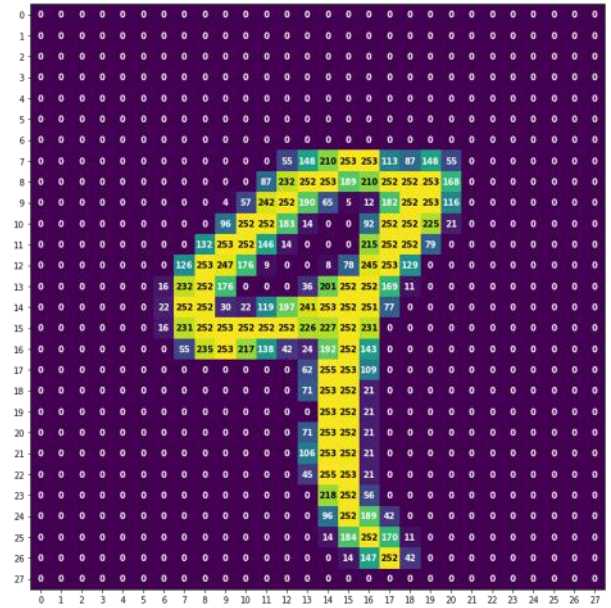
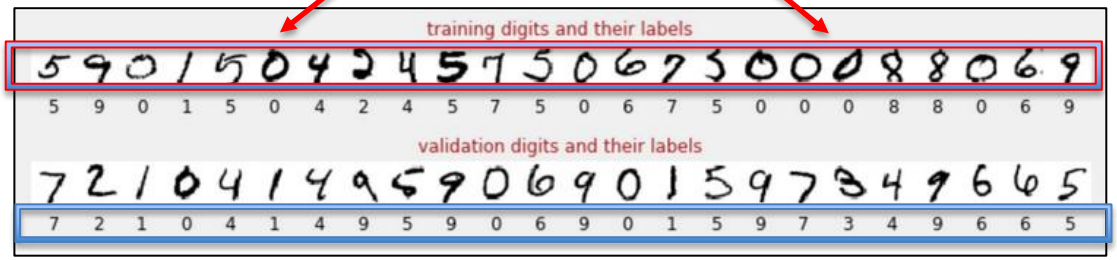
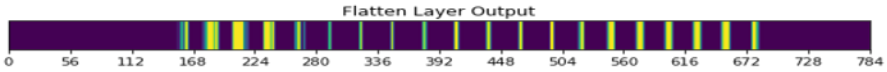


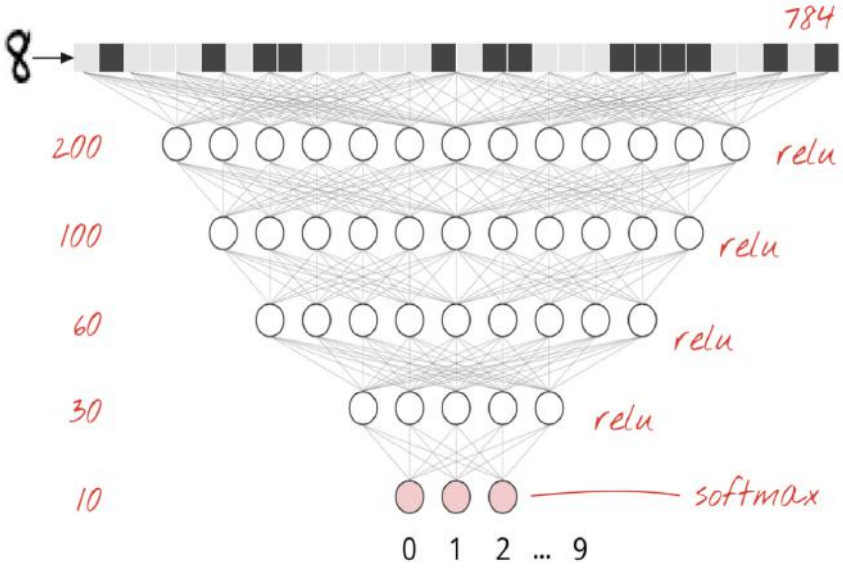
Image : input

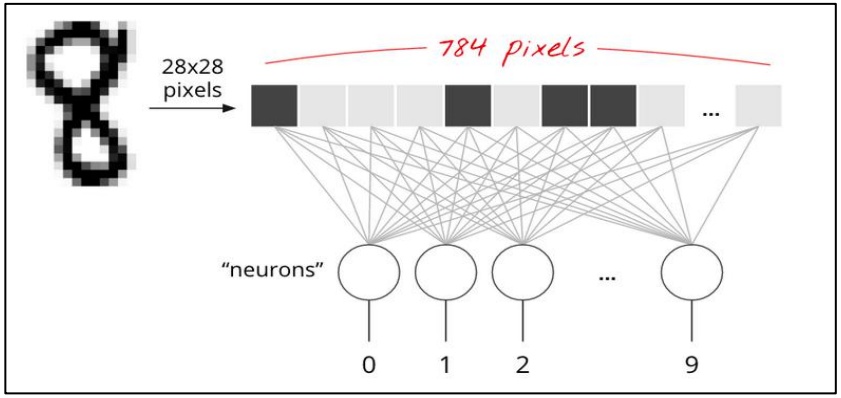


Labels : target

Flatten the 28 x 28 matrix
to a vector of 28 x 28 elements 784 elements







TensorFlow, Keras and deep learning, without a PhD
https://codelabs.developers.google.com/codelabs/cloud-tensorflow-mnist#0

Simple MNIST MLP Neural Network

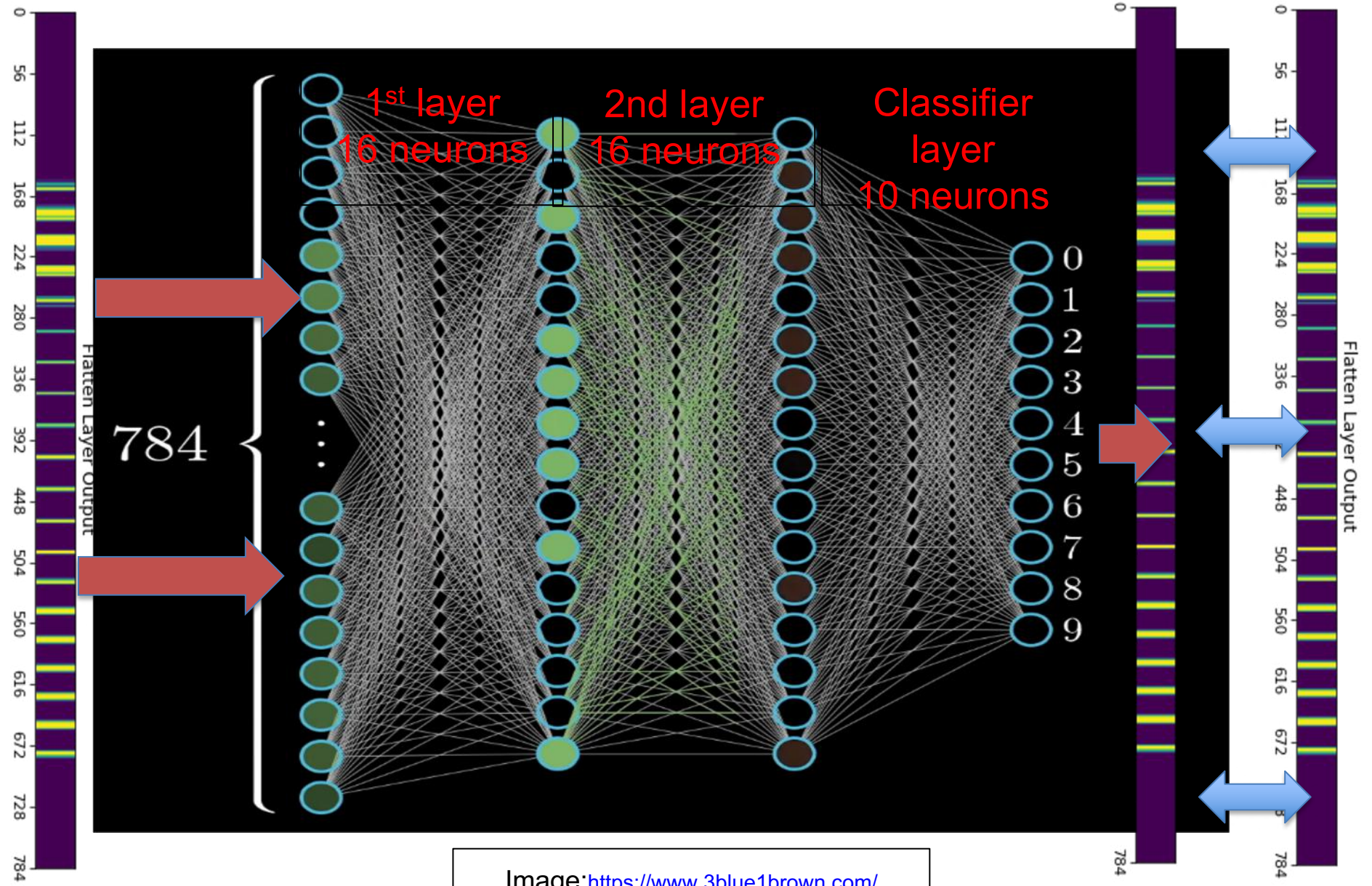
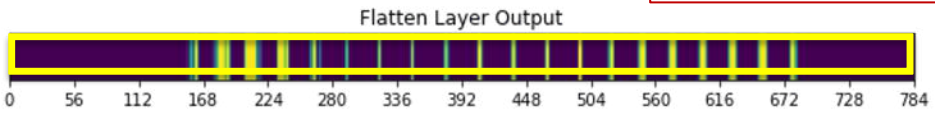
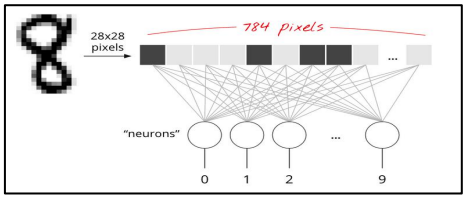


Image:https://www.3blue1brown.com/

Summary : Flow of MLP Dense layer NN

1 image = a vector of 784





An image of 28x28 pixels

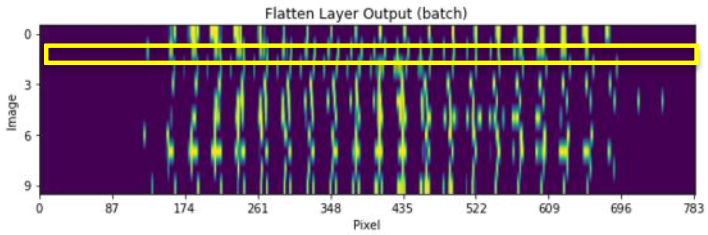
Input to 1st layer

10 images : batch of 10
10 x 784 matrix

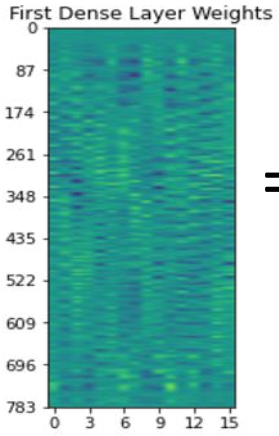
16 neurons
784 x 16 W matrix

10 x 16 output matrix

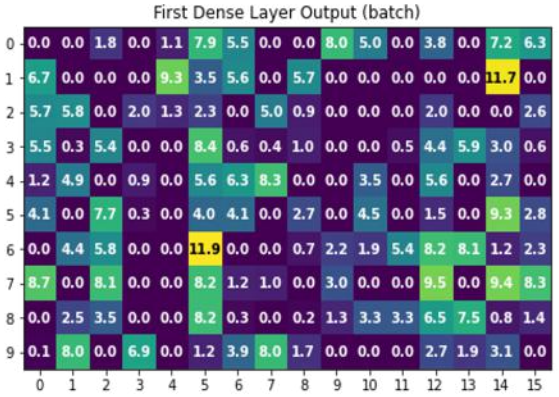
Input to 2nd layer



*

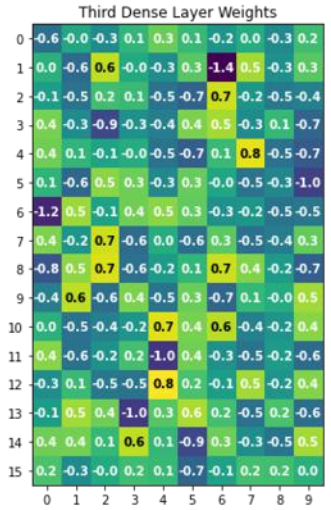


=



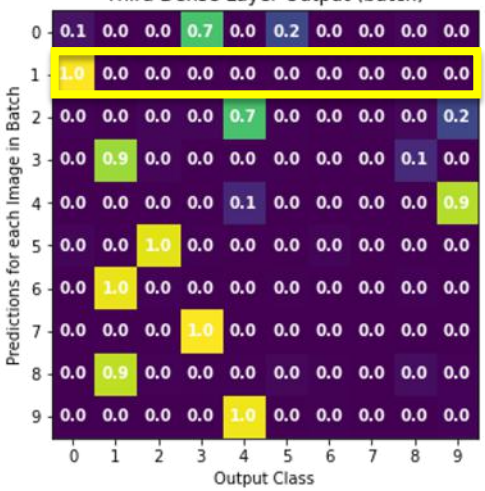
10 x 16 matrix

*

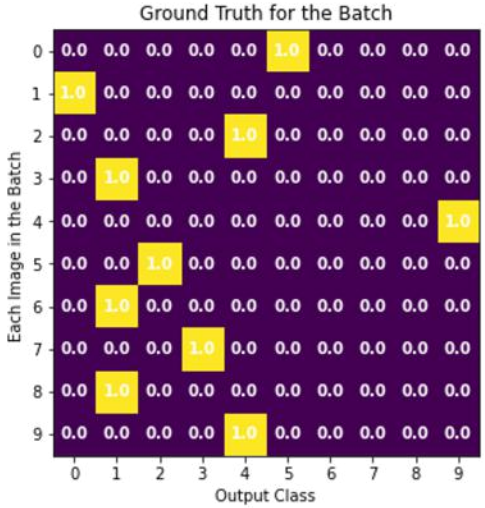


16 neurons
16 x 10
W matrix

=



10 x 10 output matrix



10 x 10 labeled matrix

Tensorflow Example

```
import tensorflow as tf

mnist = tf.keras.datasets.mnist

(x_train, y_train), (x_test, y_test) = mnist.load_data()
x_train, x_test = x_train / 255.0, x_test / 255.0

model = tf.keras.models.Sequential([
  tf.keras.layers.Flatten(input_shape=(28, 28)),
  tf.keras.layers.Dense(16, activation='relu'),
 tf.keras.layers.Dense(16, activation='relu'),
  tf.keras.layers.Dense(10, activation='softmax')
])
```

Tensorflow is imported

Sets the mnist dataset to variable "mnist"

Loads the mnist dataset

Builds the layers of the model
4 layers in this model

```
model.compile(optimizer='sdg', loss='SparseCategoricalCrossentropy',
metrics=['accuracy'])

model.summary()

logdir = os.path.join("logs", datetime.datetime.now().strftime("%Y%m%d-
%H%M%S"))
tensorboard_callback = tf.keras.callbacks.TensorBoard(logdir, histogram_freq=1)
```

Loss Function

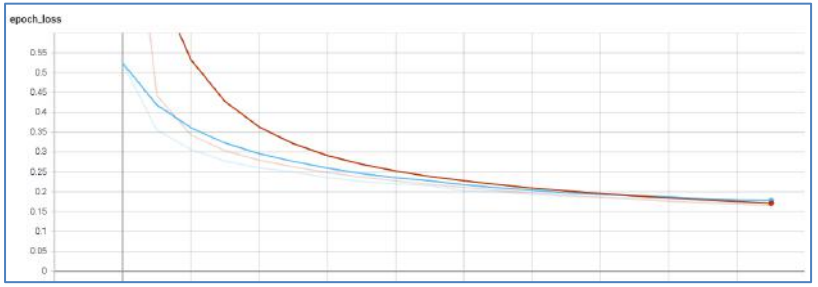
Compiles the model with the SGD optimizer

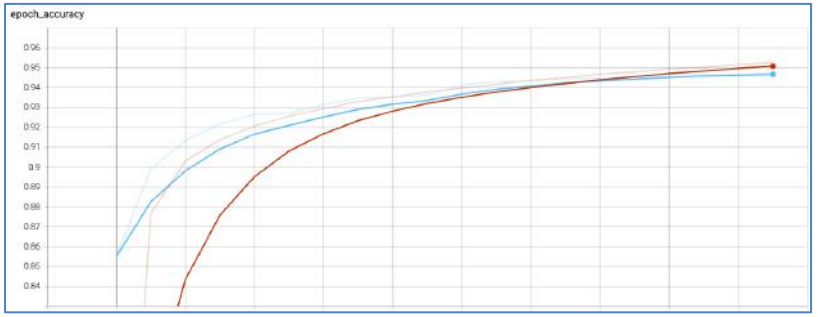
Print summary

Use tensorborad

```
model.compile(optimizer='sgd',
              loss='SparseCategoricalCrossentropy',
              metrics=['accuracy'])

model.summary()

logdir = os.path.join("logs", datetime.datetime.now().strftime("%Y%m%d-%H%M%S"))
tensorboard_callback = tf.keras.callbacks.TensorBoard(logdir, histogram_freq=1)

model.fit(x_train, y_train, epochs=20, batch_size=50,validation_data=(x_test, y_test),
          callbacks=[tensorboard_callback])

model.evaluate(x_test,  y_test, verbose=2)

%tensorboard --logdir logs
```

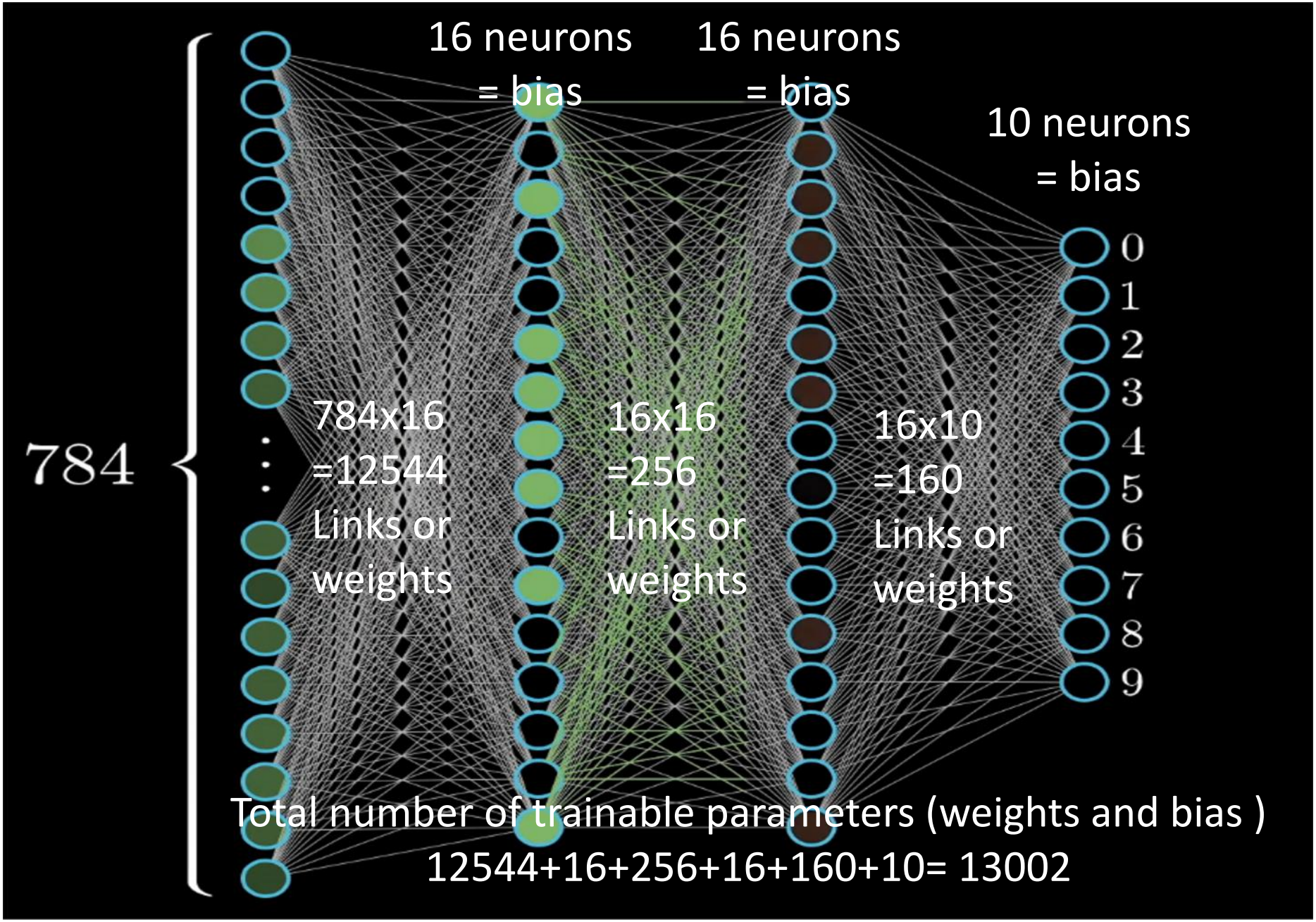
```
2.4.1
The tensorboard extension is already loaded. To reload it, use:
  %reload_ext tensorboard
Model: "sequential_1"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
flatten_1 (Flatten)          (None, 784)               0
_________________________________________________________________
dense_3 (Dense)              (None, 16)                12560
_________________________________________________________________
dense_4 (Dense)              (None, 16)                272
_________________________________________________________________
dense_5 (Dense)              (None, 10)                170
=================================================================
Total params: 13,002
Trainable params: 13,002
Non-trainable params: 0
```





Simple MLP Network

Image:https://www.3blue1brown.com/



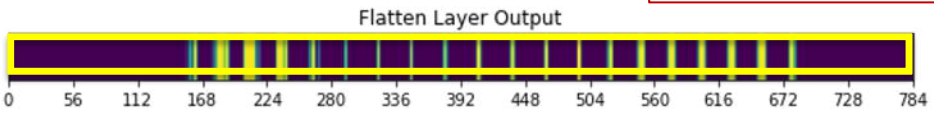
Tensorflow Example

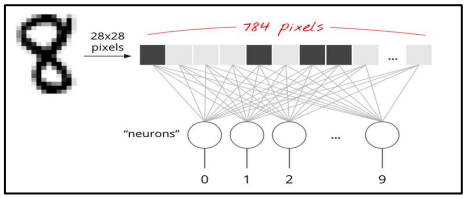
The model is trained in about 10 seconds completing 5 epochs with a Nvidia GTX 1080 GPU and has an accuracy of around 97-98%

```
2020-07-29 19:53:40.592860: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1108]      0
2020-07-29 19:53:40.592871: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1121] 0:   N
2020-07-29 19:53:40.593073: I tensorflow/stream_executor/cuda/cuda_gpu_executor.cc:981] successful NUMA node read from
SysFS had negative value (-1), but there must be at least one NUMA node, so returning NUMA node zero
2020-07-29 19:53:40.593535: I tensorflow/stream_executor/cuda/cuda_gpu_executor.cc:981] successful NUMA node read from
SysFS had negative value (-1), but there must be at least one NUMA node, so returning NUMA node zero
2020-07-29 19:53:40.593973: I tensorflow/stream_executor/cuda/cuda_gpu_executor.cc:981] successful NUMA node read from
SysFS had negative value (-1), but there must be at least one NUMA node, so returning NUMA node zero
2020-07-29 19:53:40.594387: I tensorflow/core/common_runtime/gpu/gpu_device.cc:1247] Created TensorFlow device (/job:lo
calhost/replica:0/task:0/device:GPU:0 with 7219 MB memory) -> physical GPU (device: 0, name: GeForce GTX 1080, pci bus
id: 0000:26:00.0, compute capability: 6.1)
2020-07-29 19:53:40.623075: I tensorflow/compiler/xla/service/service.cc:168] XLA service 0x55d981198b80 initialized fo
r platform CUDA (this does not guarantee that XLA will be used). Devices:
2020-07-29 19:53:40.623096: I tensorflow/compiler/xla/service/service.cc:176]   StreamExecutor device (0): GeForce GTX
1080, Compute Capability 6.1
2020-07-29 19:53:52.215159: I tensorflow/stream_executor/platform/default/dso_loader.cc:44] Successfully opened dynamic
 library libcublas.so.10
Epoch 1/5
1875/1875 [==============================] - 2s 1ms/step - loss: 0.2982 - accuracy: 0.9127
Epoch 2/5
1875/1875 [==============================] - 2s 1ms/step - loss: 0.1433 - accuracy: 0.9571
Epoch 3/5
1875/1875 [==============================] - 2s 1ms/step - loss: 0.1097 - accuracy: 0.9663
Epoch 4/5
1875/1875 [==============================] - 2s 1ms/step - loss: 0.0893 - accuracy: 0.9730
Epoch 5/5
1875/1875 [==============================] - 2s 1ms/step - loss: 0.0777 - accuracy: 0.9750
313/313 - 0s - loss: 0.0727 - accuracy: 0.9776
```

Summary : Flow of MLP Dense layer NN

1 image = a vector of 784

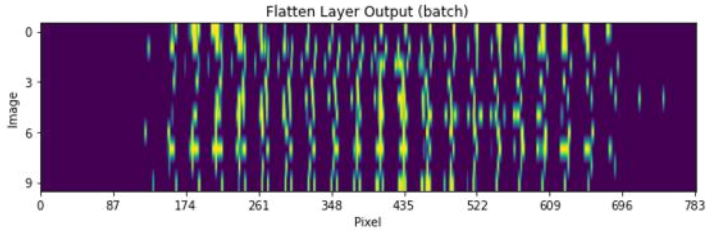




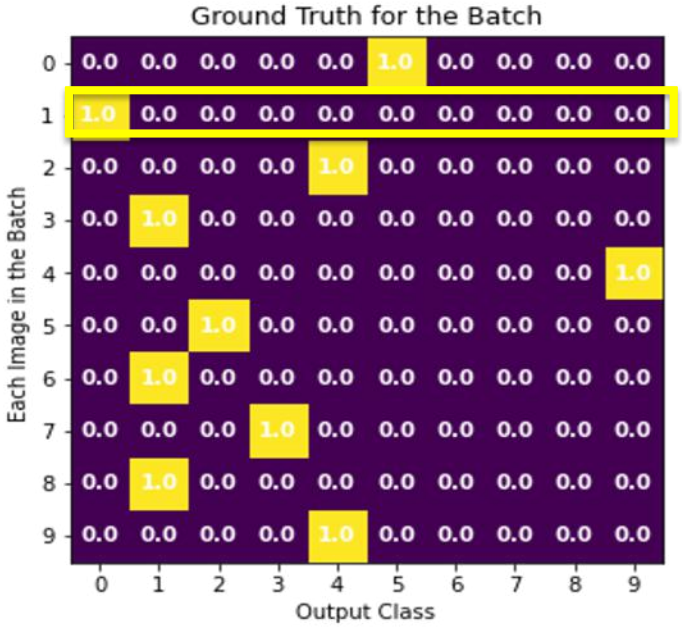
An image of 28x28 pixels

Input to 1st layer

10 images : batch of 10
10 x 784 matrix

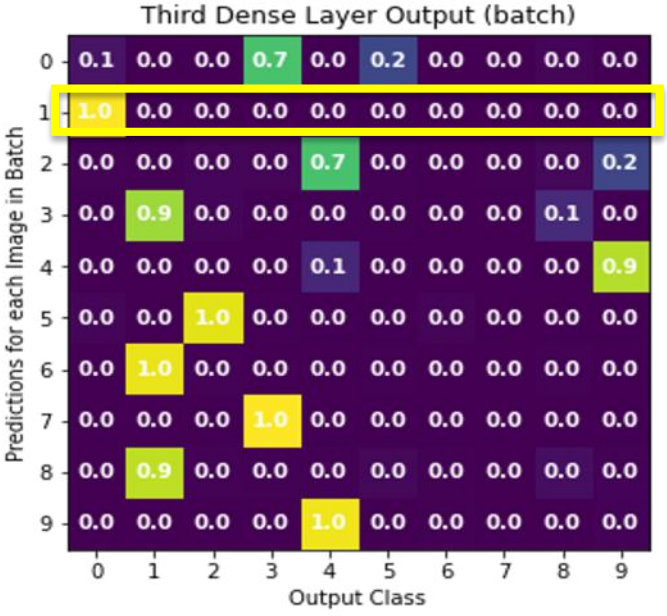


10 x 10 labeled matrix

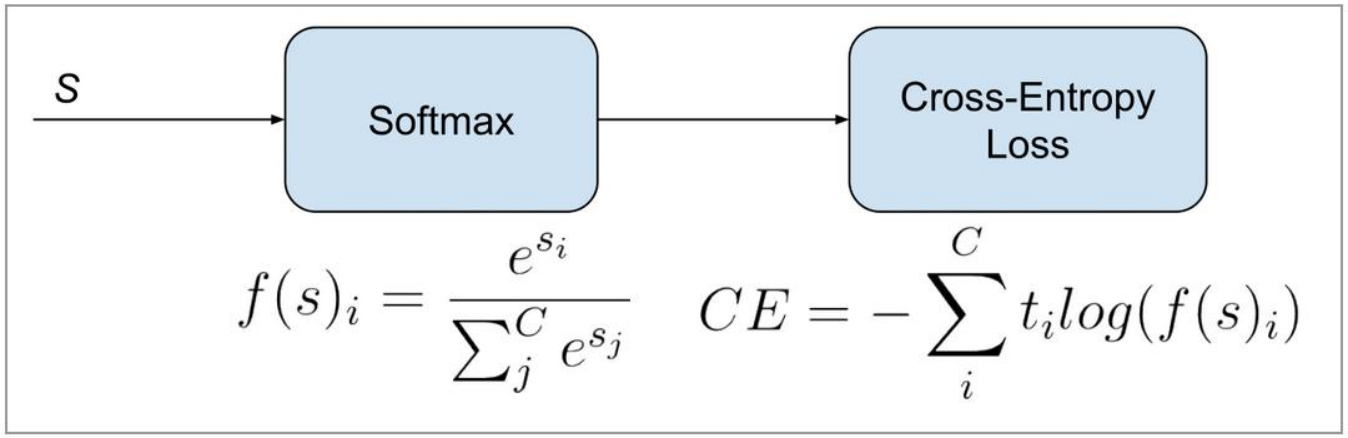


Comparing Computed NN results To Labeled results (target)

10 x 10 output matrix

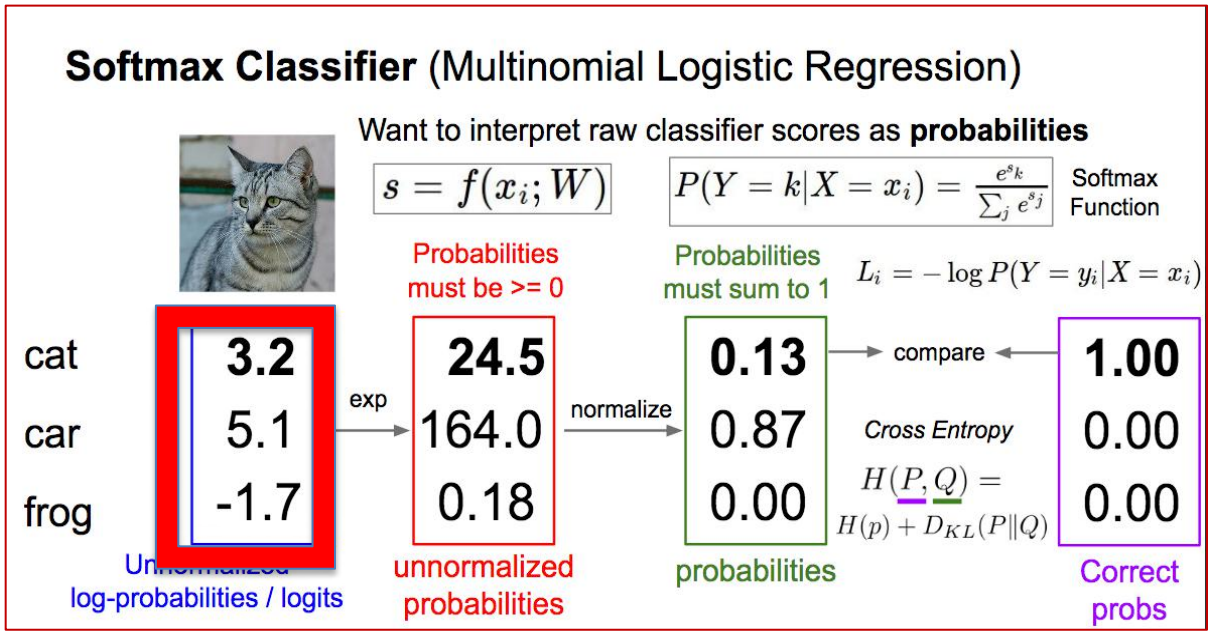


Categorical (Softmax) Cross Entropy Loss (Statistical Learning)



T = 1

ti and si are the groundtruth (label) and the computed score for each class i in C.



e^3.2 = 24.5

e^5.1 = 164.0

e^-1.7 = 0.18

188.68

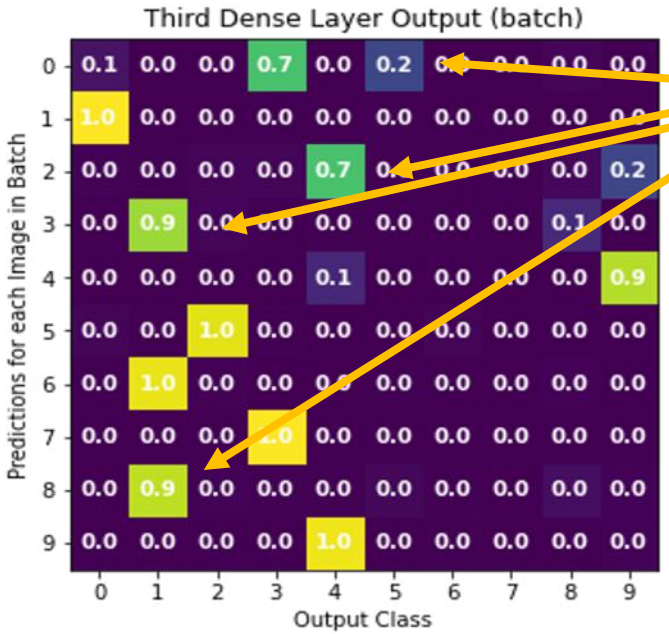
Software = 24.5 / 188.68 = 0.13

Software = 164 / 188.68 = 0.87

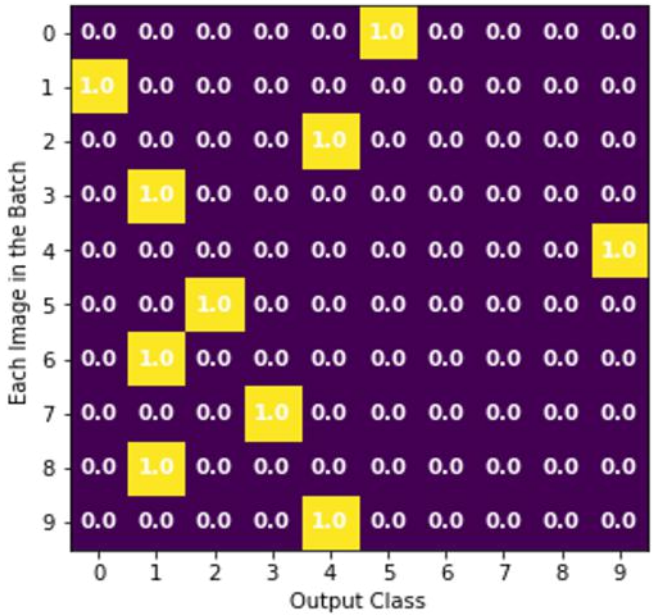
Software = -1.7 / 188.68 = 0.00

https://gombru.github.io/2018/05/23/cross_entropy_loss/

Evaluating the Error



Z



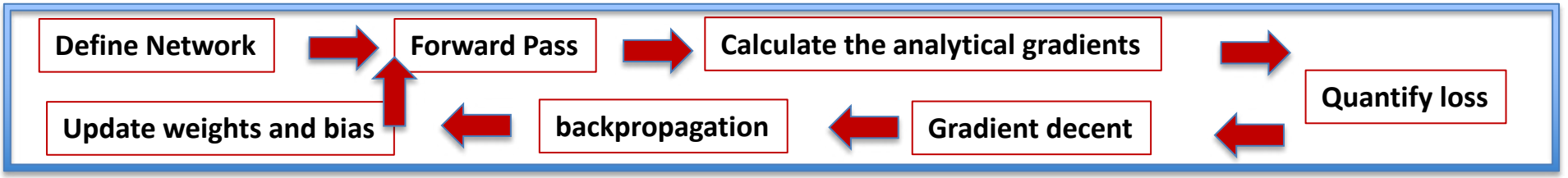
Categorical Cross Entropy Loss

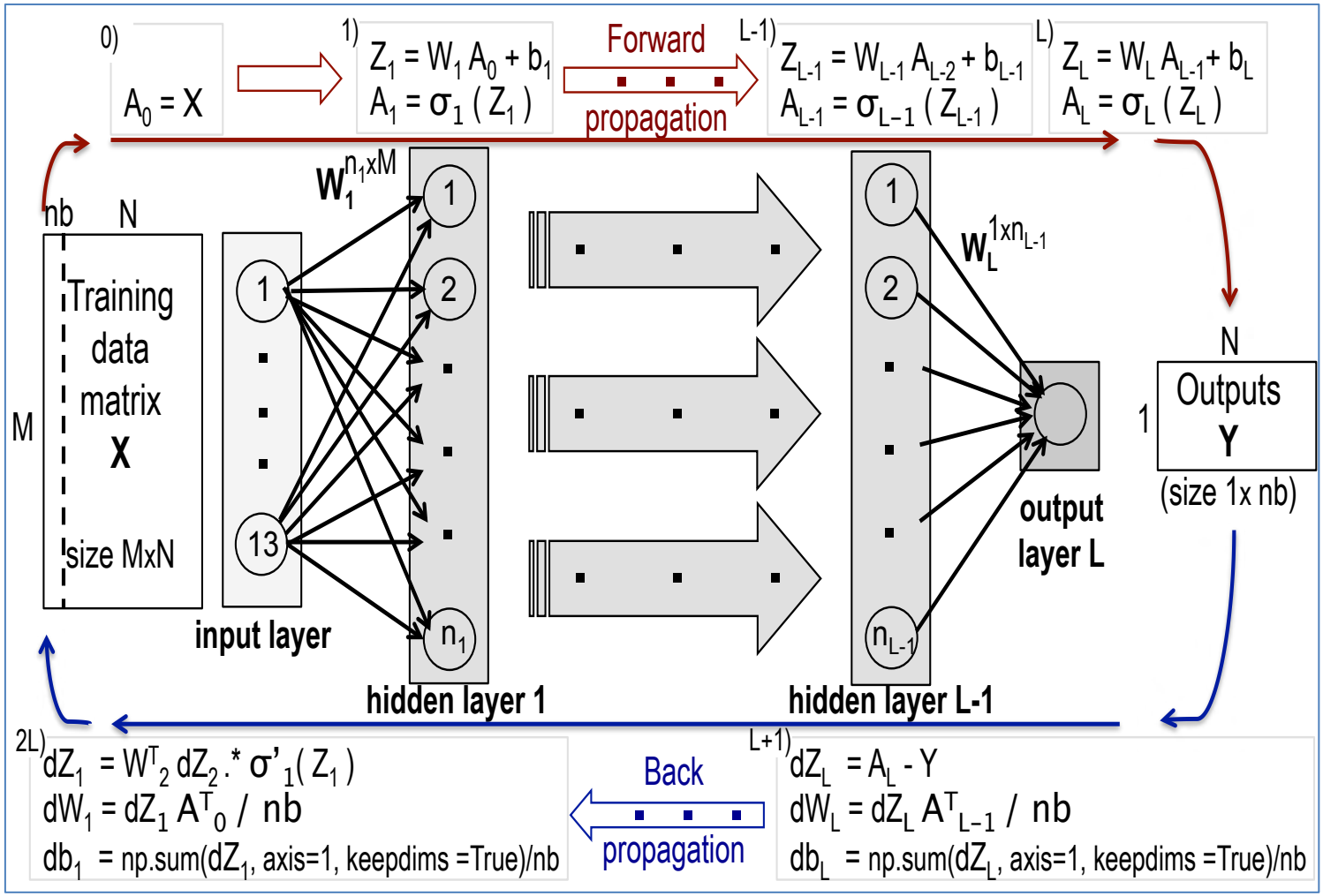
$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

L (i) = - log0.2 = 1.61	Image 0 = 5
- log1.0 = 0	Image 1 = 0
- log0.7 = 0.356	Image 2 = 4
- log0.9 = 0.105	Image 3 = 1
- log0.9 = 0.105	Image 4 = 9
- log1.0 = 0	Image 5 = 2
- log1.0 = 0	Image 6 = 1
- log1.0 = 0	Image 7 = 3
- log0.9 = 0.105	Image 8 = 2
- log1.0 = 0	Image 9 = 4

Summary : Flow of NN

```
model.fit(x_train, y_train, epochs=3, batch_size=10)
```





DNN Model

Applications

+

Data Ensemble + Input

+

It's all about

linear algebra calculations

DNN in particular

+

Algorithms, Software

+

Output + Data Analysis

+

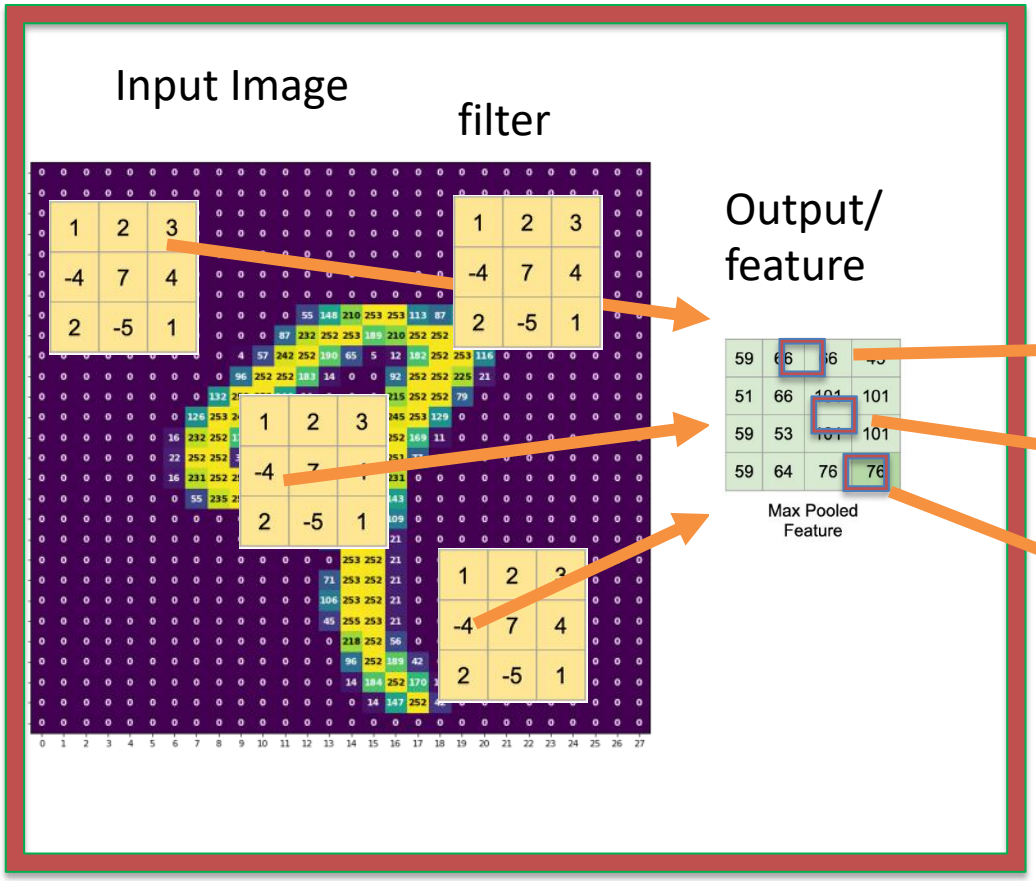
Hardware

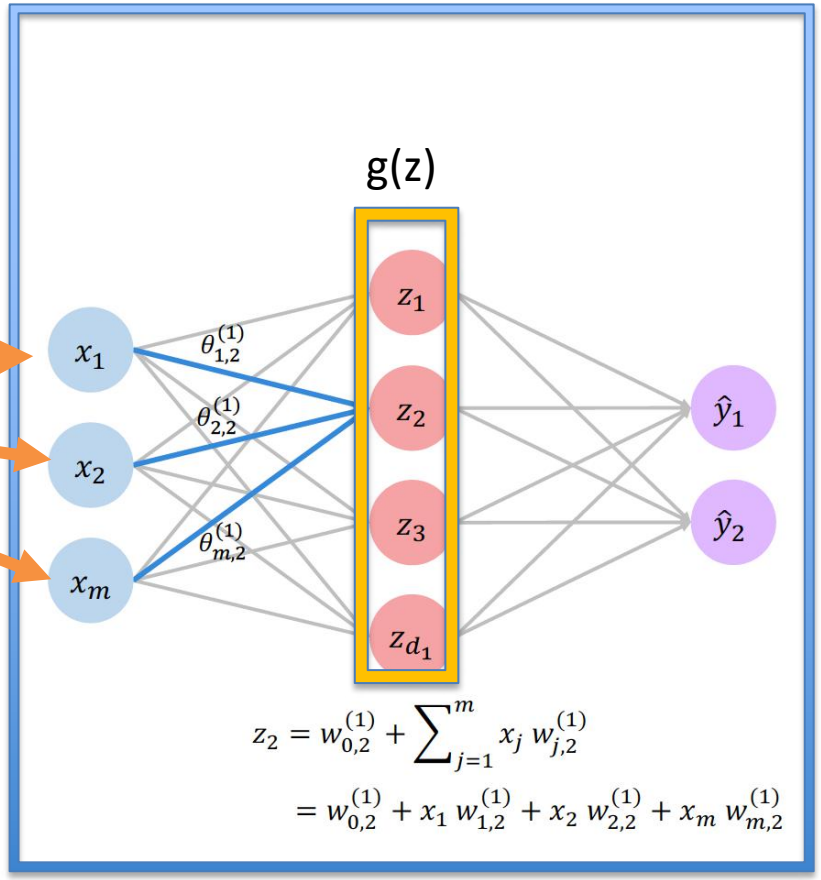
Nonlinear Parametric Data Model

Convolutional Neural Network
Same Ideas of MLP

STEP 1 : Model Definition
STEP 2 : Cost Function
Step 3 : Optimization Scheme
Step 4 : Numerical Implementation
Step 5 : Evaluation

Step 1: Parametric Model : Convolutional Neural Network (CNN)
Convolutional filter + Connected Neural Network





Convolutional filtering

Multilayer Perceptron

Convolutional Filter Computation

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

*

Horizontal

1	1	1
0	0	0
-1	-1	-1

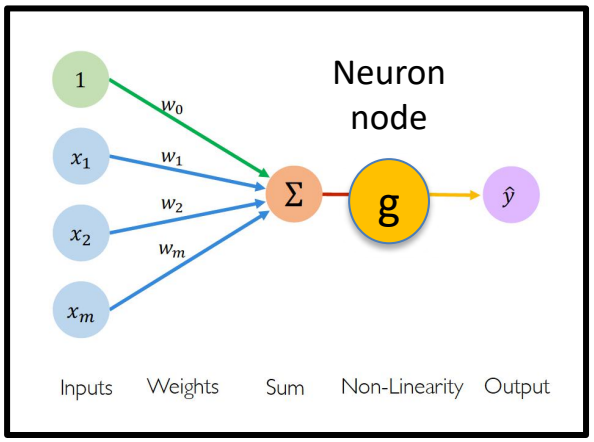
=

0	0	0	0
30	10	-10	-30
30	10	-10	-30
0	0	0	0

Vertical

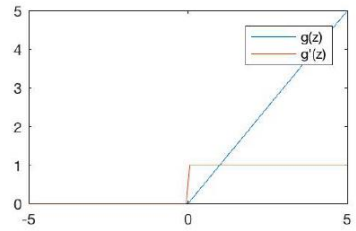
1	0	-1
1	0	-1
1	0	-1

(10,10,10)*(1,1,1)+(10,10,10)*(0,0,0)+(10,10,10)*(-1,-1,-1)=0 , element (1,1)
Elementwise multiplication



$$\hat{y} = g(w_0 + \boldsymbol{X}^T \boldsymbol{W})$$

where: $\boldsymbol{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$ and $\boldsymbol{W} = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix}$



$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$

tf.nn.relu(z)

Softmax function : Recognizing cats, dogs, and baby chicks


3


1


2


0


3


2


0


1

How Convolutional Neural Networks work by Brandon Rohrer https://www.youtube.com/watch?v=FmpDIaiMIeA

Convolutional Neural Network
Same Ideas

STEP 1 : Model Definition

STEP 2 : Cost Function

Step 3 : Optimization Scheme

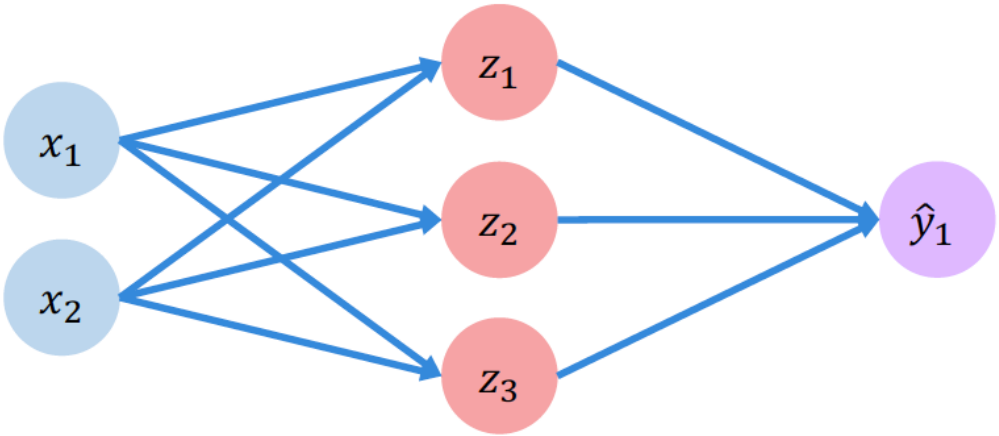
Step 4 : Numerical Implementation

Step 5 : Evaluation

Cross Entropy Loss (Categorical)

Cross entropy loss *can be used with models that output a probability between 0 and 1*

$$\boldsymbol{X} = \begin{bmatrix} 4, & 5 \\ 2, & 1 \\ 5, & 8 \\ \vdots & \vdots \end{bmatrix}$$



$$f(x) = \begin{bmatrix} 0.1 \\ 0.8 \\ 0.6 \\ \vdots \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ 0 \\ 1 \\ \vdots \end{bmatrix}$$

$$J(\boldsymbol{W}) = \frac{1}{n}\sum_{i=1}^{n} \underbrace{y^{(i)}}_{\text{Actual}} \log \underbrace{\left(f\left(x^{(i)}; \boldsymbol{W}\right)\right)}_{\text{Predicted}} + (1 - \underbrace{y^{(i)}}_{\text{Actual}}) \log \underbrace{\left(1 - f\left(x^{(i)}; \boldsymbol{W}\right)\right)}_{\text{Predicted}}$$

```
loss = tf.reduce_mean( tf.nn.softmax_cross_entropy_with_logits(model.y, model.pred) )
```

Convolutional Neural Network
Same Ideas

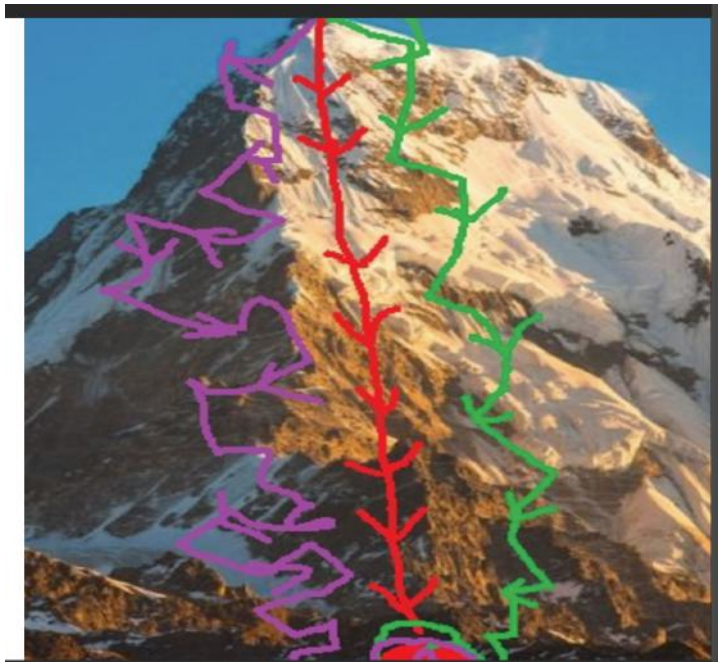
STEP 1 : Model Definition

STEP 2 : Cost Function

Step 3 : Optimization Scheme

Step 4 : Numerical Implementation

Step 5 : Evaluation



Optimization : Gradient Descent :

Batch Gradient Descent
Mini-batch Gradient Descent
Stochastic Gradient Descent

(X = W, weight)

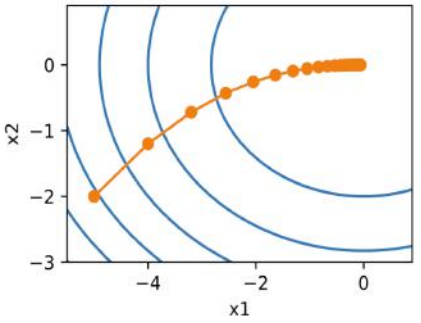
W+ = W – (learning rate) * dJ/dw

Algorithm

- Choose initial $\mathbf{x}_0$
- At time $t = 1, \ldots, T$

$$\mathbf{x}_t = \mathbf{x}_{t-1} - \eta \nabla f(\mathbf{x}_{t-1})$$

- η is called learning rate



Algorithm

1. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Compute gradient, $\frac{\partial J(W)}{\partial W}$
4. Update weights, $W \leftarrow W - \eta \frac{\partial J(W)}{\partial W}$
5. Return weights

```
weights = tf.random_normal(shape, stddev=sigma)
```

```
grads = tf.gradients(ys=loss, xs=weights)
```

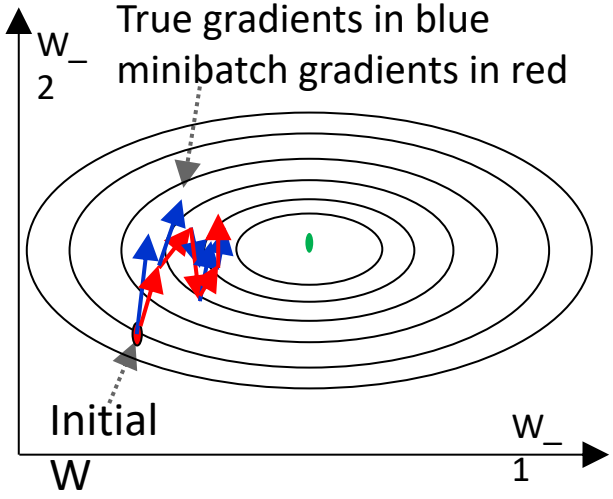
```
weights_new = weights.assign(weights - lr * grads)
```

Optimization: Batch SGD

Instead of computing a gradient across the entire dataset with size N, divides the dataset into a fixed-size subset ("minibatch") with size m, and computes the gradient for each update on this smaller batch:

(N is the dataset size, m is the minibatch size)

$$\mathbf{g} = \frac{1}{m'}\sum_{i=1}^{m'} \nabla_{\boldsymbol{\theta}} E_i(\mathbf{x}^{(n_i)}, y^{(n_i)}, \boldsymbol{\theta}),$$
$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \eta\mathbf{g},$$



Epoch step

Given: training set $\{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)\}$
Initialize all $\Theta^{(l)}$ randomly (NOT to 0!)
Loop // each iteration is called an epoch
  Loop // each iteration is a mini-batch
    Set $\Delta^{(l)}_{i,j} = 0 \quad \forall l, i, j$ (Used to accumulate gradient)
    Sample m training instances $\mathcal{X} = \{(\mathbf{x}'_1, y'_1), \ldots, (\mathbf{x}'_m, y'_m)\}$ without replacement
    For each instance in $\mathcal{X}$, $(\mathbf{x}_k, y_k)$:
      Set $\mathbf{a}^{(1)} = \mathbf{x}_k$
      Compute $\{\mathbf{a}^{(2)}, \ldots, \mathbf{a}^{(L)}\}$ via forward propagation
      Compute $\boldsymbol{\delta}^{(L)} = \mathbf{a}^{(L)} - y_k$
      Compute errors $\{\boldsymbol{\delta}^{(L-1)}, \ldots, \boldsymbol{\delta}^{(2)}\}$
      Compute gradients $\Delta^{(l)}_{ij} = \Delta^{(l)}_{ij} + a^{(l)}_j \delta^{(l+1)}_i$
    Compute mini-batch regularized gradient $D^{(l)}_{ij} = \begin{cases} \frac{1}{m}\Delta^{(l)}_{ij} + \lambda\Theta^{(l)}_{ij} & \text{if } j \neq 0 \\ \frac{1}{m}\Delta^{(l)}_{ij} & \text{otherwise} \end{cases}$
    Update weights via gradient step $\Theta^{(l)}_{ij} = \Theta^{(l)}_{ij} - \alpha D^{(l)}_{ij}$
  Until all training instances are seen
Until weights converge or max #epochs is reached

Batch iteration

Convolutional Neural Network
Same Ideas

STEP 1 : Model Definition
STEP 2 : Cost Function
Step 3 : Optimization Scheme
Step 4 : Numerical Implementation
Step 5 : Evaluation

Training

"Training a neural network"

What does it mean?

What does the network train?

minimize error of the computed values against the labeled values (loss function)

What are the training parameters

W and b

How does it work?

optimization based on gradient descent algorithm

go back and forth in the NN model to adjust for w and b => need derivatives w.r.t. w and b

Go through forward calculation

training with backpropagation

b. Propagate backward through the network:

The weight is updated based on the gradient of E_t

$$\boldsymbol{w}(t+1) = \boldsymbol{w}(t) - \lambda_t \nabla_{\boldsymbol{w}} E_t$$

where λ_t is the learning rate (step size).

STEP 4 : Supervised Learning Finding the parameters : Training Process

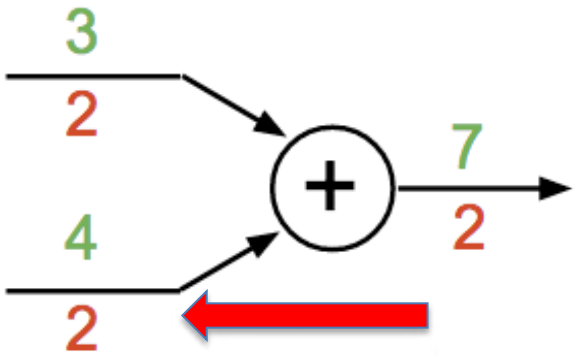
Go through forward calculation, training with backpropagation

1. Cost Function (L) = CE
2. Need dL/dw, dL/db
3. w, connection links in MLP
4. b, each neuron has a bias
5. What are w in the convolutional filtering process?
   Values in the filter
6. What are b in the convolutional filtering process?
   Each filter has a bias

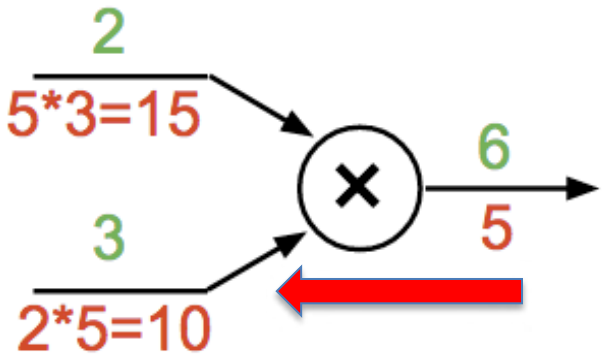
Forward and Backpropagation Operators (MLP)

Forward path : backward path

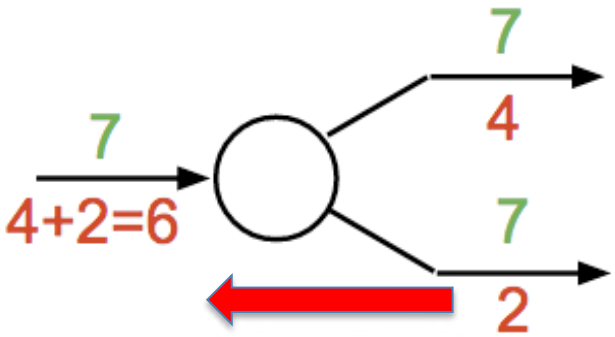
add gate: gradient distributor



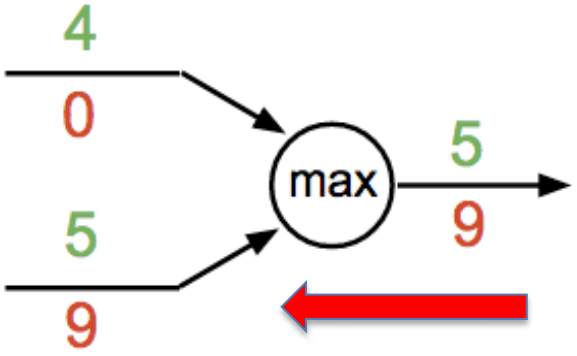
mul gate: "swap multiplier"



copy gate: gradient adder



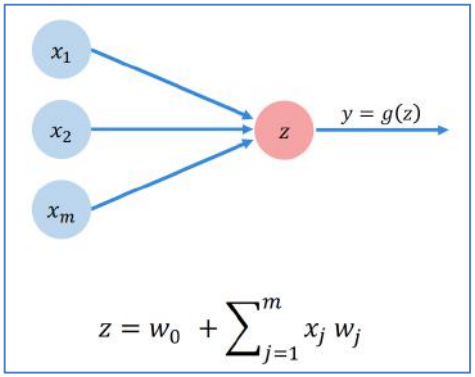
max gate: gradient router



http://cs231n.stanford.edu/slides/2020/lecture_4.pdf

Forward and backward computation operators

Z = (x1*w1 + x2*w2) + b = (-1.0*-2.0 + -2.0*3.0) + -3.0 = 4.0 – 3.0 = 1.0



Sigmoid function

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Y = σ (z) = 1 / (1 + e $^{-Z}$)
= 1 / (1 + e $^{-1}$) = 0.731

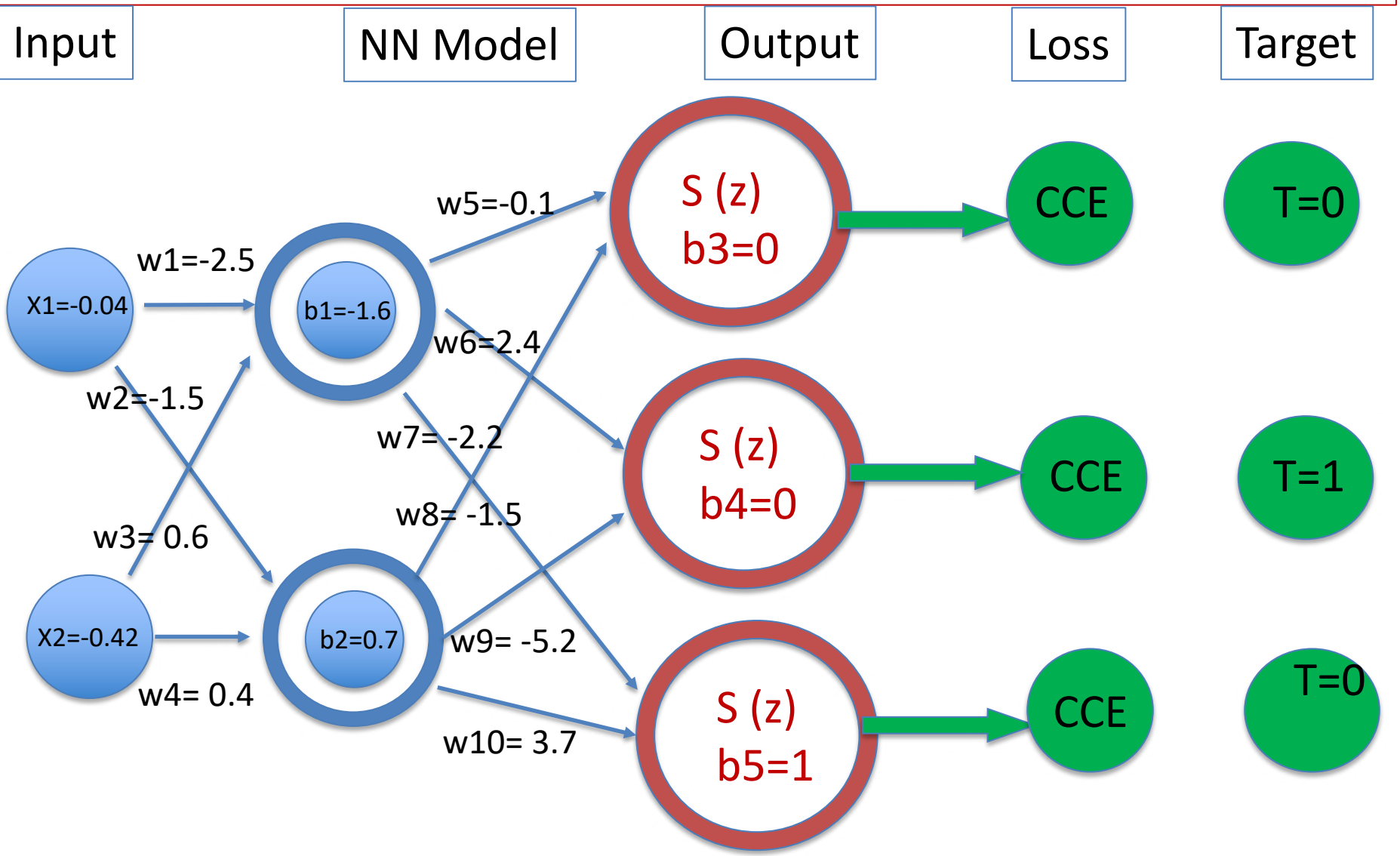
Z = (x1*w1 + x2*w2) + b

multiple operator :
downstream gradient =
upstream gradient * input value

add operator :
Downstream gradient =
upstream gradient

function operator :
downstream gradient =
Upstream gradient * local function gradient

Exercise : Represent the following NN model as a computational graph and build a python code to update w and b

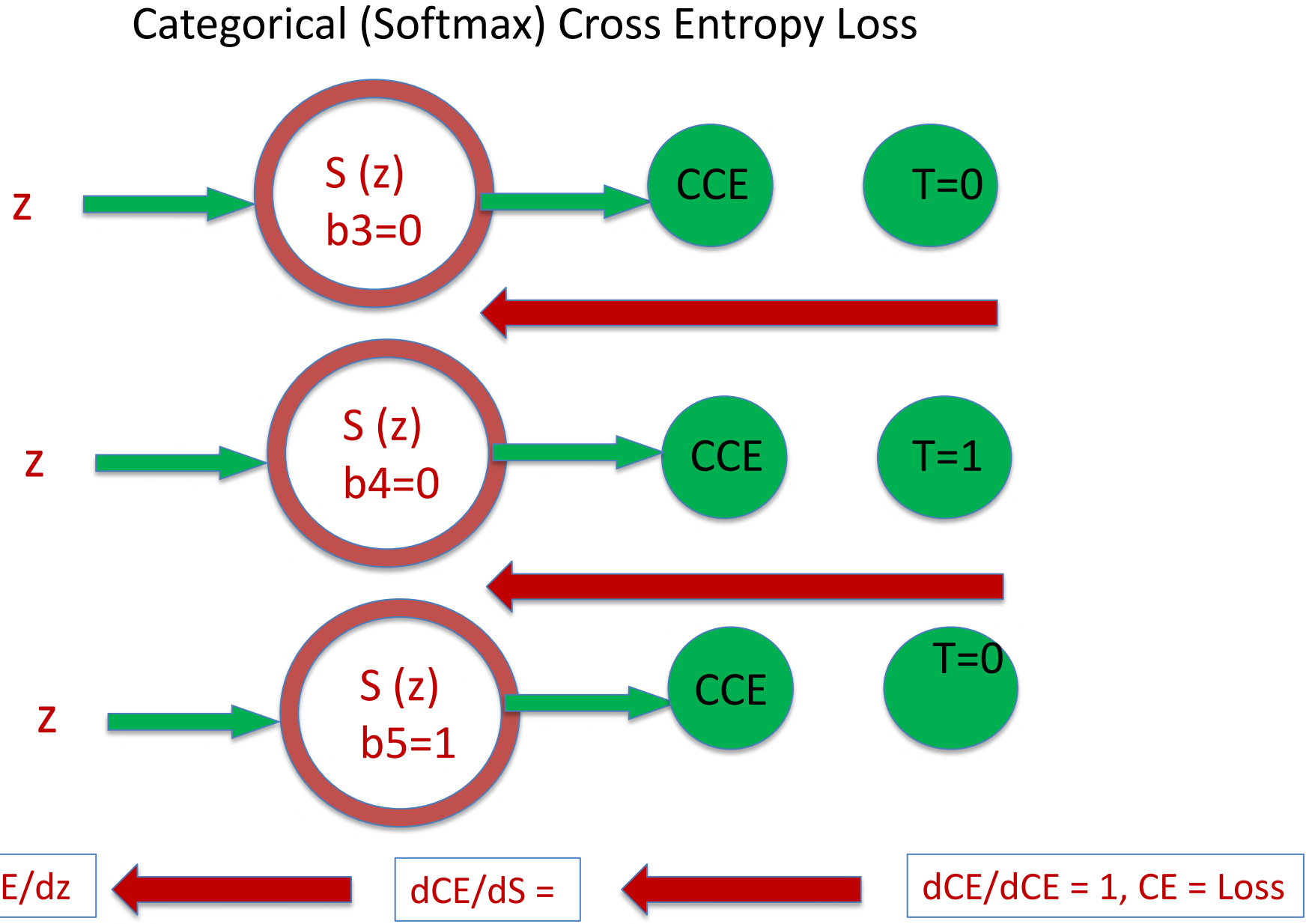


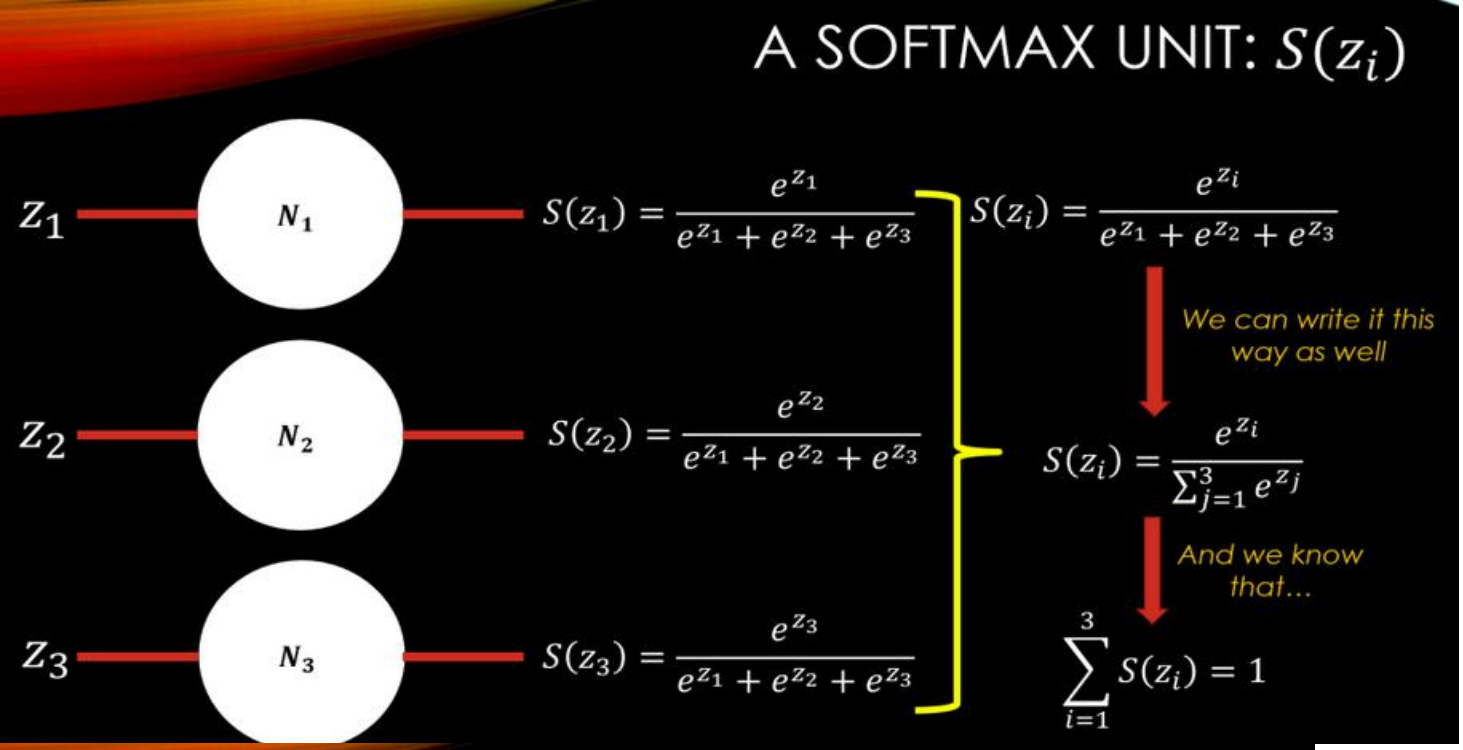
CE : Categorical Cross Entropy Loss
Classification
0, 1, 2, …., 9 MNIST Example
Image Recognition

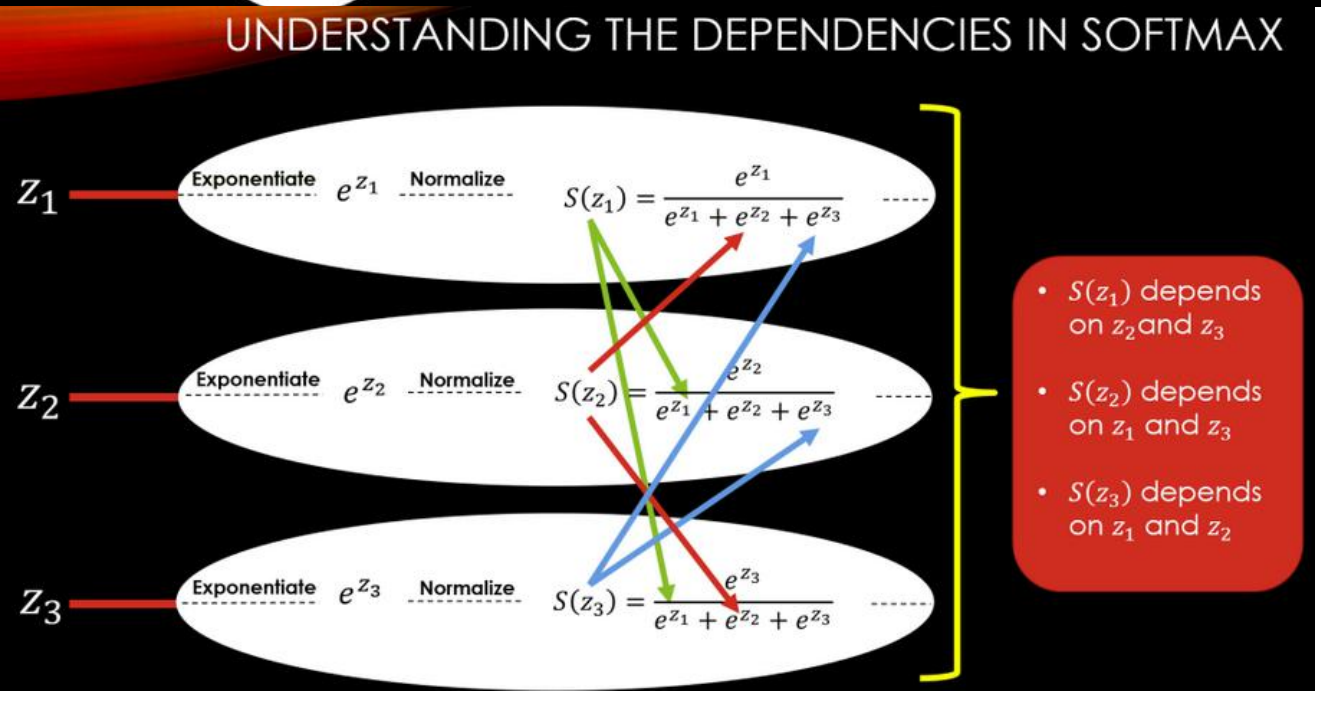
Cross Entropy Loss = CL
Need d(CL)/dwi for backpropagation

For Images
Convolutional Neural Network

Exercise : Represent the following NN model as a computational graph and build a python code to update w and b







https://www.mldawn.com/the-derivative-of-softmaxz-function-w-r-t-z/

WHAT IS $\frac{\partial S(z_1)}{\partial z_1}$ EQUAL TO?

$$\frac{\partial S(z_1)}{\partial z_1} = \frac{\left[\frac{\partial e^{z_1}}{\partial z_1} \times (e^{z_1} + e^{z_2} + e^{z_3})\right] - \left[\frac{\partial (e^{z_1} + e^{z_2} + e^{z_3})}{\partial z_1} \times e^{z_1}\right]}{(e^{z_1} + e^{z_2} + e^{z_3})^2}$$

$$\frac{\partial S(z_1)}{\partial z_1} = \frac{[e^{z_1} \times \Sigma] - [e^{z_1} \times e^{z_1}]}{\Sigma^2} = \frac{e^{z_1}(\Sigma - e^{z_1})}{\Sigma \times \Sigma} = \frac{e^{z_1}}{\Sigma} \times \frac{(\Sigma - e^{z_1})}{\Sigma} = S(z_1) \times (1 - S(z_1))$$

$$\boldsymbol{\frac{\partial S(z_1)}{\partial z_1} = S(z_1) \times (1 - S(z_1))}$$

WHAT IS $\frac{\partial S(z_2)}{\partial z_1}$ EQUAL TO?

$$\frac{\partial S(z_2)}{\partial z_1} = \frac{\left[\frac{\partial e^{z_2}}{\partial z_1} \times (e^{z_1} + e^{z_2} + e^{z_3})\right] - \left[\frac{\partial (e^{z_1} + e^{z_2} + e^{z_3})}{\partial z_1} \times e^{z_2}\right]}{(e^{z_1} + e^{z_2} + e^{z_3})^2}$$

$$\frac{\partial S(z_1)}{\partial z_1} = \frac{[0 \times \Sigma] - [e^{z_1} \times e^{z_2}]}{\Sigma^2} = \frac{-e^{z_1} \times e^{z_2}}{\Sigma \times \Sigma} = \frac{-e^{z_1}}{\Sigma} \times \frac{e^{z_2}}{\Sigma} = -S(z_1) \times S(z_2)$$

$$\boldsymbol{\frac{\partial S(z_2)}{\partial z_1} = -S(z_1) \times S(z_2)}$$

WHAT IS $\frac{\partial S(z_3)}{\partial z_1}$ EQUAL TO?

$$\frac{\partial S(z_3)}{\partial z_1} = \frac{\left[\frac{\partial e^{z_3}}{\partial z_1} \times (e^{z_1} + e^{z_2} + e^{z_3})\right] - \left[\frac{\partial (e^{z_1} + e^{z_2} + e^{z_3})}{\partial z_1} \times e^{z_3}\right]}{(e^{z_1} + e^{z_2} + e^{z_3})^2}$$

$$\frac{\partial S(z_3)}{\partial z_1} = \frac{[0 \times \Sigma] - [e^{z_1} \times e^{z_3}]}{\Sigma^2} = \frac{-e^{z_1} \times e^{z_3}}{\Sigma \times \Sigma} = \frac{-e^{z_1}}{\Sigma} \times \frac{e^{z_3}}{\Sigma} = -S(z_1) \times S(z_3)$$

$$\boldsymbol{\frac{\partial S(z_3)}{\partial z_1} = -S(z_1) \times S(z_3)}$$

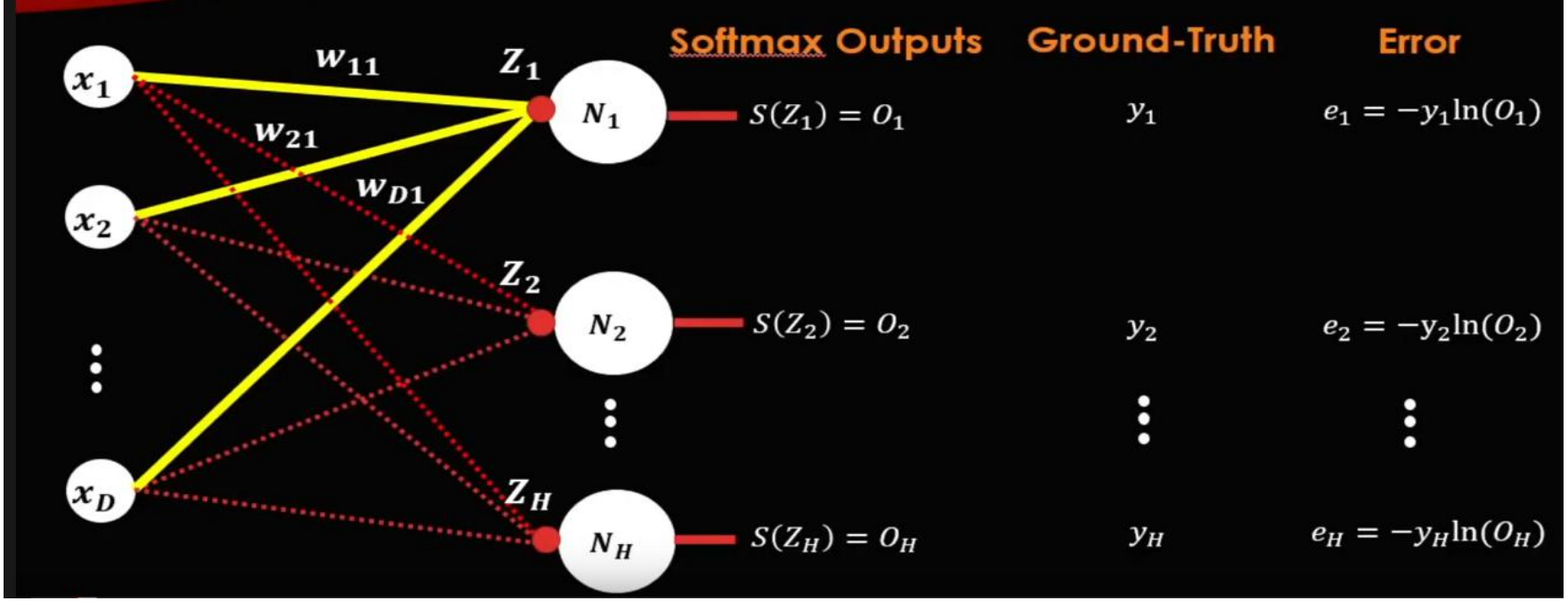
$$\frac{\partial S_i}{\partial x_j} = S_i(\delta_{ij} - S_j)$$

CONCLUSION FOR N OUTPUT

$$S(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{N} e^{z_j}}$$

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

$$\boldsymbol{\frac{\partial S(z_i)}{\partial z_j} = \begin{cases} S(z_i) \times (1 - S(z_i)) & if\ i = j \\ -S(z_i) \times S(z_j) & if\ i \neq j \end{cases}}$$



$$E = \sum_{i=1}^{H} e_i \quad e_i = -y_i \ln(O_i) \quad E = -\sum_{i=1}^{H} y_i \ln(O_i)$$

Computing Gradients

$$\frac{\partial E}{\partial Z_k} = \frac{\partial(-\sum_{i=1}^{H} y_i \ln(O_i))}{\partial Z_k}$$

Derivative of SUM = SUM of Derivatives

$$\frac{\partial E}{\partial Z_k} = -\sum_{i=1}^{H} \frac{\partial(y_i \ln(O_i))}{\partial Z_k}$$

Rule of Independence: Ground-Truth Comes Out of the Derivative

$$\frac{\partial E}{\partial Z_k} = -\sum_{i=1}^{H} y_i \frac{\partial \ln(O_i)}{\partial Z_k}$$

O_i is not a direct function of Z_k! So we will have to use the Chain Rule

https://www.youtube.com/watch?v=znqbtL0fRA0 - backpropagation through cross entropy and software – ML Dawn

- According to the chain rule, we know that: $\frac{\partial \ln(O_i)}{\partial Z_k} = \frac{\partial \ln(O_i)}{\partial O_i} \times \frac{\partial O_i}{\partial Z_k}$

- By replacing this in $\frac{\partial E}{\partial Z_k} = -\sum_{i=1}^{H} y_i \frac{\partial \ln(O_i)}{\partial Z_k}$ we will get:

We need the derivative of Softmax w.r.t its inputs

$$\frac{\partial E}{\partial Z_k} = -\sum_{i=1}^{H}\left[y_i \frac{\partial \ln(O_i)}{\partial O_i} \times \frac{\partial O_i}{\partial Z_k}\right]$$

($\frac{1}{O_i}$)

$$\frac{\partial \ln(O_i)}{\partial O_i} = \frac{1}{O_i}$$

$$\frac{\partial E}{\partial Z_k} = -\sum_{i=1}^{H}\left[\frac{y_i}{O_i} \times \frac{\partial O_i}{\partial Z_k}\right]$$

- So, let's separate the SUM into 2 parts:

1. ***All the terms for which $i \neq k$***
2. ***The term for which $i = k$***

- As a result $\frac{\partial E}{\partial Z_k} = -\sum_{i=1}^{H}\left[\frac{y_i}{O_i} \times \frac{\partial O_i}{\partial Z_k}\right]$ will change as follows:

$$\frac{\partial E}{\partial Z_k} = -\left[\sum_{i \neq k}^{H}\left[\frac{y_i}{O_i} \times -O_i \times O_k\right] + \frac{y_k}{O_k} \times O_k \times (1 - O_k)\right]$$

($\frac{\partial O_i}{\partial Z_k}$ when $i \neq k$; $\frac{\partial O_i}{\partial Z_k}$ when $i = k$)

All terms for which $i \neq k$ | The term For which $i = k$

https://www.youtube.com/watch?v=znqbtL0fRA0 – backpropagation through cross entropy and software – ML Dawn

- There are things that we can simplify:

Independent of the index k

$$\frac{\partial E}{\partial Z_k} = -\left[\sum_{i \neq k}^{H}\left[\frac{y_i}{O_i} \times -O_i \times O_k\right] + \left[\frac{y_k}{O_k} \times O_k \times (1 - O_k)\right]\right] \Rightarrow \frac{\partial E}{\partial Z_k} = -\left[\sum_{i \neq k}^{H}[-y_i \times O_k] + [y_k \times (1 - O_k)]\right]$$

so O_k comes out of the SUM

$$\Rightarrow \frac{\partial E}{\partial Z_k} = -\left[-O_k \sum_{i \neq k}^{H} y_i + [y_k \times (1 - O_k)]\right]$$

- As our ground-truth vectors are one-hot encoding vectors $y = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, then:

$$\sum_{i=1}^{H} y_i = 1$$

- As a result, we can play a trick and re-write $\sum_{i \neq k}^{H} y_i$:

$$\sum_{i \neq k}^{H} y_i = \sum_{i=1}^{H} y_i - y_k = 1 - y_k$$

https://www.youtube.com/watch?v=znqbtL0fRA0 - backpropagation through cross entropy and software – ML Dawn

- So, when using Binary Cross-Entropy Error Function with Softmax() output function, the derivative of the error w.r.t the inputs to the Softmax() function is computed as follows:

$$\frac{\partial E}{\partial Z_k} = O_k - y_k$$

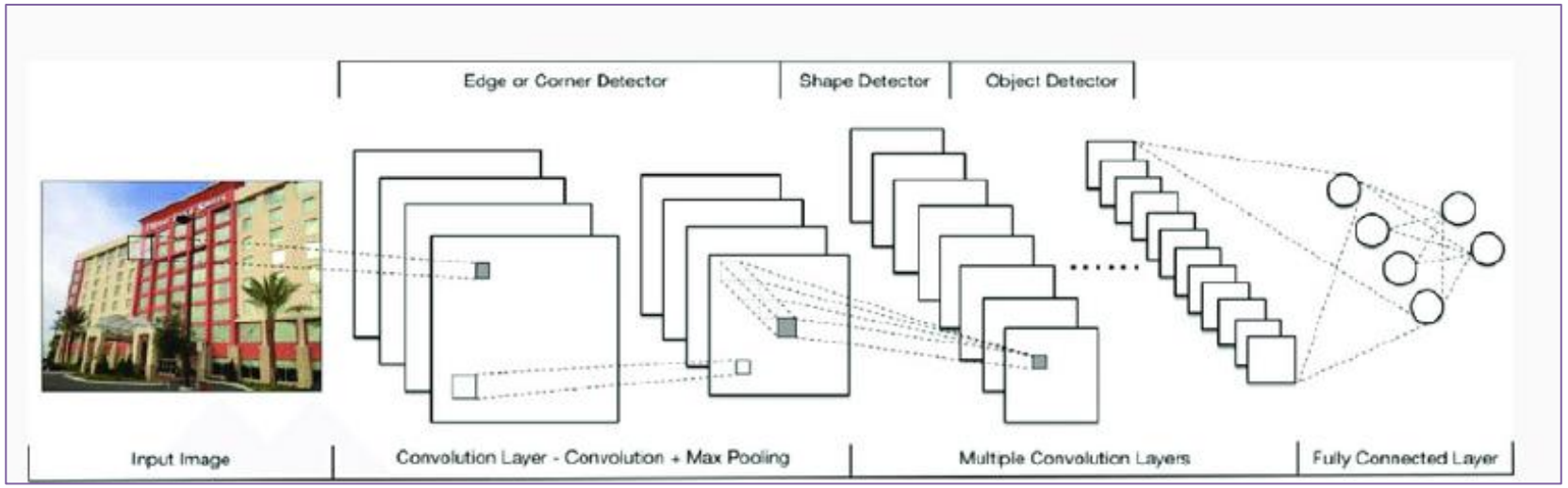
- Where y_k is the k^{th} element of the one-hot encoding ground truth vector y, and it is either 0 or 1
- And O_k is the Softmax() function defined as: $O_k = \frac{e^{z_k}}{\sum_{i=1}^{H} e^{z_i}}$

https://www.youtube.com/watch?v=znqbtL0fRA0 - backpropagation through cross entropy and software – ML Dawn

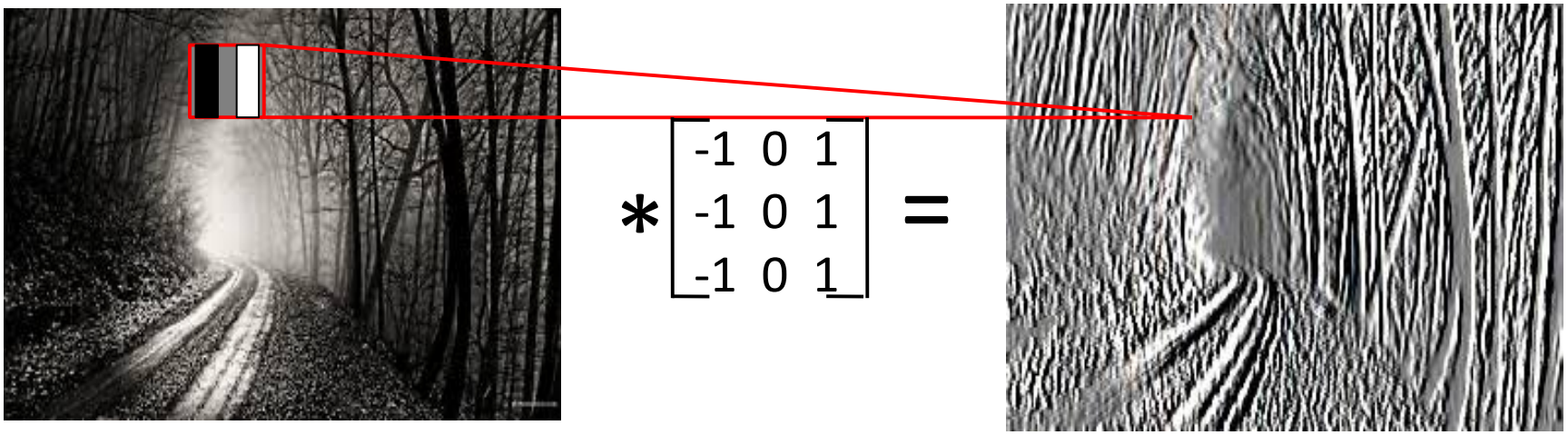
Convolutional Neural Network

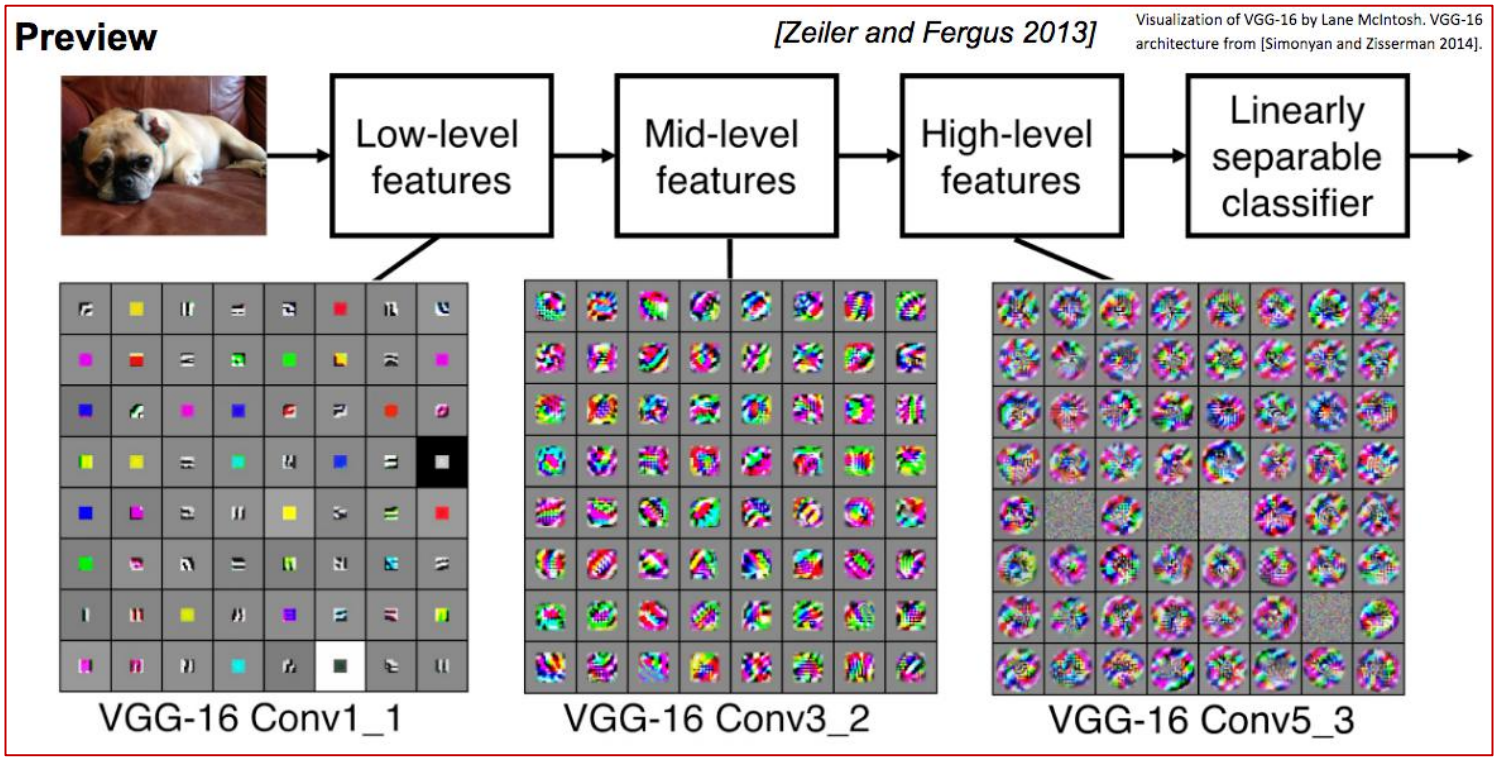
- ✓ **Convolutional Neural Network Introduction**
- ✓ **CNN example, Convolution operations**
- ✓ **MM in convolution operations**

- ✓ **CNN framework :**a class of neural network where the input is modified by a filter
- ✓ Multiple filters in convolutional networks are applied to different slices of an image, mapping them one by one, and identifying different features of an input image

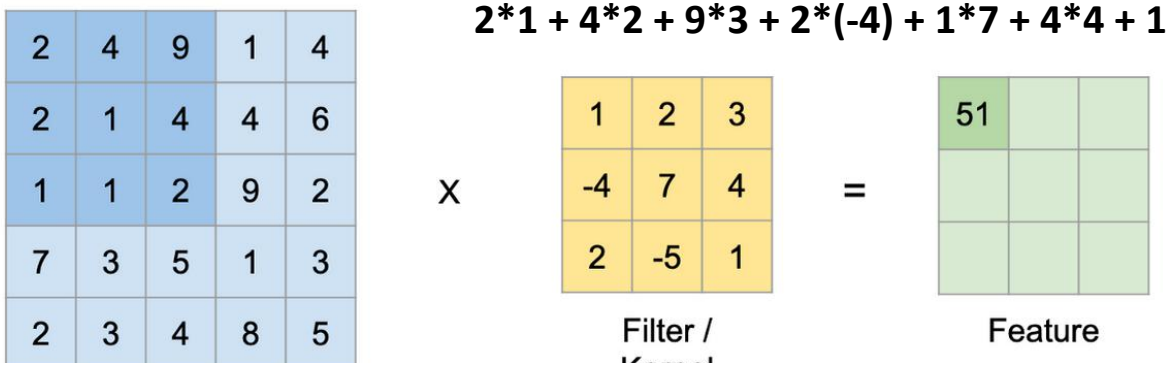


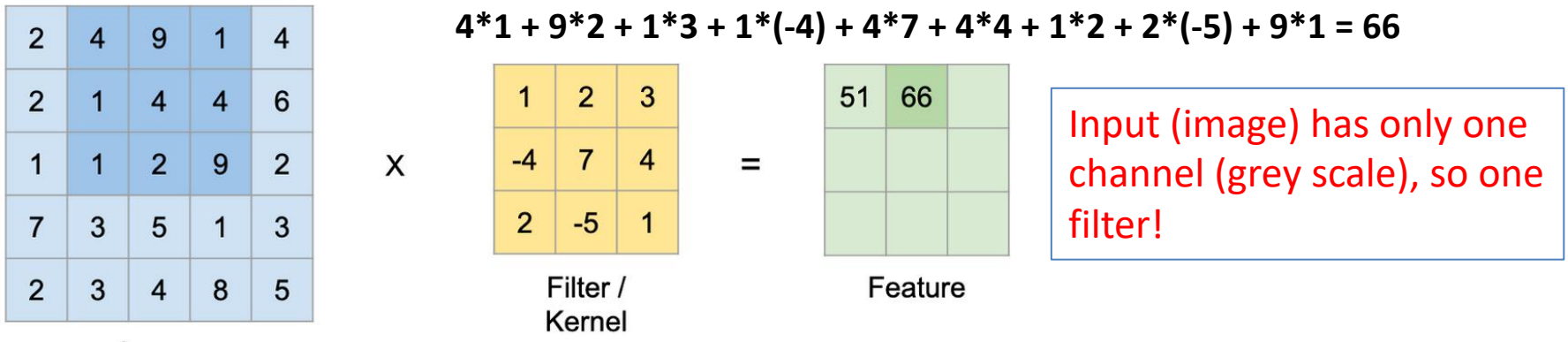
CNN filters


-1 0 1
-1 0 1
-1 0 1


Preview
[Zeiler and Fergus 2013]
Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].
Low-level features
Mid-level features
High-level features
Linearly separable classifier
VGG-16 Conv1_1
VGG-16 Conv3_2
VGG-16 Conv5_3

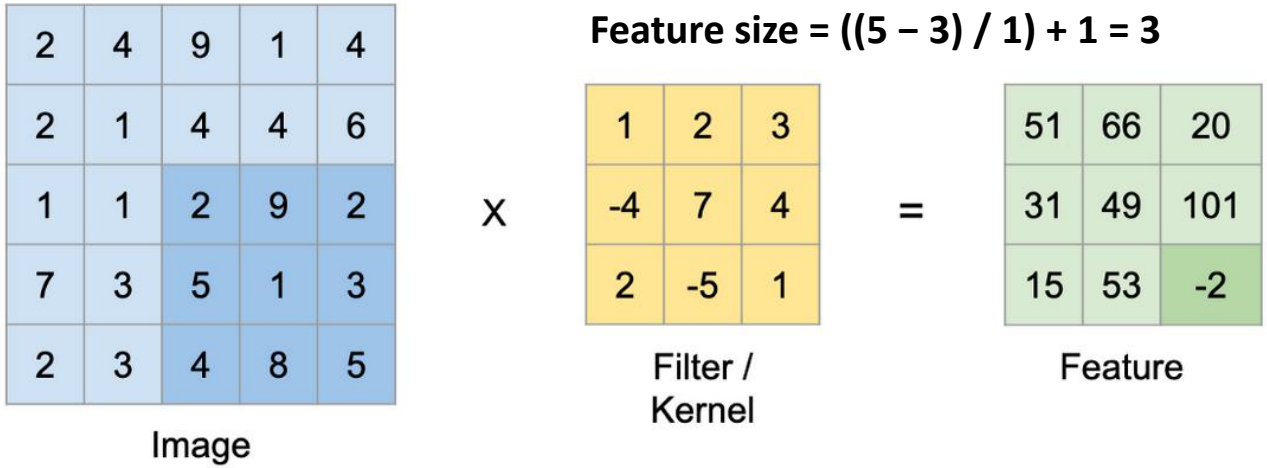
https://towardsdatascience.com/convolution-neural-networks-a-beginners-guide-implementing-a-mnist-hand-written-digit-8aa60330d022





Feature size = ((Image size − Kernel size) / Stride) + 1

Feature size = ((5 − 3) / 1) + 1 = 3



Feature size = ((Image size + 2 * Padding size − Kernel size) / Stride)+1

Feature size = ((5 + 2 * 1 − 3) / 1) + 1= 5

0	0	0	0	0	0	0
0	2	4	9	1	4	0
0	2	1	4	4	6	0
0	1	1	2	9	2	0
0	7	3	5	1	3	0
0	2	3	4	8	5	0
0	0	0	0	0	0	0

Image

X

1	2	3
-4	7	4
2	-5	1

Filter / Kernel

=

21	59	37	-19	2
30	51	66	20	43
-14	31	49	101	-19
59	15	53	-2	21
49	57	64	76	10

Feature

21	59	37	-19	2
30	51	66	20	43
-14	31	49	101	-19
59	15	53	-2	21
49	57	64	76	10

Convolved Feature

21	59	37	-19	2
30	51	66	20	43
-14	31	49	101	-19
59	15	53	-2	21
49	57	64	76	10

Convolved Feature

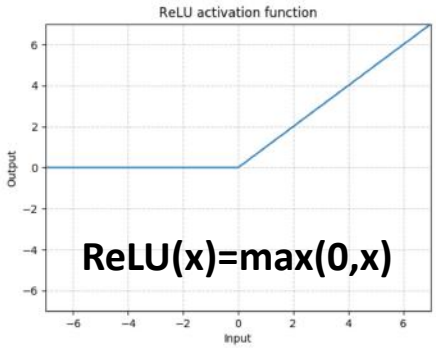
59	66		

Max Pooled Feature

59	66	66	43
51	66	101	101
59	53	101	101
59	64	76	76

Max Pooled Feature

MaxPool(Relu(x)) = Relu(MaxPool(x))



The procedure satisfies the communicative property and can be used either way. In practice ReLU activation function is applied right after a convolution layer and then that output is max pooled. Change negative numbers to zero, positive numbers stay the same

-?

0

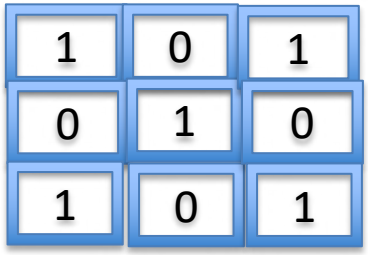
CNN : Forward Path
A Simple Example

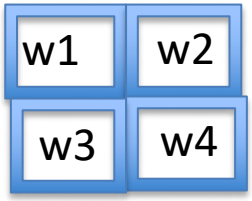
Example exercise

Compute the updated values of filter weights after backpropagation of the following problem -- 3x3 input matrix, 2x2 filter, stride 1, no padding, flatten it and use it as input connecting to a two-neuron layer, then pass the results to a softmax binary classification of two neurons.

3a) (0.5%) What is the values of the output at the end of the forward path

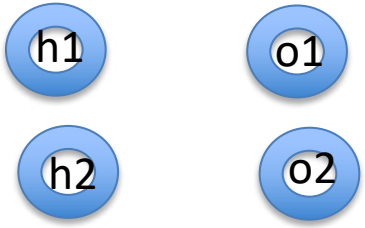
3b) (1.5%) What are the updated values of the weight and bias after backpropagation





b1 = 0.1

w1=0.9
w2=0.1
w3=0.1
w4=0.9



Apply ReLU activation function after the convolution step

Flatten the value after applying the ReLU activation function

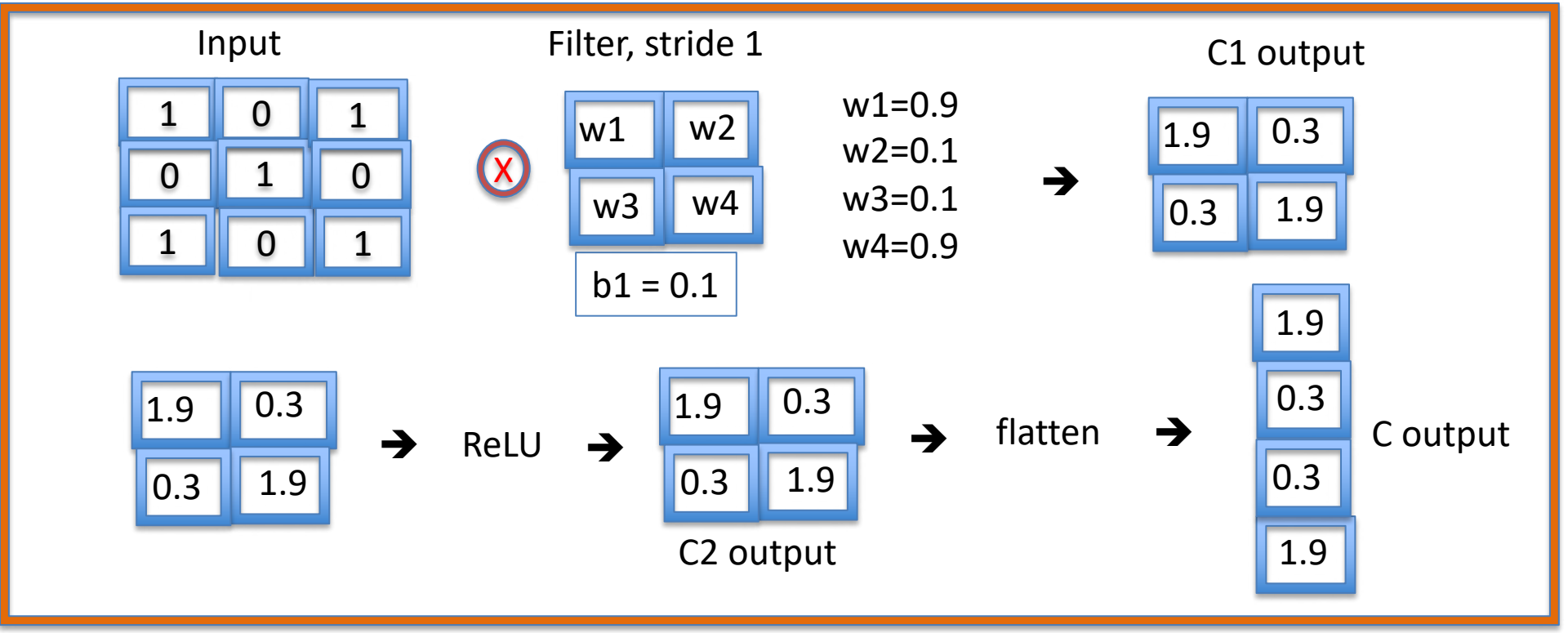
Assuming the weight of each link to this layer is w = 0.1 to 0.8, b2=0.2

Fully connecting to a layer of two neurons

Assuming the weight of each link is w = 0.1 to 0.4, b3=0.3

Fully Connected layer of 2 softmax neurons

Labeled output of two classes are 1.0 and 0.00



ReLU (1xw1+0xw2+0xw3+1xw4+b1 =1x0.9+0x0.1+0x0.1+1x0.9+0.1=1.9) = C1

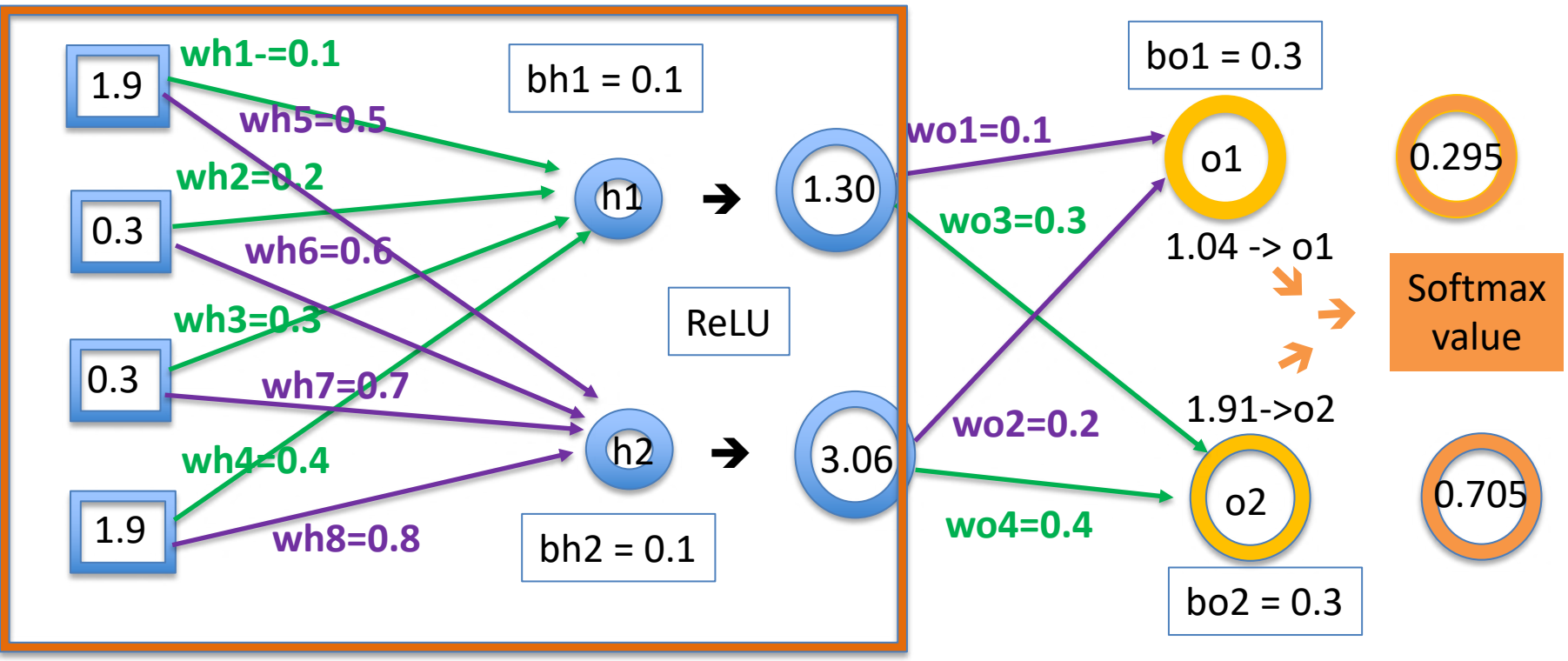
ReLU (0xw1+1xw2+1xw3+0xw4+b1=0x0.9+1x0.1+1x0.1+0x0.9+0.1=0.3) = C2

ReLU (0xw1+1xw2+1xw3+0xw4+b1=0x0.9+1x0.1+1x0.1+0x0.9+0.1=0.3) = C3

ReLU (1xw1+0xw2+0xw3+1xw4+b1 =1x0.9+0x0.1+0x0.1+1x0.9+0.1=1.9) = C4

1.9
0.3
0.3
1.9

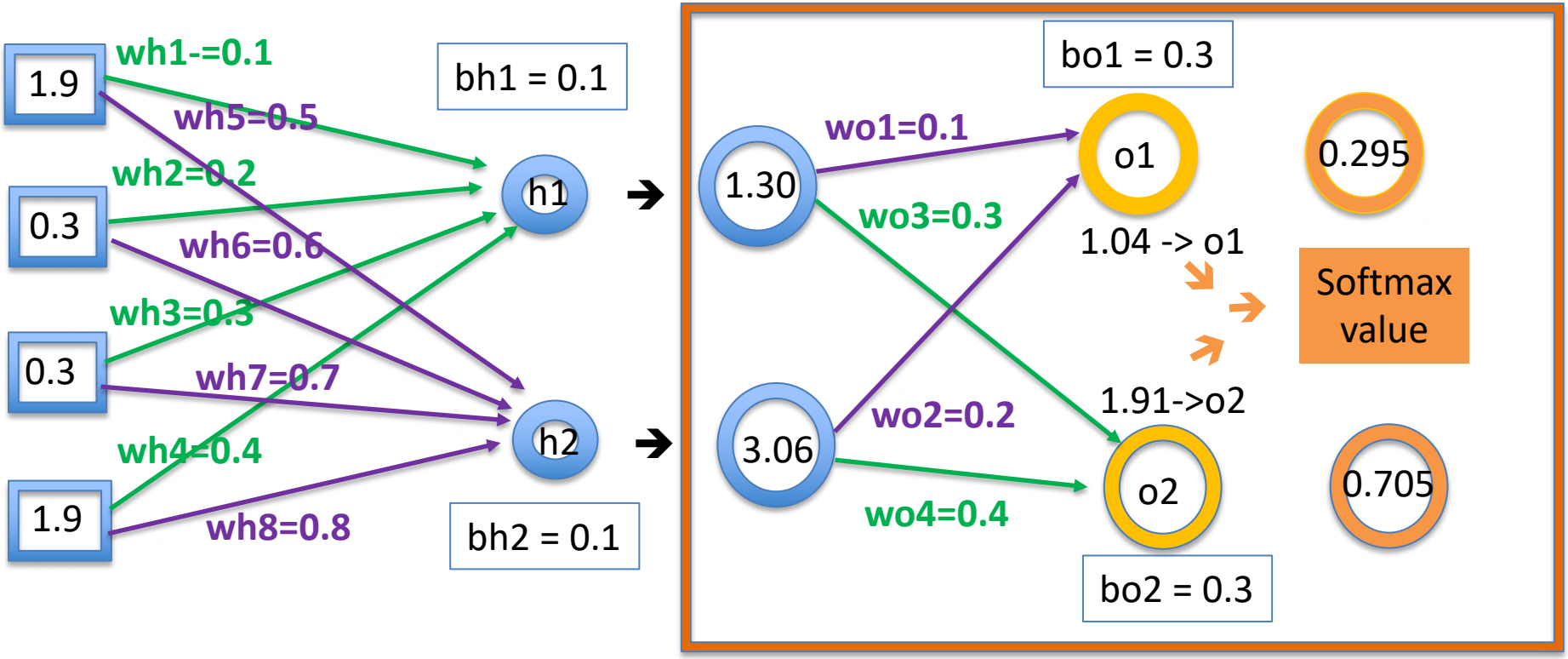
C output



C1xwh1+ C2xwh2 + C3xwh3 x C4xwh4 + bh1 = 1.9x0.1+0.3x0.2+0.3x0.3+1.9x0.4+0.1= 1.3 = h1

C1xwh5+ C2xwh6 + C3xwh7 x C4xwh8 + bh2 = 1.9x0.5+0.3x0.6+0.3x0.7+1.9x0.8+0.1= 3.06 = h2

ReLU { h1, h2 } = {H1, H2} = {1.3, 3.06}



Softmax (H1 x wo1+ H2 x wo2 + bo1 = 1.3 x 0.1+ 3.06 x 0.2 +0.3 = 1.042) = O1

Softmax (H1 x wo3+ H2 x wo4 + bo2 = 1.3 x 0.3+ 3.06 x 0.4 +0.3 = 1.914) = O2

Softmax function

$$\sigma(j) = \frac{\exp(\mathbf{w}_j^\top \mathbf{x})}{\sum_{k=1}^{K} \exp(\mathbf{w}_k^\top \mathbf{x})} = \frac{\exp(z_j)}{\sum_{k=1}^{K} \exp(z_k)}$$

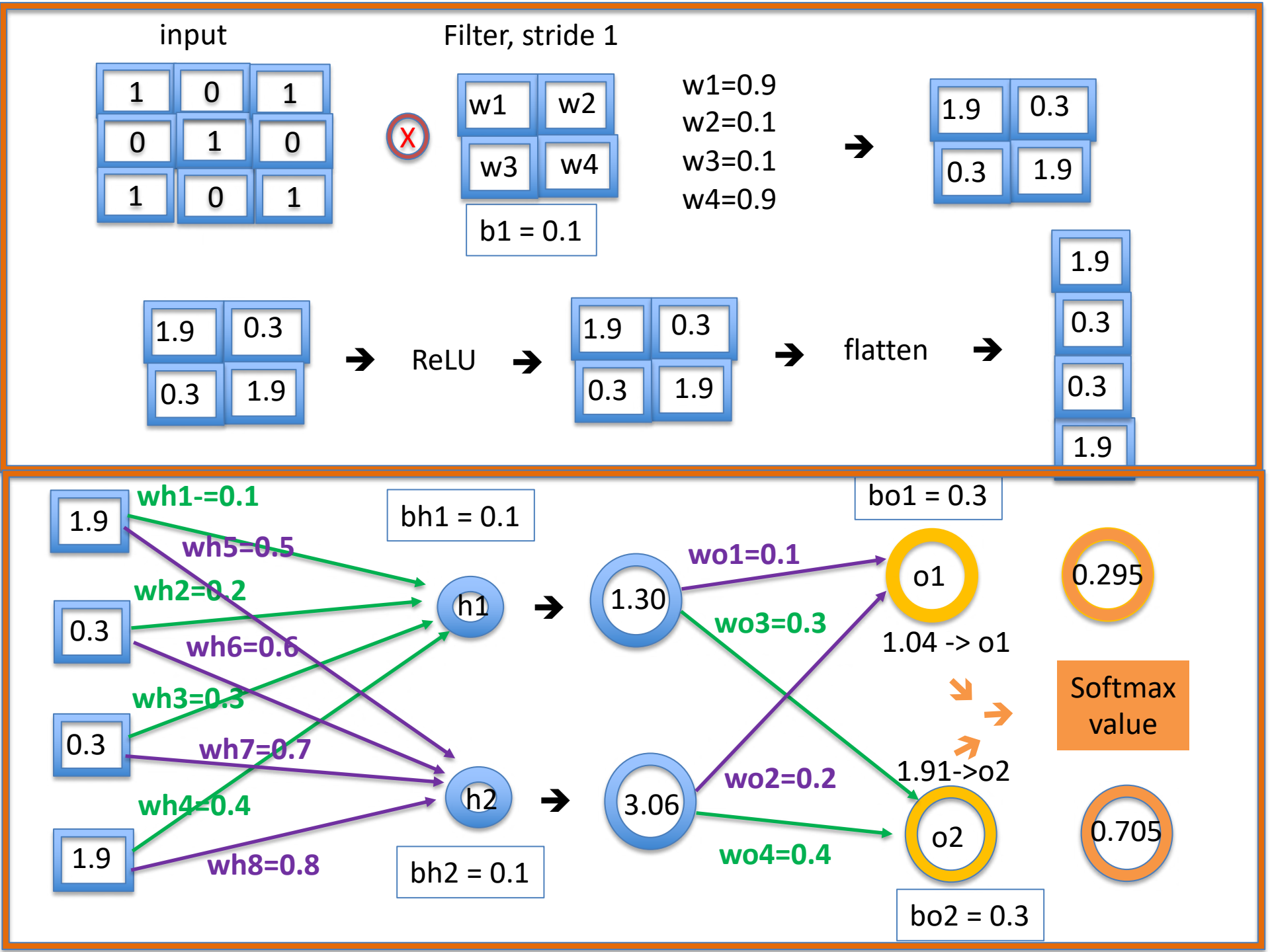
exp(1.042) = 2.8348

exp(1.914) = 6.7802

exp(1.042) + exp(1.914) = 9.615

O1 = 0.295

O1 = 0.705


input
Filter, stride 1
1 0 1
0 1 0
1 0 1
X
w1 w2
w3 w4
b1 = 0.1
w1=0.9
w2=0.1
w3=0.1
w4=0.9
1.9 0.3
0.3 1.9
1.9 0.3
0.3 1.9
ReLU
1.9 0.3
0.3 1.9
flatten
1.9
0.3
0.3
1.9
1.9
wh1-=0.1
wh5=0.5
wh2=0.2
wh6=0.6
wh3=0.3
wh7=0.7
wh4=0.4
wh8=0.8
0.3
0.3
1.9
bh1 = 0.1
h1
h2
bh2 = 0.1
1.30
3.06
wo1=0.1
wo3=0.3
wo2=0.2
wo4=0.4
bo1 = 0.3
o1
o2
bo2 = 0.3
1.04 -> o1
1.91->o2
Softmax value
0.295
0.705

CNN Arithmetic, multiple input channels (RGB image)

Input : W=5, H=5, D=3 (Channel)

Filter : K=2 (number), F=3 x3 (size), S (Stride)=2, P (Zero Padding) = 1

(W-F+S)/3 + P = output volume size
Output : W=3, H=3, D=2 (Channel)

Input Volume (+pad 1) (7x7x3)

x[:,:,0]

0	0	0	0	0	0	0
0	1	0	1	0	1	0
0	1	2	0	1	0	0
0	1	0	1	1	2	0
0	0	0	2	1	2	0
0	1	2	2	1	1	0
0	0	0	0	0	0	0

x[:,:,1]

0	0	0	0	0	0	0
0	0	1	0	1	2	0
0	0	0	1	0	1	0
0	1	0	1	1	1	0
0	2	2	2	1	1	0
0	1	0	2	2	1	0
0	0	0	0	0	0	0

x[:,:,2]

0	0	0	0	0	0	0
0	0	2	0	1	1	0
0	2	1	2	1	1	0
0	2	2	0	1	2	0
0	0	2	1	2	1	0
0	2	2	2	1	1	0
0	0	0	0	0	0	0

Filter W0 (3x3x3)

w0[:,:,0]

0	-1	1
0	1	1
-1	0	1

w0[:,:,1]

0	0	1
0	1	1
1	-1	0

w0[:,:,2]

0	1	0
-1	-1	0
-1	1	0

Bias b0 (1x1x1)

b0[:,:,0]

1

Filter W1 (3x3x3)

w1[:,:,0]

-1	-1	1
1	1	0
1	-1	1

w1[:,:,1]

-1	-1	1
-1	-1	-1
0	1	0

w1[:,:,2]

-1	0	-1
0	-1	0
0	-1	0

Bias b1 (1x1x1)

b1[:,:,0]

0

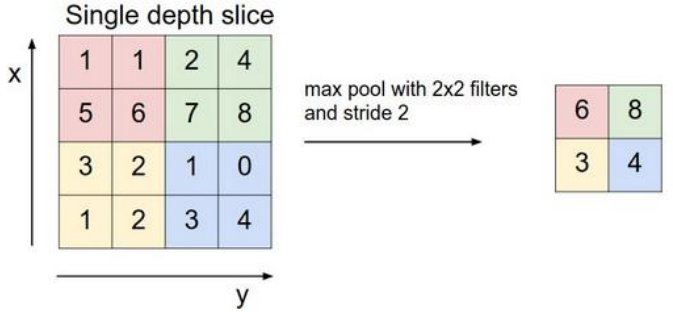
Output Volume (3x3x2)

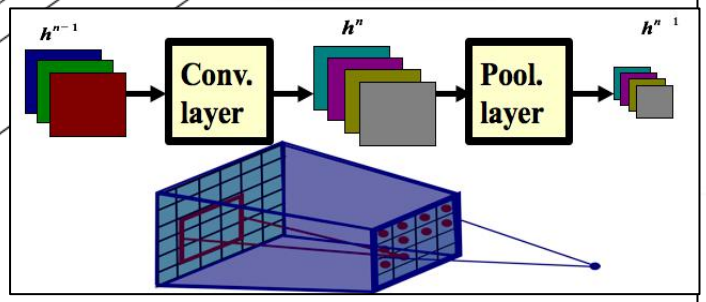
o[:,:,0]

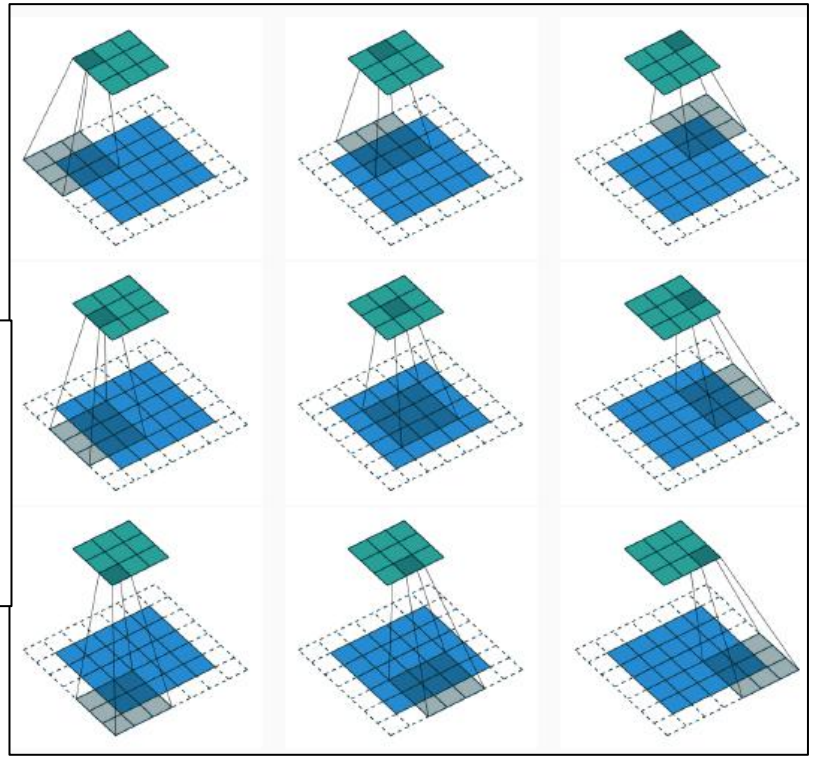
7	0	0
2	6	0
5	5	0

o[:,:,1]

-1	1	-2
0	-5	-5
-4	-10	-9

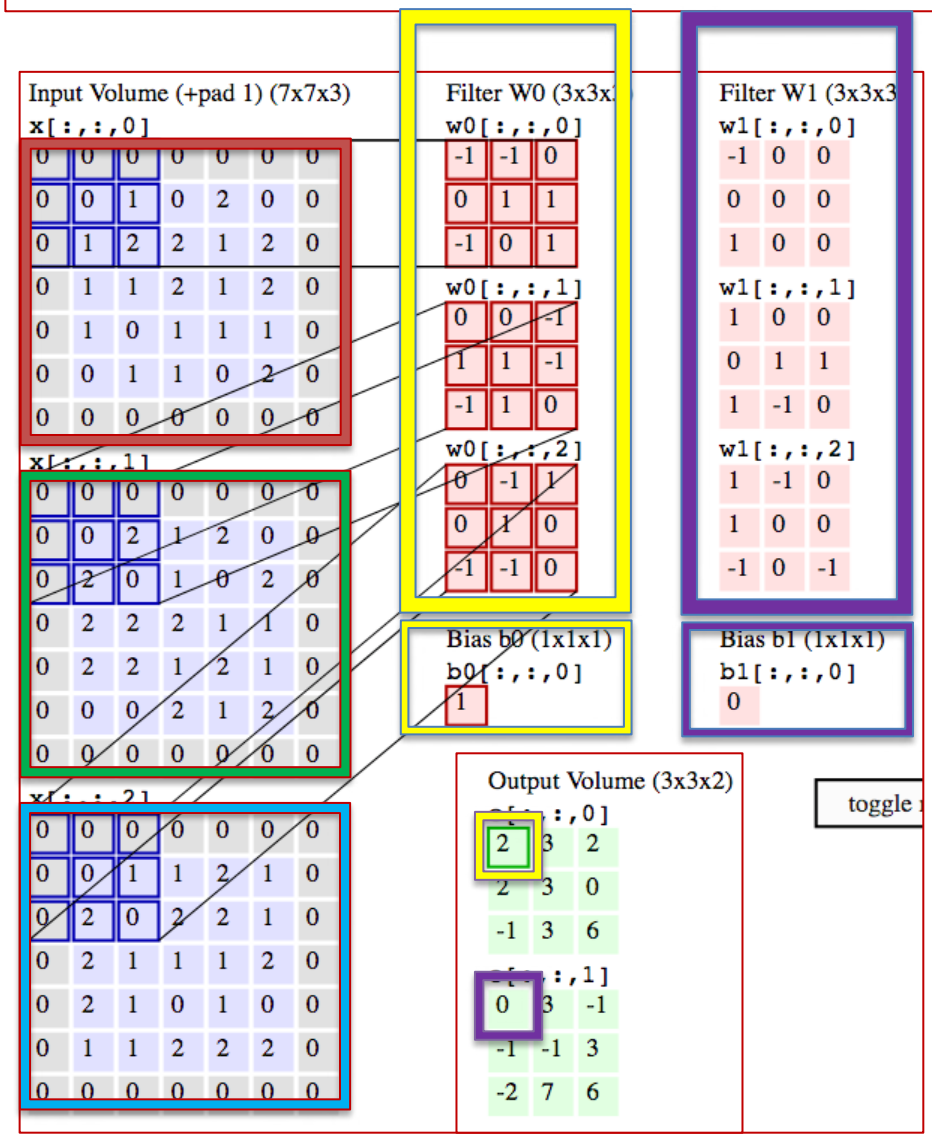


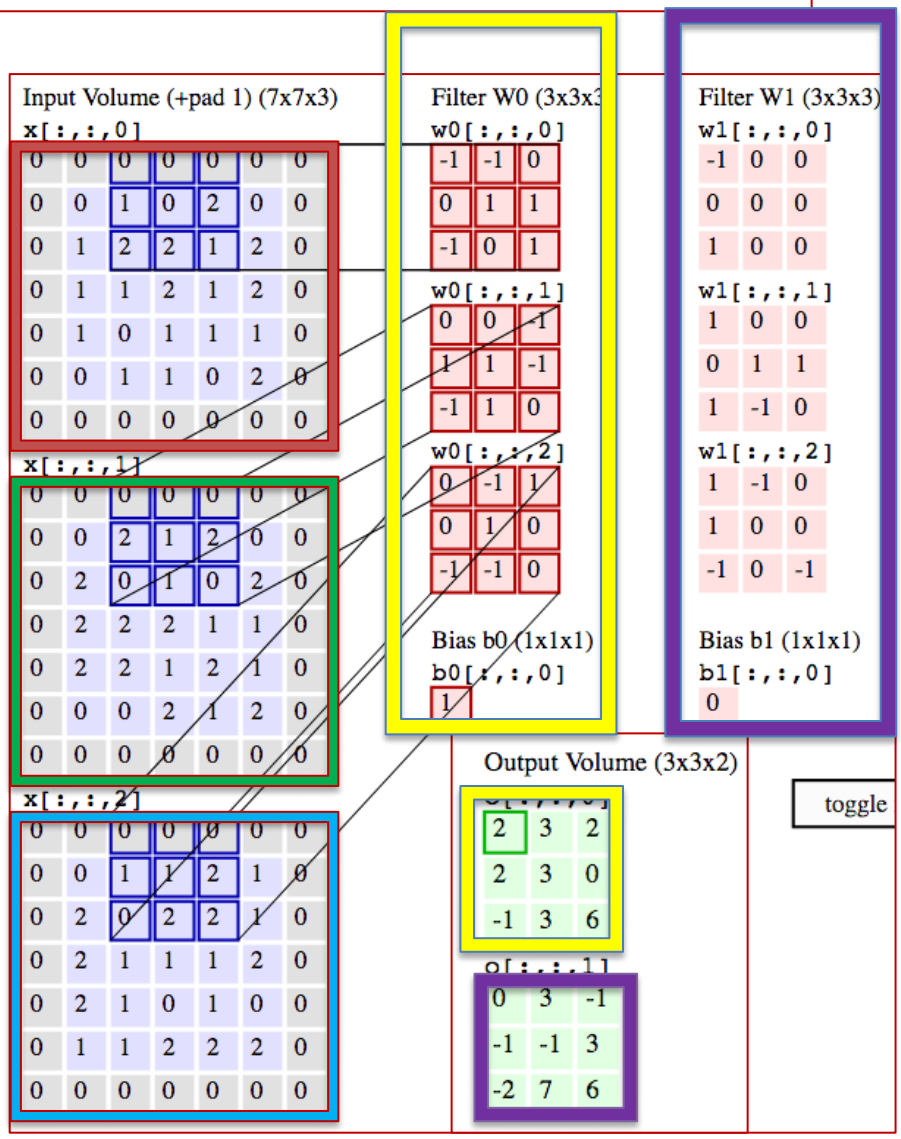




https://cs231n.github.io/convolutional-networks/

Input volume : $W1=5, H1=5, D1=3$, CONV layer parameters = $K=2, F=3, S=2, P=1$.





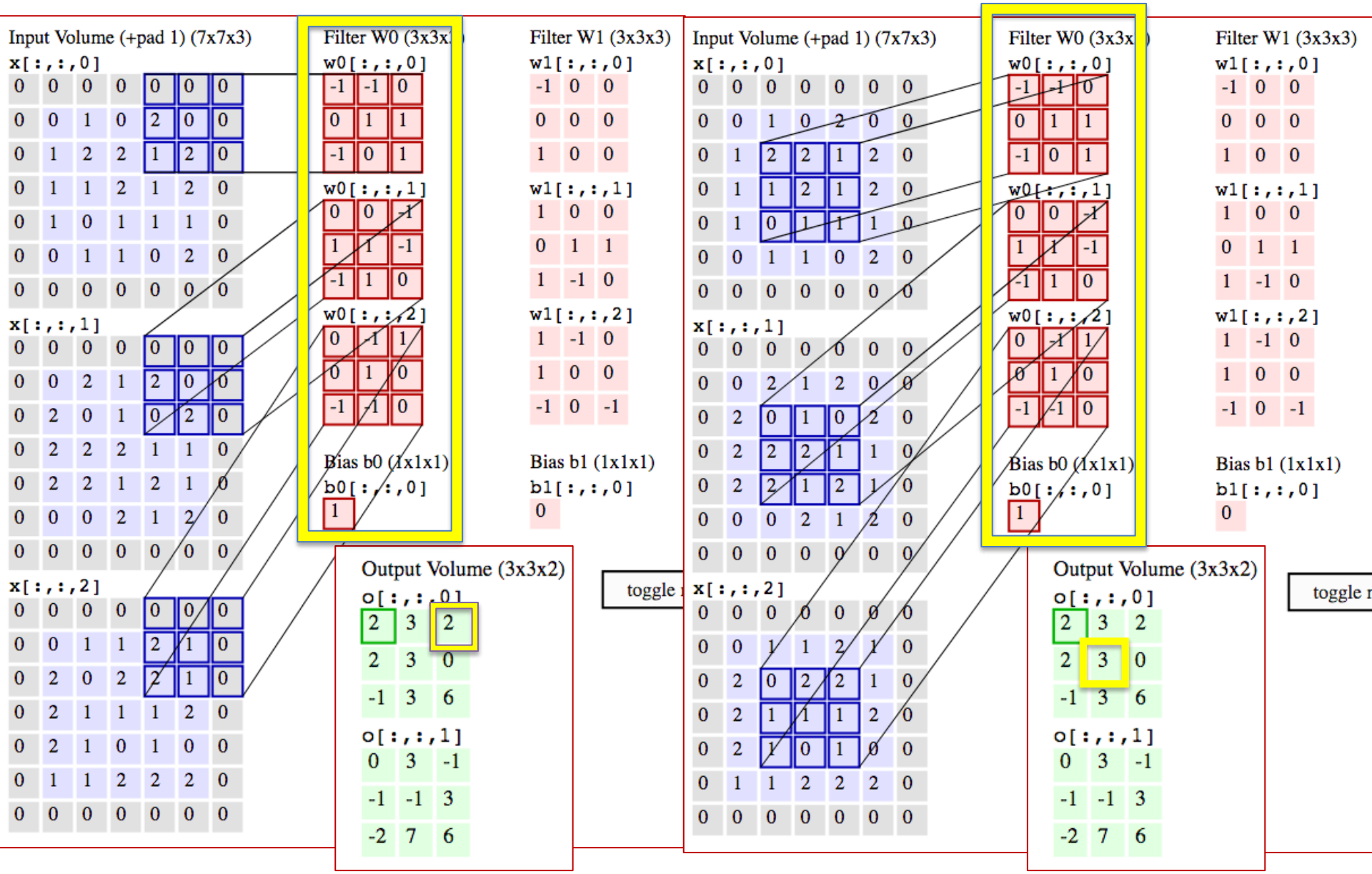
(0 x -1 + 0 x -1 + 0 x 0) + (0 x 0 + 0 x 1 + 1 x 1) + (0 x -1 + 1 x 0 + 2 x 1) = 3

(0 x 0 + 0 x 0 + 0 x -1) + (0 x 1 + 0 x 1 + 2 x -1)+ (0 x -1 + 2 x 1 + 0 x 0) = 0

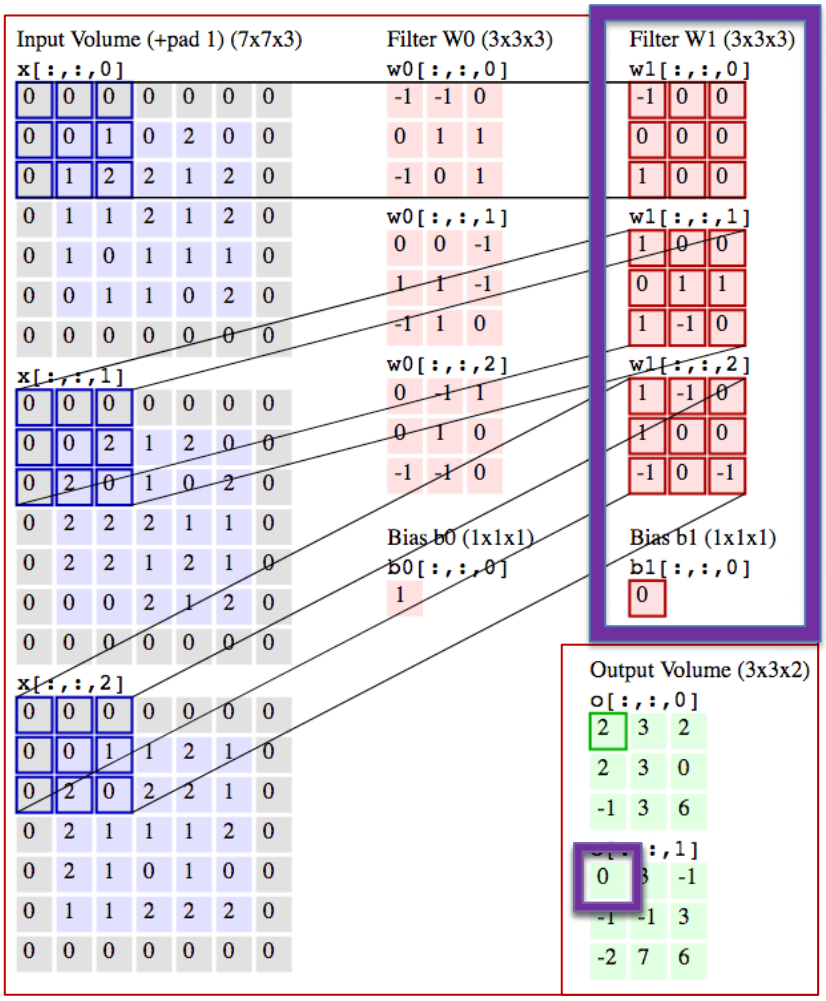
(0 x 0 + 0 x -1 + 0 x -1) + (0 x 0 + 0 x 1 + 1 x 0)+ (0 x -1 + 2 x -1 + 0 x 0) = -2

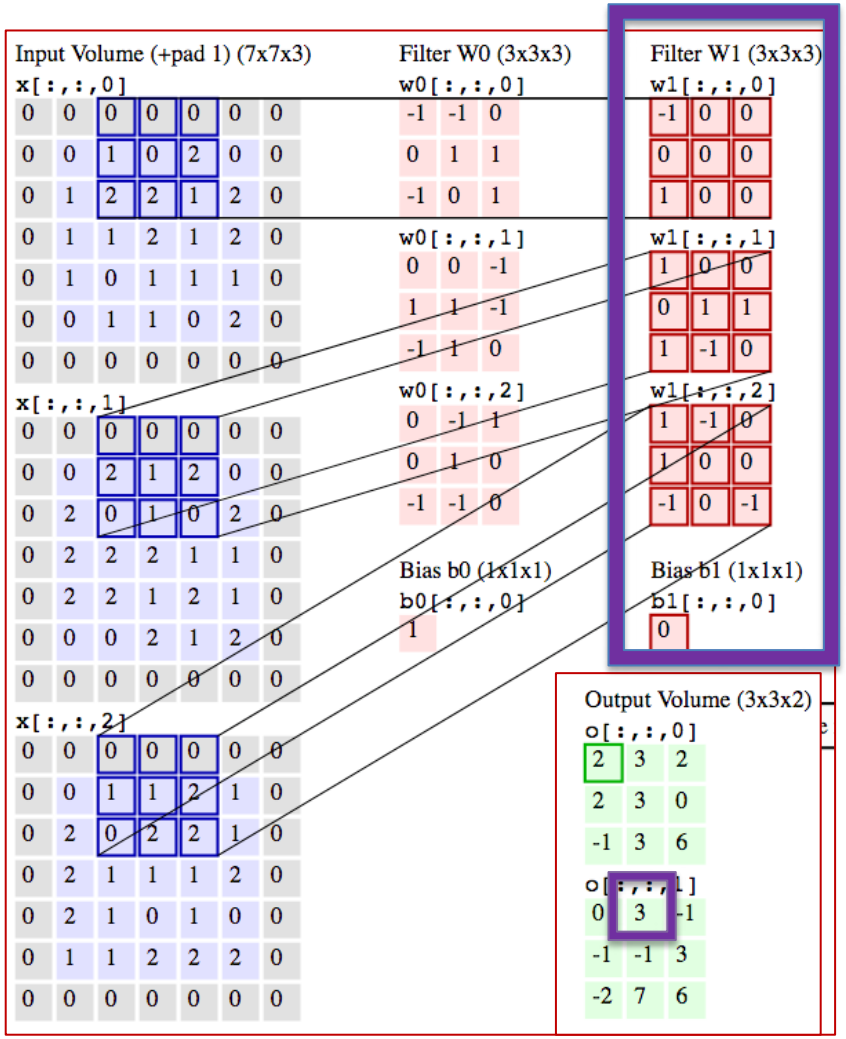
1 + b0 (1) = 2

CNN Filter Operations - RGB



CNN Filter Operations - RGB





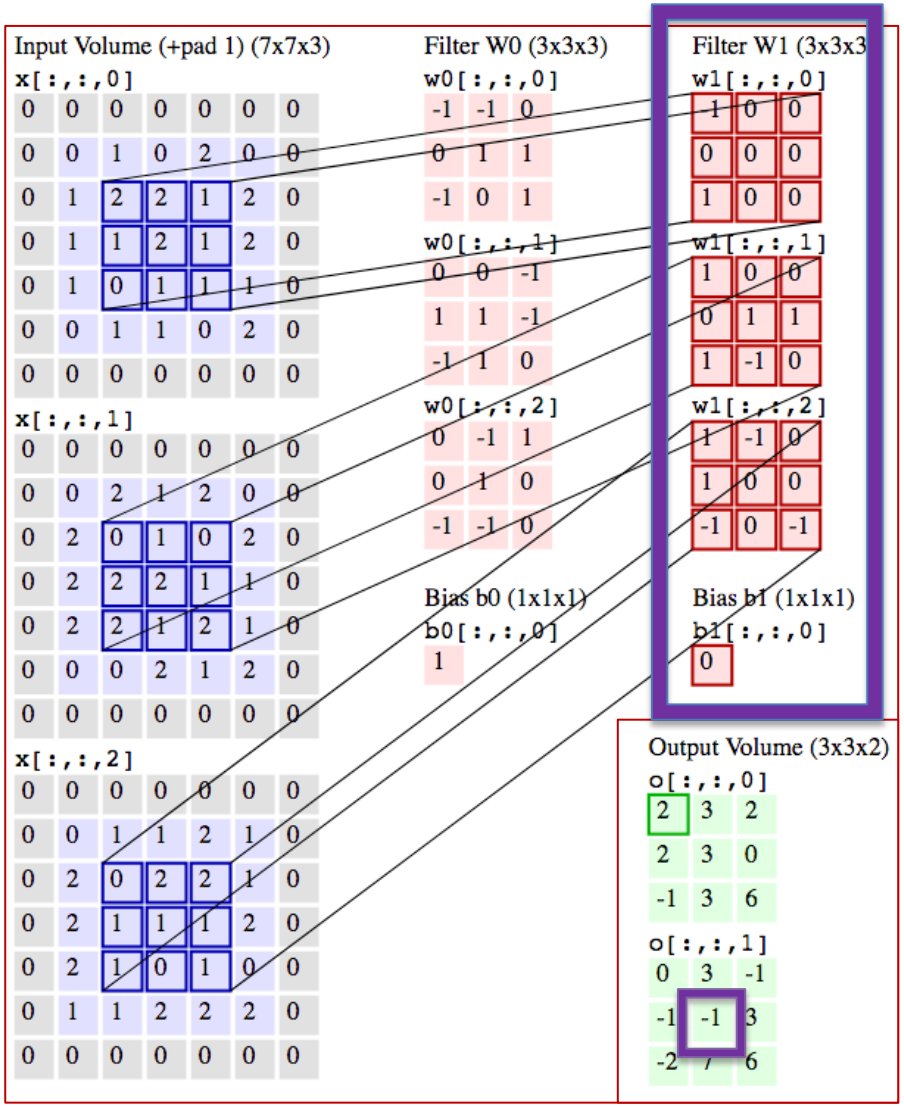
(0 x -1 + 0 x 0 + 0 x 0) + (0 x 0 + 0 x 0 + 1 x 0) + (0 x 1 + 1 x 0 + 2 x 0) = 0

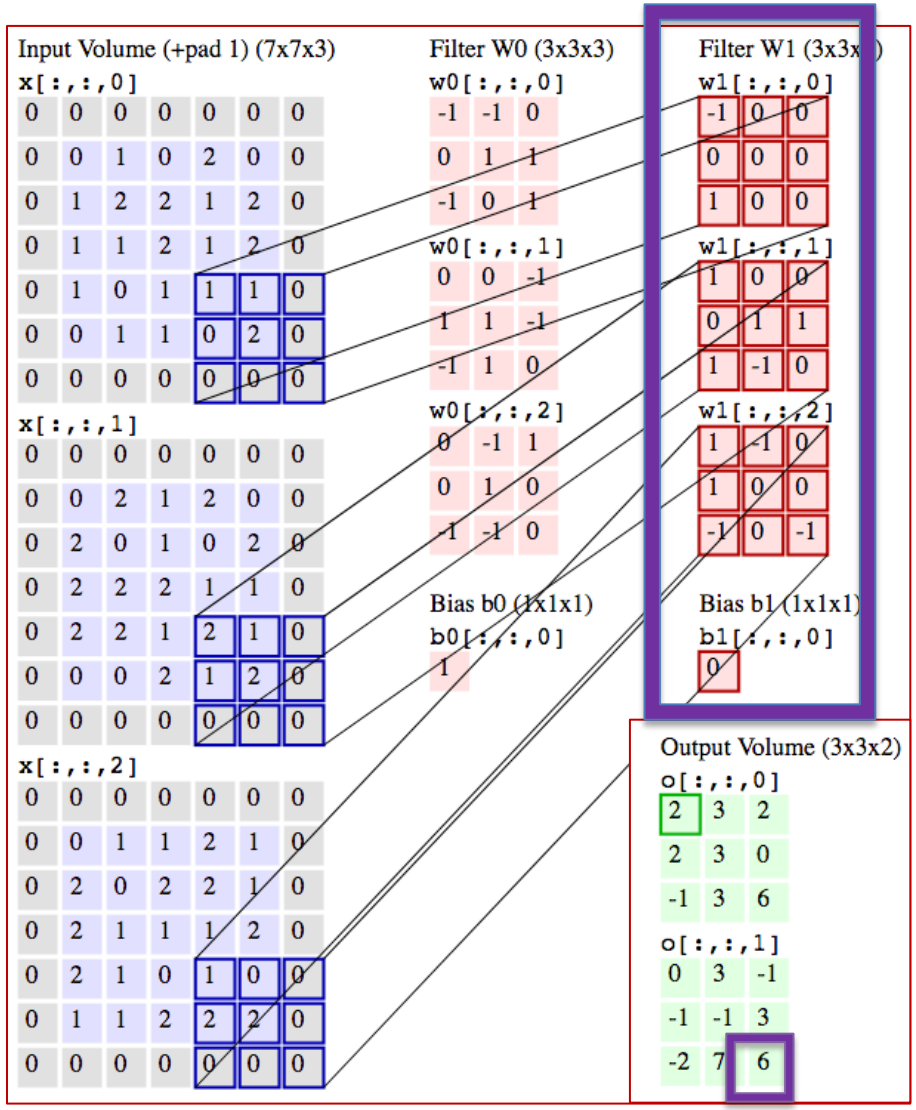
(0 x 1 + 0 x 0 + 0 x 0) + (0 x 0 + 0 x 1 + 2 x 1)+ (0 x 1 + 2 x -1 + 0 x 0) = 0

(0 x 1 + 0 x -1 + 0 x 0) + (0 x 1 + 0 x 0 + 1 x 0)+ (0 x -1 + 2 x 0 + 0 x -1) = 0

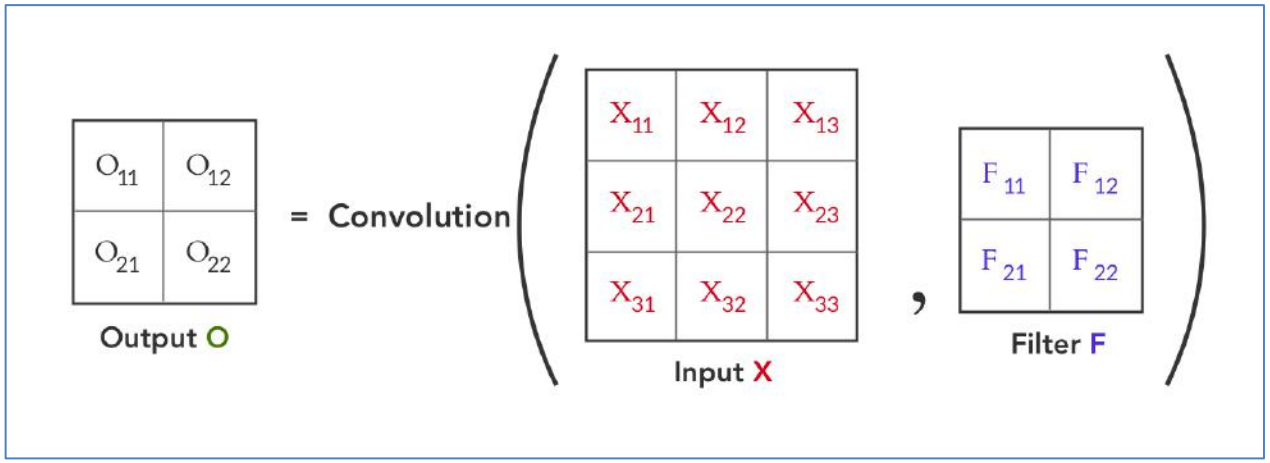
0 + b1 (0) = 0

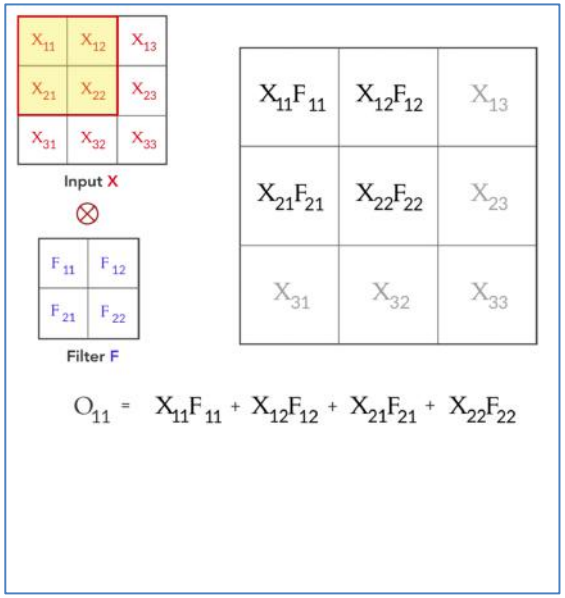
CNN Filter Operations - RGB

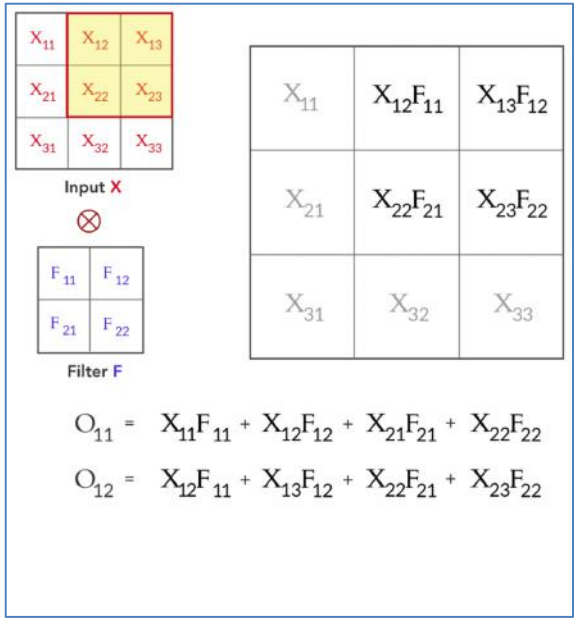


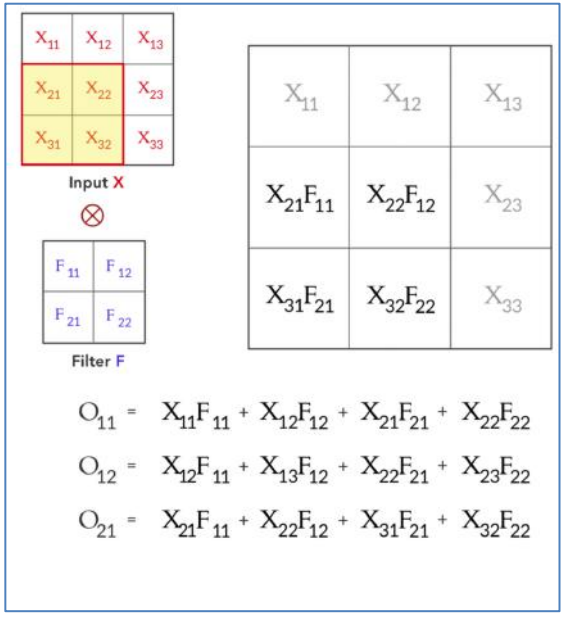


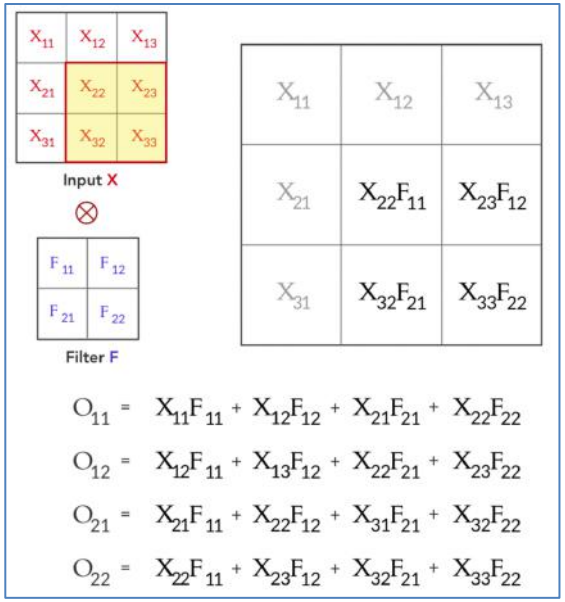
Forward path





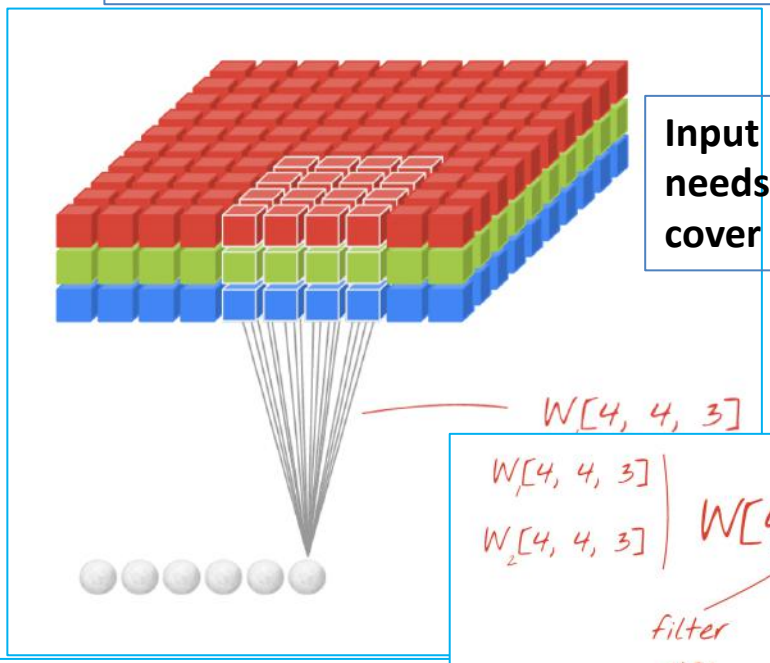




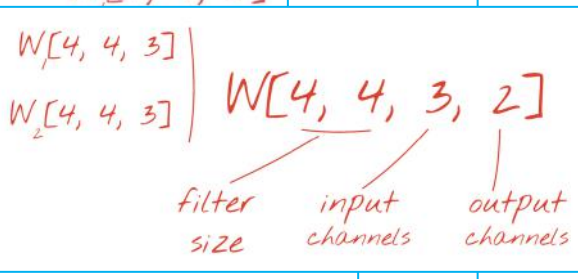


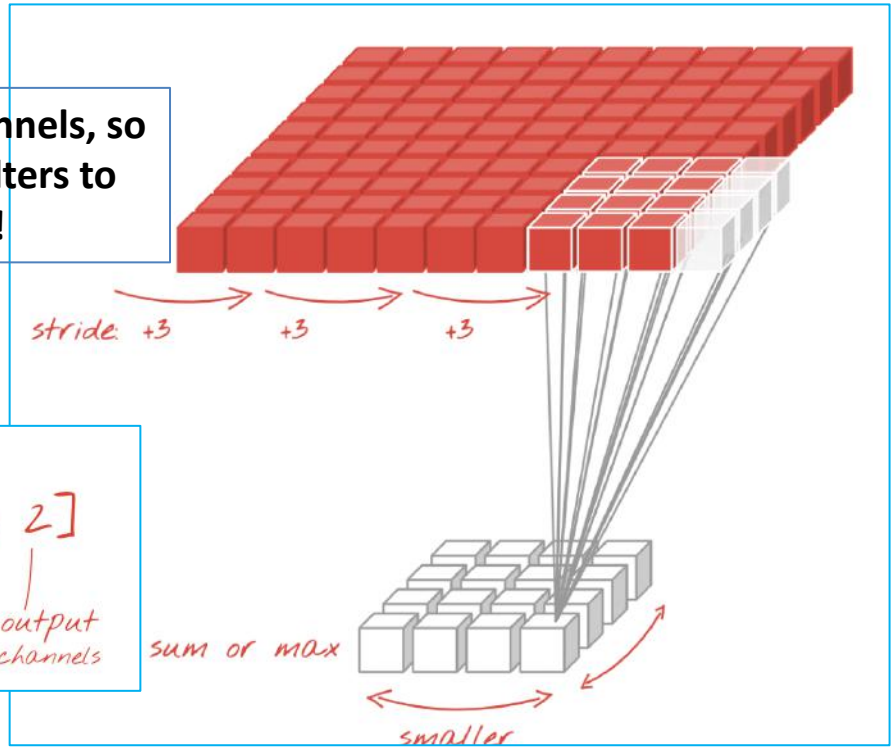
https://medium.com/@pavisj/convolutions-and-backpropagations-46026a8f5d2c

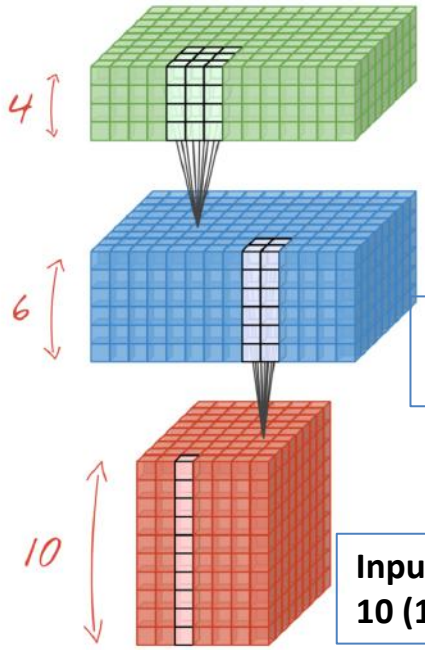
Multiple channels in inputs (outputs from previous convolution steps)



Input has 3 channels, so needs 3 (4x4) filters to cover the input!







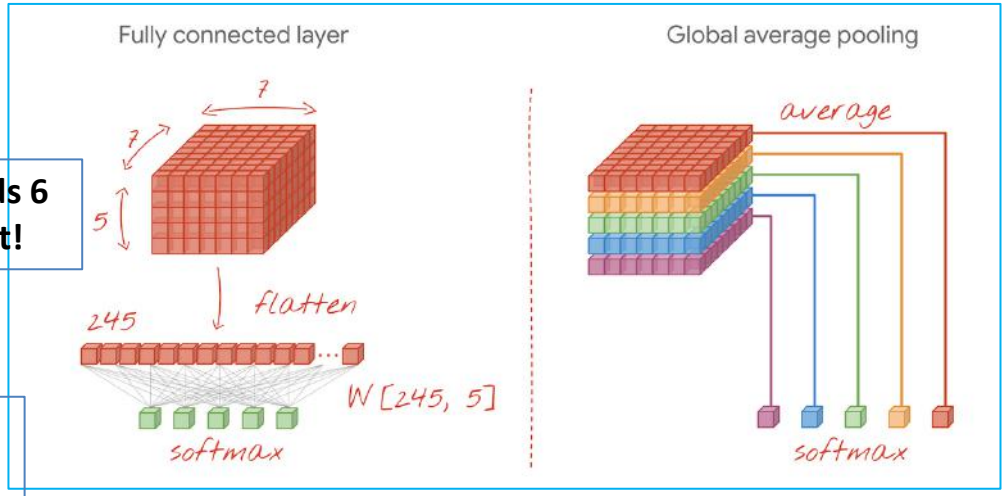
W1[3, 3, 4, 6]

W2[2, 2, 6, 10]

Input has 6 channels, so needs 6 (2x2) filters to cover the input!

W2[1, 1, 10, ...]
stride 2

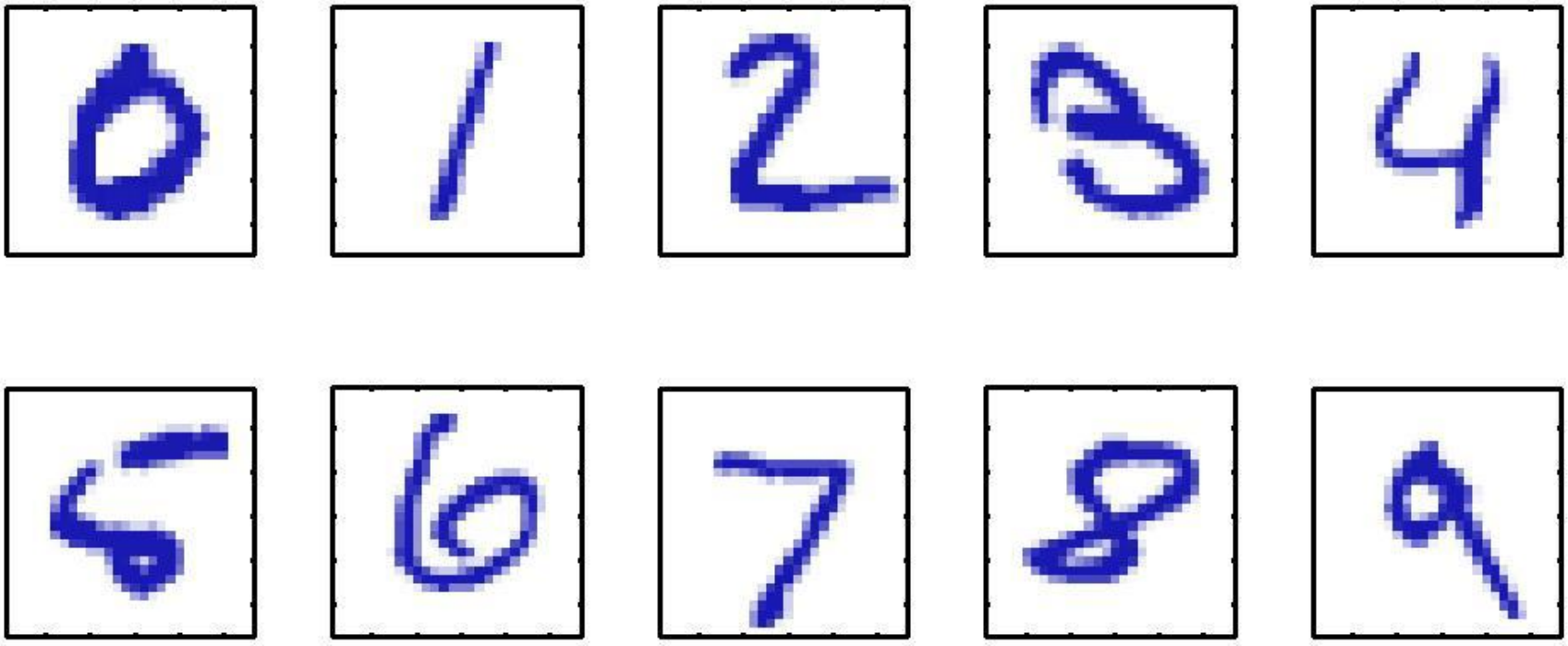
Input has 10 channels, so needs 10 (1x1) filters to cover the input!



Example: how can computer see images?

Handwritten Digit Recognition (MNIST data set)

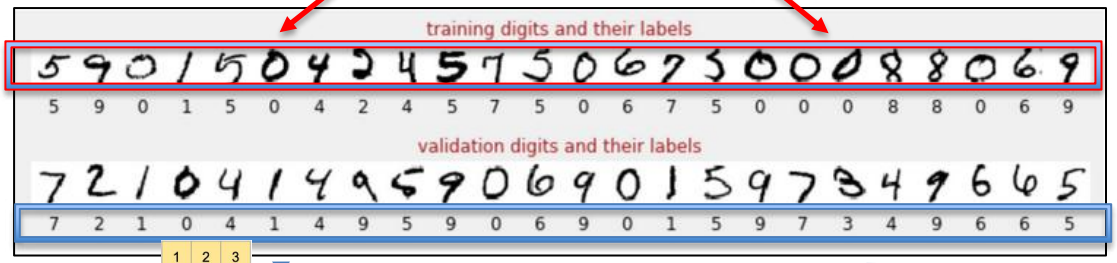
The **MNIST database** (*Modified National Institute of Standards and Technology database*) is a large database of handwritten digits that is commonly used for training various image processing systems. The database is also widely used for training and testing in the field of machine learning.[4][5] It was created by "re-mixing" the samples from NIST's original datasets. The creators felt that since NIST's training dataset was taken from American Census Bureau employees, while the testing dataset was taken from American high school students, it was not well-suited for machine learning experiments. Furthermore, the black and white images from NIST were normalized to fit into a 28x28 pixel bounding box and anti-aliased, which introduced grayscale levels. The MNIST database contains 60,000 training images and 10,000 testing images. – from Wikipedia



MNIST Example (28x28 pixels image)

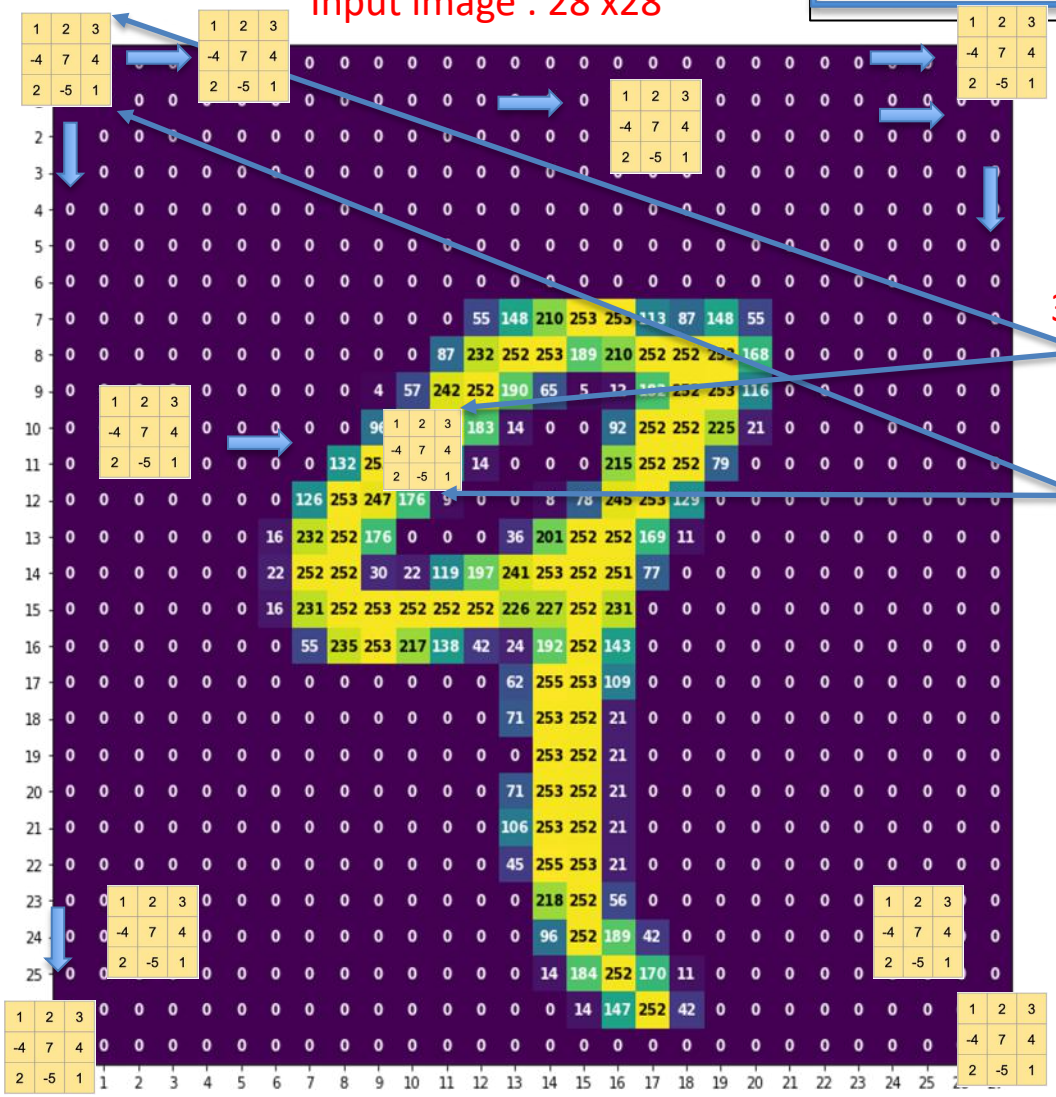
Handwritten digits in the MNIST dataset are 28x28 pixel grayscale images, use the original image (28x28 matrix).

Image : input



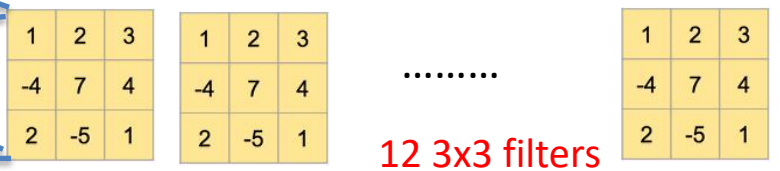
Labels : target

Input image : 28 x28

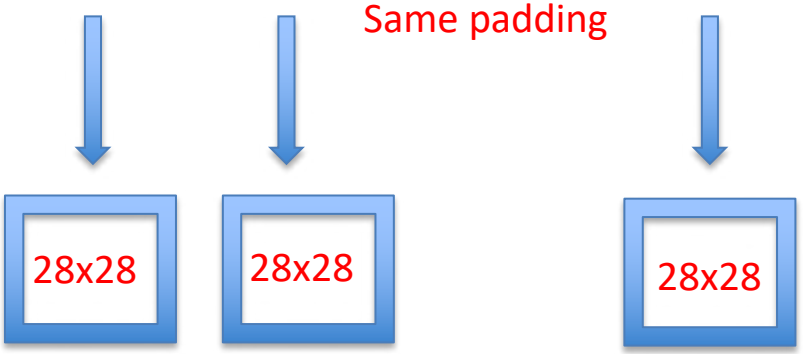


12 filters, each one acts on an image until it is fully covered.

3 x 3 filter (kernel)



12 3x3 filters
Same padding

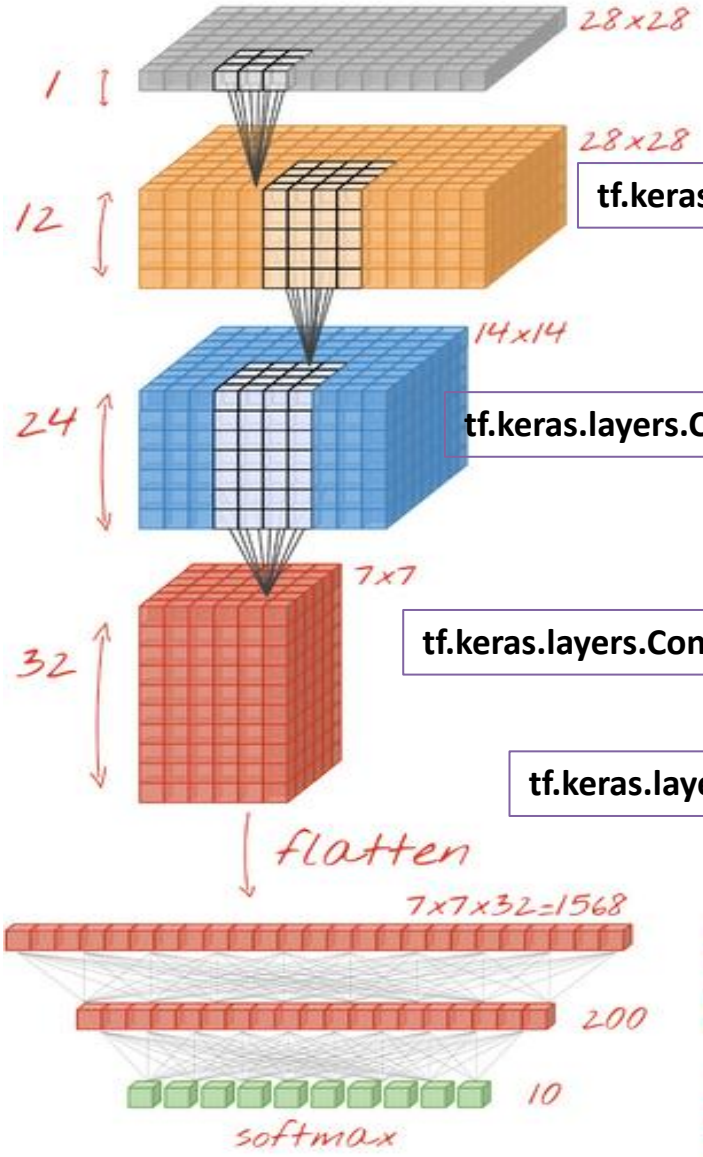


12 Output features : each 28 x28 matrix

Convolution NN : MNIST 28x28 Pixels

[FW, FH, C, H] = [filter size ? X ?, input channel (Depth), output channel (no. of filter)]

tf.keras.layers.Reshape(input_shape=(28*28,), target_shape=(28, 28, 1))



Convolutional 3x3 filters=12

$W_1[3, 3, 1, 12]$

tf.keras.layers.Conv2D(kernel_size=3, filters=12, padding='same', activation='relu')

Convolutional 6x6 filters=24

$W_2[6, 6, 12, 24]$ stride 2

tf.keras.layers.Conv2D(kernel_size=6, filters=24, padding='same', activation='relu',strides=2)

Convolutional 6x6 filters=32

$W_2[6, 6, 24, 32]$ stride 2

tf.keras.layers.Conv2D(kernel_size=6, filters=32, padding='same', activation='relu', strides=2)

tf.keras.layers.Flatten()

https://codelabs.developers.google.com/codelabs/cloud-tensorflow-mnist/#0

Dense layer

$W_4[1568, 200]$

tf.keras.layers.Dense(200, activation='relu')

Softmax dense layer

$W_5[200, 10]$

tf.keras.layers.Dense(10, activation='softmax')

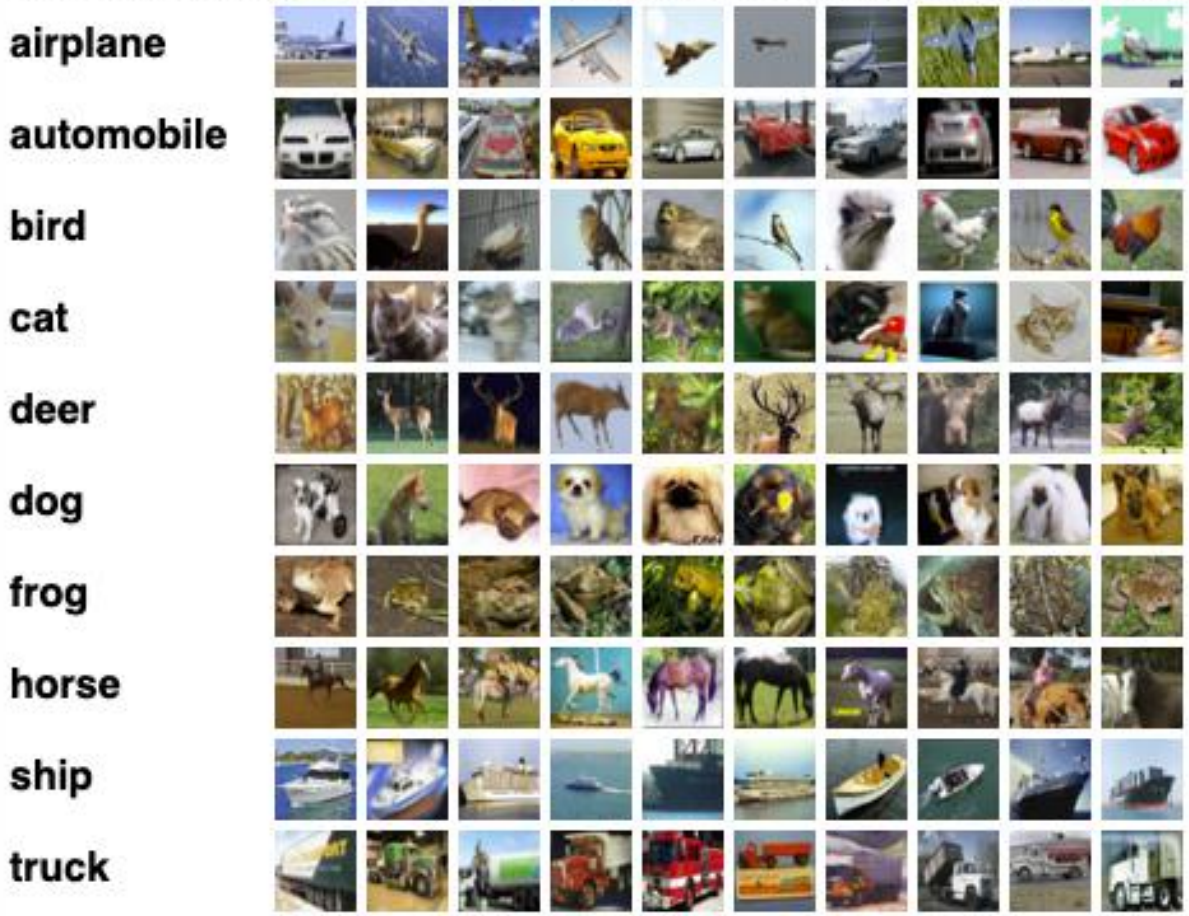
CNN : example – MNIST

There are three main types of layers in a CNN: Convolutional Layer, Pooling Layer, and Fully-Connected Layer.

- ✓ INPUT [28x28x1] will hold the raw pixel values of the image, in this case an image of width 28, height 28 in grey scale (0-255)
- ✓ Apply twelve (12) 3x3 filters to one layer (channel) - W[3,3,Channel=1,number of filters=12]
- ✓ CONV layer will compute the output of neurons that are connected to local regions in the input, each computing a dot product between their weights and a small region they are connected to in the input volume. This may result in volume of [28x28x12] when we use 12 filters. Volume size = [28 width x height = 28 , 12 channels (depth)]
- ✓ RELU layer will apply an elementwise activation function, such as the $max(0,x)$
- ✓ Apply 24 6x6 filters to 12 layers (channel) with stride 2 - W[6,6,12,24] stride 2
- ✓ This may result in volume of [14x14x24] when we use 24 filters, stride 2.
- ✓ RELU layer will apply an elementwise activation function, such as the $max(0,x)$
- ✓ Apply 32 6x6 filters to 24 layers (channel) with stride 2 - W[6,6,24,32] stride 2
- ✓ This may result in volume of [7x7x32] when we use 32 filters, stride 2.
- ✓ RELU layer will apply an elementwise activation function, such as the $max(0,x)$
- ✓ Flatten the volume (tensor) to be a vector of 7x6x32=1568 elements
- ✓ Use 200 neurons with the input vector, Fully connected layer, W [1558, 200], output=1 x 200
- ✓ FC (i.e. fully-connected) layer will compute the class scores, resulting in volume of size [1x1x10], where each of the 10 numbers correspond to a class score.
- ✓ In this way, ConvNets transform the original image layer by layer from the original pixel values to the final class scores.

The **CIFAR-10 dataset** (Canadian Institute For Advanced Research) is a collection of images that are commonly used to train machine learning and computer vision algorithms. The CIFAR-10 dataset contains 60,000 32x32 color images in 10 different classes. There are 6,000 images of each class. Since the images in CIFAR-10 are low-resolution (32x32), this dataset can allow researchers to quickly try different algorithms to see what works. CIFAR-10 is a labeled subset of the 80 million tiny images dataset. When the dataset was created, students were paid to label all of the images.

Here are the classes in the dataset, as well as 10 random images from each:



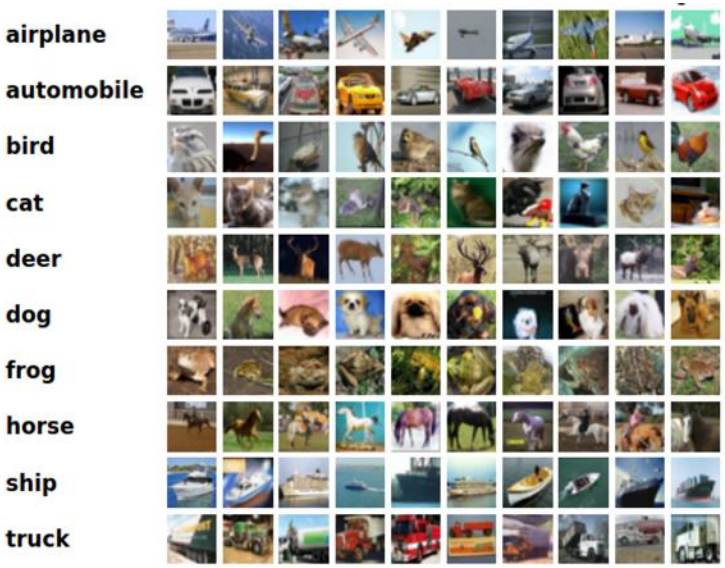
https://www.cs.toronto.edu/~kriz/cifar.html

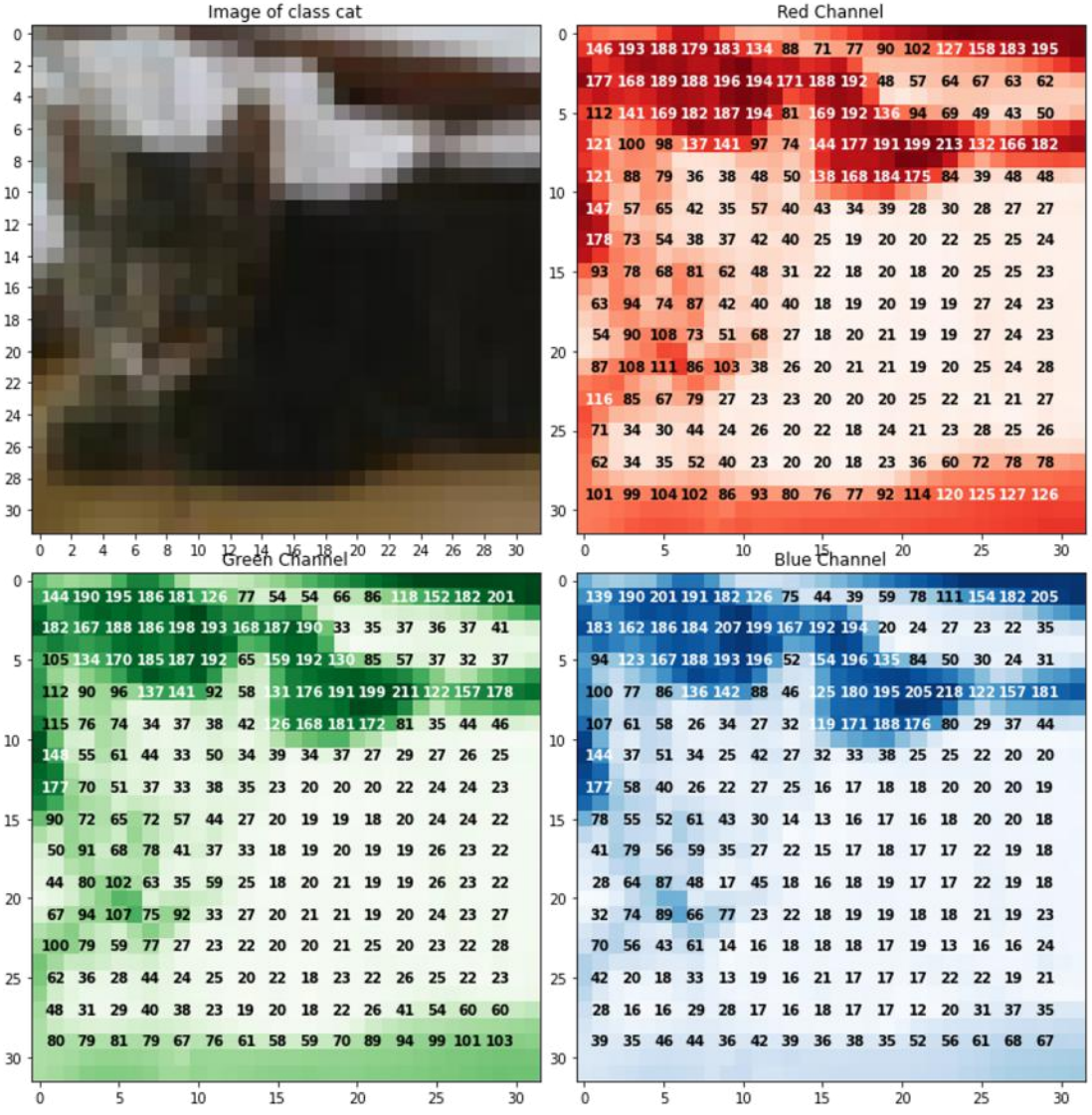
TensorFlow 2.0 : CIFAR 10 Dataset

Loading the CIFAR10 Dataset

```
import tensorflow as tf
cifar10 = tf.keras.datasets.cifar10
#download the images
(x_train, y_train), (x_test, y_test)
= cifar10.load_data()
```

- ✓ **Each Image is a 32x32 image of one of 10 objects**
- ✓ **There are 50,000 training images and 10000 testing images**






```
#normalize
x_train, x_test = x_train / 255.0, x_test / 255.0
```

TensorFlow 2.0

The CNN Network

```
model = tf.keras.models.Sequential([
tf.keras.layers.Conv2D(4, (3, 3), activation='relu',
input_shape=(32, 32, 3)),
tf.keras.layers.MaxPooling2D((2, 2)),
tf.keras.layers.Conv2D(4, (3, 3), activation='relu'),
tf.keras.layers.Flatten(),
tf.keras.layers.Dense(16, activation='relu'),
tf.keras.layers.Dense(10, activation='softmax')
])

loss_fn =
tf.keras.losses.SparseCategoricalCrossentropy(from_logi
ts=True)

model.compile(optimizer='adam',
              loss=loss_fn,
              metrics=['accuracy'])
model.summary()
model.fit(
 x_train,
 y_train,
 epochs=3,
 batch_size = 10)
```

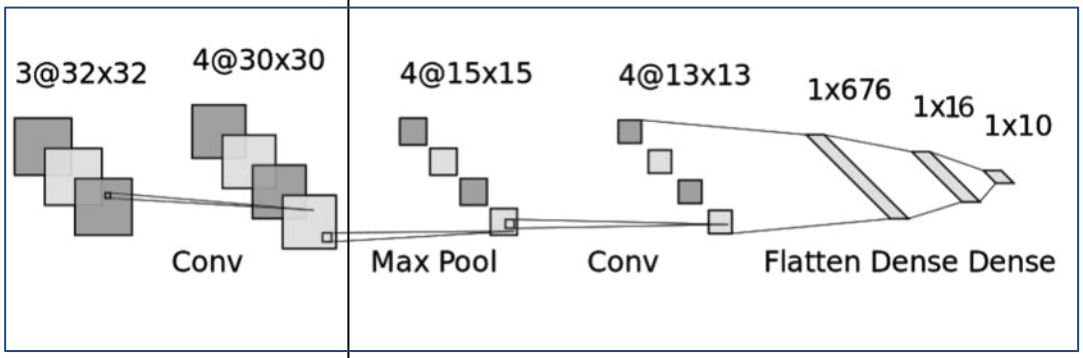
Let's use this simple network to demonstrate what TensorFlow is doing behind the scenes

Batch Size: how many images do we pass through the network for each iteration (10)

Epochs: how many times we pass over the entire dataset (3)

 # Iterations per Epoch = number of images / batch size (50,000/10=5,000)

Simple CNN Network



CNN : example – CIFIR 10

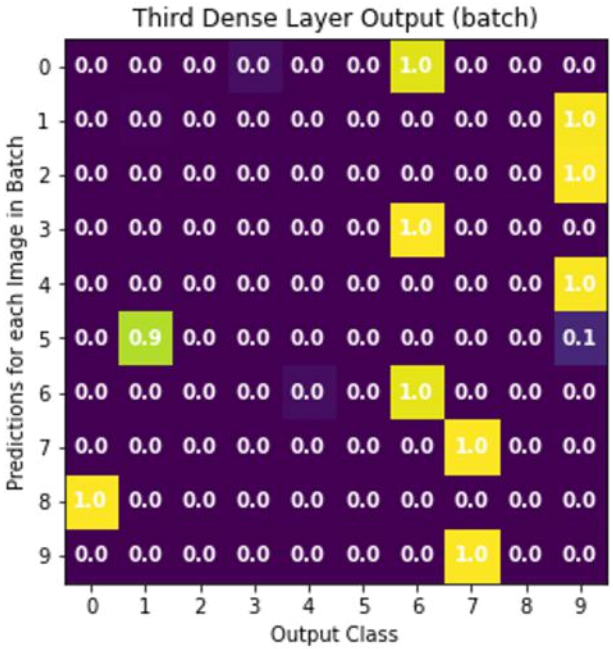
There are three main types of layers in a CNN: Convolutional Layer, Pooling Layer, and Fully-Connected Layer. A simple CNN for the CIFIT-10 classification could have the architecture [INPUT - CONV - RELU - POOL - FC]. In more detail:

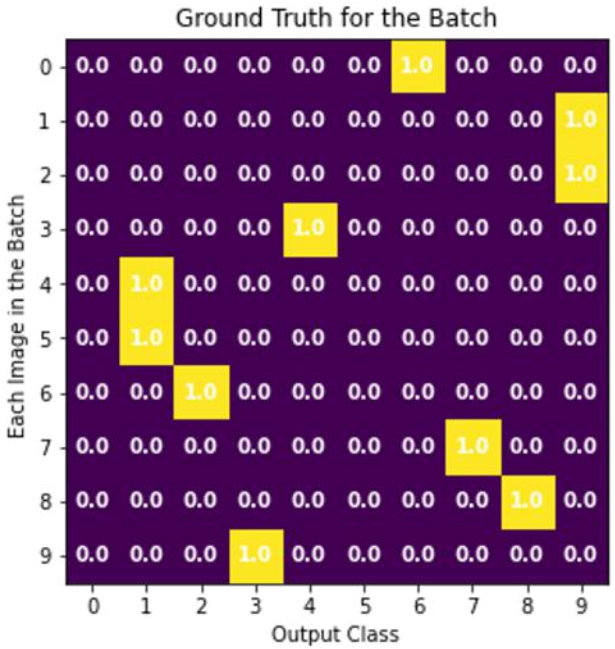
- ✓ INPUT [32x32x3] will hold the raw pixel values of the image, in this case an image of width 32, height 32, and with three color channels R,G,B.
- ✓ CONV layer will compute the output of neurons that are connected to local regions in the input, each computing a dot product between their weights and a small region they are connected to in the input volume. This may result in volume [30x30x4] if 4 sets of filters are used, each set has 3 filters corresponding to 3 (R, G, B) input channels.
- ✓ ReLU layer will apply an elementwise activation function, such as the $max(0,x)$, thresholding at zero. This leaves the size of the volume unchanged ([32x32x4]).
- ✓ Max POOL layer of (2,2) will perform a downsampling operation along the spatial dimensions (width, height), resulting in volume such as [15x15x4].
- ✓ Input is (15x15x4), filters are 4 x (3,3) this may result in volume [13x13x4] if we decided to use 4 filters.
- ✓ RELU layer will apply an elementwise activation function, such as the $max(0,x)$, thresholding at zero. This leaves the size of the volume unchanged ([13x13x4]).
- ✓ Flatten the volume (tensor) to be a vector of 13x13x4=1x674 elements
- ✓ Use 16 neurons with the input vector, Fully connected layer, W [676, 16], output=1 x 16
- ✓ FC (i.e. fully-connected) layer will compute the class scores, resulting in volume of size [1x10], where each of the 10 numbers correspond to a class score, such as among the 10 categories of CIFAR-10. As with ordinary Neural Networks and as the name implies, each neuron in this layer will be connected to all the numbers in the previous volume.
- ✓ In this way, ConvNets transform the original image layer by layer from the original pixel values to the final class scores.

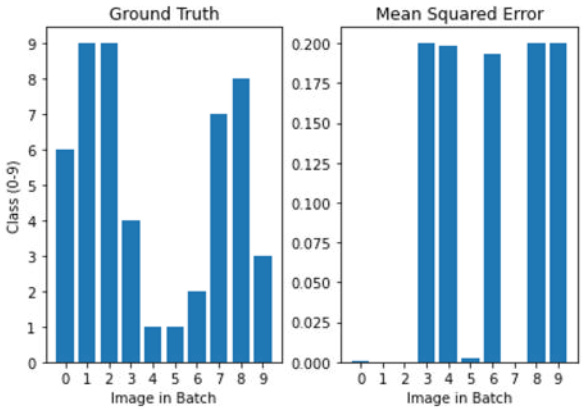
TensorFlow 2.0 : CNN for CIFAR10

Evaluation

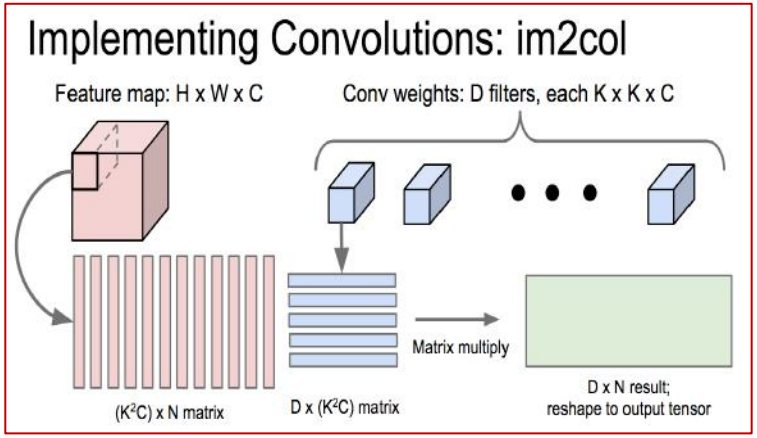
```
tf.losses.mean_squared_error(y_true_tf, y_pred_tf)
```



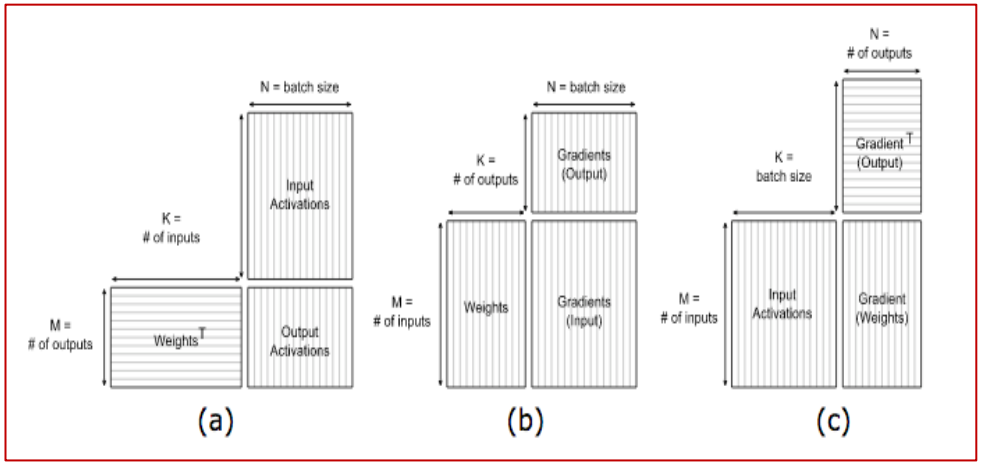




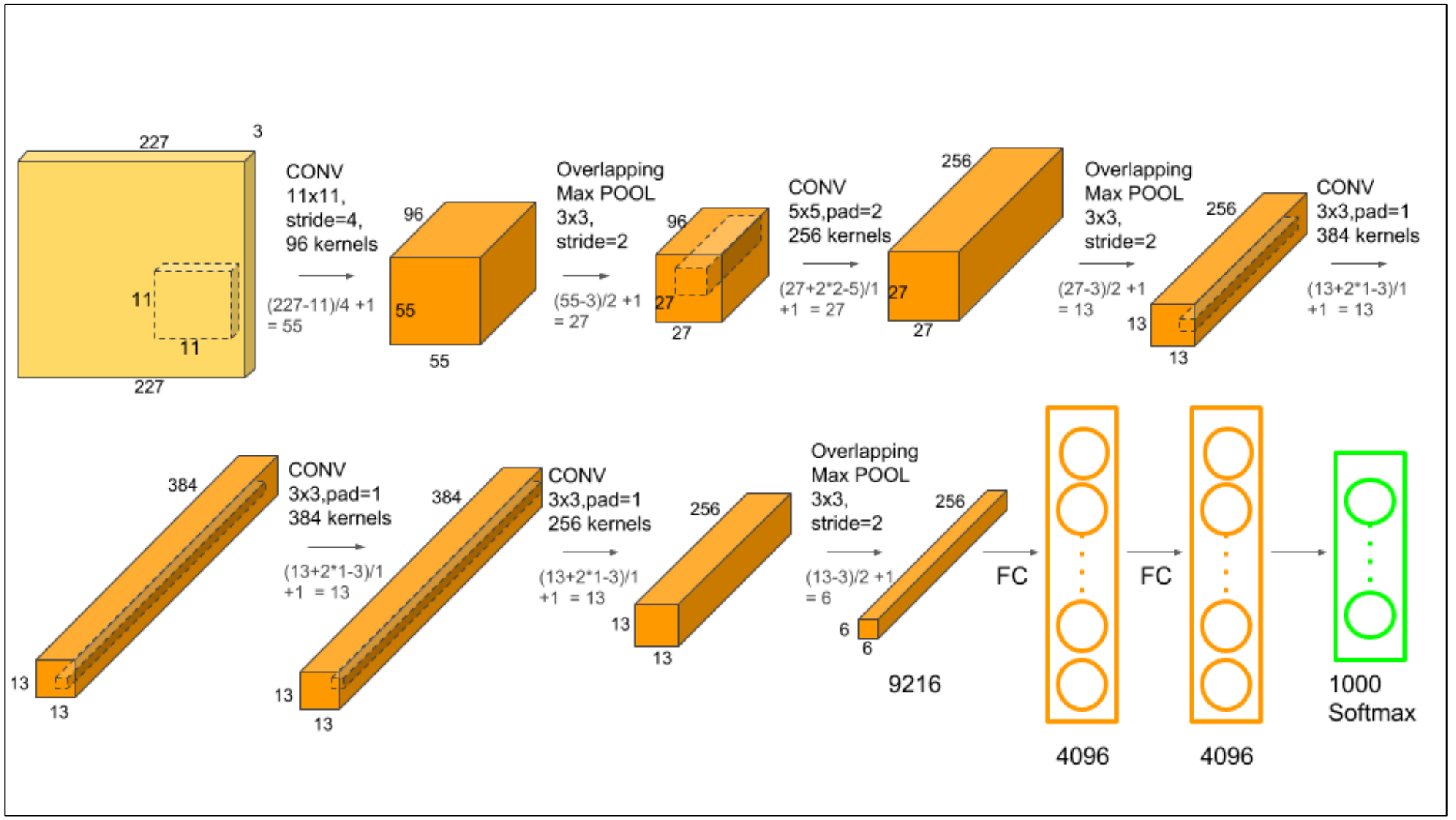
$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \tilde{y}_i)^2$$



Matrix Multiplication : MM it is !!



AlexNet :Number of Parameters



https://www.learnopencv.com/number-of-parameters-and-tensor-sizes-in-convolutional-neural-network/

Number of Parameters

- ✓ **Input**: Color images of size 227x227x3. The AlexNet paper mentions the input size of 224×224 but that is a typo in the paper.
- ✓ **Conv-1**: The first convolutional layer consists of 96 kernels of size 11×11 applied with a stride of 4 and padding of 0.
- ✓ **MaxPool-1**: The maxpool layer following Conv-1 consists of pooling size of 3×3 and stride 2.
- ✓ **Conv-2**: The second conv layer consists of 256 kernels of size 5×5 applied with a stride of 1 and padding of 2.
- ✓ **MaxPool-2**: The maxpool layer following Conv-2 consists of pooling size of 3×3 and a stride of 2.
- ✓ **Conv-3**: The third conv layer consists of 384 kernels of size 3×3 applied with a stride of 1 and padding of 1.
- ✓ **Conv-4**: The fourth conv layer has the same structure as the third conv layer. It consists of 384 kernels of size 3×3 applied with a stride of 1 and padding of 1.
- ✓ **Conv-5**: The fifth conv layer consists of 256 kernels of size 3×3 applied with a stride of 1 and padding of 1.
- ✓ **MaxPool-3**: The maxpool layer following Conv-5 consists of pooling size of 3×3 and a stride of 2.
- ✓ **FC-1**: The first fully connected layer has 4096 neurons.
- ✓ **FC-2**: The second fully connected layer has 4096 neurons.
- ✓ **FC-3**: The third fully connected layer has 1000 neurons

https://www.learnopencv.com/number-of-parameters-and-tensor-sizes-in-convolutional-neural-network/

Number of Parameters

Size of the Output Tensor (Image) of a Conv Layer

O = Size (width) of output image.

I = Size (width) of input image.

K = Size (width) of kernels used in the Conv Layer

N = Number of kernels.

S = Stride of the convolution operation.

P = Padding.

The size (O) of the output image is given by

$$O = \frac{227 - 11 + 2 \times 0}{4} + 1 = 55$$

$$O = \frac{I - K + 2P}{S} + 1$$

Number of Parameters of a Conv Layer

W_c = Number of weights of the Conv Layer.

B_c = Number of biases of the Conv Layer.

P_c = Number of parameters of the Conv Layer.

K = Size (width) of kernels used in the Conv Layer.

N = Number of kernels.

C = Number of channels of the input image.

$$W_c = K^2 \times C \times N$$
$$B_c = N$$
$$P_c = W_c + B_c$$

$$W_c = 11^2 \times 3 \times 96 = 34,848$$
$$B_c = 96$$
$$P_c = 34,848 + 96 = 34,944$$

Number of Parameters of a MaxPool Layer = 0

Size of Output Tensor (Image) of a MaxPool Layer

O = Size (width) of output image.

I = Size (width) of input image.

S = Stride of the convolution operation.

P_s = Pool size.

The size (O) of the output image is given by

$$O = \frac{55 - 3}{2} + 1 = 27$$

$$O = \frac{I - P_s}{S} + 1$$

https://www.learnopencv.com/number-of-parameters-and-tensor-sizes-in-convolutional-neural-network/

Number of Parameters

The total number of parameters in AlexNet is the sum of all parameters in the 5 Conv Layers + 3 FC Layers. It comes out to a whopping **62,378,344**! The table below provides a summary.

Layer Name	Tensor Size	Weights	Biases	Parameters
Input Image	227x227x3	0	0	0
Conv-1	55x55x96	34,848	96	34,944
MaxPool-1	27x27x96	0	0	0
Conv-2	27x27x256	614,400	256	614,656
MaxPool-2	13x13x256	0	0	0
Conv-3	13x13x384	884,736	384	885,120
Conv-4	13x13x384	1,327,104	384	1,327,488
Conv-5	13x13x256	884,736	256	884,992
MaxPool-3	6x6x256	0	0	0
FC-1	4096x1	37,748,736	4,096	37,752,832
FC-2	4096x1	16,777,216	4,096	16,781,312
FC-3	1000x1	4,096,000	1,000	4,097,000
Output	1000x1	0	0	0
Total				**62,378,344**

W_{ff} = Number of weights of a FC Layer which is connected to an FC Layer.
B_{ff} = Number of biases of a FC Layer which is connected to an FC Layer.
P_{ff} = Number of parameters of a FC Layer which is connected to an FC Layer.
F = Number of neurons in the FC Layer.
F_{-1} = Number of neurons in the previous FC Layer.

$$W_{ff} = F_{-1} \times F$$
$$B_{ff} = F$$
$$P_{ff} = W_{ff} + B_{ff}$$

In the above equation, $F_{-1} \times F$ is the total number of connection weights from neurons of the previous FC Layer the neurons of the current FC Layer. The total number of biases is the same as the number of neurons (F).

Example: The last fully connected layer of AlexNet is connected to an FC Layer. For this layer, $F_{-1} = 4096$ and $F = 1000$. Therefore,

$$W_{ff} = 4096 \times 1000 = 4,096,000$$
$$B_{ff} = 1,000$$
$$P_{ff} = W_{ff} + B_{ff} = 4,097,000$$

W_{cf} = Number of weights of a FC Layer which is connected to a Conv Layer.
B_{cf} = Number of biases of a FC Layer which is connected to a Conv Layer.
O = Size (width) of the output image of the previous Conv Layer.
N = Number of kernels in the previous Conv Layer.
F = Number of neurons in the FC Layer.

$$W_{cf} = O^2 \times N \times F$$
$$B_{cf} = F$$
$$P_{cf} = W_{cf} + B_{cf}$$

Example: The first fully connected layer of AlexNet is connected to a Conv Layer. For this layer, $O = 6$, $N = 256$ and $F = 4096$. Therefore,

$$W_{cf} = 6^2 \times 256 \times 4096 = 37,748,736$$
$$B_{cf} = 4096$$
$$P_{cf} = W_{cf} + B_{cf} = 37,752,832$$

https://www.learnopencv.com/number-of-parameters-and-tensor-sizes-in-convolutional-neural-network/

Training

"Training a neural network"

What does it mean?

What does the network train?

minimize error of the computed values against the labeled values (loss function)

What are the training parameters

W, b ,**and values of the filters (Wf)**

How does it work?

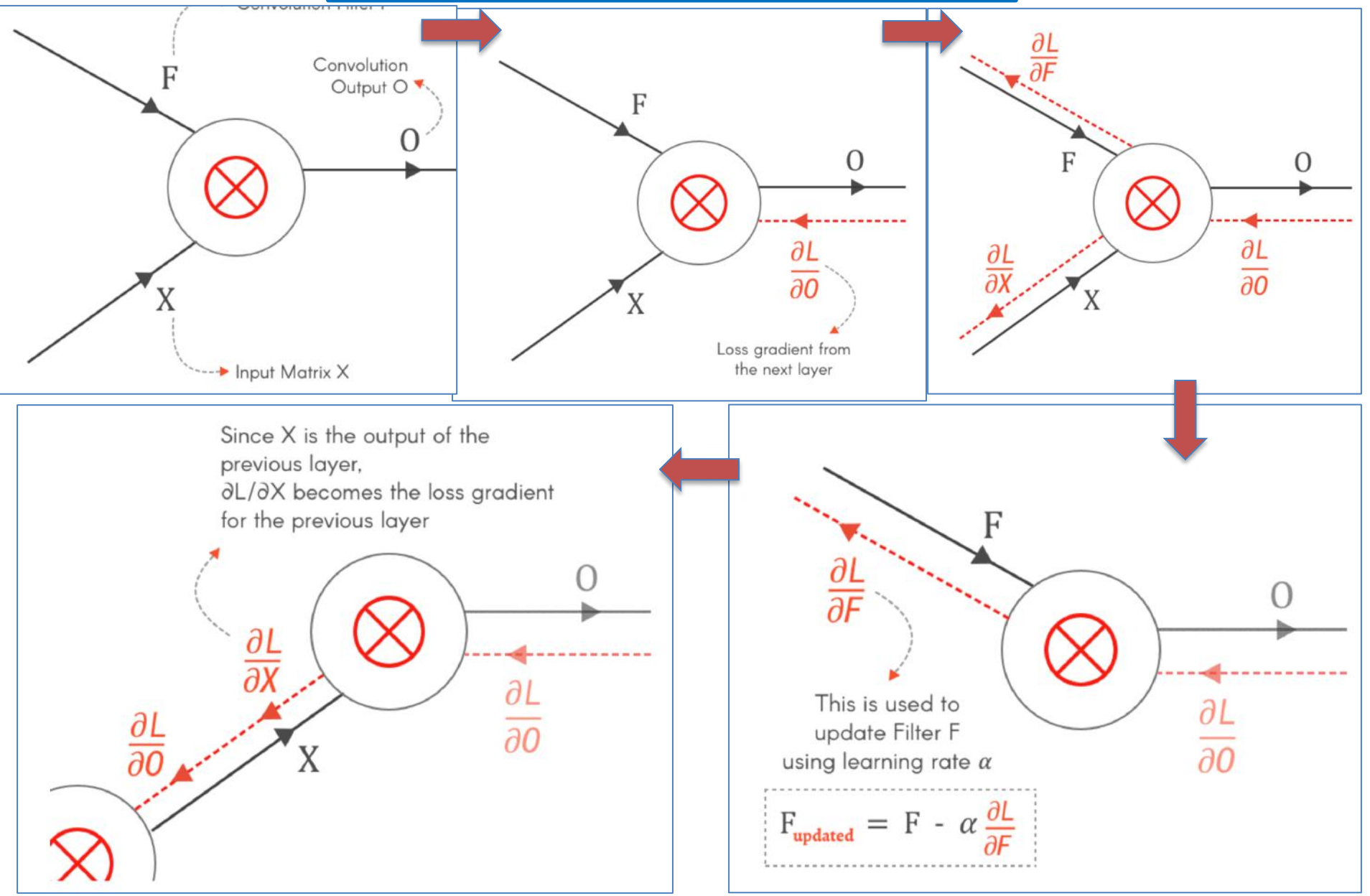
optimization based on gradient descent algorithm

go back and forth in the NN model to adjust for w and b => need derivatives w.r.t. parameters

Go through forward calculation

training with backpropagation

Forward path : backward path



https://medium.com/@pavisj/convolutions-and-backpropagations-46026a8f5d2c

Derivatives of CE Loss w.r.t z

Zj = wi*xi + bj

$$S(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{N} e^{z_j}}$$

z1 → 1.04 → exp → 2.83 → S(z1) → 0.295

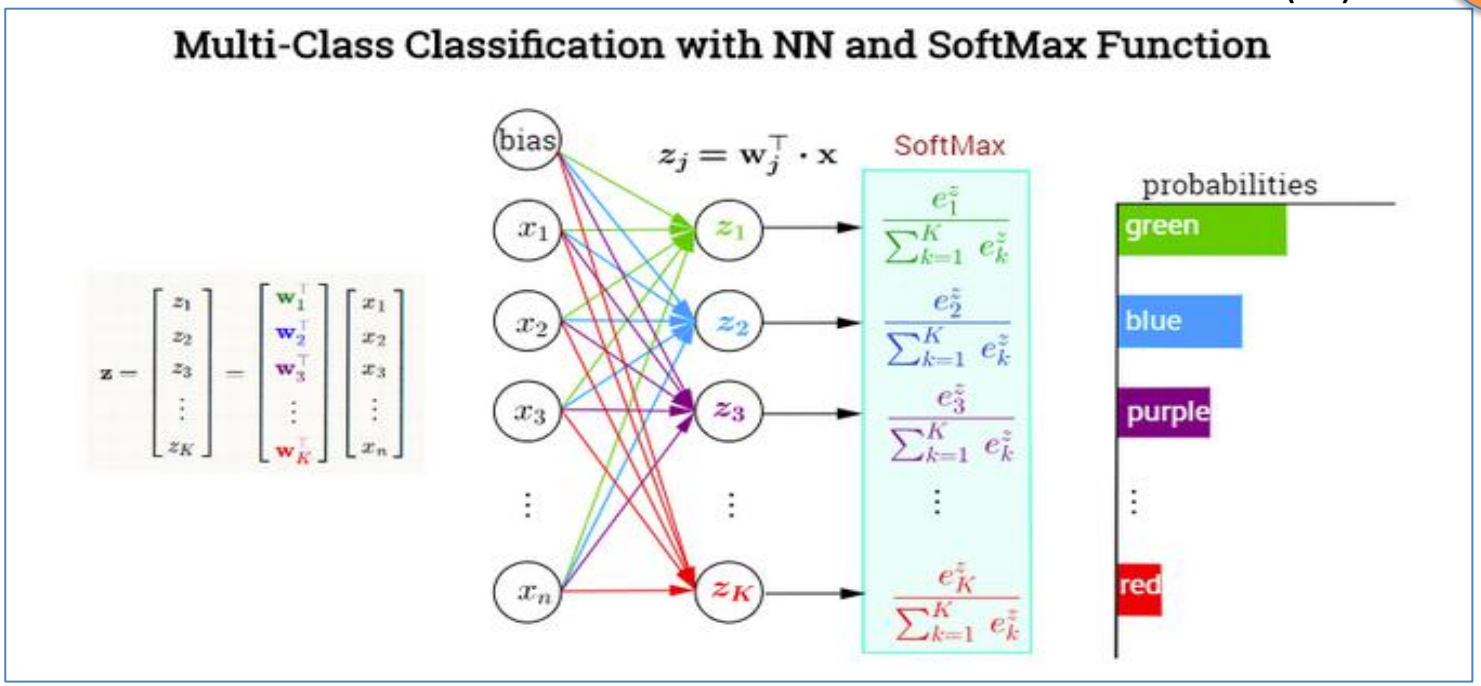
exp(1.04) = 2.83

exp(1.91) = 6.75

exp(1.042) + exp(1.914) = 9.58

Softmax value

z2 → 1.91 → 6.75 → S(z2) → 0.705



dE/dz=d(E=cross entropy)/d(z=w^Tx+b) = d(yi(lnoi)/dz = (oi – yi)

- So, when using Binary Cross-Entropy Error Function with Softmax() output function, the derivative of the error w.r.t the inputs to the Softmax() function is computed as follows:

$$\frac{\partial E}{\partial Z_k} = O_k - y_k$$

- Where y_k is the k^{th} element of the one-hot encoding ground truth vector y, and it is either 0 or 1
- And O_k is the Softmax() function defined as: $O_k = \frac{e^{z_k}}{\sum_{i=1}^{H} e^{z_i}}$

https://www.youtube.com/watch?v=znqbtL0fRA0 - backpropagation through cross entropy and software – ML Dawn

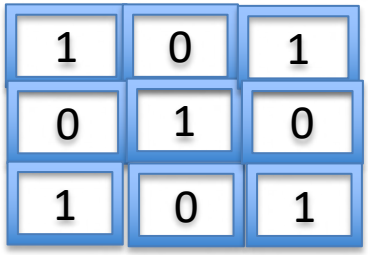
CNN : Backward Path Calculation
A Simple Example

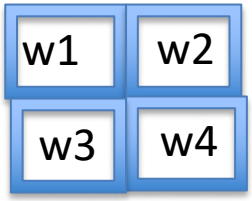
Example exercise

Compute the updated values of filter weights after backpropagation of the following problem -- 3x3 input matrix, 2x2 filter, stride 1, no padding, flatten it and use it as input connecting to a two-neuron layer, then pass the results to a softmax binary classification of two neurons.

3a) (0.5%) What is the values of the output at the end of the forward path

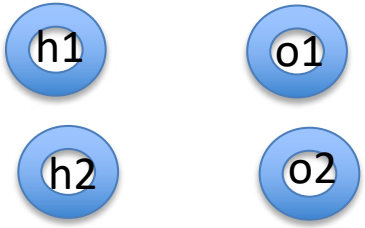
3b) (1.5%) What are the updated values of the weight and bias after backpropagation





b1 = 0.1

w1=0.9
w2=0.1
w3=0.1
w4=0.9



Apply ReLU activation function after the convolution step

Flatten the value after applying the ReLU activation function

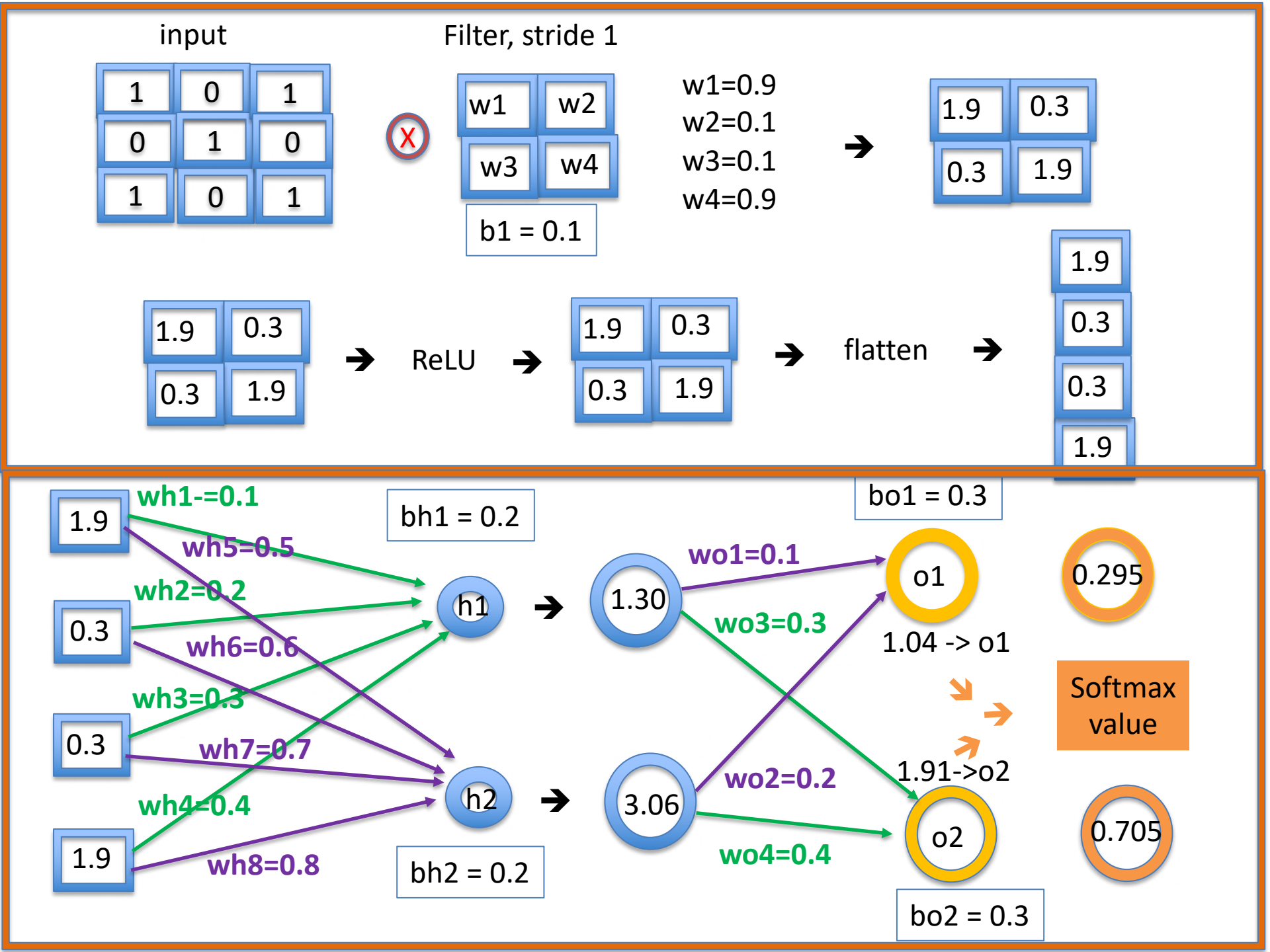
Assuming the weight of each link to this layer is w = 0.1 to 0.8, b2=0.2

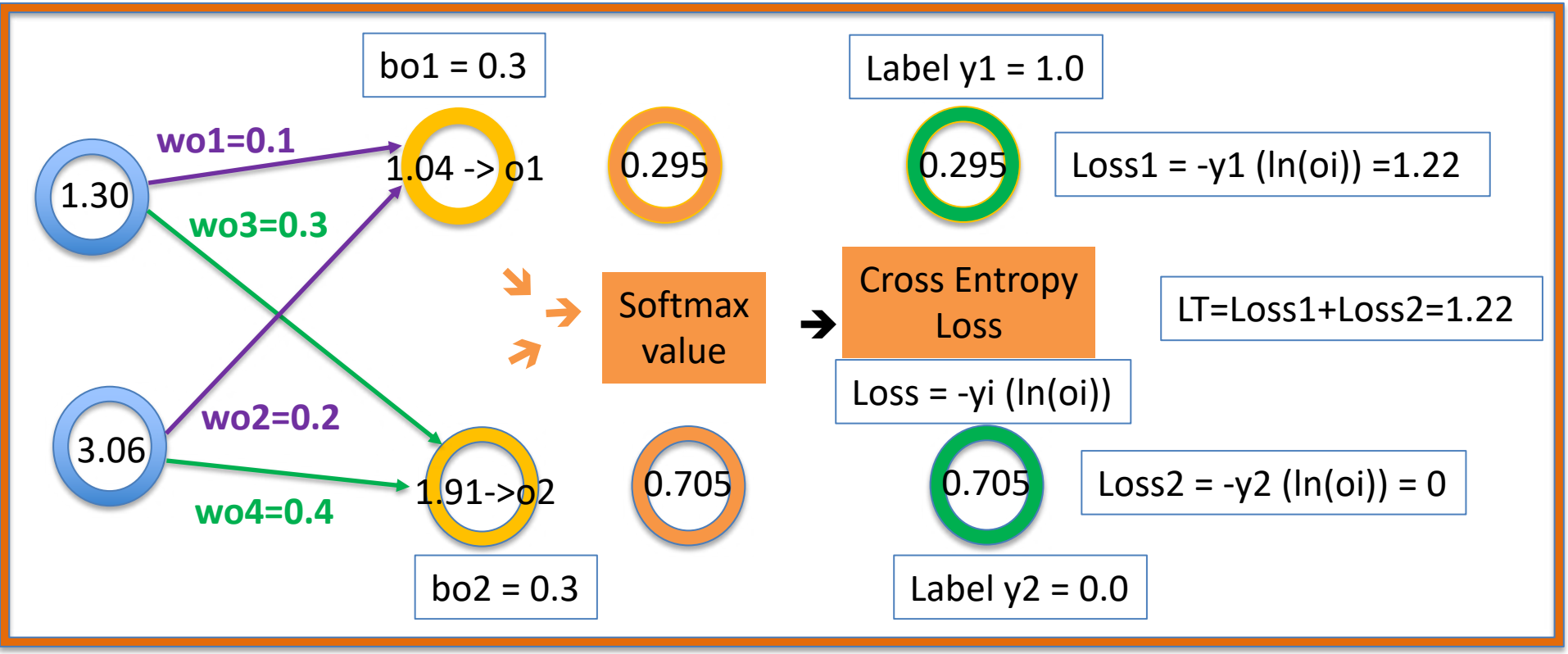
Fully connecting to a layer of two neurons

Assuming the weight of each link is w = 0.1 to 0.4, b3=0.3

Fully Connected layer of 2 softmax neurons

Labeled output of two classes are 1.0 and 0.00


input
Filter, stride 1
1 0 1
0 1 0
1 0 1
X
w1 w2
w3 w4
b1 = 0.1
w1=0.9
w2=0.1
w3=0.1
w4=0.9
1.9 0.3
0.3 1.9
1.9 0.3
0.3 1.9
ReLU
1.9 0.3
0.3 1.9
flatten
1.9
0.3
0.3
1.9
wh1-=0.1
wh5=0.5
wh2=0.2
wh6=0.6
wh3=0.3
wh7=0.7
wh4=0.4
wh8=0.8
1.9
0.3
0.3
1.9
bh1 = 0.2
h1
h2
bh2 = 0.2
1.30
3.06
wo1=0.1
wo3=0.3
wo2=0.2
wo4=0.4
bo1 = 0.3
o1
o2
bo2 = 0.3
1.04 -> o1
1.91->o2
0.295
0.705
Softmax value



CNN : Backward Path

Learning rate = n = 0.5

$$\frac{\partial S(z_i)}{\partial z_j} = \begin{cases} S(z_i) \times (1 - S(z_i)) & \text{if } i = j \\ -S(z_i) \times S(z_j) & \text{if } i \neq j \end{cases}$$

$$\frac{\partial E}{\partial Z_k} = O_k - y_k$$

weights bias

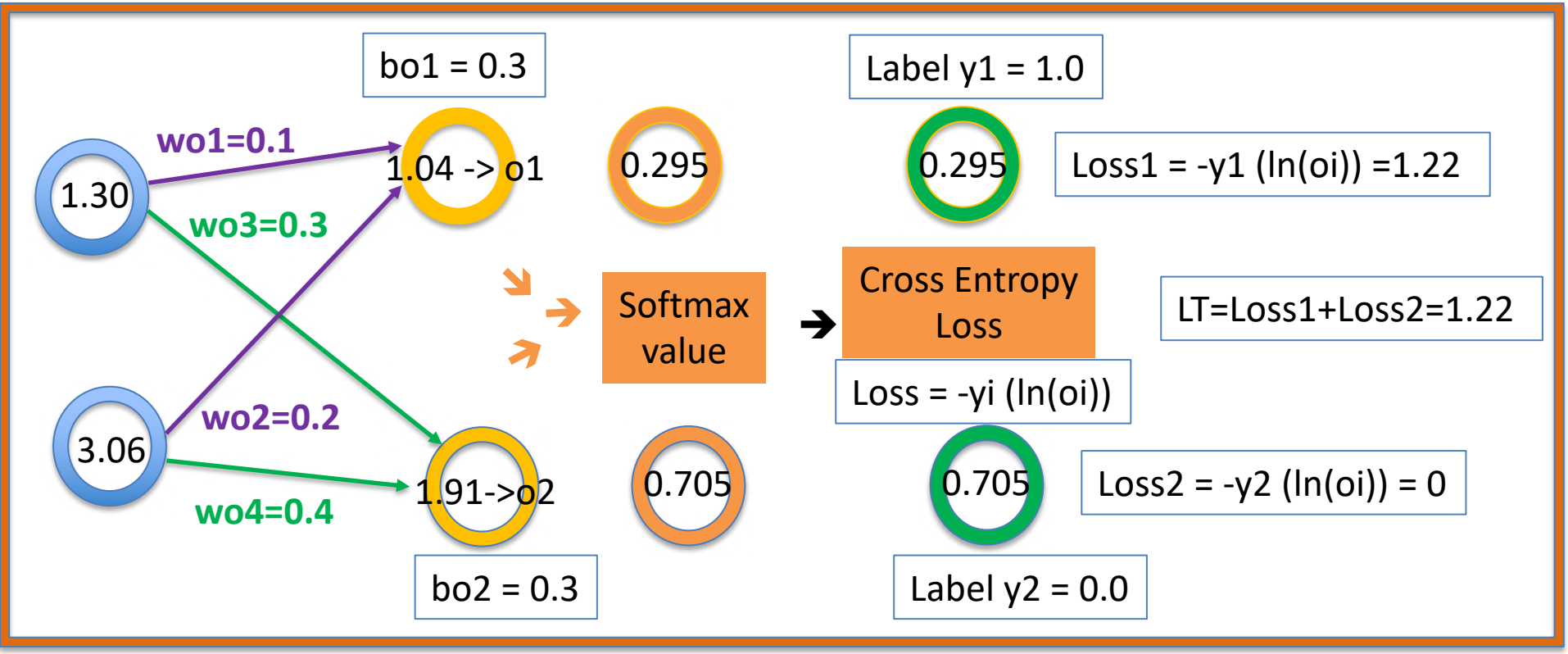
w+ = w – n * (dE/dw)
dE/dw = (dE/dS) * (dS/dw)

Softmax value

dL/dS

Cross Entropy Loss (L)

dL/dL = 1



CNN : Backward Path – jumps over Cross Entropy and Softmax all together

Learning rate = n = 0.5

$$\frac{\partial E}{\partial Z_k} = O_k - y_k$$

Z = Z(w, b)

S = S(z)

weights bias

w+ = w – n * (dE/dw)
dE/dw = (dE/dS) * (dS/dw)

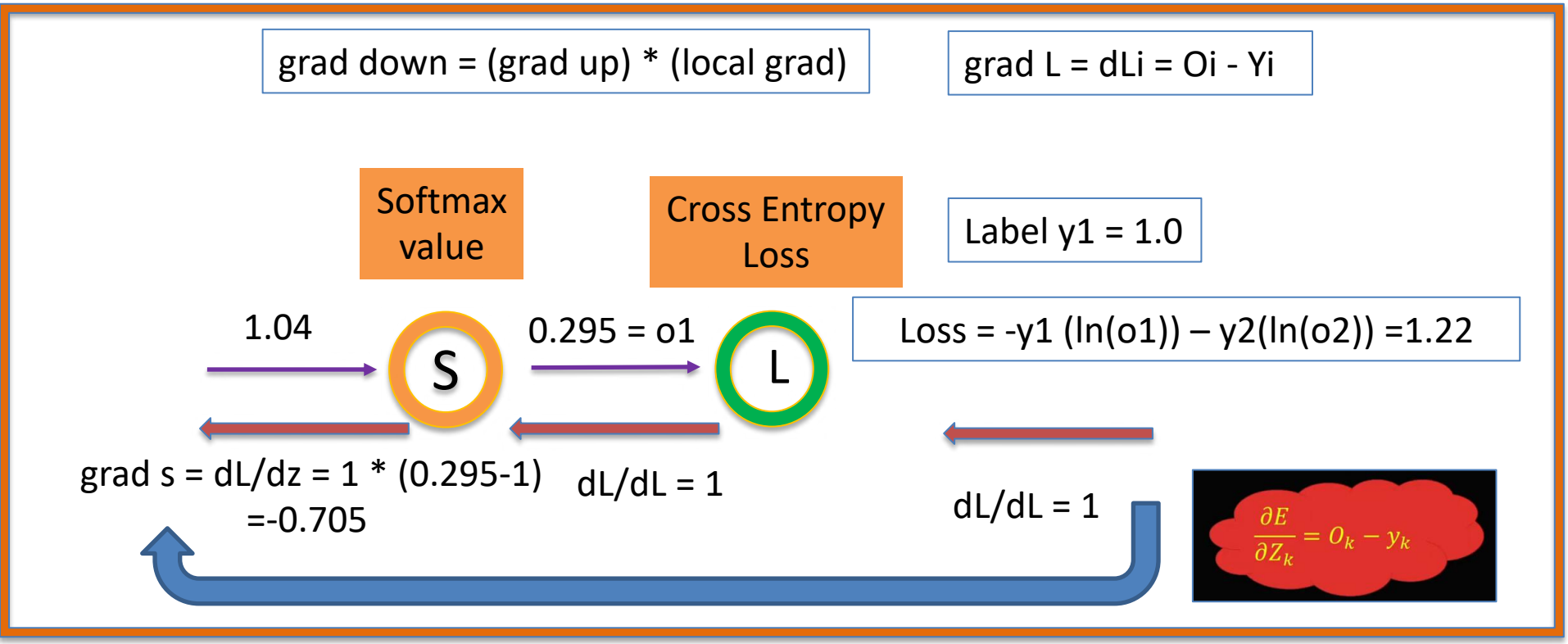
Softmax value

dL/dS

Cross Entropy Loss (L)

dL/dL = 1

Flow Back from the Total Loss to Softmax to Z (S1)



Jump from cross entropy and softmax (E , S) to (weight and bias) Z (o1), Y1 =1

E=L ; dE/dz = dL/dz = (dL/ds * ds/dz)

dE/dZ = local grad = O1 – Y1

grad S = grad down = (grad up * local grad)

= 1 * (0.295-1) = -0.705

$$\frac{\partial E}{\partial Z_k} = O_k - y_k$$

Flow Back from CCE Loss to Softmax to W and b (S1)

grad down = (grad up) * (local grad)

wo1=0.1

1.30

wo2=0.2

-0.705

3.06

-0.705

bo1 = 0.3

-0.705

+

1.04

S

Softmax value

Cross Entropy Loss

L

dL/dw ; dL/db

grad wo1=gradS * 1.30
grad wo2=gradS * 3.06
grad bo1 =-0.705

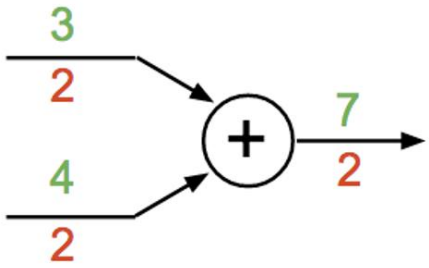
gradS = 1 * (0.295-1)
=-0.705

$$\frac{\partial E}{\partial Z_k} = O_k - y_k$$

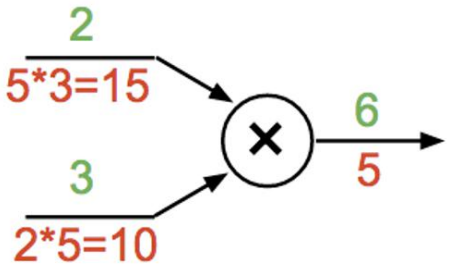
Jump from S(o1) to wo1, wo2, bo1

Same as MLP example. use multiplication operators to propagate gradient back

add gate: gradient distributor

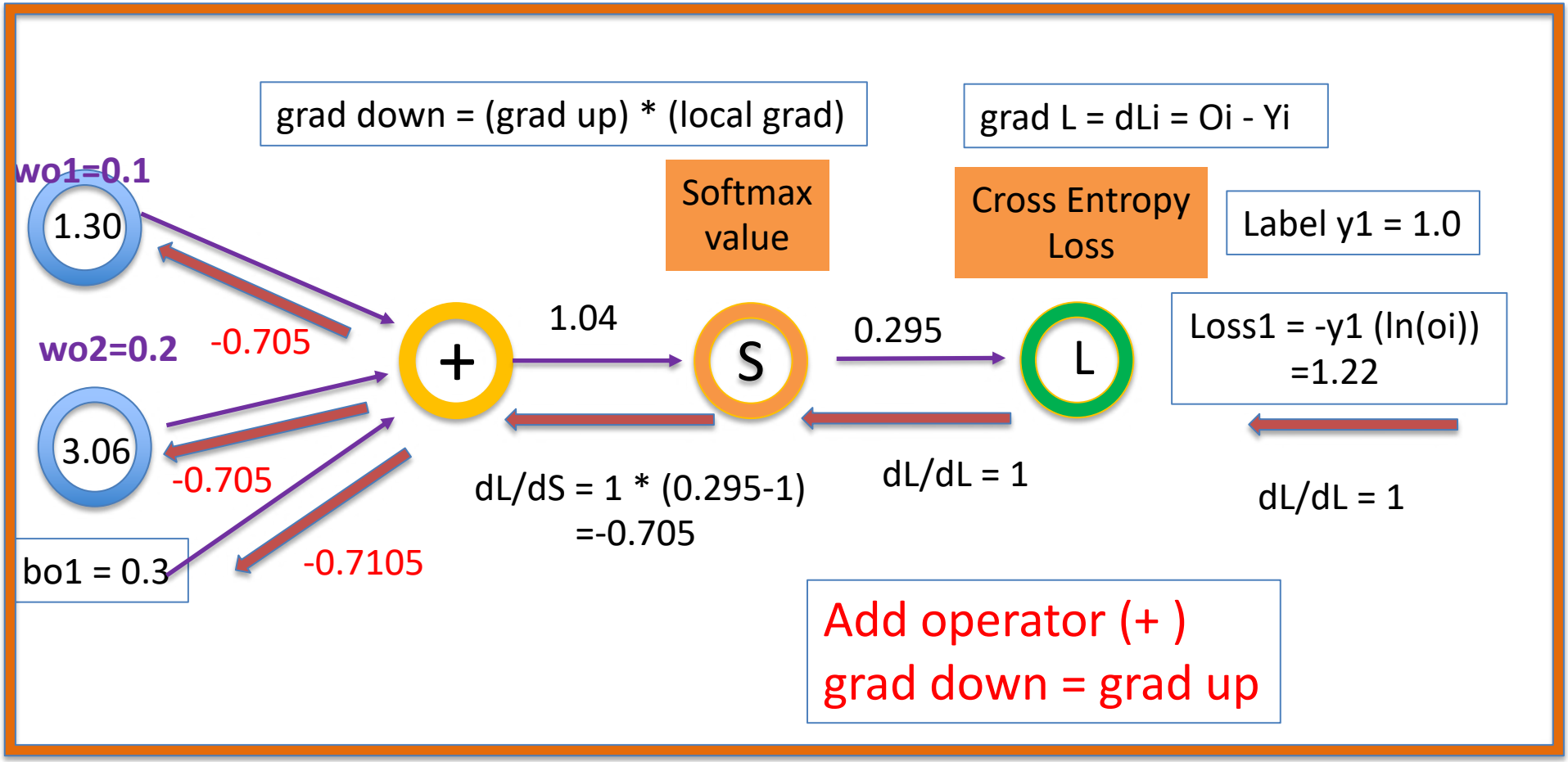


mul gate: "swap multiplier"



Flow Back from Cross Entropy and Softmax to W and b

L -> L1 -> S1 (o1) -> wo1, wo2, bo1



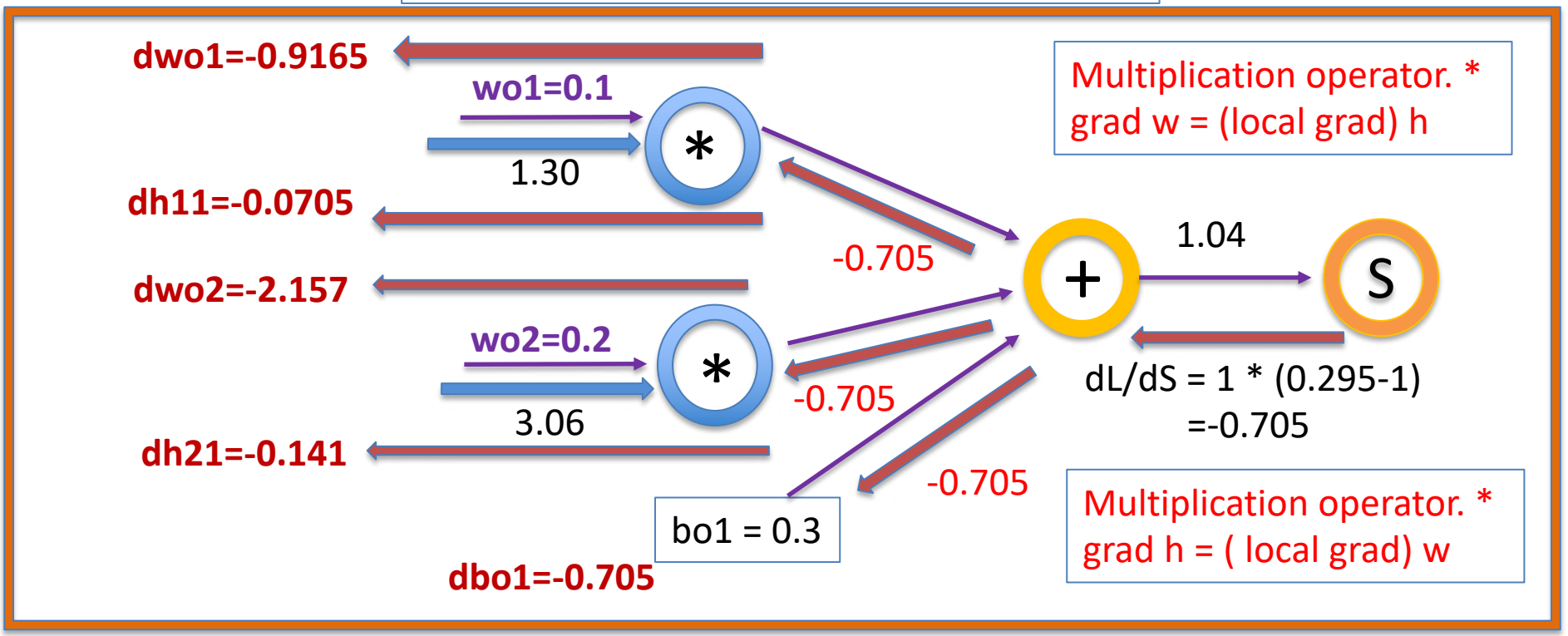
Multiplication operator. (*)

grad w = (local grad) h

Multiplication operator. (*)

grad h = (local grad) w

Update values of wo1, wo2, bo1



In here, i =1 (o1),
j = 1,2: h1, h2

d (L/dwo1) = dL/d(w11)
= -0.705*1.30 = -0.9165

d (L/dwo2) = dL/d(w12)
= -0.705*3.06= -2.1573

d (L/dbo1) = dL/d(b1)
= dL/dS = Oi-Yi=-0.705

wo1+= 0.1– (0.5)*(-0.9165) =0.558

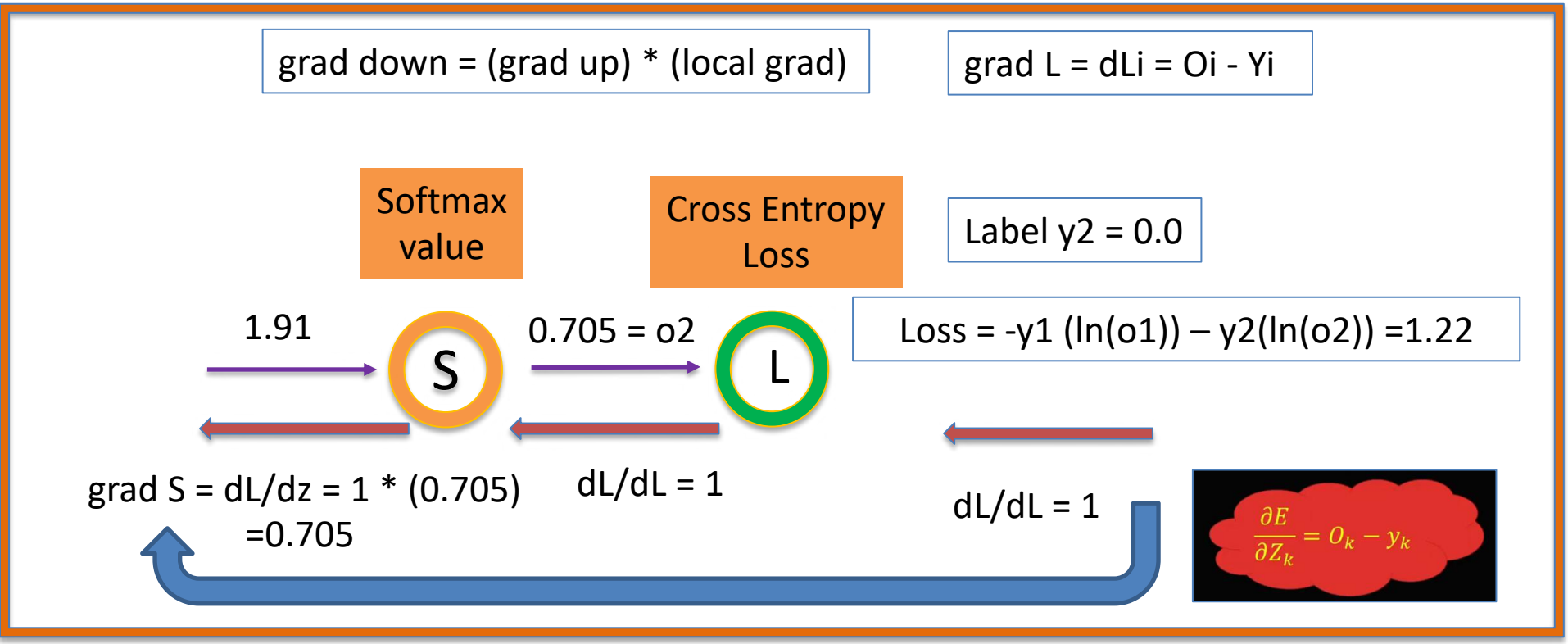
wo2+= 0.2– (0.5)*(-2.1573) =1.278

bo1+= 0.3– (0.5)*(-0.705) =0.652

dh11= (-0.0705)

dh21= (-0.141)

Flow Back from the CEE Loss to Softmax to Z : S2



Jump from cross entropy and softmax (E, S) to weight and bias Z (o2), Y2=0

E=L ; dE/dz = dL/dz = (dL/ds * ds/dz)

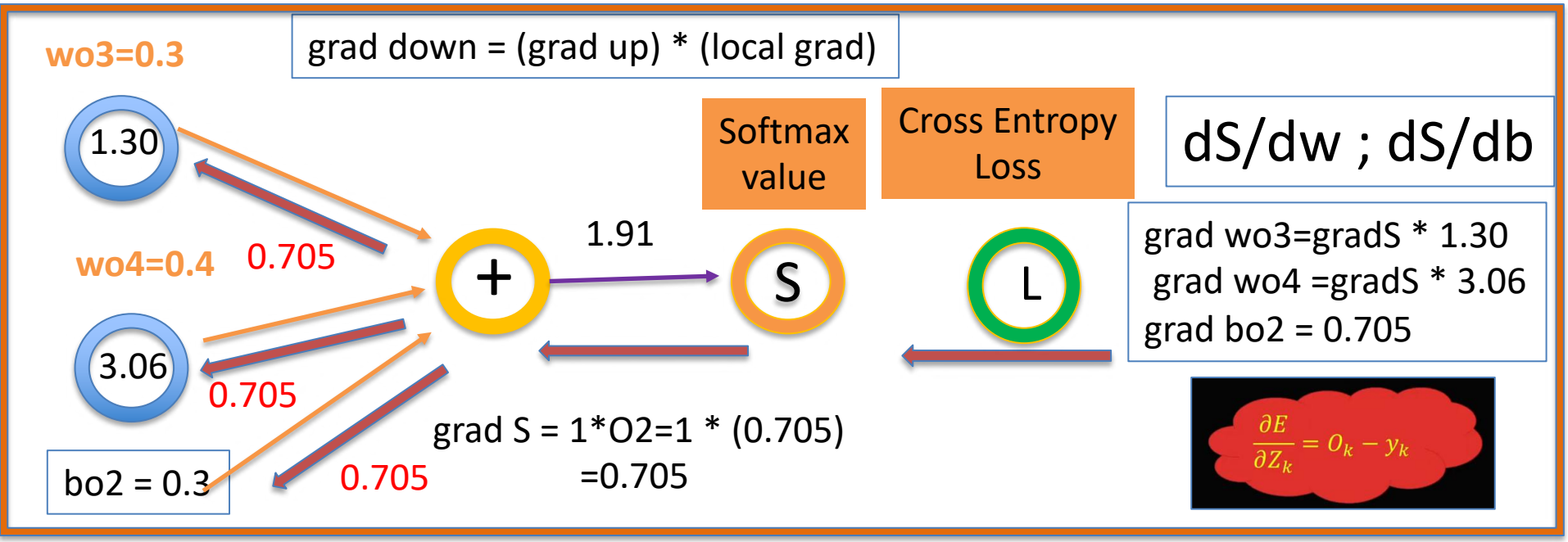
dL/dz = local grad = O2 – Y2 = O2

grad S = grad down = (grad up * local grad)

= 1 * (0.705) = 0.705

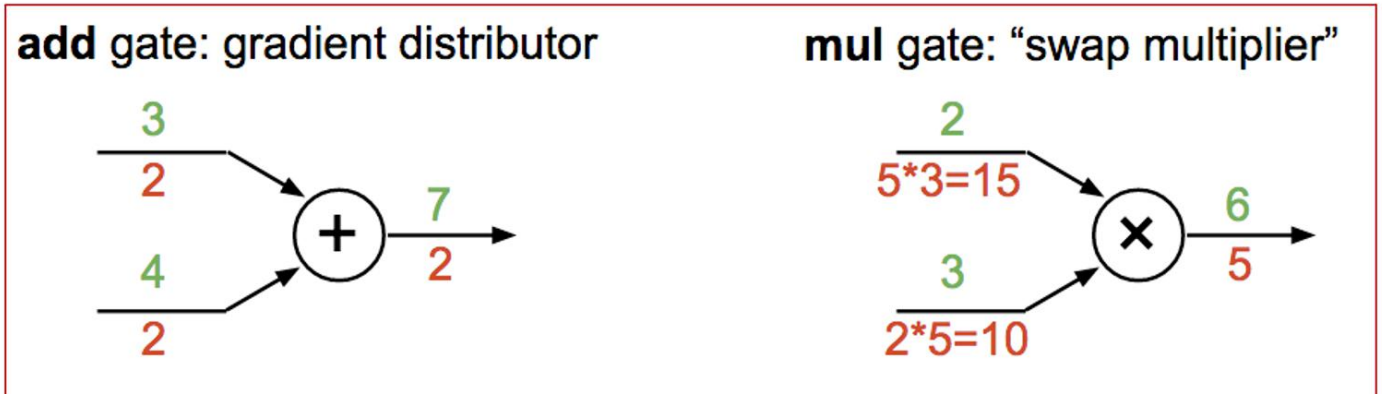
$$\frac{\partial E}{\partial Z_k} = O_k - y_k$$

Flow Back from Softmax to W and b (S2)



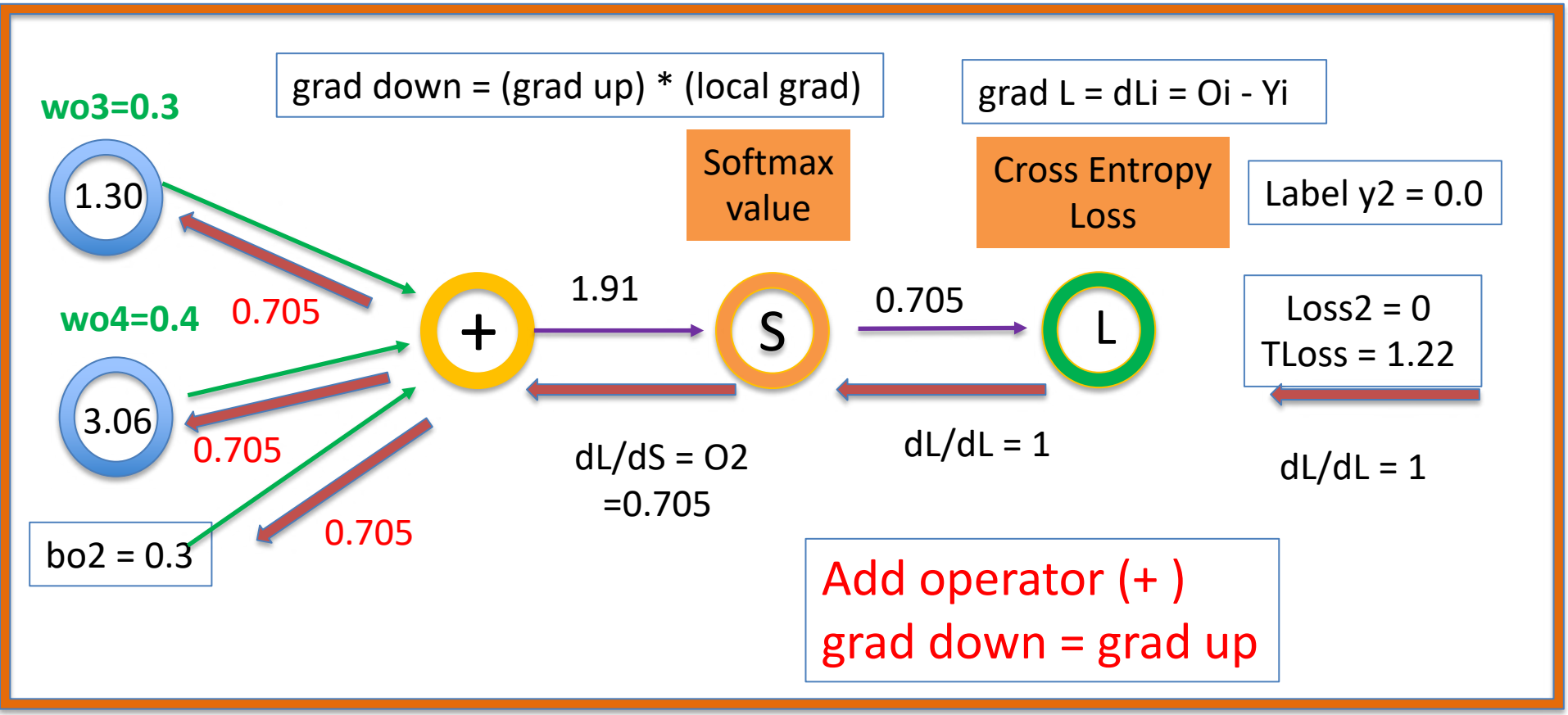
Jump from S(o2) to wo3, wo4, bo2

Same as MLP example. use multiplication operators to propagate gradient back



Flow Back from Cross Entropy and Softmax to W and b

L -> L2 -> S2 (o2) -> wo3, wo4, bo2



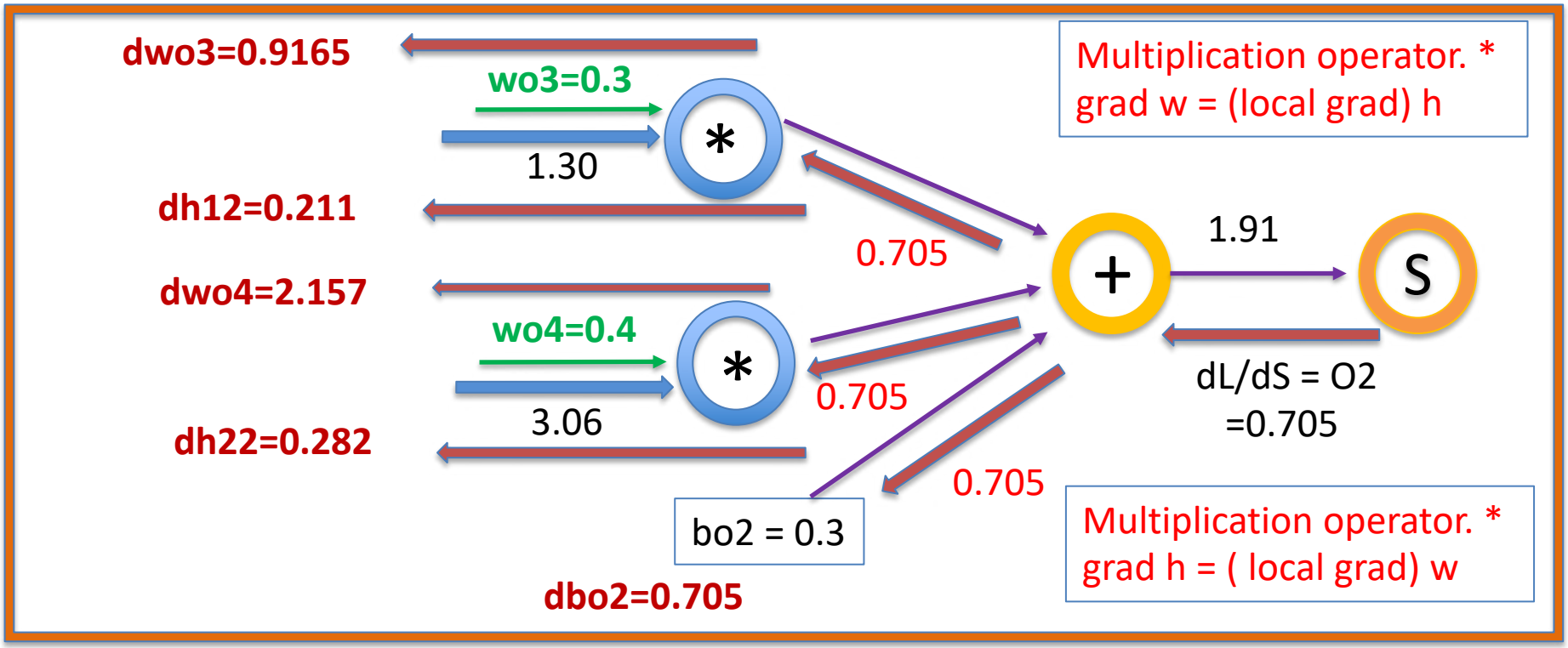
Multiplication operator. (*)

grad w = (local grad) h

Multiplication operator. (*)

grad h = (local grad) w

Update values of wo3, wo4, bo2



In here , i =2 (o2)
j = 1,2: h1, h2

d (L/dwo3) = dL/d(w21)
= 0.705*1.30 = 0.9165

d (L/dwo4) = dL/d(w22)
= 0.705*3.06 = 2.157

d (L/dbo1) = dL/d(b1)
= dL/dS = Oi-Yi=0.705

wo3+= 0.3– (0.5)*(0.9165) =-0.158

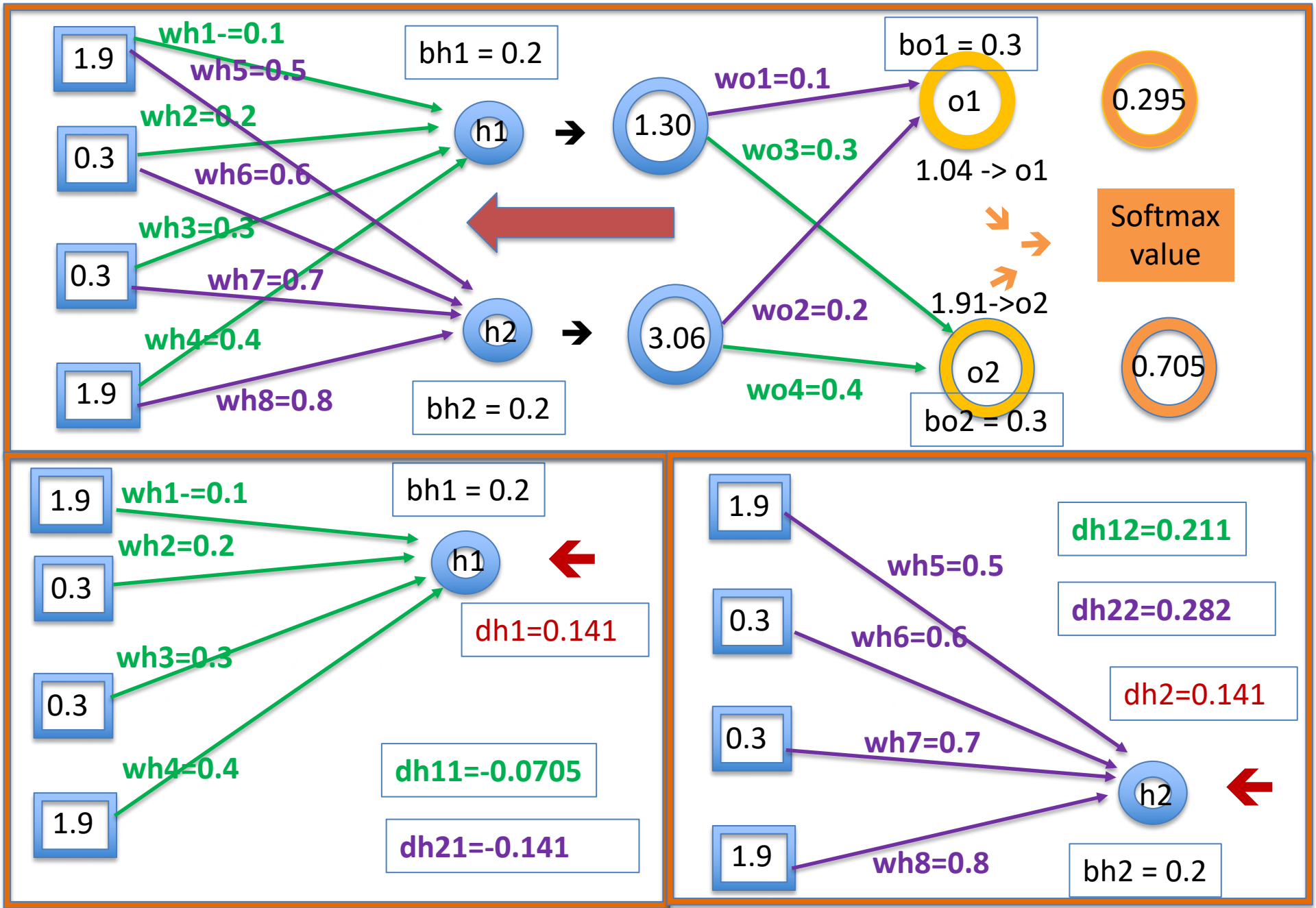
wo4+= 0.4– (0.5)*(2.157) =-0.678

bo2+= 0.3– (0.5)*(0.705) =-0.352

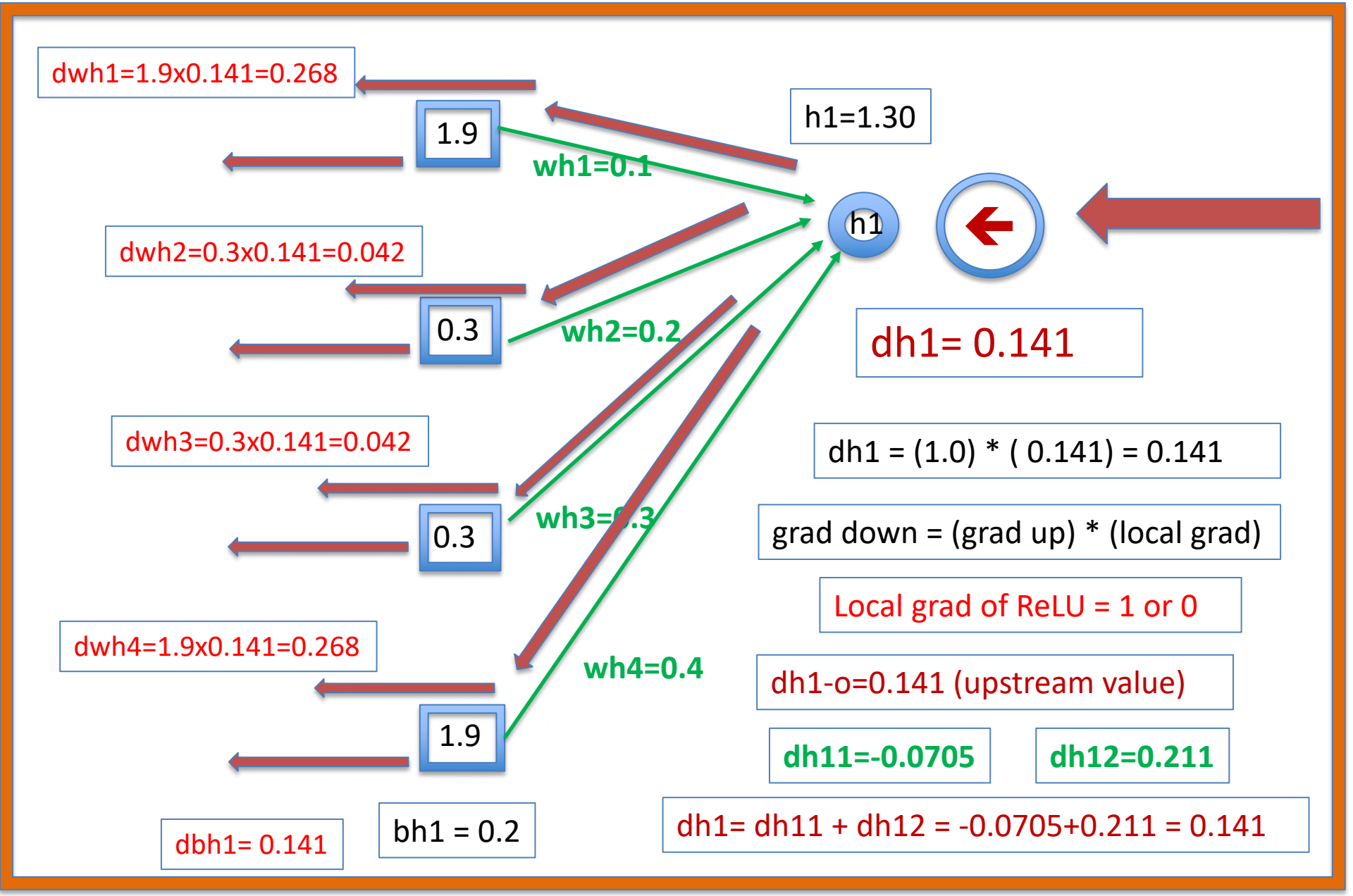
dh12= (0.211)

dh22= (0.282)

Flow back from o1 to the hidden NN layer (h) , MLP calculation



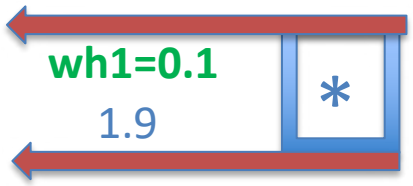
Gradients after h1,ReLU function (h1 neuron)



Update values of wh1, wh2, wh3, wh4, bh1

-0.034 = wh1+=0.1-0.5x0.268

dwh1=1.9x0.141=0.268

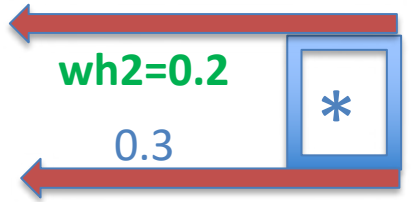


dC1h1=0.1x0.141=0.0141

dh1= 0.141

0.179 = wh2+=0.2-0.5x0.042

dwh2=0.3x0.141=0.042

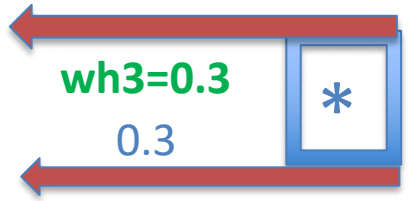


dC2h1=0.2x0.141=0.0282

dh1= 0.141

0.279 = wh3+=0.3-0.5x0.042

dwh3=0.3x0.141=0.042

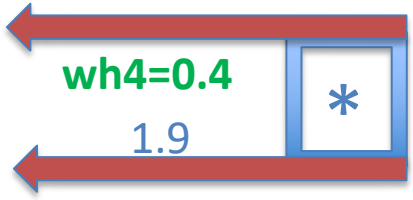


dC3h1=0.3x0.141=0.0423

dh1= 0.141

0.266 = wh4+=0.4-0.5x0.268

dwh4=1.9x0.141=0.268



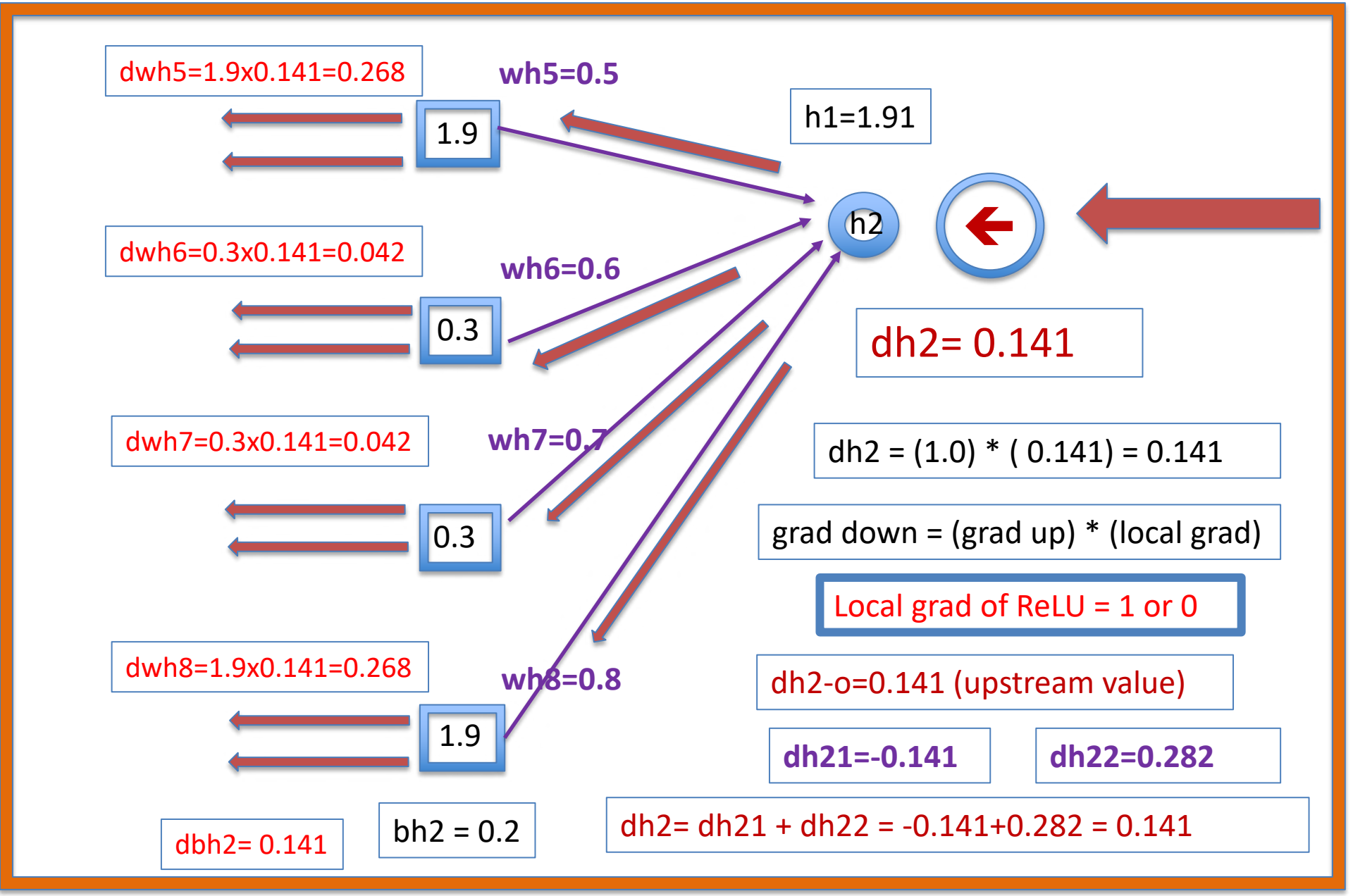
dC4h1=0.4x0.141=0.0564

dh1= 0.141

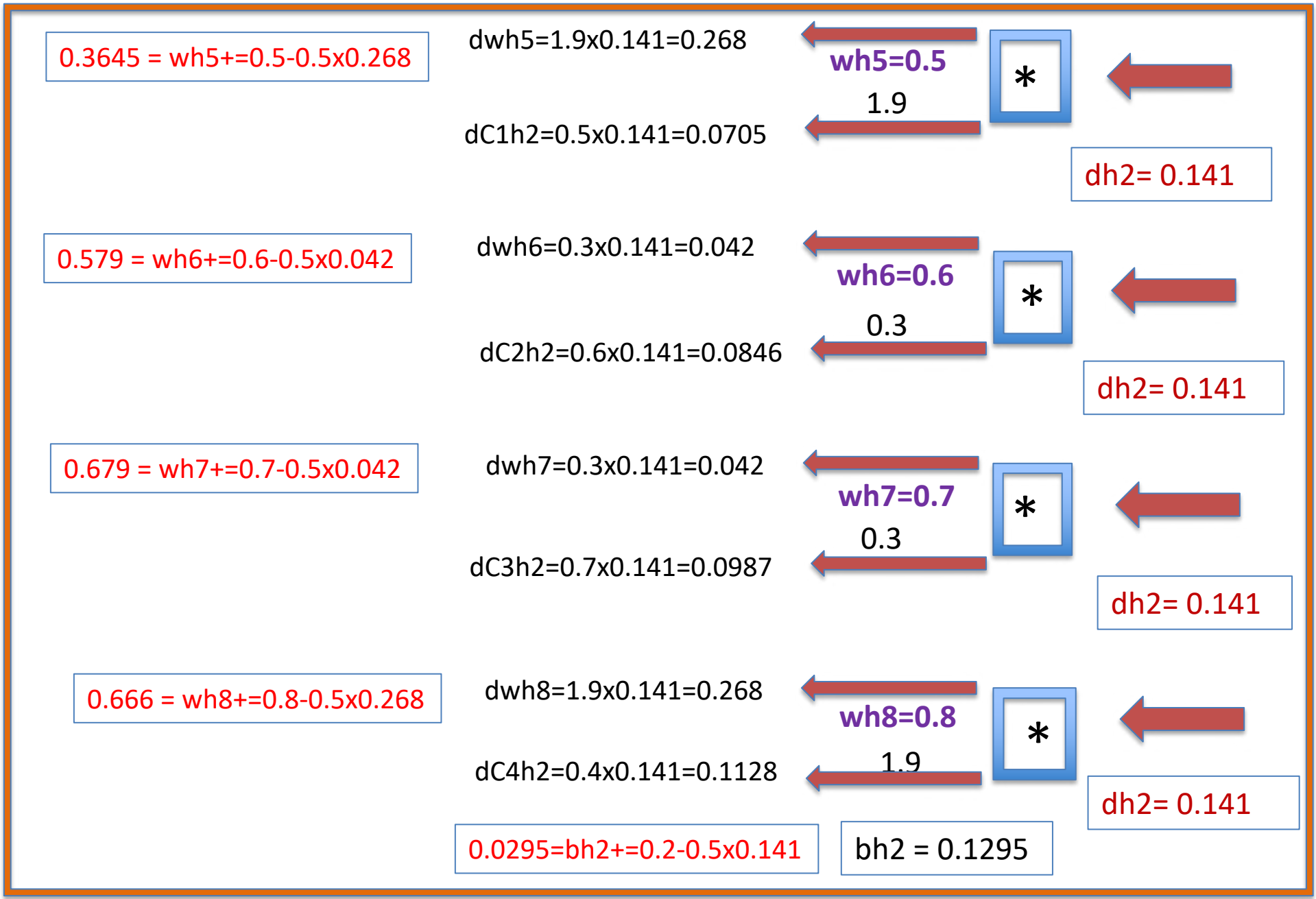
0.0295=bh1+=0.2-0.5x0.141

bh1 = 0.1295

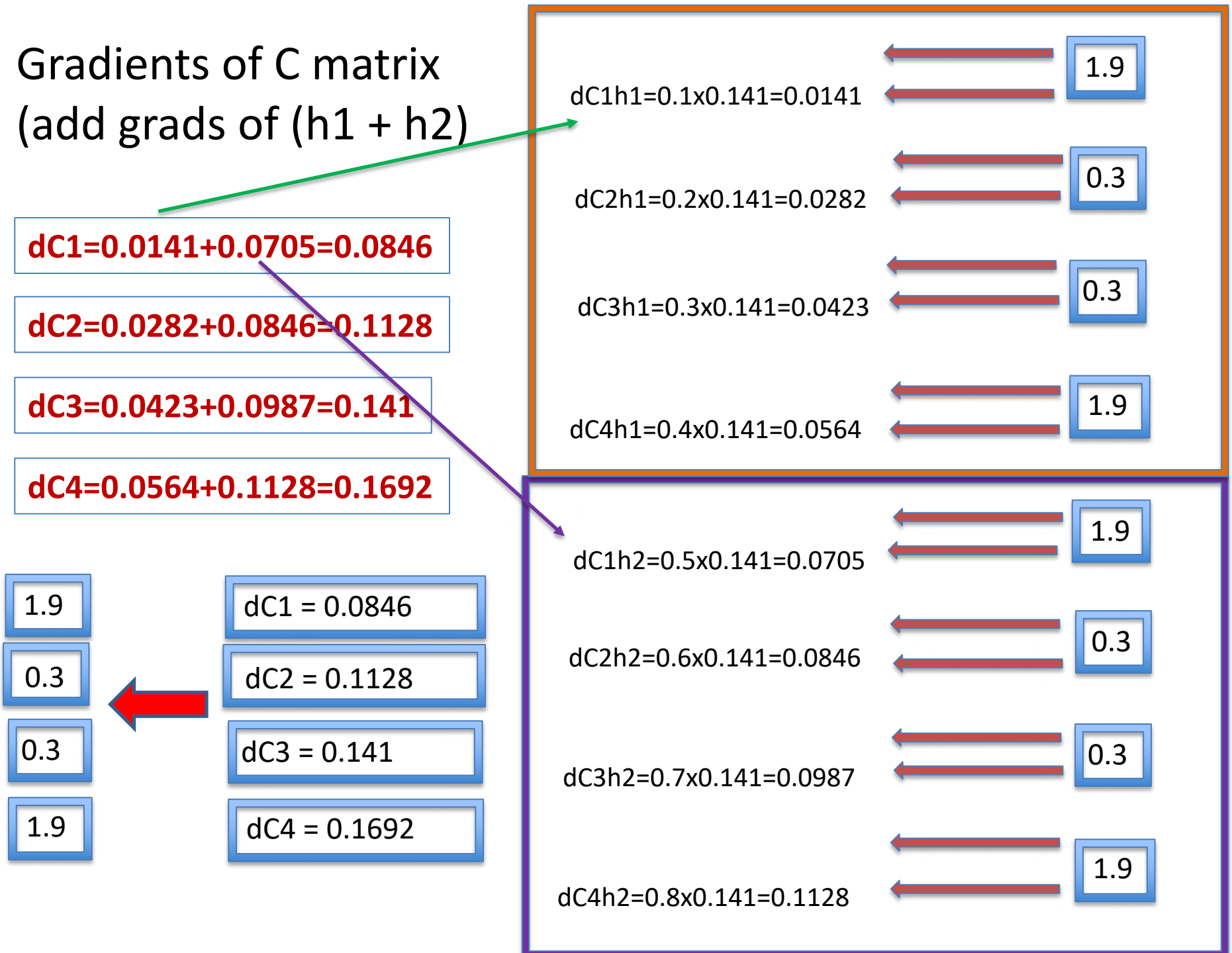
Gradients after h2, ReLU function (h2 neuron)

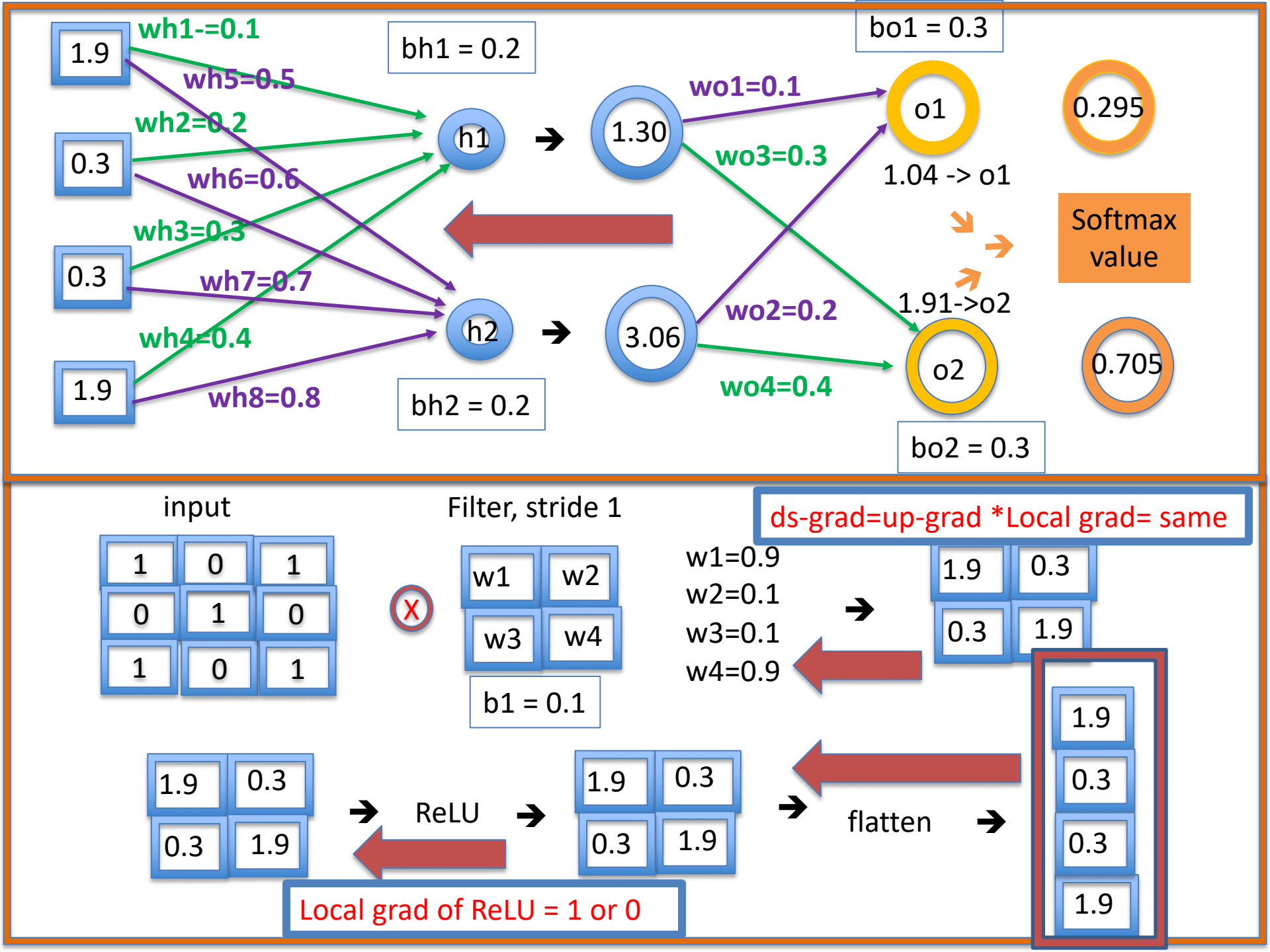


Update values of wh5, wh6, wh7, wh8, bh2



Gradients of C matrix (add grads of (h1 + h2)




1.9
0.3
0.3
1.9
wh1-=0.1
wh5=0.5
wh2=0.2
wh6=0.6
wh3=0.3
wh7=0.7
wh4=0.4
wh8=0.8
bh1 = 0.2
h1
h2
bh2 = 0.2
1.30
3.06
wo1=0.1
wo3=0.3
wo2=0.2
wo4=0.4
bo1 = 0.3
o1
o2
bo2 = 0.3
0.295
1.04 -> o1
1.91->o2
0.705
Softmax value
input
1 0 1
0 1 0
1 0 1
X
Filter, stride 1
w1 w2
w3 w4
b1 = 0.1
ds-grad=up-grad *Local grad= same
w1=0.9
w2=0.1
w3=0.1
w4=0.9
1.9 0.3
0.3 1.9
1.9
0.3
0.3
1.9
flatten
1.9 0.3
0.3 1.9
ReLU
1.9 0.3
0.3 1.9
Local grad of ReLU = 1 or 0

https://medium.com/@pavisj/convolutions-and-backpropagations-46026a8f5d2c

Local Gradients → (A)

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Finding derivatives with respect to F_{11}, F_{12}, F_{21} *and* F_{22}

$$\frac{\partial O_{11}}{\partial F_{11}} = X_{11} \quad \frac{\partial O_{11}}{\partial F_{12}} = X_{12} \quad \frac{\partial O_{11}}{\partial F_{21}} = X_{21} \quad \frac{\partial O_{11}}{\partial F_{22}} = X_{22}$$

Similarly, we can find the local gradients for O_{12}, O_{21} *and* O_{22}

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{11}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{11}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{11}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{11}}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{12}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{12}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{12}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{12}}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{21}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{21}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{21}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{21}}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{22}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{22}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{22}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{22}}$$

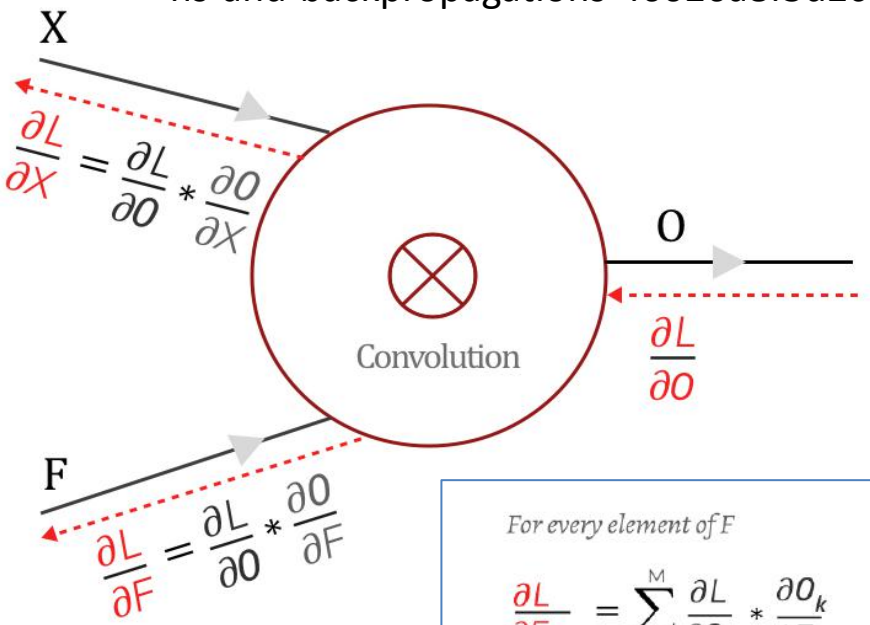
a convolution operation between input X and loss gradient ∂L/∂O

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * X_{11} + \frac{\partial L}{\partial O_{12}} * X_{12} + \frac{\partial L}{\partial O_{21}} * X_{21} + \frac{\partial L}{\partial O_{22}} * X_{22}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * X_{12} + \frac{\partial L}{\partial O_{12}} * X_{13} + \frac{\partial L}{\partial O_{21}} * X_{22} + \frac{\partial L}{\partial O_{22}} * X_{23}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * X_{21} + \frac{\partial L}{\partial O_{12}} * X_{22} + \frac{\partial L}{\partial O_{21}} * X_{31} + \frac{\partial L}{\partial O_{22}} * X_{32}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * X_{22} + \frac{\partial L}{\partial O_{12}} * X_{23} + \frac{\partial L}{\partial O_{21}} * X_{32} + \frac{\partial L}{\partial O_{22}} * X_{33}$$



For every element of F

$$\frac{\partial L}{\partial F_i} = \sum_{k=1}^{M} \frac{\partial L}{\partial O_k} * \frac{\partial O_k}{\partial F_i}$$

$\frac{\partial O}{\partial X}$ & $\frac{\partial O}{\partial F}$ *are local gradients*

$\frac{\partial L}{\partial z}$ *is the loss from the previous layer which has to be backpropagated to other layers*

$$\frac{\partial L}{\partial F} = \frac{\partial L}{\partial O} * \frac{\partial O}{\partial F}$$

- $\frac{\partial L}{\partial F}$: Gradient to update Filter F
- $\frac{\partial L}{\partial O}$: Loss Gradient from previous layer
- $\frac{\partial O}{\partial F}$: Local Gradients

$$\begin{bmatrix} \frac{\partial L}{\partial F_{11}} & \frac{\partial L}{\partial F_{12}} \\ \frac{\partial L}{\partial F_{21}} & \frac{\partial L}{\partial F_{22}} \end{bmatrix} = \text{Convolution} \left(\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} \right)$$

∂L/∂F is nothing but the convolution between Input X and Loss Gradient from the next layer ∂L/∂O

where

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} = \text{Input X} \qquad \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} = \frac{\partial L}{\partial O}$$ Loss gradient from previous layer

Local Gradients: (B)

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Differentiating with respect to X_{11}, X_{12}, X_{21} *and* X_{22}

$$\frac{\partial O_{11}}{\partial X_{11}} = F_{11} \quad \frac{\partial O_{11}}{\partial X_{12}} = F_{12} \quad \frac{\partial O_{11}}{\partial X_{21}} = F_{21} \quad \frac{\partial O_{11}}{\partial X_{22}} = F_{22}$$

Similarly, we can find local gradients for O_{12}, O_{21} *and* O_{22}

For every element of X_i

$$\frac{\partial L}{\partial X_i} = \sum_{k=1}^{M} \frac{\partial L}{\partial O_k} * \frac{\partial O_k}{\partial X_i}$$

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial O_{11}} * F_{11}$$

$$\frac{\partial L}{\partial X_{12}} = \frac{\partial L}{\partial O_{11}} * F_{12} + \frac{\partial L}{\partial O_{12}} * F_{11}$$

$$\frac{\partial L}{\partial X_{13}} = \frac{\partial L}{\partial O_{12}} * F_{12}$$

$$\frac{\partial L}{\partial X_{21}} = \frac{\partial L}{\partial O_{11}} * F_{21} + \frac{\partial L}{\partial O_{21}} * F_{11}$$

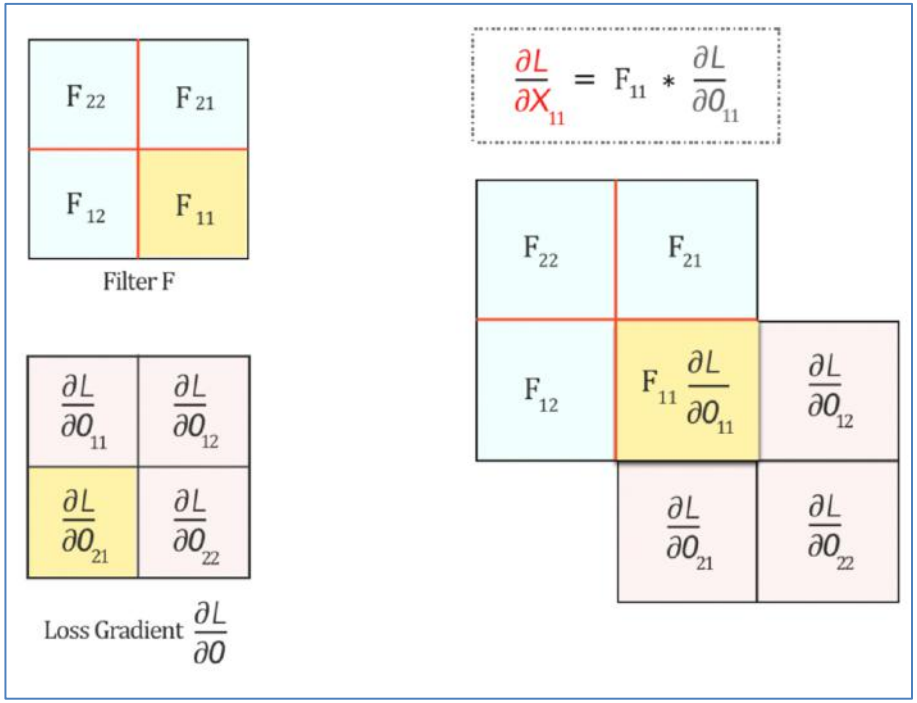
$$\frac{\partial L}{\partial X_{22}} = \frac{\partial L}{\partial O_{11}} * F_{22} + \frac{\partial L}{\partial O_{12}} * F_{21} + \frac{\partial L}{\partial O_{21}} * F_{12} + \frac{\partial L}{\partial O_{22}} * F_{11}$$

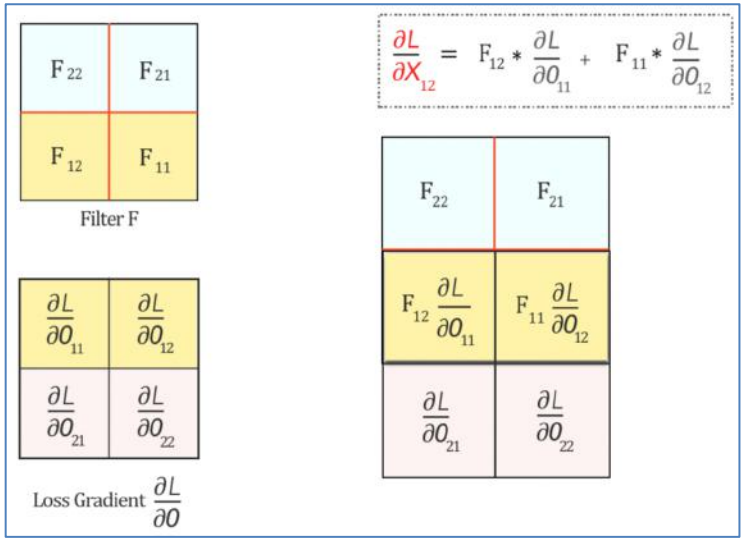
$$\frac{\partial L}{\partial X_{23}} = \frac{\partial L}{\partial O_{12}} * F_{22} + \frac{\partial L}{\partial O_{22}} * F_{12}$$

$$\frac{\partial L}{\partial X_{31}} = \frac{\partial L}{\partial O_{21}} * F_{21}$$

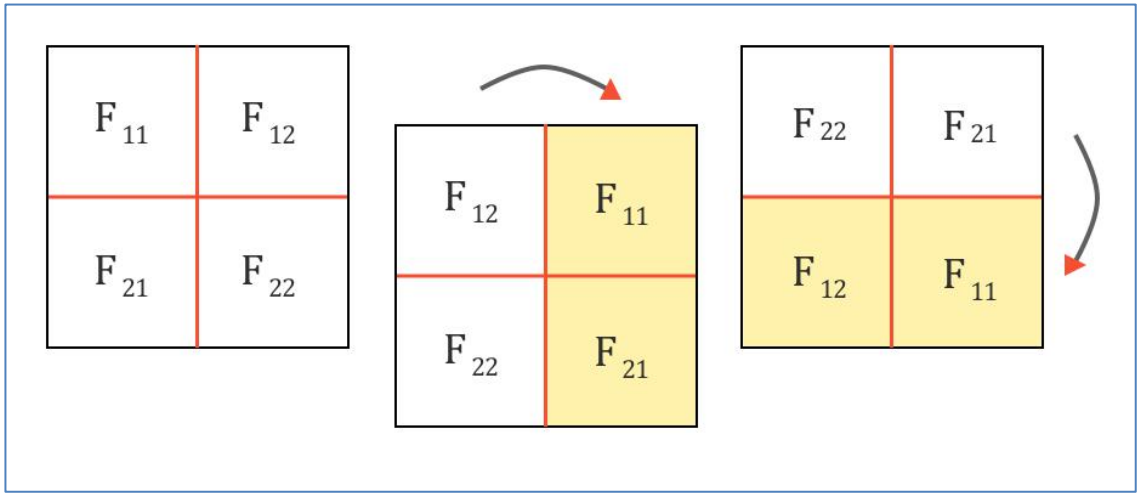
$$\frac{\partial L}{\partial X_{32}} = \frac{\partial L}{\partial O_{21}} * F_{22} + \frac{\partial L}{\partial O_{22}} * F_{21}$$

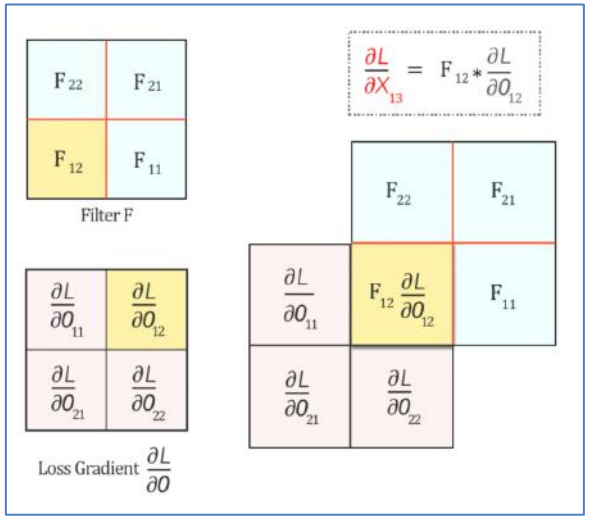
$$\frac{\partial L}{\partial X_{33}} = \frac{\partial L}{\partial O_{22}} * F_{22}$$





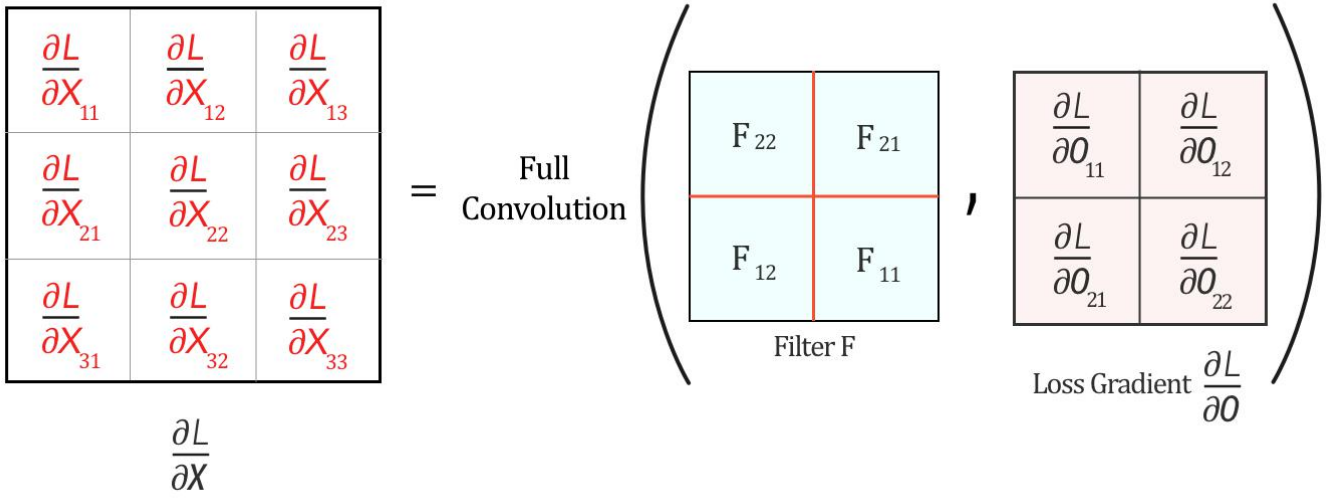
∂L/∂X can be represented as 'full' convolution between a 180-degree rotated Filter F and loss gradient ∂L/∂O





Forward path : backward path

Forward path : backward path



Backpropagation in a Convolutional Layer of a CNN

Finding the gradients:

$$\frac{\partial L}{\partial F} = \text{Convolution}\left(\text{Input } X, \text{ Loss gradient } \frac{\partial L}{\partial O}\right)$$

$$\frac{\partial L}{\partial X} = \text{Full Convolution}\left(180^\circ \text{rotated Filter } F, \text{ Loss Gradient } \frac{\partial L}{\partial O}\right)$$

∂L/∂X can be represented as 'full' convolution between a 180-degree rotated Filter F and loss gradient ∂L/∂O

EXERCISES

Run this code on the webpage:

https://codelabs.developers.google.com/codelabs/cloud-tensorflow-mnist/#12

Exercise

a) A neural network is used to classify a dataset composed of 5000 grayscale images of 0 to 9 digits. Each of the image has 32 x 32 pixels. The neural network contains 4 layers. The first layer is the input layer, the second layer has 20 neurons, the third one has 15 neurons, the last one is the softmax neuron of 10 classes. Please specify clearly the dimension of the matrices of each layer (both input weights and output) of the training process given a batch size of 50?

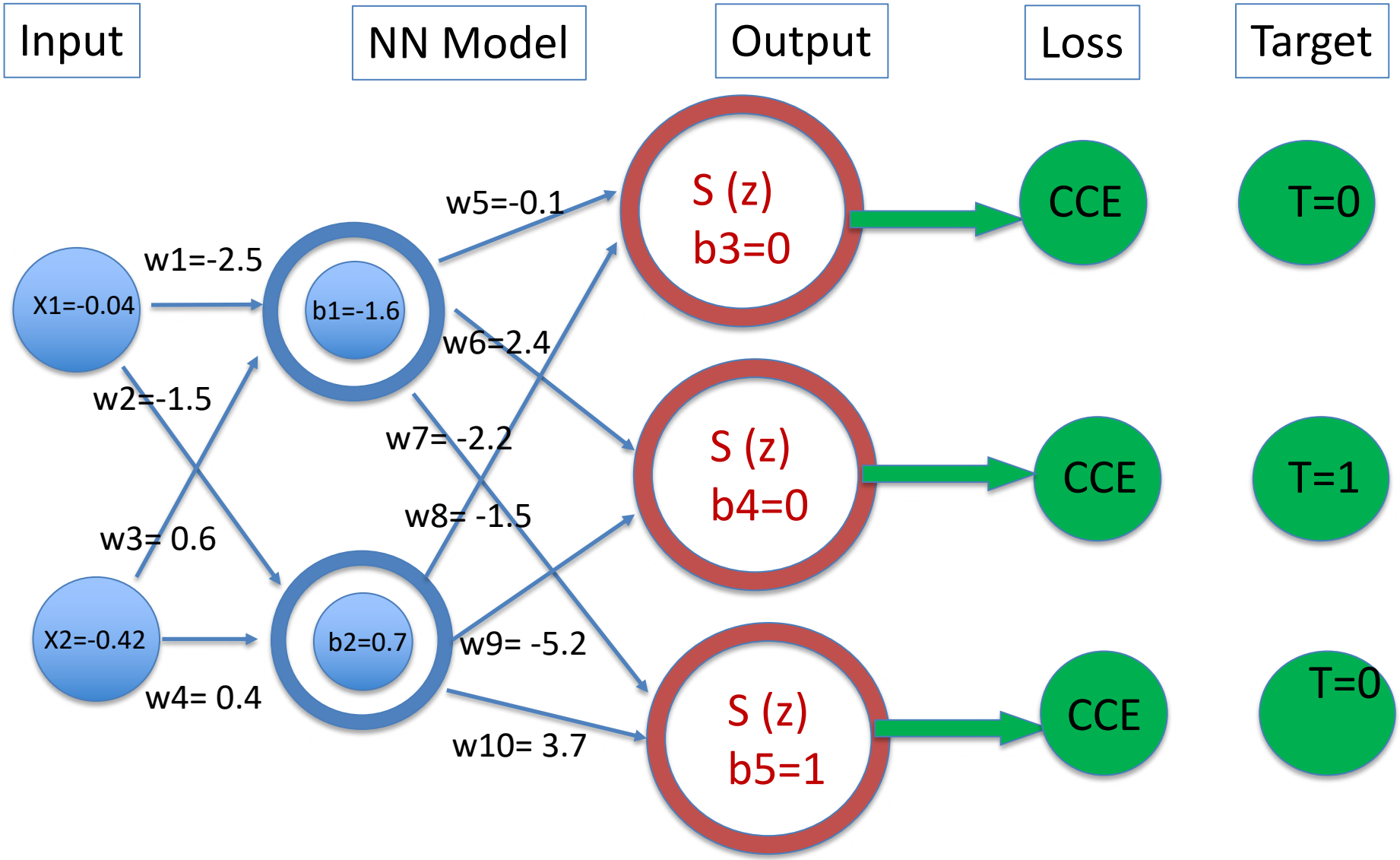
Layer one : input vector
Layer two:
- input matrix and
- weight matrix,
- output matrix

Layer three:
- input matrix
- weight matrix,
- output matrix

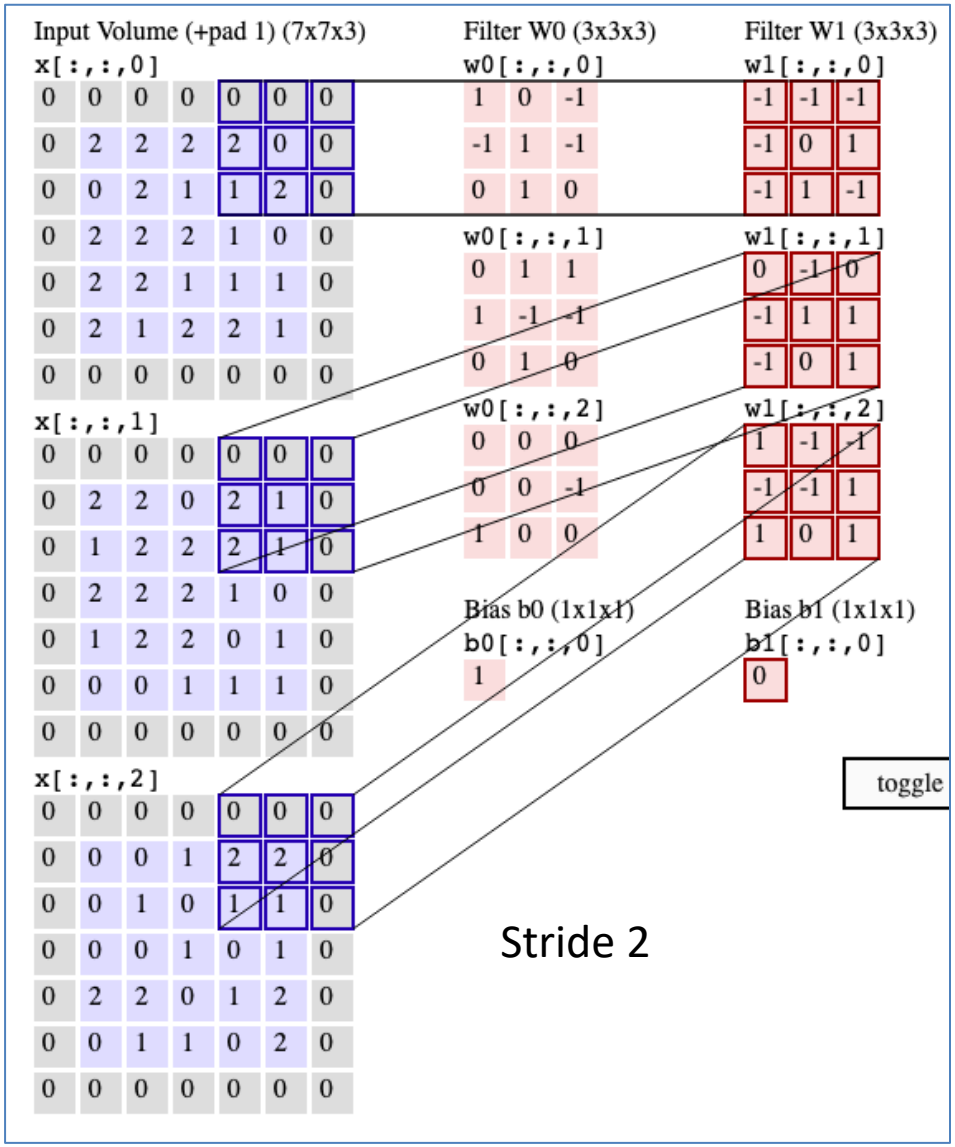
Layer four:
- input matrix,
- weight matrix, and
- output

b) How many batch iterations are needed to finish one epoch ?

Exercises : Represent the following NN model as a computational graph and build a python code to update w and b



Categorical (Softmax) Cross Entropy Loss



Overlapping max pooling of 2x2, stride 1

Flatten Fully Connected layer of neurons

Fully Connected layer of 2 softmax neurons

Exercise

(Q1) Forward path calculation of CNN

(a) Compute the values of the output matrices after the convolution step.

(b) Compute the output values after the ReLU operation.

(c) Compute the values after the max pooling operation

(d) List the values of the vector after it is flattened based on the row major counting system.

(e) Given the NN, what is the total number of the trainable parameters?

(Q2) Build a python code to compute the values of (Q1), compute also the outputs.

Homework Question

Given the CNN network shown below, an RGB image size of 5x5, filter size of 3x3, padding of 1, stride of 2:

a) Compute the single value of the output after the convolution of the highlighted regions.

b) What is the total number of unknown parameters of this network after an overlapping max pooling of 2x2 of stride 1, followed by a FC layer and a softmax output layer?

0	0	0	0	0	0	...
0	156	155	156	158	158	...
0	153	154	157	159	159	...
0	149	151	155	158	159	...
0	146	146	149	153	158	...
0	145	143	143	148	158	...
...	...	...	...	...	...	...

Input Channel #1 (Red)

0	0	0	0	0	0	...
0	167	166	167	169	169	...
0	164	165	168	170	170	...
0	160	162	166	169	170	...
0	156	156	159	163	168	...
0	155	153	153	158	168	...
...	...	...	...	...	...	...

Input Channel #2 (Green)

0	0	0	0	0	0	...
0	163	162	163	165	165	...
0	160	161	164	166	166	...
0	156	158	162	165	166	...
0	155	155	158	162	167	...
0	154	152	152	157	167	...
...	...	...	...	...	...	...

Input Channel #3 (Blue)

image = RGD
Im size = 5x5

filter = 3x3

padding = 1

stride = 2

-1	-1	1
0	1	-1
0	1	1

Kernel Channel #1

1	0	0
1	-1	-1
1	0	-1

Kernel Channel #2

0	1	1
0	1	0
1	-1	1

Kernel Channel #3

Bias = 2

Overlapping max pooling of 2x2, stride 1

Flatten Fully Connected layer of neurons

Fully Connected layer of 2 softmax neurons

Homework

Question #4

CIFAR stands for Canadian Institute For Advanced Research and 10 refers to 10 classes. The CIFIR 10 dataset consists of 60000 32x32 color images in 10 classes, with 6000 images per class. There are 50000 training images and 10000 test images.

Convert CIFIR-10 images to greyscale images or reshape the CIFIR-10 images to vectors similar to that of MNIST dataset.

Modify the simple MLP TensorFlow tutorial which uses the MNIST data to train the modified CIFIR 10 dataset. Plot the loss and accuracy of the training and testing results.

Helpful web links:
https://www.tensorflow.org/tutorials/quickstart/beginner
https://www.tensorflow.org/api_docs/python/tf/image/rgb_to_grayscale
https://datascience.stackexchange.com/questions/24459/how-to-give-cifar-10-as-an-input-to-mlp

Acknowledgements and References

This portion of the tutorial contains many extracted materials from many online websites and courses. Listed below are the major sites. This portion of the materials is not intended for public distribution. Please visit the sites websites for detail contents. If you are beginner users of DNN, I suggest to read the following list of websites in its order.

1) http://neuralnetworksanddeeplearning.com

2) https://codelabs.developers.google.com/codelabs/cloud-tensorflow-mnist

3) https://www.deeplearning.ai/deep-learning-specialization/

4) http://cs231n.stanford.edu/

5) MIT 6.S191, https://www.youtube.com/watch?v=njKP3FqW3Sk

6) https://livebook.manning.com/book/deep-learning-with-python/about-this-book/

7) https://www.fast.ai/

8) https://petewarden.com/2015/04/20/why-gemm-is-at-the-heart-of-deep-learning/

9) https://www.nersc.gov/users/training/gpus-for-science/gpus-for-science-2020/

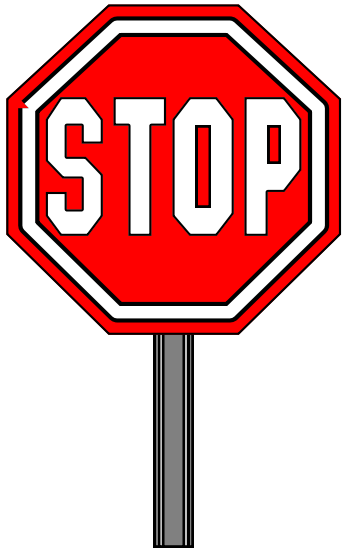
10) https://oneapi-src.github.io/oneDNN/

11) http://www.cs.cornell.edu/courses/cs4787/2020sp/

12) More sites and free books are listed in www.jics.utk.edu/actia → ML

13) https://machinelearningmastery.com

The End



- The End!