COMPUTER SCIENCE TRIPOS Part IA – 2022 – Paper 2

1 Digital Electronics (ijw24)

(*a*) Show using Boolean algebra

(*i*) $\overline{X}.Y \oplus X.\overline{Y} = \overline{X}.Y + X.\overline{Y}$

(*ii*) $\overline{X}.\overline{Y} \oplus X.Y = \overline{X}.\overline{Y} + X.Y$

[4 marks]

(*b*) Using the results in Part (*a*) or otherwise, express the four-variable function

$$F(A, B, C, D) = \overline{A}.\overline{B}.\overline{C} + \overline{A}.\overline{C}.\overline{D} + \overline{A}.B.C.D + A.B.\overline{C}.D + A.C.\overline{D} + A.\overline{B}.C$$

as the Exclusive OR of three 2-variable AND terms, i.e., $F = X_1.X_2 \oplus X_3.X_4 \oplus X_5.X_6$ where each X_i is either one of the four variables or its complement.

[4 marks]

(*c*) (*i*) Simplify the four-variable function

$$G(A, B, C, D) = \sum(0, 2, 6, 7, 8, 9, 10, 13, 15)$$

using the Quine-McCluskey (Q-M) method. The numbers in the summation are the decimal representations of the minterms of G (where A represents the most-significant bit of the equivalent binary representation).

[9 marks]

(*ii*) How many equal-complexity solutions exist in total? Justify your answer.

[3 marks]

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2 Digital Electronics (ijw24)

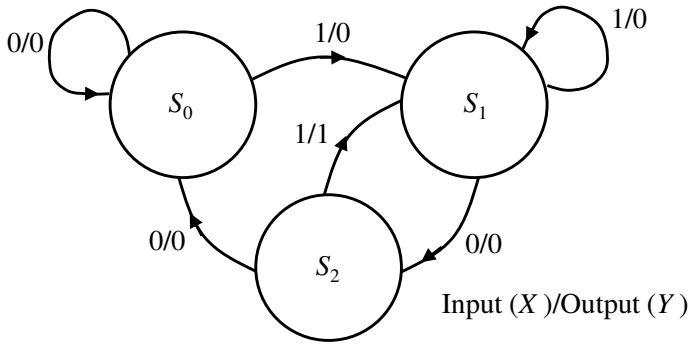
(*a*) A (fictional) edge-triggered UV flip-flop has inputs U and V and output Q. Its state-transition table is given by:

Current state (Q)	Next state (Q')			
$UV =$	00	01	10	11
0	0	1	0	1
1	0	0	0	1

(*i*) Draw the state-transition diagram for the Q output. [3 marks]

(*ii*) For an implementation based on a D-type flip-flop, determine the simplified Boolean equation in sum-of-products form for the next-state (Q') logic. [2 marks]

(*b*) Consider the following state machine:



(*i*) Assuming that the machine starts in state S_0 and that the input data sequence at input (X) is appropriately synchronised with the state machine clock, determine the next-state and output sequences for the input sequence 0101011011011. What operation does the machine perform? [5 marks]

(*ii*) For an implementation based on two D-type flip-flops (labelled A and B), determine simplified Boolean expressions for the next-state and output combinational logic, assuming the state assignment $S_0 = 00$, $S_1 = 01$ and $S_2 = 10$ is used, where a state is labelled $Q_A Q_B$ in terms of the flip-flop outputs. [4 marks]

(*iii*) For an alternative one-hot implementation based on D-type flip-flops, determine expressions for the next-state and output logic. [4 marks]

(*iv*) What feature, inherent in the proposed state-machine design, may give rise to problems at the output Y? How might this be addressed? [2 marks]