Problem Chosen	2025 MCM/ICM Summary Sheet	Team Control Number
C		9562

The Key to Victory: The Application of Momentum in Tennis

Summary

In tennis matches, momentum reflects a player's strength and drive, playing a crucial role in analyzing the match's progress. This paper introduces a momentum quantification and prediction model, and a match turning point prediction model to quantify player performance, predict match dynamics, and help players formulate strategies.

Firstly, we selected 8 indicators and processed data from the first 31 matches. After conducting Spearman correlation analysis, **principal component analysis (PCA)** was used to determine 3 principal components quantifying the player's attack, defense, and stability. By weighting these components, we obtained momentum and established a **momentum quantification and prediction model**. We used the model to visualize players' momentum during the 1701 finals. The results indicated that the model accurately predicted the match, aligning well with actual dynamics.

Secondly, based on the first part's results, we analyzed momentum sizes and the number of cases in four categories: match outcomes (win/loss). By calculating test statistics and performing **Pearson's chi-square test**, we examined the correlation between momentum and winning probabilities. This disproved the view that momentum fluctuations and match outcomes were random. The results showed a significant statistical relationship between momentum and match outcomes.

Next, after reviewing the literature, we set the momentum difference threshold at **0.18** to determine when match dynamics shift. Using the difference in 8 indicators between the players as independent variables and the classification variable reflecting momentum changes as the dependent variable, we established an **Adaboost classifier**. After 10-fold cross-validation to evaluate the model's performance, we created a **match turning point prediction model**. This trained classifier, applied to the 1701 finals, captured momentum turning points with **91.45%** accuracy. Based on the results, we provided recommendations for coaches and players.

To test the model's broader applicability, we collected data from the 2024 Wimbledon Women's Singles Final and applied both models. The results showed **87.64%** prediction accuracy for match outcomes and **88.45%** for turning points. Considering gender differences in momentum, we adjusted the threshold and found the highest prediction accuracy at **90.32%** when set to **0.1**. We also analyzed the model's generalizability.

Finally, we evaluated the model, discussing its strengths, weaknesses, and improvements. By setting the fluctuation range for the momentum threshold, we visualized the match outcome prediction accuracy, confirming the model's robustness and stability. Based on these findings, we wrote a memorandum explaining momentum's role and provided recommendations for coaches and players.

Keywords: tennis; momentum quantification model; Chi-square test; PCA; Adaboost classification

Contents

1 Introduction

1.1 Problem Background

Tennis is a sport with a long history. Many people engage in tennis as a form of exercise during their leisure time. Tennis players from different countries and regions compete in tournaments, building deep friendships and showcasing the spirit of sportsmanship and unity. As a result, tennis has become a global platform for cultural exchange. The four Grand Slam tournaments refer to the Australian Open, the French Open, the Wimbledon Championships, and the US Open. Among these, the 2023 Wimbledon Championships are undoubtedly the grand event that tennis enthusiasts and sports fans around the world are eagerly awaiting.

Some believe that changes and momentum transfer influence the spin, speed, and direction of the ball. Tennis players control changes and momentum transfer by adjusting their hitting techniques and match strategies, which can ultimately affect the outcome of the game. Although tennis players often appear energetic during matches, it is difficult to assess the exact role momentum plays in a game, as well as the magnitude of its influence. Therefore, is there a quantifiable momentum model that can be used to help players achieve better match results?

Figure 1: Tennis

1.2 Restatement of the Problem and Our Work

- **Task 1:** Construct a dynamic model to calculate the performance of both players in a tennis match that incorporates the advantage of serving.

  Our work: Firstly, we referenced the match data of players from the Tennis Professional Association to identify **eight metrics** that describe the performance of the players. Then, we applied **Principal Component Analysis (PCA)** for dimensionality reduction to extract **three principal components (aggressiveness, defense, and stability)**, which were quantified as a "momentum" metric to describe the momentum shifts during the match. Finally, through visualization techniques, we presented the real-time momentum trends during the 2023 Wimbledon Gentlemen's Final and validated the model's accuracy by comparing it with the actual match outcomes.

- **Task 2:** Verify if momentum has an impact on tennis match outcomes, and whether the effect is statistically significant.
  Our work: Based on the momentum model developed in task1, we further explored the relationship between momentum and match outcomes. By analyzing data from 117 matches, we correlated the momentum values with match results and performed a **Chi-squared test** to assess the statistical significance of the relationship between momentum and victory or defeat.

- **Task 3:** Identify key factors that may trigger momentum shifts and use collected data to predict momentum transitions.
  Our work: First, through data processing and literature review, we identified **threshold values for momentum changes** to determine when a significant shift in momentum occurred. We then employed **the AdaBoost classification algorithm**, using the eight metrics as independent variables and the occurrence of momentum shifts as the dependent variable. Finally, we used the model for real-time prediction of momentum shifts during matches and found that it accurately identified turning points in momentum. Based on the results, we provided pre-match preparation and tactical advice for coaches and players.

- **Task 4:** Evaluate the model's ability to predict match swings and its performance across different tournaments, courts, and other sports.
  Our work: We **applied the model** to the 2024 Wimbledon Women's Singles Final (Krejčíková B. vs Paolini J.) to validate its applicability in women's matches. We also outlined **potential improvements for the model**, such as adjusting the momentum difference threshold, recalibrating the principal component weights using women's match data, and adding additional metrics (e.g., longest consecutive points, return-of-serve success rate) to enhance prediction accuracy.

A more intuitive understanding of our process can be seen in the flowchart below for a clearer understanding of our approach.

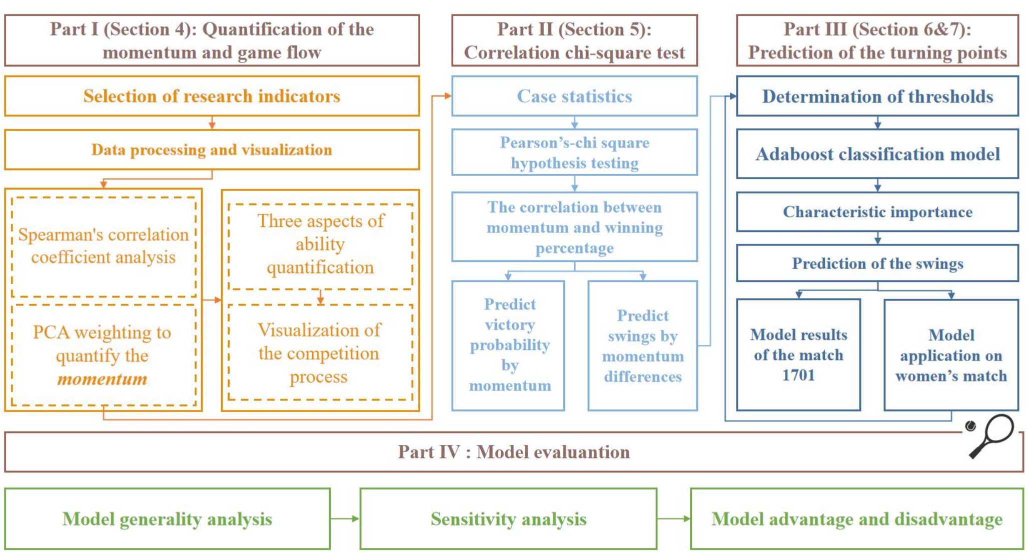

Figure 2: Flowchart of our work

2 Assumptions

- **Assumption1**: A player's performance on the court is rarely influenced by external factors such as court conditions, weather, or crowd atmosphere.
  Justification1: A player's performance is primarily determined by their technical skills and match strategy. External factors, such as court type, weather changes, or crowd interference, have a relatively small impact on the overall outcome of the match.

- **Assumption2**: The performance of each player in this match is rarely influenced by the results of previous matches and is instead focused on the current match.
  Justification2: Professional players possess strong psychological regulation skills, allowing them to focus on the current match and avoid letting past results interfere with their performance.

- **Assumption3**: The provided data and selected indicators can objectively reflect a player's attacking, defensive abilities, and stability, thereby indicating the player's momentum in the match.
  Justification3: The chosen indicators are widely recognized as key performance metrics in tennis. They provide a comprehensive assessment of a player's performance. For example, the double fault rate reflects the risk of errors, while the ACE rate indicates the intensity of a player's attacks.

- **Assumption4**: The trend of momentum changes can reflect a player's advantage or disadvantage in the match.
  Justification4: The changes in momentum are closely linked to a player's psychological state and actual performance, as recognized in sports psychology.

- **Assumption5**: The setting of a momentum difference threshold can accurately identify key turning points in the match.
  Justification5: The momentum difference reflects the instantaneous change in the skill gap between the two players during the match. By establishing a threshold, we can capture significant shifts in momentum that indicate key turning points, such as when one player gains a substantial advantage or when there is a dramatic change in the match dynamics.

3 Notifications

Symbols	Description
S_{attack}	principal component of aggression
S_{defend}	principal component of defence
$S_{stability}$	principal component of stability
P_i	momentum of the ith player
V_i	a victory probability of the ith player

Other symbols will be described as they are used.

4 Momentum Quantification and Prediction Models

4.1 Selection of Research Indicators

In tennis, the factors influencing the outcome of a match can be examined from various perspectives. The official website of the Association of Tennis Professionals (ATP) lists match data[1] for each player, including 16 statistical indicators for individual matches.

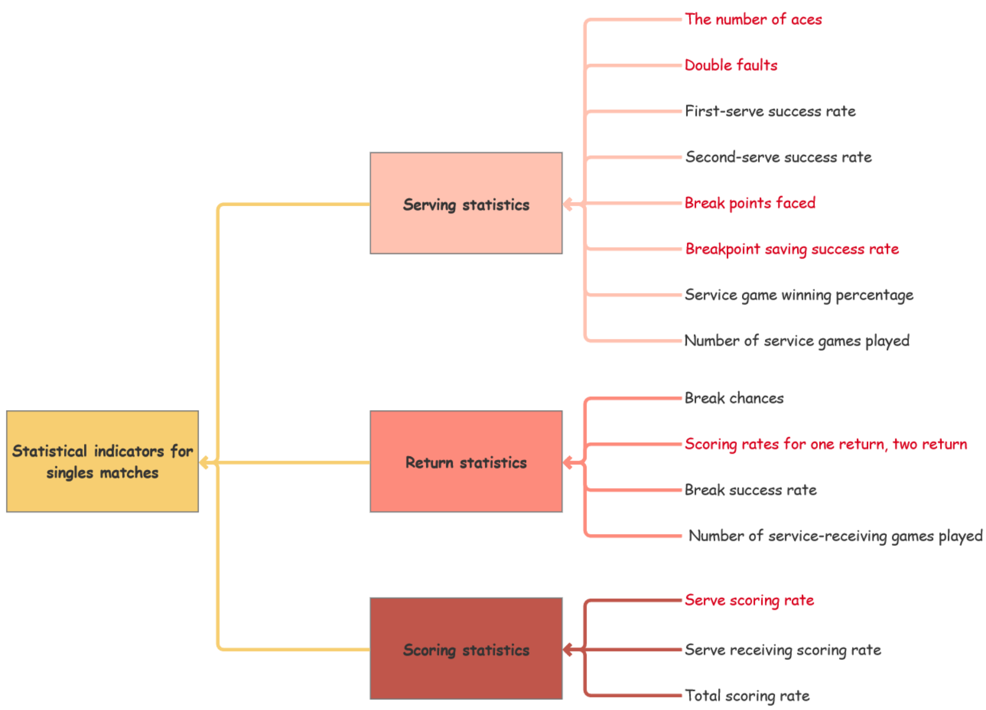

Figure 3: Selection of research indicators

These indicators provide a comprehensive analysis of a player's performance in various aspects of the game, offering insights into their strengths and weaknesses during the match. By analyzing these statistics, we can better understand how each factor contributes to the overall outcome of a tennis match.

Based on the 16 research indicators mentioned above, this study focuses on 7 key indicators for in-depth analysis, as highlighted in red in figure 3. These indicators are: the number of aces, double faults, break points faced, break point saving success rate, scoring rates for first and second serve returns, and serve scoring rate.We can find that break point, double fault, serve score, and other aspects play an important role in evaluating the performance of tennis players. Therefore, we refer to these significant indicators above and select eight indicators to describe the performance of tennis players.

The following is an explanation of the 8 specialized terms used in these indicators:

- Double fault rate(X1):

$$Double\ fault\ rate = \frac{double\ fault}{total\ serves} \tag{1}$$

The double fault rate refers to the percentage of times a player commits two consecutive serving errors during a match. This indicator reflects the player's serving stability, with a higher double fault rate indicating greater instability in their serving performance.

- Unforced error rate(X2):

$$Unforced\ error\ rate = \frac{unforced\ errors}{total\ errors} \tag{2}$$

An unforced error refers to a mistake made by a player due to poor judgment or technical inadequacies, without external pressure from the opponent. When calculating the unforced error rate, the total number of errors is replaced by the opponent's points won as a result of these errors.

- Serve score rate(X3):

$$Serve\ score\ rate = \frac{serve\ win}{total\ points\ won} \tag{3}$$

The serve scoring rate refers to the percentage of points a player wins during their service games. A higher serve scoring rate indicates better serving performance, as the player is able to either win points directly through their serve or use their serve to control the pace of the match.

- ACE rate(X4):

$$ACE\ rate = \frac{ACE\ win}{total\ points\ won} \tag{4}$$

An ACE refers to a serve where the ball is hit with such speed or placed at such an angle that the opponent is unable to return it, resulting in an immediate point for the server. The ACE scoring rate describes the aggressiveness of a player's serve, with more ACEs indicating that the player's serve is a strong threat to their opponent.

- Net score rate(X5):

$$Net\ score\ rate = \frac{net\ play\ win}{net\ approaches} \tag{5}$$

The serve-and-volley strategy typically requires players to move quickly to the net after serving, using volleys or drop shots to dictate the pace of the match. A higher net points won rate indicates that the player is effectively utilizing their advantage at the net.

- Break rate(X6):

$$Break\ rate = \frac{successful\ breaks}{break\ points\ earned} \tag{6}$$

A break point refers to the opportunity a player has to break their opponent's serve during the opponent's service game. The break point conversion rate indicates the percentage of break points a player successfully converts into points. A higher conversion rate means the player is more effective at capitalizing on crucial opportunities during key moments in the match.

- Winner rate(X7):

$$Winner\ rate = \frac{winning\ shots}{total\ points\ won} \tag{7}$$

A winner is a successful shot that prevents the opponent from returning the ball, resulting in an immediate point. The winner success rate describes a player's ability to hit winning shots through aggressive play during the match.

- Victory probability(X8):

Using a set as the unit of analysis, the total points for both players are calculated through data preprocessing. By dividing one player's total score by the sum of both players' total scores, we obtain an approximate probability of victory for that player.

4.2 Data Processing

4.2.1 The Method of Data Processing

This study, based on the dataset "$Wimbledon\_featured\_matches$" provided in the attachment, organizes the data for each match. Data cleaning and outlier treatment were performed, and eight indicators (X1~X8) for each player in every set were calculated. Since most of the indicators listed in this paper are in percentage form, there could be instances where the denominator is zero, meaning that the event did not occur at all during the set. To ensure the rigor of the study, such outliers were assigned a value of 0.5.

4.2.2 The Visualization of Data

For the Wimbledon event discussed in the topic, this study selects the fifth set of the match as the object of analysis. The indicators X1 to X8 are analyzed, and the results are visualized, as shown in figure 4.

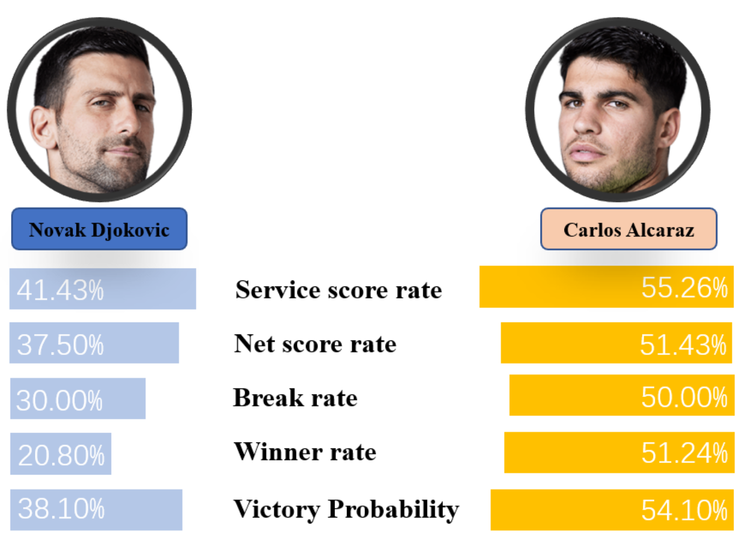

Figure 4: Alcaraz VS Djokovic

As shown in figure 4, Alcaraz's performance indicators are superior to Djokovic's, indicating that Alcaraz performed better in the fifth set of the match. In fact, he ultimately won the match and demonstrated his ability to surpass Djokovic at key moments, securing his first Wimbledon men's singles title.

When analyzing match statistics such as service score rate, break rate, and winner rate, we can observe that these figures provide a detailed snapshot of a player's performance at specific moments during the match. However, to fully understand what these numbers mean for their overall success, we must consider the existence of momentum. Therefore, in the following section, we use principal component analysis (PCA) to study the relationship between the selected indexes and momentum.

4.3 Principal Component Analysis (PCA) for Momentum Determination

Principal Component Analysis (PCA) is a statistical method that transforms the original variables into a new set of uncorrelated composite variables. These new variables, called principal components, are linear combinations of the original variables and are ordered based on the amount of variance they explain in the data[2]. To perform dimensionality reduction on the eight variables mentioned above, the following steps are involved:

Step 1: Normality Test. The normality test checks whether the data for each variable follows a normal distribution. Based on the test results, it was found that all variables except for $Unforced\ error\ rate$, $Winner\ rate$, and $Victory\ probability$ do not follow a normal distribution.

Step 2: Spearman Correlation Analysis. Since not all variables follow a normal distribution, we use Spearman's rank correlation to assess the relationships between the variables. Spearman's correlation is a non-parametric test that measures the strength and direction of association between two ranked variables.

The results of the Spearman correlation analysis are shown in the following figure.

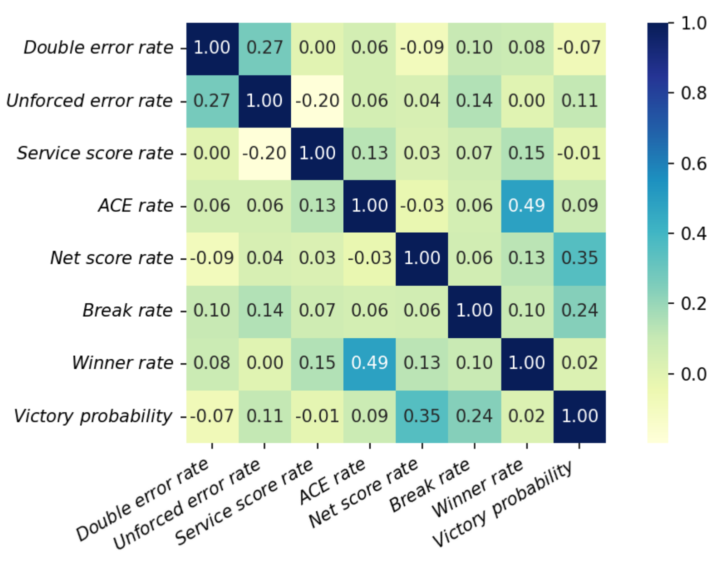

Figure 5: Spearman correlation analysis

To further quantify momentum in the match, Kaiser-Meyer-Olkin (KMO) Test and Bartlett's Test of Sphericity are performed on the standardized data of the eight indicators to determine if Principal Component Analysis (PCA) can be conducted.

KMO Test: The KMO value measures the adequacy of the data for PCA. A KMO value greater than 0.7 indicates that the data is suitable for PCA. Values closer to 1 suggest strong correlations among variables, making PCA more effective.

Bartlett's Test of Sphericity: Bartlett's test checks whether the correlation matrix is significantly different from an identity matrix, which would indicate that variables are uncorrelated. If the p-value is less than 0.05, we reject the null hypothesis and conclude that the variables are correlated, making PCA appropriate. If the p-value is greater than 0.05, the null hypothesis is not rejected, indicating that the variables may be independent and not suitable for PCA.

Table 1: Test Value

KMO		0.722
Bartlett's test for sphericity	Approximate Chi-square	169.150
	Degrees of freedom	28
	Significance	0.000***

As shown in Table 1, the data passed both the Kaiser-Meyer-Olkin (KMO) test and Bartlett's Test of Sphericity, indicating that the data is suitable for Principal Component Analysis (PCA). The results of the analysis are visualized in the Scree plot in Figure 6, which shows the eigenvalues of the principal components.Table 2 summarizes the explained variance for each principal component.

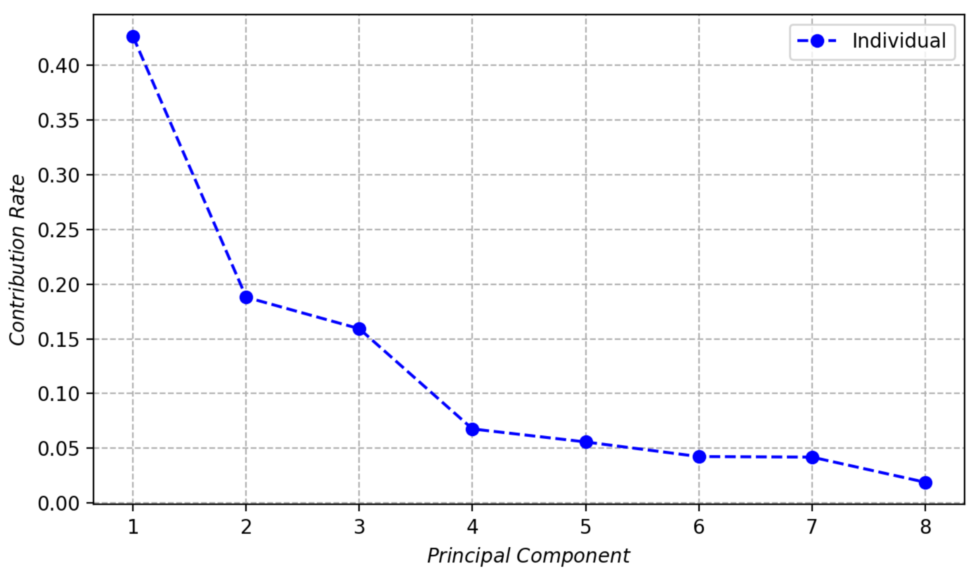

Figure 6: Scree plot

Table 2: Explained Total Variance

Explained Total Variance		
Component	Variance Explained (%)	Cumulative Variance Explained (%)
1	42.622	0.42622
2	18.802	0.61424
3	15.922	0.77346
4	6.761	0.84107
5	5.573	0.8968
6	4.238	0.93918
7	4.189	0.98107
8	1.893	1

As shown in Table 2, the cumulative contribution rate of the first three principal components reaches 77.346%. Additionally, from Figure 7, it is evident that there is a sharp "elbow" from the third to the fourth principal component, which suggests that the first three components explain the majority of the variance in the data.

From the factor loading matrix heatmap in Figure 7, the coefficients and color depth indicate the strength of the correlation between each principal component and the variables. The first principal component is positively correlated with all the indicators, with $Winner\ rate$ and $ACE\ rate$ having the highest factor loadings. Therefore, the first principal component is termed the "Aggressiveness Component" (S_{attack}).The second principal component shows strong correlations with $Net\ score\ rate$ and $Victory\ probability$, suggesting that it is related to the player's defense in controlling the match. This component is termed the "Defensive Component" (S_{defend}).The third principal component is strongly positively correlated with $Double\ error\ rate$ and $Unforced\ error\ rate$. Thus, the third principal component is called the "Stability Component" ($S_{stability}$).

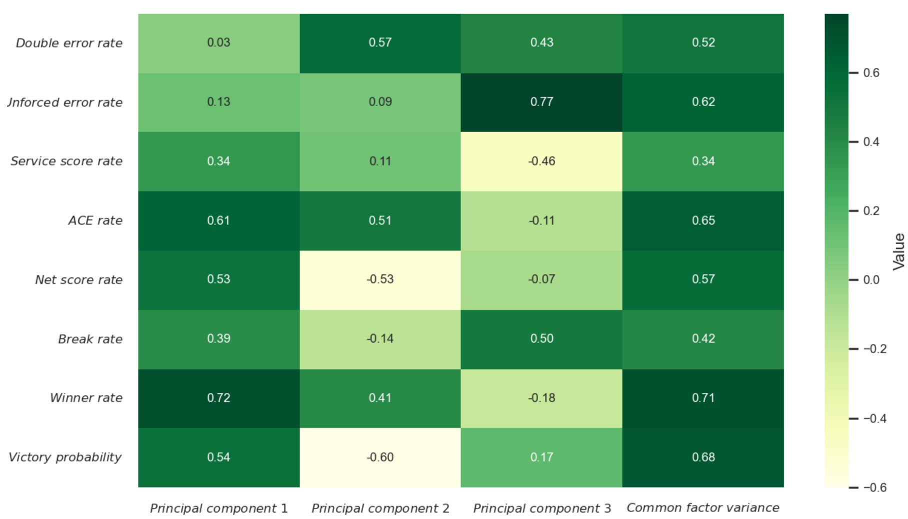

Figure 7: Factor Loading Matrix Heatmap

Table 3: Component Matrix Table

Indicator	Component 1	Component 2	Component 3
Double error rate	0.018	0.399	0.327
Unforced error rate	0.072	0.06	0.585
Service score rate	0.194	0.073	-0.351
ACE rate	0.349	0.355	-0.085
Net score rate	0.304	-0.371	-0.054
Break rate	0.224	-0.1	0.377
Winner rate	0.41	0.282	-0.14
Victory probability	0.311	-0.418	0.126

The component matrix is shown in the table above. Table 3 illustrates the factor score coefficients (principal component loadings) for each component, which are used to calculate the component scores and derive the factor formulas. The calculation formula is as follows:

Linear Combination Coefficients * (Variance Explained / Cumulative Variance Explained).

The factor weight scores are then obtained by normalizing this result. Factor loading coefficients divided by the corresponding eigenvalue, which corresponds to the coefficients in the component matrix.The final relationships between the three principal components and the respective indicators are as follows:

$$S_{attack} = 0.018X_1 + 0.072X_2 + 0.194X_3 + 0.349X_4 + 0.304X_5 + 0.224X_6 + 0.41X_7 + 0.311X_8 \quad (8)$$

$$S_{defend} = 0.399X_1 + 0.06X_2 + 0.073X_3 + 0.355X_4 - 0.371X_5 - 0.1X_6 + 0.282X_7 - 0.418X_8 \quad (9)$$

$$S_{stability} = 0.327X_1 + 0.585X_2 - 0.351X_3 - 0.085X_4 - 0.054X_5 + 0.377X_6 - 0.14X_7 + 0.126X_8 \quad (10)$$

Calculating the variance explained by the three principal components, and defining it as the "momentum" in the topic, as follows:

$$P = 0.3705S_{attack} + 0.3225S_{defend} + 0.3070S_{stability} \quad (11)$$

4.4 Inspection of the Model and Visualization of the Competition Process

To validate the accuracy of the model, data from match 1701 is used here. Using the momentum formula derived from the evaluation system mentioned above, a real-time "momentum" trend graph of the match process was generated, showcasing the model's predictive ability in the tournament, as shown in Figure 8. We choose set1, set3, and set5 to analyze the process.

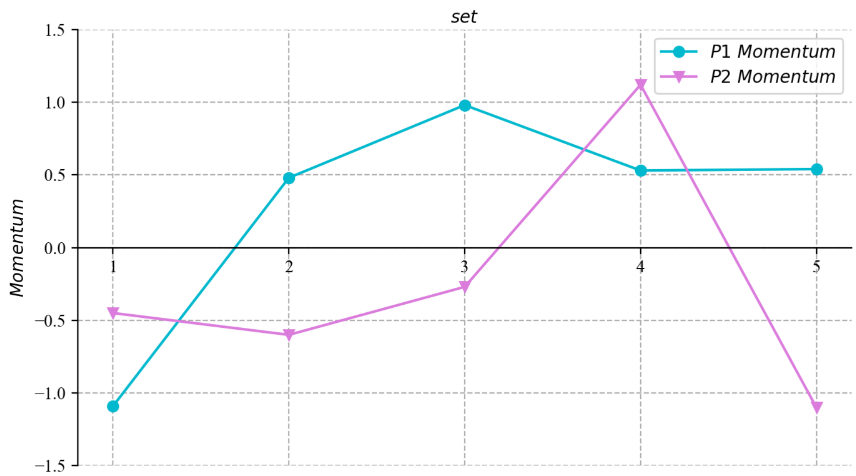

Figure 8: Real-time "momentum" trend during the match process

Set 1: Djokovic Dominates (6-1)

Alcaraz (P1) has a low momentum, approaching -1.0. This suggests that in the early stages of the match, Alcaraz may have felt pressure or failed to adapt quickly to the pace of the game. Meanwhile, Djokovic (P2), one of the all-time greats in Grand Slams, performed well, with high momentum. This aligns with the fact that Djokovic won the first set convincingly.

Set 3: Alcaraz Completely Takes Control (6-1)

Alcaraz's momentum rises again, nearing 1.0, reaching a high level, suggesting that he has established a clear advantage in the match. At this point, his confidence may have reached its peak. Meanwhile, Djokovic's momentum shows little improvement, possibly due to experiencing a break of serve or multiple errors, preventing him from maintaining his advantage. The model results align with the fact that Alcaraz won the third set.

Set 5: Alcaraz Wins (6-4)

Alcaraz's momentum remains at a high level but fluctuates slightly. He remained calm in key moments, demonstrating strong mental toughness and resilience, successfully controlling the pace of the match. Djokovic's momentum noticeably declines. The model results are consistent with Alcaraz winning the fifth set.

From the fluctuations shown in the line chart in Figure 8, it is evident that Alcaraz displayed strong momentum in the latter half of the match, particularly in the third set and mid-way through the fourth set. By adjusting his tactics and maintaining his confidence, he successfully overcame challenges and steadily maintained his advantage. In contrast, Djokovic showed some decline in momentum, particularly between the third and fifth sets, indicating that he failed to recover effectively at certain points.

Based on the above analysis, it can be concluded that a player's momentum within a set can have a significant impact on the outcome of the match, and momentum changes throughout the match. These aspects will be further explored in subsequent sections of the our paper.

5 Correlation Test Based on Chi-square Distribution

Based on the previous calculation results, it is evident that "momentum" plays an important role in tennis matches. However, whether this impact is statistically significant still warrants investigation. A tennis coach proposed a hypothesis: that changes in momentum during the match are not significantly related to the player's victory or loss, and are instead completely random. Under this hypothesis, we can use statistical methods to test the impact of "momentum" on the win probability and assess its role in the match outcome.

For the data of 117 sets, we separately calculate the momentum values and the corresponding match outcomes (whether the player won or not). For momentum where P1 > P2, we consider P1 to have higher momentum, and then we analyze the relationship between the momentum size and whether P1 wins the set. The resulting data is shown below:

Table 4: statistical data

	P1 wins	P1 fails	sum
strong momentum	57	20	77
weak momentum	8	32	40
sum	65	52	117

Table 4 shows the relationship between larger and smaller momentum groups and winning or losing in 117 matches. Since both momentum size and winning or losing are categorical variables, methods for numerical data analysis such as t-tests are not applicable. The chi-squared test, which is specifically used for categorical data, is therefore chosen to study the correlation between momentum and winning.

The chi-squared test is a data correlation analysis method based on the chi-squared distribution. The basic idea is to compare the fit between the theoretical frequency and the actual frequency, mainly used to assess the difference between the observed and expected frequencies. The steps for conducting a chi-squared test include: calculating the expected frequency for each category, comparing it with the observed frequency, and finally computing the chi-squared value based on the ratio.

Step 1: Define the Null Hypothesis and Alternative Hypothesis.

Null Hypothesis (H_0): There is no significant relationship between momentum (the amount of momentum) and winning.

Alternative Hypothesis (H_1): There is a significant relationship between momentum (the amount of momentum) and winning.

Step 2: Calculate the Expected Frequencies $E_{i,j}$:

In the chi-squared test, the formula to calculate the expected frequency $E_{i,j}$ is:

$$E_{i,j} = \frac{\left(\mathrm{sum}_i^{Row} \times sum_j^{\mathrm{Column}}\right)}{Num^{tota;}} \tag{12}$$

In this context, sum_i^{Row} refers to the total frequency of the i-th row, which is the sum of all observed frequencies in the i-th row. sum_j^{Column} refers to the total frequency of the j-th column, which is the sum of all observed frequencies in the j-th column. Num^{total} refers to the total sample size, i.e., the grand total of all observations in the dataset.

Step 3: The calculation formula for the Pearson chi-squared test is:

$$X^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{(O_{i,j} - E_{i,j})^2}{E_{i,j}} \tag{13}$$

In this context, i and j represent the rows and columns of the contingency table, respectively. $O_{i,j}$ represents the observed value for the i-th row and j-th column. $E_{i,j}$ represents the expected frequency for the i-th row and j-th column.The overall chi-squared statistic, X^2, in the chi-squared test is the weighted sum of the differences (the discrepancy between the observed frequency and the expected frequency) for each cell, with each discrepancy being standardized according to the expected frequency.

Table 5: Pearson's chi-square

Case number	Degree of freedom	Pearson's chi-square	Significance
117	1	31.119	P<0.01

Based on the results of the chi-squared test, there is a significant statistical relationship between the size of momentum and winning or losing. Since the p-value is less than 0.05, we reject the null hypothesis and conclude that momentum has a significant impact on the win probability.

From the above calculations, we can conclude that there is a significant correlation between momentum and the outcome of a tennis match. Specifically, momentum has an impact on whether competitors achieve success, and it is not completely random.

6 Prediction of the Direction of the Game

The direction of a tennis match is difficult to predict, as a player's momentum can change at any point during the game. Therefore, it is essential to select appropriate indicators to help determine the course of the match, specifically to identify when the match transitions from favoring one player to favoring the other.

6.1 Determining the Threshold – Identifying Momentum Shifts

To accurately describe the momentum changes and the related progress of the match, we continue to use the dataset obtained in Task 1 for further processing. By calculating the differences in the eight indicators and momentum for both players, we can obtain the differences in each set of the match, as well as the momentum differences between the two players.

To determine if a momentum shift occurs, we need to establish a threshold α. We processed the momentum differences for each set in the dataset and calculated the standard deviation. Referencing the parameters from [3], we set the momentum shift threshold as 1.5 times the standard deviation. Ultimately, the threshold was determined to be 0.18.If the sequence is$|P_1 - P_2| \geq 0.18$, it indicates that there is a significant momentum difference between the two players, suggesting a potential shift in the direction of the match. Based on this, we can define a categorical variable L to represent the classification of the match's momentum change.Next, we will use the eight indicators as independent variables and the categorical variable L as the dependent variable to perform Adaboost classification.

6.2 Adaboost Classification

6.2.1 Model Introduction

Traditional time series analysis methods may fail to ensure that the data passes stationarity tests, potentially resulting in a loss of critical information. On the other hand, while LASSO regression offers advantages in variable selection and sparsity control, its capacity to model nonlinear features is limited. Given these constraints, the Adaboost algorithm emerges as a more suitable choice for this study. Its mechanism for adjusting weights is independent of linear assumptions between variables, making it particularly effective in capturing complex relationships. Therefore, we adopt the Adaboost algorithm to perform the regression analysis for this study.

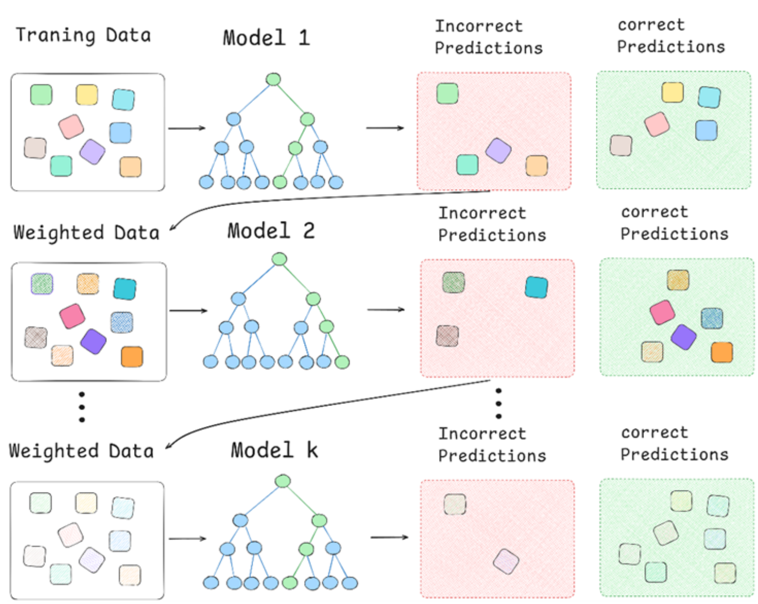

Figure 9: Adaboost algorithm

6.2.2 The Construction of an Adaboost Classification Model

The Adaboost algorithm operates through iterative updates of weights and the sequential adjustment of weak learners. [4]First, we initialize the weights, calculate the maximum error(E_t) and the relative error(e_{ti}), then compute the overall error rate of the weak learner(e_t). Based on the overall error, we determine the weight coefficient(α_t), further adjust the weights, update the sample weights($w_{t+1,i}$), and finally normalize the weights to obtain the strong learner($H(x)$).The following figure is a description of the flow of the Adaboost algorithm.

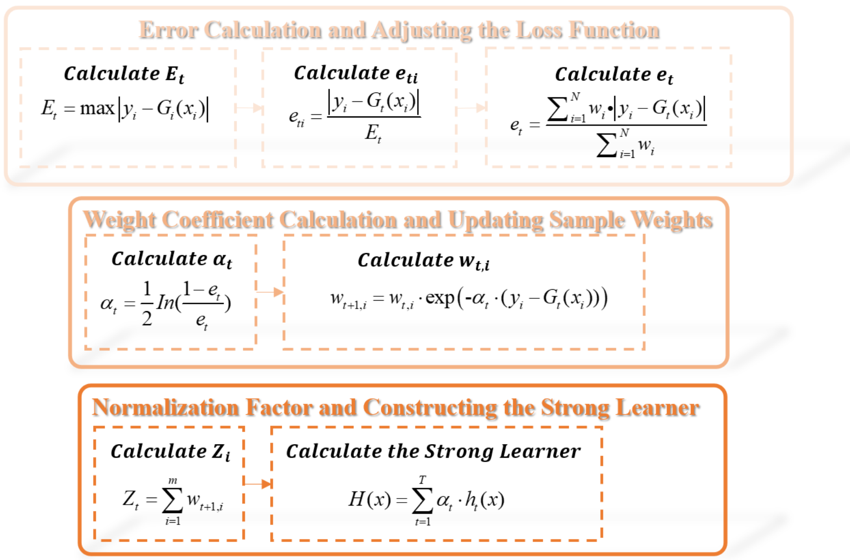

Figure 10: the flow of the Adaboost algorithm

The strong learner is derived by selecting the weak learner corresponding to the median value of the aggregated predictions. This approach mitigates the influence of outlier weak learners, ensuring more stable regression predictions. By iteratively combining weak learners through the above steps, we obtain an optimal strong learner that is well-suited for predicting momentum difference in a tennis match.

6.3 Solution and Demonstration of Model Results

6.3.1 Solution of Model Results

We utilized a decision tree classifier as the base learner for the Adaboost algorithm. The dataset was divided using a 10-fold cross-validation approach, ensuring robust evaluation of the model's performance across different data splits. A learning rate of 1 was adopted, and the data were shuffled to reduce potential bias. The final training results are summarized as follows.

We have obtained the feature importance of the eight selected indicators regarding the momentum difference between the two players, as shown in Figure 11.

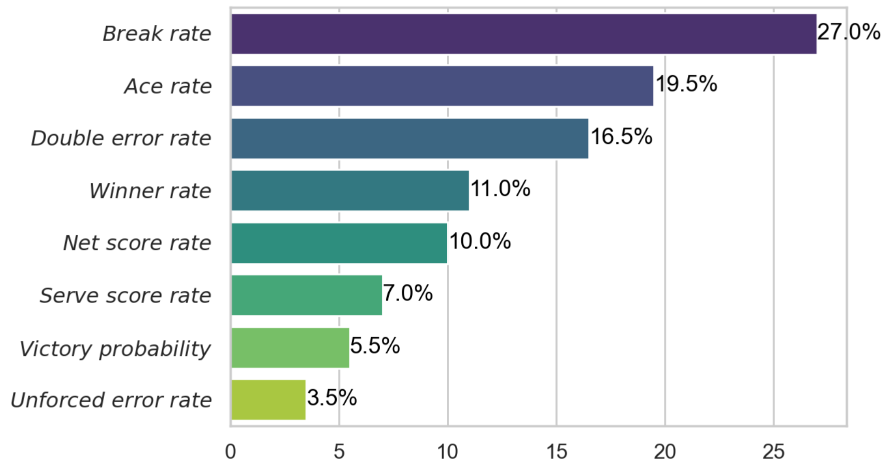

Figure 11: Feature Importance Weight Table

We found that the break point success rate (27.00%) and first serve point win rate (19.50%) have the greatest contribution to predicting the momentum difference. These two features have the strongest influence within the entire model. This suggests that a player's ability to successfully break serve or hit winning serves significantly boosts their morale, thereby enhancing their control over the match and providing a psychological advantage.

F1 Score is the harmonic mean of precision and recall, used to evaluate the overall accuracy of a classifier. Its formula is as follows:

$$F1 = 2 \times \frac{P \times R}{P + R} \tag{14}$$

The F1 score is particularly useful for evaluating classifiers in cases of imbalanced data, as it takes both precision and recall into account, avoiding the bias that may arise from relying on a single metric.

As shown in Table 6, the model has a very high fit on the training data and also achieves favorable results on the test set. Another way to evaluate the model's performance is through the ROC curve, which is assessed by the classifier's True Positive Rate (TPR) (i.e., recall) and False Positive Rate (FPR) (i.e., false positive rate). The ROC curve for the prediction results of the Adaboost regression is shown in Figure 12.

Table 6: model evaluation metrics summary table

	accuracy	recall	precision	F1
training set	0.963	0.963	0.965	0.963
test set	0.917	0.917	0.917	0.916

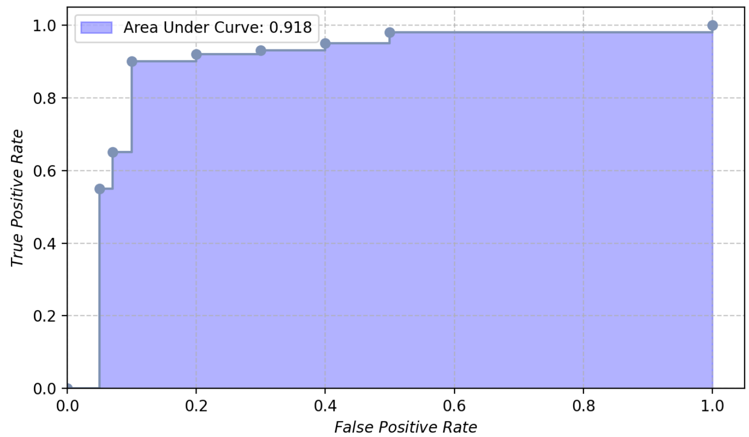

Figure 12: ROC curve

The area under the ROC curve (AUC) is 0.918, which is close to 1, indicating that the model can effectively distinguish between the positive and negative classes, demonstrating strong classification ability. In conclusion, our model exhibits high prediction accuracy and is highly effective in identifying momentum turning points during the match.

6.3.2 Demonstration of Model Results

We still select the final match 1701 and use the model to describe and predict the entire match's progress. The results are shown in Figure 13.

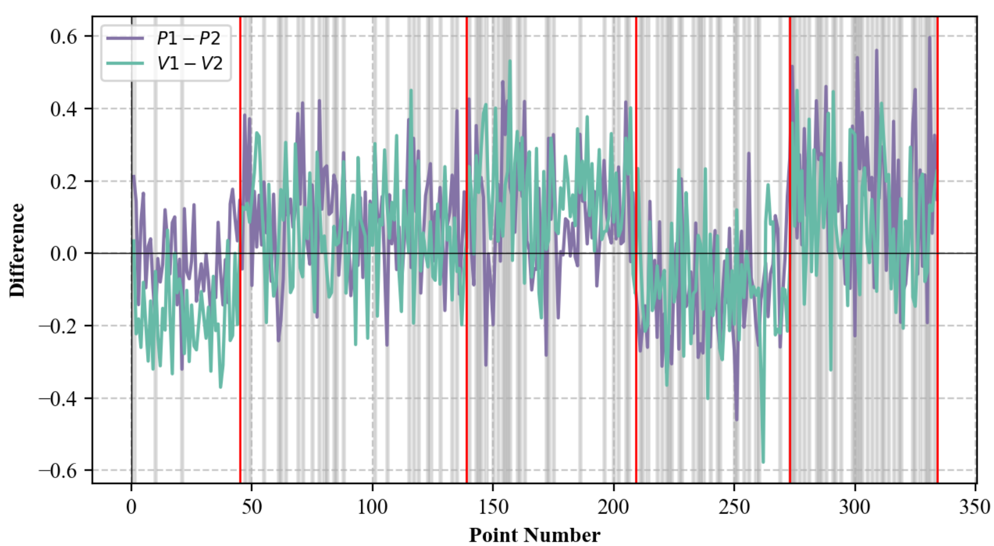

Figure 13: Momentum Difference and Winning Probability Change in the Competition Process

Figure 13 primarily illustrates the changes in momentum difference and winning probability throughout the course of the match. The gray vertical bars represent sections where the absolute value of the momentum difference exceeds 0.18, while the red line indicates the end of each set. It can be observed that the predicted results align closely with the actual match progression. For Player 1 (p1), the model predicts losses in the first and fourth sets, and victories in the second, third, and fifth sets. Additionally, the changes in the difference of winning probabilities generally lag slightly behind the changes in momentum difference. The accuracy of predicting momentum shift points by the model reached 91.45%.

6.4 Suggestions Based on Our Model

Based on the above conclusions and the graphical results, we can propose the following recommendations:

- **Pre-Match Preparation**: Compare momentum changes between different players

  The model provides a comparison of historical momentum changes between different players. When preparing to face a new opponent, coaches should study the opponent's past momentum change patterns. This information can help formulate strategies tailored to exploit the opponent's strengths and weaknesses during the match.

- **In-Match Tactics**: Exploit weaknesses and mental toughness training

  Exploiting Opponent's Weaknesses: If the opponent tends to lose momentum after serving errors, players should focus on improving their return game and looking for break point opportunities. Patience is key, and players should wait for the right moment to capitalize on the opponent's weaknesses.

  Mental Toughness Training: After understanding the predictability of momentum changes, players should train their psychological resilience. The model's results indicate that break points and ACEs have the largest impact on momentum turning points in the match. When facing these unfavorable situations, players should stay calm and continue to focus on the rest of the match, maintaining mental composure.

7 Application of the Model in Women's Tennis Matches

On July 13, 2024, at 13:00 UTC, on Centre Court in London, United Kingdom, Krejčíková B. faced Paolini J. in the Women's Singles Final of the 2024 Wimbledon Championships. The Czech player, the 2021 Roland-Garros singles champion, defeated the Italian Paolini J. with a score of 6-2, 2-6, 6-4, securing her second Grand Slam singles title and her 12th Grand Slam title across all disciplines.

We selected this Wimbledon Women's Singles match as the subject for our model study to verify the accuracy of its application. As in the Problem 1, we used the same 8 indicators to compute the principal components and derived the momentum (data source: https://www.wtatennis.com/). The analysis was performed on a per-set basis, highlighting the performance of both players in terms of attack, defense, and stability, as shown in Figure 14.p1 refers to Krejčíková B. and p2 refers to Paolini J.

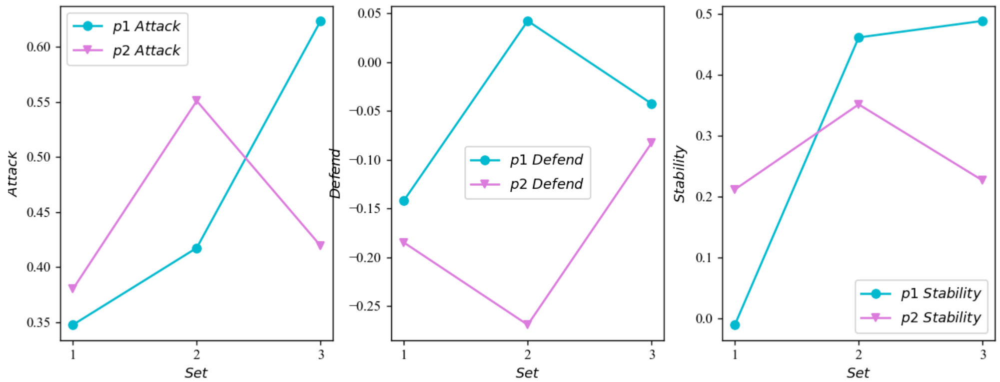

Figure 14: players' scores for attack defense and stability

As shown in Figure 14, Krejčíková B. exhibited stronger defense, while her attack and stability fluctuated more significantly. In terms of actual match performance, her aggressive momentum—such as direct service points, highest consecutive points, break point success rate, and winner point success rate—was higher than Paolini J.'s. However, her double fault rate and unforced error rate were also higher, indicating slightly weaker stability.

For the entire match (which includes 3 sets), the momentum difference and win rate difference were plotted against the score point progression, with the Adaboost classification algorithm from Problem 3 used to predict momentum turning points. The results, as shown in Figure 15, revealed that the Adaboost prediction accuracy for turning points was 88.45%, and the accuracy of predicting the match outcome based on momentum was 87.64%.The red line separates the three sets of the tennis match, and the gray areas indicate regions where momentum turning points are likely to occur.

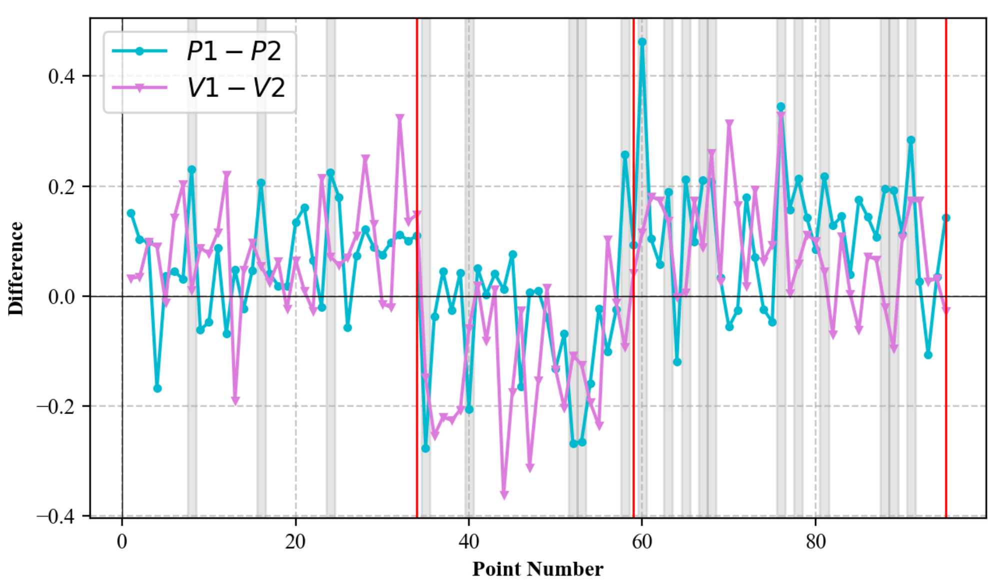

Figure 15: Visualization of the 2024 Wimbledon women's singles final data

As shown in Figure 15, momentum is closely related to win rate. In the first two sets of the match, momentum fluctuated relatively little, while in the final set, momentum experienced more significant fluctuations, resulting in a larger variation in win rate. This illustrates the intensity of the decisive set, where the situation could shift at any moment.

A comparison of Figure 14 and Figure 15 reveals the following analysis:

- Considering gender differences, women generally have lower physical endurance than men. Therefore, a smaller momentum difference threshold could be chosen, and the model could be retrained to more accurately predict momentum turning points in female matches.
- The impact of various abilities on performance in tennis matches may differ between men and women. For instance, in men's matches, attacking power might have a greater influence, whereas in women's matches, stability may play a more significant role. As a result, the weights of the principal components may differ between genders.

For (1), we set the threshold to 0.1 and retrained the model for turning point prediction, resulting in Figure 16 . The Adaboost prediction accuracy for turning points was 90.32%, while the accuracy for predicting the match outcome remained unchanged.

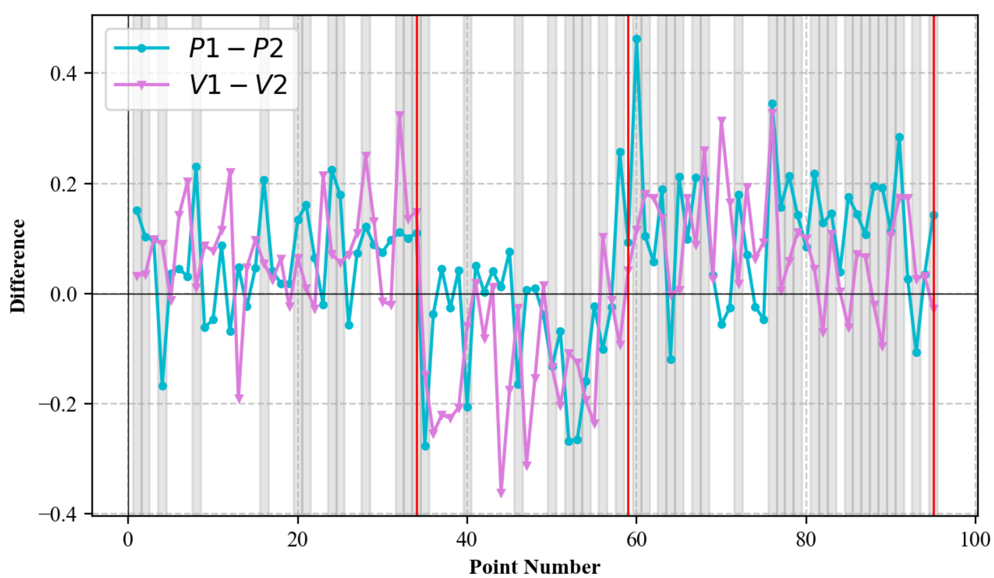

Figure 16: Visualization of the 2024 Wimbledon women's singles final data

For (2), a large dataset of women's singles matches is needed to determine the weights of the three principal components, thereby improving the accuracy of match outcome prediction. Additionally, beyond the 8 indicators mentioned, other factors could be considered, such as the highest consecutive points, which also reflect momentum size.

In summary, to make the model more applicable for predicting turning points and outcomes in tennis matches, we propose the following improvements:

- Introduce gender classification variables and establish a similar model based on women's singles match data.
- Expand the selection of indicators, such as the highest consecutive points, return-of-service points, and groundstroke point-winning rates.
- Select an appropriate momentum difference threshold based on the type of match.

8 Sensitivity Analysis

In Section 6, in the analysis of the actual case, we found that the momentum difference between female athletes is generally smaller than that of male athletes. Therefore, we promptly adjusted the threshold, which improved the accuracy of the model.

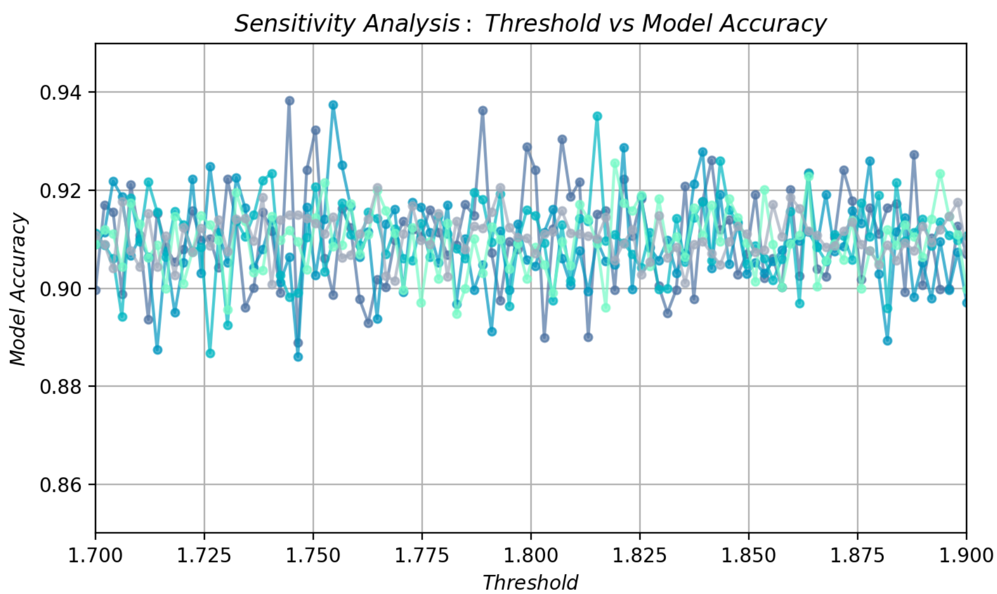

Figure 17: Sensitivity analysis

To test the sensitivity of the model in predicting momentum turning points in the match, we introduced a 10% small perturbation to the threshold α. Then, based on the changed threshold, we recalculated the classification variable P used for the model's classification results, predicted the turning points in the dataset, and re-evaluated the prediction accuracy. The results are shown in the figure below.

The changes in the model's prediction accuracy are not significant. When the threshold is altered, the accuracy fluctuates between 89% and 93%. Additionally, adding noise to the model's indicators does not cause a significant change in the prediction accuracy.

The analysis shows that our model is both highly sensitive and stable, with minimal impact from initial value perturbations. The results of the sensitivity analysis are meaningful and indicate that the model performs consistently under small changes in the threshold.

9 Model Evaluation

9.1 Strengths

- **Scientifically rational metric selection**: The eight selected metrics fully reflect the performance of the players during the match, covering aggressiveness, defense, and stability. These metrics lay a reliable foundation for the momentum model.

- **Effective dimensionality reduction using Principal Component Analysis (PCA)**: The principal components extracted through PCA explain a large portion of the data's variance. Momentum can be quantified by a weighted combination of these three principal components with reasonable weighting coefficients.

- **Innovative use of machine learning to predict turning points**: The use of the AdaBoost algorithm ensures that the prediction accuracy of turning points remains high, maintaining robustness and stability. The model also provides quantitative support for match flow analysis.

- **Strong model scalability**: The model is also applicable to women's singles tennis matches, demonstrating its general applicability and versatility.

9.2 Weaknesses

- **Exclusion of psychological and environmental factors**: The model focuses primarily on objective data metrics and does not take into account psychological factors or the influence of the match environment on player performance.

- **Limited match data**: The model could benefit from a larger dataset for better prediction accuracy and greater generalizability. Further data collection would enhance the model's performance.

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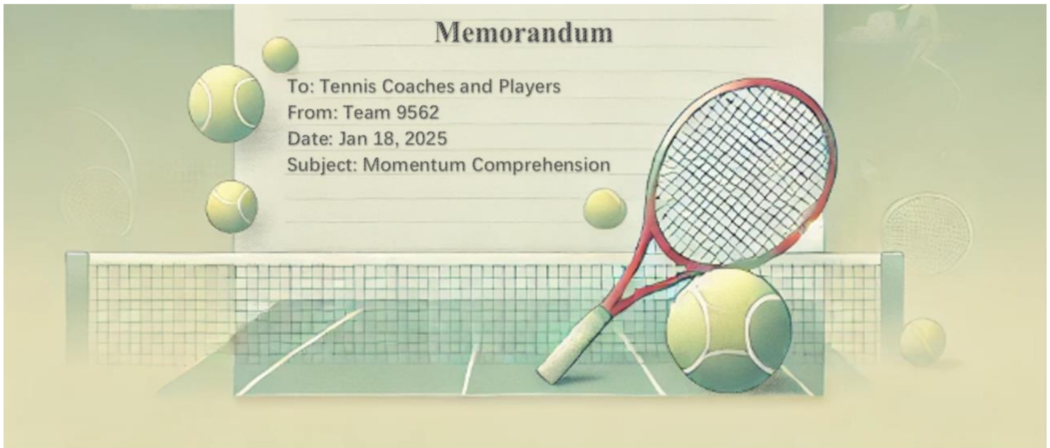

Based on our team’s modeling and analysis, we have summarized a report for you. This may aid in your guidance and training of tennis players. Our report will unfold in three parts.

Firstly, here is **the summary of our results**: Our team used eight indicators such as break score rate and ACE score rate, used principal component analysis (PCA) to reduce the dimensionality of the data, extracted three principal components (aggression, defensiveness, and stability), and finally quantified the "momentum" index naturally. We also established an Adaboost classification model and analyzed the relationship between player-related indicators and whether the momentum of the two sides of the game changed, and found the most important relevant indicators of momentum change. Furthermore, through the Chi-square test, we conclude a strong correlation between the momentum of tennis players and their win percentage.

Secondly, based on our evaluation, we give our recommendations:

- **Analyze your opponent's momentum patterns:** Study your opponent's momentum changes in previous matches, especially after breaks, service errors, or consecutive points. Use this information to develop targeted tactics.
- **Take advantage of the opponent's weakness:** When the opponent makes a double fault or unforced error, the player should seize the opportunity to strengthen the attack and strive for a break or consecutive points.
- **Control the pace of the game:** At key moments (such as break points or tiebreaks), players should maintain momentum by serving and defending steadily, avoiding mistakes due to impatience.

Thirdly, the training strategies: In tennis matches, to enhance a player’s momentum, coaches should have players strengthen their serve training, focusing on variations in speed, placement, and spin while preparing multiple serving strategies to adjust according to the opponent; improve the quality of returns, forcing opponents to make errors and creating break opportunities; cultivate quick decision-making skills, knowing how to exploit the opponent’s weaknesses and continuously pressuring to gain opportunities for consecutive scoring; maintain a positive attitude in the face of errors, not letting mistakes affect confidence; and enhance physical training to improve players’ movement speed and endurance.