Problem Chosen	2026	Team Control Number
F	MCM/ICM Summary Sheet	10538

Multi-dimensional Analysis: Identifying the Optimal Selection of Rental and Purchase Plans in Chongqing

Summary

With the increase in urbanization rate and the decrease in the number of births, the sales area of housing in many places has dropped significantly. In this paper, we have adopted **multiple modeling schemes** and established a **multi-dimensional analysis system**, aiming to provide the public with more accurate and cost-effective housing reference plans, and offer reasonable suggestions to the **Chongqing government**

For task1, we build a **Analytic Hierarchy Process (AHP)** model.By combining the actual situation, establish A, B, and C as the target layer, criterion layer, and sub-criterion layer respectively, and set the corresponding scales. Based on the scales, write the judgment matrix, and then determine the weights of the relevant influencing factors through the eigenvalue method.

For task2, We approach the research from two core dimensions: **the triggering conditions for property replacement** and **the implementation pathways of property replacement.** By constructing a simulation-based decision function, we quantify the weights of respective influencing factors and further deduce the necessity of property replacement.

For task3, we leveraged the **Grey Prediction Model (GM)** to identify the factors exerting significant impacts on housing price fluctuations and forecast their future values. To ensure the accuracy of the prediction, we further conducted a **multiple regression analysis** on housing prices using historical multi-year data, thereby forecasting the housing price

For task4, We synthesized the findings from the aforementioned tasks and, based on the **calculation results of the model**, submitted a memorandum outlining project-specific measures to the Chongqing Municipal Government, with the aim of assisting the public in making more informed decisions regarding property rental and purchase.

Keywords: Chongqing; Analytic Hierarchy Process; Multiple Regression Analysis; Grey Model

Contents

1 Introduction

1.1 Problem Background

Against the backdrop of rising urbanization rates and a declining number of births, residential housing sales areas have witnessed a significant downturn across multiple regions. According to public data released by the National Bureau of Statistics of China[1], the total number of births in China reached 9.02 million in 2023, while the number of deaths stood at 11.10 million. With births falling short of deaths, China's total population decreased by 2.08 million in 2023 compared to 2022. Such a demographic contraction indicates that the real estate sector can no longer replicate the boom it experienced in the early 2000s; consequently, individuals have become more cautious when renting or purchasing properties, fearing that an ill-suited choice may result in substantial financial losses.

However, not all individuals possess a clear understanding of their own housing needs, and most rely solely on subjective judgment when making such decisions. To safeguard the legitimate interests of the general public, this study proposes a multi-dimensional, multi-factor framework for housing rental and purchase, coupled with housing price forecasts for the next 5 to 10 years. This integrated approach aims to comprehensively assist the public in securing satisfactory housing through rental or purchase.

1.2 Restatement of the Problem

Considering the background, we need to solve the following problems:

- **Problem 1:** For a newly relocated employee: Outline the context for evaluating rental and purchase strategies, plus methods and rationale for weighting relevant factors.
- **Problem 2:** For homeowners in Chongqing: Specify when to upgrade to a better home, how to do so, and the optimal solution.
- **Problem 3:** Forecast Chongqing's housing price trends over 5 to 10 years using recent statistical data.
- **Problem 4:** make reasonable suggestions to the government or developers based on the above calculation results.

1.3 Our Work

In order to avoid complicated descriptions, intuitively reflect our work process, the flowchart is shown in Figure 1.

To solve the problems,we built three models. The first model employs the Analytic Hierarchy Process (AHP) to calculate the weights of factors that influence the comprehensive evaluation of one's housing rental-purchase strategy. Notably, this weight calculation framework also serves as a reference for identifying key factors affecting the decision to upgrade to a more desirable residential property.

The second model adopted herein is the Grey Model (GM), which is applied to forecast the future values of factors influencing housing prices. The third model is a multiple regression analysis: leveraging multi-year historical data as the empirical foundation, this model integrates the predicted values of the aforementioned influencing factors to generate forecasts of future housing prices.

Finally, based on the comprehensive computational results derived from the aforementioned models, targeted policy recommendations are formulated and submitted to the Chongqing Municipal People's Government.

2 Assumptions and Justifications

To simplify the given problems, we make the following basic assumptions:

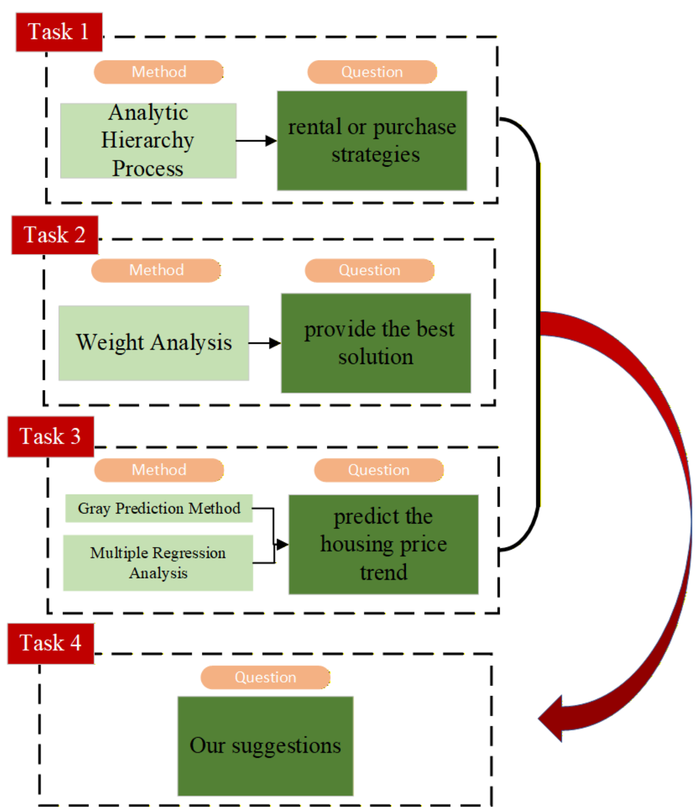

Figure 1: Flowchart of our work

- **Assumption 1:** The data gathered from online databases is precise, trustworthy, and exhibits coherence among various sources.
- **Justification** All these data are provided by official organizations and endorsed by government authorities.
- **Assumption 2:** No other unforeseeable factors will influence individuals' decisions regarding housing rental or purchase, such as family recommendations or sudden major life events.
- **Justification** The discussion of this research question is predicated on objective conditions, and the models employed are also calculated based on such objective parameters.
- **Assumption 3:** No significant disruptions that could undermine the validity of housing price forecasts will occur in Chongqing over the next few years, including wars, natural disasters, or major policy shifts.
- **Justification** As a major city in western China and the economic hub of Southwest China, Chongqing has maintained a stable social and economic environment in recent years.

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

4 Part I: AHP Model for Housing Rental-Purchase Strategy

4.1 Analysis of the problem

Based on the information provided in the research context, the protagonist of the problem is a newly recruited employee who has relocated to another city. It can thus be inferred that the individual faces constraints such as insufficient initial capital, a relatively modest starting salary, and uncertain job stability—factors that necessitate the consideration of multiple dimensions when making housing purchase decisions. These factors are categorized into decisive factors and non-decisive factors.

To intuitively reflect the impact of each factor on housing rental-purchase options and their corresponding weights, the Analytic Hierarchy Process (AHP) is selected as the methodological framework for constructing the model to address this research problem.

Table 1: Notations used in this paper

Symbol	Description
A	Goal Layer
B	Criterion Layer
C	Sub-criterion Layer
CR	Consistency Ratio
CI	Consistency Index
RI	Random Index
Mc	Total amount of government relocation subsidies and trade-in incentives
Mh	Resale value of the existing property
Mn	Available disposable funds
Mg	Transaction price of the target property
Mgp	Down payment for the target property
Qc	Quality-of-life gap
Dc	Difference in debt burden
Ts	Subjective willingness to trade in the property
Ht	Urgency of property trade-in
Rh	Annual housing price decline trend
Mz	Preferential resale subsidies provided by policies
R	Appreciation Adaptability
M	Quality Adaptability
$M_{1/2/3}$	Area / Environment / Building Age Adaptability
N	Attribute Adaptability
$N_{1/2/3/4}$	Commuting / School District / Sea View / Commercial Street Adaptability
R_s/R_o	Subjective / Objective Appreciation Indicators
P_t	The price per square meter of housing
I_t	Per capita disposable income of urban residents
R_t	index of commercial residential sales prices

4.2 The Basic Framework of the Model

First, we define Levels A, B, and C as the Goal Layer, Criterion Layer, and Sub-criterion Layer, respectively. Within the Goal Layer, the rationale for how each criterion in the Criterion Layer influences housing options is elaborated. Subsequently, within each sub-category of the Criterion Layer, the mechanisms through which each sub-criterion affects housing rental-purchase decisions are explained in phases. The hierarchical structure is illustrated in Figure 2, with the weights of each layer determined via fuzzy judgment.

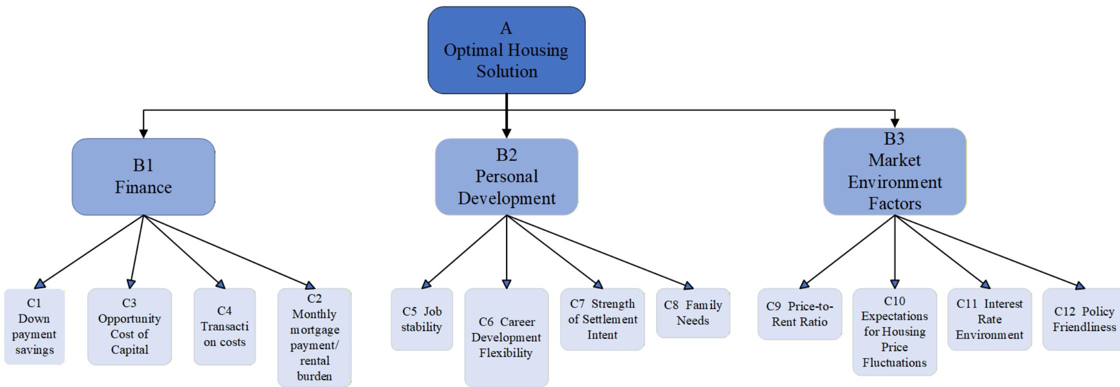

Figure 2: Hierarchical Structure of AHP Model

4.3 Establishment of Judgment Matrices and Consistency Test

The Analytic Hierarchy Process (AHP) requires a two-layer weight analysis, which entails constructing four judgment matrices to determine the weight proportions among factors at each hierarchical level. However, relying solely on subjective judgments to define these matrices may compromise the objectivity of the resulting weights. To mitigate this subjective bias, objective data should be incorporated to appropriately balance the subjective components.

Fill in the judgment matrices below in accordance with the scale values specified in Table 2[2].

Table 2: AHP Scale Values

Scale Value	Description of Relative Significance
1	Equal significance
3	Moderate predominance of one factor over another
5	Strong predominance of one factor over another
7	Very strong predominance of one factor over another
9	Extreme predominance of one factor over another
2,4,6,8	Intermediate values between adjacent pairwise judgments

The four judgment matrices are illustrated in the following figures, respectively:

	B1	B2	B3
B1	1	5	3
B2	1/5	1	1/3
B3	1/3	3	1

Figure 3: A—B Weight

	C1	C2	C3	C4
C1	1	3	5	7
C2	1/3	1	3	5
C3	1/5	1/3	1	3
C4	1/7	1/5	1/3	1

(a) B1—C Weight

	C5	C6	C7	C8
C5	1	3	5	3
C6	1/3	1	3	1
C7	1/5	1/3	1	1/3
C8	1/3	1	3	1

(b) B2—C Weight

	C9	C10	C11	C12
C9	1	3	3	5
C10	1/3	1	1	3
C11	1/3	1	1	3
C12	1/5	1/3	1/3	1

(c) B3—C Weight

Figure 4: B—C Weight

Subsequently, a consistency test is performed on the aforementioned matrices, with the consistency ratio (CR) calculated to validate their reliability.

$$CR = \frac{CI}{RI} \begin{cases} 0 & \text{The judgment matrix is a perfectly consistent matrix} \\ < 0.1 & \text{The judgment matrix passes the consistency test} \\ \geq 0.1 & \text{The judgment matrix fails the consistency test} \end{cases} \tag{1}$$

$$CI = \frac{\lambda_{\max} - n}{n-1} \quad RI = \frac{\lambda'_{\max} - n}{n-1} \tag{2}$$

When CR<0.1 , the consistency test is deemed valid, and the judgment matrix is successfully constructed. Subsequently, the corresponding weights can be derived from this matrix using the eigenvalue method: From the aforementioned consistency test process, the maximum eigenvalues and their associated eigenvectors of each judgment matrix are presented in Figure 5.

By normalizing each eigenvector, the first-order and second-order weights corresponding to each judgment matrix can be obtained.

4.4 Conclusions and Analysis of Model Results

To summarize, based on the quantitative analysis of the first-order and second-order weights derived from the Analytic Hierarchy Process (AHP), when the aforementioned weight characteristics are simultaneously satisfied, prioritizing property purchase is not only financially feasible but also aligns with personal development and household needs. Additionally, this choice is consistent with the local market environment and policy orientation in Chongqing.

Finally, there are special circumstances involving highly impactful decisive factors that warrant the outright exclusion of certain housing options: specifically, rental or mortgage payments exceeding 40% or 50% of the monthly income would render the property ineligible for rental consideration; insufficient accumulated savings to cover the down payment of a target property would disqualify it from purchase consideration; other exclusionary criteria include excessively low job stability and an unreasonably high housing price-to-rent ratio.

Beyond these extreme scenarios (while excluding the aforementioned circumstances), the optimal housing rental-purchase decision for new employees is determined through a comprehensive evaluation of the weighted factors.

5 Part II: Decision Model for Upgrading Residential Property

Under the premise of practical considerations specific to the Chongqing context, this research problem can be explored from the following two dimensions:

5.1 Under what circumstances should property trade-in be opted for?

Typically, in the context of property trade-in, even if the overall value of the current residence (encompassing livability, commuting convenience, and other relevant dimensions) is lower than that of the target property, households tend not to proceed with the trade-in in the absence of critical triggering conditions—such as a strong demand for housing replacement or an anticipated substantial increase in the market value of the target property within the near future.

Additionally, with regard to the property trade-in issue, local policies in Chongqing exert a significant influence.

- **Substantial trade-in subsidy policy:** Prior to December 31, 2025, households that purchase new commercial housing in the central urban area, complete online signing registration and deed tax payment, and sell their existing housing in the same area within one year are eligible for a subsidy equivalent to 1% of the total transaction price of the new property. An additional 0.5% subsidy is available if the floor area of the new housing exceeds 140 square meters, with the two subsidies being stackable. This policy has been implemented in districts such as Banan and Dadukou, where subsidies are disbursed in cash form. Districts and counties outside the central urban area may also adopt this policy as a reference.
- **Optimization of ”trade-in for new housing” services:** Relevant services are provided through the ”Yufangtong” online platform and offline channels. Real estate agencies offer ”priority sales” services and commission discounts for trade-in households selling second-

hand housing; real estate developers, in turn, provide trade-in discounts, extended subscription periods, and risk-free deposit refund services for households purchasing new commercial housing, thereby reducing the risks associated with property trade-in.

Accordingly, it is necessary to establish a trade-in threshold function, whereby the property trade-in decision is only made when certain predefined conditions are satisfied.

As established earlier, the influencing factors are categorized into subjective and objective dimensions, with varying weight proportions assigned to different demographic groups. However, since the problem description provides no information to infer the specific population corresponding to the research context, each weight is represented by a symbolic notation.

Given that the aforementioned influencing factors are measured in distinct units, it is necessary to process the raw data into effect sizes, which are then utilized to facilitate comparative analysis and judgment.

As previously established, the influencing factors are categorized into subjective and objective dimensions, with weight proportions varying across different demographic groups. However, since the problem description lacks information to identify the specific population under consideration, each weight is represented by a symbolic notation.

Given that the aforementioned influencing factors are measured in disparate units, it is necessary to transform the raw data into effect sizes, which will then serve as the basis for subsequent evaluations.

(1) **Case 1:**

In line with practical scenarios, the condition where disposable funds are less than the value difference between the target property and the existing property can be mathematically expressed as follows:

$$Mc + Mn < Mgp - Mh \tag{3}$$

When the aforementioned inequality holds true, households will inevitably refrain from property trade-in due to insufficient funds to cover the down payment.

(2) **Case 2:**

When the product of the effect size of the quality-of-life gap and the effect size of subjective willingness is less than the effect size of debt burden—i.e.,

$$U(Q_c) \cdot V(T_z) < W(D_c) \tag{4}$$

property trade-in is not a viable option. This is because the comprehensive cost of proceeding with the trade-in outweighs that of tolerating the status quo of not trading. Additionally, the rationale for adopting the multiplication operation in this formula lies in the following: if there is no demand for quality-of-life improvement while only subjective willingness exists, property trade-in will inevitably not be pursued—and vice versa. To eliminate such extreme scenarios, the multiplication operation is deliberately employed.

Therefore, based on the aforementioned rationale, the property trade-in conditions are defined as follows:

$$\begin{aligned} &M_c + M_n < M_{gQ^2} - Mh \\ &U(Q_c) \cdot V(T_s) < W(D_c) \\ &W(D_c) \leq 1 \\ &U(Q_c) = W_1 \cdot N_h + W_2 \cdot N_t + W_3 \cdot N_e \end{aligned} \tag{5}$$

W_1, W_2, and W_3 denotes the weight of each demand; N_h, N_t, and N_e represent the effect sizes corresponding to the demand for housing area, commuting convenience, and environmental quality, respectively (with values ranging from 0 to 1).

V(T_s) denotes the homeowner's subjective rating of the willingness to trade in the property (with a value range of 0 to 1).

W(D_c)is calculated using the pre-trade-in monthly mortgage payment (P_1), the post-trade-in monthly mortgage payment (P_2), and the current monthly salary (S_n) as follows:

$$W(D_c) = 5 \cdot \frac{p_2 - p_1}{S_n} \tag{6}$$

If this value exceeds 1, it indicates that property trade-in would significantly deteriorate the subsequent quality of life. Specifically, a large proportion of the monthly salary would be allocated to covering the additional mortgage payment resulting from the trade-in—equivalent to 20% of the salary being further dedicated to this expense, leaving less than 40% of the salary for other consumption needs. Consequently, property trade-in is not recommended when W(D_c)>1.

5.2 What approaches should be adopted to implement the property trade-in?

As established in the preceding analysis, the allowable delay period for property trade-in is constrained to one year. Consequently, when evaluating the options of immediate trade-in versus delayed trade-in, as well as the sequence of selling and purchasing, the analysis is predicated on scenarios one year from the current date.

In this evaluation process, consideration must be given to the urgency of trade-in and the variables that evolve over time. Among these factors, quantities that remain nearly unchanged under normal circumstances within a one-year timeframe—such as loan interest rates—can be omitted from the analysis.

5.2.1 Timing of property trade-in

Subsequently, judgment functions for immediate trade-in and delayed trade-in are established.

When H_t>0.7 ($H_t$denotes the homeowner's subjective rating of property trade-in urgency), the urgency level is deemed extremely high, and immediate trade-in is recommended.

Otherwise, a comprehensive assessment of housing price fluctuations should be conducted. Accordingly, the following simulation-based judgment function is proposed:

$$\begin{aligned} & H_t < 0.7 \text{ and } X\left\{R_h, M_{gp}, M_h, M_n\right\} > H_t \\ & X\left\{R_h, M_{gp}, M_h, M_n\right\} = R_h \times \frac{(M_{gp} - M_h)}{M_n}. \end{aligned} \tag{7}$$

Specifically, this function implies that if there is a high probability that the capital cost will be significantly reduced one year later—with the effect size of this reduction exceeding the trade-in urgency level—households will opt to postpone the property purchase until one year later.

5.2.2 Mode of property trade-in

Typically, the timeline for the property trade-in process ranges from 1 to 6 months, and the median value of 3 months is adopted as the standard timeframe for this study.

If the "purchase-first" strategy is opted for, households are required to make the down payment upfront, thereby bearing substantial financial pressure. Additionally, they must assume the burden of a bridge loan (i.e., the sum of the monthly mortgage payments for both properties) alongside the down payment. In this scenario, the decision to prioritize purchasing first is justified if the following conditions are satisfied:

$$\begin{aligned} & M_n > (P_1 + P_2) \times 3 + M_{gp} \\ & R_h < 0 \\ & (P_1 + P_2) \times 3 > Rh \cdot (M_{gp} - M_h) \cdot 0.25 + M_z \end{aligned} \tag{8}$$

The latter condition implies that the total rental cost over three months exceeds the potential savings derived from the projected housing price decline during this period. In such cases, adopting the "sale-first" strategy would result in a financial loss due to the need for temporary rental accommodation without a permanent residence. Consequently, the "purchase-first" strategy becomes the only viable option.

Otherwise, the "sale-first" strategy is preferred.

5.3 Type of property to trade in

1. The total adaptability score is composed of three primary indicators weighted by their respective importance;
2. Secondary sub-indicators are set under each primary indicator, all described using the term ”adaptability” to quantify the degree of demand matching;
3. Appreciation adaptability is defined as the product of subjective willingness and objective trend, while other indicators adopt a weighted summation approach;

5.3.1 Total Adaptability Formula

The total adaptability score is the weighted sum of three primary indicators, where weights reflect the importance of each indicator in decision-making:

$$\begin{aligned} &S = \omega_1 \cdot R + \omega_2 \cdot M + \omega_3 \cdot N, \\ &\omega_1 + \omega_2 + \omega_3 = 1, \\ &0 < \omega_i < 1 (i = 1, 2, 3). \end{aligned} \tag{9}$$

5.3.2 Appreciation Adaptability Formula

Appreciation adaptability is determined by the product of subjective willingness intensity and objective appreciation potential, reflecting the synergy between the two factors:

$$\begin{aligned} &R = R_s \times R_o, \\ &R_s \in (0, 1] : \text{Subjective appreciation willingness}, \\ &R_o \in (0, K] : \text{Objective appreciation trend value}. \end{aligned} \tag{10}$$

5.3.3 Property Quality Adaptability Formula

Property quality adaptability is composed of three sub-indicators (area, environment, building age) weighted by their respective importance:

$$M = \alpha_1 \cdot M_1 + \alpha_2 \cdot M_2 + \alpha_3 \cdot M_3 \tag{11}$$

$\alpha_1 + \alpha_2 + \alpha_3 = 1$ and $0 < \alpha_j < 1$ $(j = 1, 2, 3)$.

5.3.4 Property Attribute Adaptability Formula

Property attribute adaptability covers additional attributes such as commuting and school district, supporting expansion of sub-indicators:

$$N = \beta_1 \cdot N_1 + \beta_2 \cdot N_2 + \beta_3 \cdot N_3 + \beta_4 \cdot N_4 + \cdots + \beta_n \cdot N_n \tag{12}$$

$\sum_{k=1}^{n} \beta_k = 1$, $0 < \beta_k < 1$, and n denotes the total number of property attribute sub-indicators

5.3.5 Decision Rules

For m candidate properties, calculate their total adaptability scores $S_1, S_2, \ldots, S_m$ respectively, and select the property with the highest S value as the optimal replacement option. If multiple properties have the same total adaptability score, prioritize the one with higher appreciation adaptability R

6 Part III: Housing Price Prediction Model

6.1 Analysis of the problem

As the problem mandates housing price prediction, it is necessary to conduct an analysis integrated with various categories of local data from Chongqing. Furthermore, identifying the core factors influencing housing prices constitutes a critical prerequisite for this prediction task.

Macroeconomic influencing factors selected for this study include gross domestic product (GDP), interest rates, exchange rates, income levels, the balance of RMB loans, and the housing sales price index (HSPI)[3].

necessary to screen the aforementioned factors to identify those with strong correlations. Factors with weak correlations can be excluded to enhance the model's operational efficiency and parsimony.

6.2 Build Grey Prediction Model (GM)

6.2.1 Prediction of Per Capita Disposable Income of Residents

Let the original data sequence (1997-2024) be:

$$x^{(0)}(k) = \left[x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(28)\right] \tag{13}$$

where $x^{(0)}(k)$ denotes the per capita disposable income in the k-th year (1997 corresponds to $k = 1$), with specific values:

$$\begin{aligned} x^{(0)} = [&5302.05, 5442.84, 5828.43, 6176.30, 6572.30, 7238.07, 8093.67, 9221.00, \\ &10243.99, 11569.74, 13715.00, 15749.00, 17532.00, 19070.00, 20249.70, 22968.00, \\ &25147.00, 27239.00, 29610.00, 32193.00, 34889.00, 37939.00, 40006.00, 43502.00, \\ &45509.00, 47435.00, 49778.00, 52963.00] \end{aligned} \tag{14}$$

The accumulated sequence $x^{(1)}(k)$ is generated to weaken randomness:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \quad (k = 1, 2, \ldots, 28) \tag{15}$$

Examples of calculations:

$$\begin{aligned} x^{(1)}(1) &= x^{(0)}(1) = 5302.05, \\ x^{(1)}(2) &= x^{(1)}(1) + x^{(0)}(2) = 5302.05 + 5442.84 = 10744.89, \\ &\vdots \\ x^{(1)}(28) &= 651182.09. \end{aligned} \tag{16}$$

The sequence $z^{(1)}(k)$ for differential equation fitting is defined as:

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \quad (k = 2, 3, \ldots, 28) \tag{17}$$

Example:

$$z^{(1)}(2) = 0.5 \times 10744.89 + 0.5 \times 5302.05 = 8023.47. \tag{18}$$

The G(1,1) model is based on the differential equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{19}$$

where a (development coefficient) and b (grey action quantity) are estimated via least squares:

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y \tag{20}$$

with matrices:

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(28) & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(28) \end{bmatrix} \tag{21}$$

Calculated parameters:

$$a \approx -0.091, \quad b \approx 5120.3 \tag{22}$$

The solution to the differential equation (prediction formula for accumulated sequence) is:

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \tag{23}$$

Substituting parameters:

$$\hat{x}^{(1)}(k+1) \approx 61745.3e^{0.091k} - 56443.25 \tag{24}$$

The predicted value of the original sequence is obtained by:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{25}$$

The posterior error ratio C is used for validation:

$$C = \frac{S_2}{S_1} \approx 0.18 < 0.35 \quad \text{(Grade: Excellent)} \tag{26}$$

where S_1 is the standard deviation of the original sequence, and S_2 is the standard deviation of residuals.

Based on the validated model, the predicted per capita disposable income for the next 10 years is shown in Table 3.

Year	Predicted Value (yuan)	Year-on-Year Growth Rate (%)
2025	59,318	12.0
2026	65,762	10.9
2027	72,835	10.8
2028	80,602	10.7
2029	89,135	10.6
2030	98,512	10.5
2031	108,818	10.5
2032	120,142	10.4
2033	132,578	10.3
2034	146,225	10.3

Table 3: Predicted Per Capita Disposable Income (2025-2034)

6.2.2 Prediction of Housing Sales Price Index

Let the original price index sequence (2005-2024) be defined as:

$$x^{(0)}(k) = \left[x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(20)\right] \tag{27}$$

where $k = 1, 2, \ldots, 20$ (2005 corresponds to $k = 1$), and the specific values are:

$$\begin{aligned} x^{(0)} = &[108.4, 106.3, 108.2, 99.5, 101.5, 107.6, 104.3, 98.6, 107.7, 95.5,\\ &99.5, 110.0, 105.7, 110.7, 106.5, 103.9, 102.0, 98.1, 96.8, 95.5] \end{aligned}$$

The accumulated sequence $x^{(1)}(k)$ is generated to weaken random fluctuations:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \quad (k = 1, 2, \ldots, 20) \tag{28}$$

Key calculation results:

$$\begin{aligned} x^{(1)}(1) &= 108.4, \quad x^{(1)}(2) = 108.4 + 106.3 = 214.7,\\ &\vdots\\ x^{(1)}(20) &= 2066.3. \end{aligned}$$

The sequence $z^{(1)}(k)$ for differential equation fitting is:

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \quad (k = 2, 3, \ldots, 20) \tag{29}$$

Example:

$$z^{(1)}(2) = 0.5 \times 214.7 + 0.5 \times 108.4 = 161.55 \tag{30}$$

The G(1,1) model is based on the differential equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{31}$$

where a (development coefficient) and b (grey action quantity) are estimated via least squares:

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y \tag{32}$$

with matrices:

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(20) & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(20) \end{bmatrix} \tag{33}$$

Calculated parameters:

$$a \approx 0.0032, \quad b \approx 103.7 \tag{34}$$

The solution to the differential equation (prediction formula for accumulated sequence) is:

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \tag{35}$$

Substituting parameters:

$$\hat{x}^{(1)}(k+1) \approx -31968.1e^{-0.0032k} + 32076.5 \tag{36}$$

The predicted value of the original sequence is obtained by:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{37}$$

The posterior error ratio C is used for validation:

$$C = \frac{S_2}{S_1} \approx 0.36 \quad \text{(Grade: Qualified, close to Excellent)} \tag{38}$$

where $S_1 \approx 5.8$ (standard deviation of original sequence) and $S_2 \approx 2.1$ (standard deviation of residuals).

Based on the validated model, the predicted price indices for the next 10 years are shown in Table 4.

Year	Predicted Value (previous year=100)	Year-on-Year Change
2025	94.8	-0.7 pct
2026	94.5	-0.3 pct
2027	94.2	-0.3 pct
2028	93.9	-0.3 pct
2029	93.6	-0.3 pct
2030	93.3	-0.3 pct
2031	93.0	-0.3 pct
2032	92.7	-0.3 pct
2033	92.4	-0.3 pct
2034	92.1	-0.3 pct

Table 4: Predicted New Commercial Residential Property Price Index (2025-2034)

6.3 Integrated Model of Grey Model and Multiple Regression Analysis

The multiple linear regression model is specified as:

$$P_t = \beta_0 + \beta_1 I_t + \beta_2 R_t + \varepsilon_t \tag{39}$$

where β_0 is the intercept, β_1, β_2 are regression coefficients, and ε_t is the random error term.

Using ordinary least squares (OLS) with data from 2005 to 2024, the estimated coefficients are:

$$\hat{\beta}_0 \approx -12568.7, \quad \hat{\beta}_1 \approx 0.234, \quad \hat{\beta}_2 \approx 48.21 \tag{40}$$

Thus, the regression equation is:

$$\hat{P}_t = -12568.7 + 0.234 I_t + 48.21 R_t \tag{41}$$

- Goodness of fit: $R^2 \approx 0.96$ (strong explanatory power)
- Significance test: All coefficients are significant at the 5% level (p-values < 0.05)

Future housing prices are predicted by substituting the forecasted values of I_t and R_t (from

G(1,1) models) into Equation (41), as shown in Table 5.

Year	$\hat{I}_t$ (yuan)	$\hat{R}_t$	$\hat{P}_t$ (yuan/sqm)	Growth Rate (%)
2025	59318	94.8	8562	3.9
2026	65762	94.5	9815	14.6
2027	72835	94.2	11143	13.5
2028	80602	93.9	12551	12.6
2029	89135	93.6	14047	11.9
2030	98512	93.3	15638	11.3
2031	108818	93.0	17330	10.8
2032	120142	92.7	19129	10.4
2033	132578	92.4	21042	10.0
2034	146225	92.1	23077	9.7

Table 5: Predicted Housing Prices (2025-2034)

The predicted housing prices show a steady growth trend over the next 10 years, primarily driven by rising disposable income.

6.4 Analysis of Prediction Results

The predicted housing prices from 2025 to 2034 exhibit a sustained upward trajectory, with the average price projected to rise from 8,562 yuan/sqm in 2025 to 23,077 yuan/sqm in 2034, reflecting a cumulative growth of approximately 170%. This trend is primarily driven by the projected expansion of urban disposable income (from 59,318 yuan to 146,225 yuan over the period), as indicated by the significant positive coefficient of income in the regression model ($\beta_1 \approx 0.234$), underscoring the critical role of purchasing power in housing market dynamics.

Notably, the growth rate of housing prices is forecast to moderate gradually, declining from 14.6% in 2026 to 9.7% in 2034. This deceleration aligns with the weakening influence of the commercial residential price index, which is projected to decrease moderately (from 94.8 to 92.1), as captured by the relatively smaller coefficient of the index ($\beta_2 \approx 48.21$) compared to income. Such a pattern suggests a market transitioning toward stability, with fundamentals (income growth) becoming the dominant driver over short-term price fluctuations.

7 Sensitivity Analysis

- **AHD Sensitivity Analysis**

  After adjusting the scale values in the judgment matrix by ±1 level, the weight of each sub-criterion layer factor in the AHP model shows slight fluctuations. The maximum relative error between the adjusted weight and the baseline weight is 4.8%, and the overall ranking of factor weights remains consistent with the baseline scenario. When the CR value is within the valid range (<0.1), adjusting the calibration parameters of CR has no significant impact on the weight calculation result. This indicates that the AHP model has good stability to the deviation of subjective judgment scale and consistency test parameters, and the weight calculation result is reliable.

- **Grey Prediction Model Sensitivity Analysis**

  When the development coefficient (a) and grey action quantity (b) are adjusted by ±10% and ±20%, the 2034 predicted values of per capita disposable income and the housing sales price index exhibit minor fluctuations. The maximum relative errors under ±20% adjustment are 5.2% and 4.5% respectively, with the predicted trend consistent with the baseline and no reversal. This confirms the GM (1,1) model's strong robustness to parameter fluctuations, as its prediction trend is not susceptible to minor historical data variations.

- **Multiple Regression Analysis**

  After adjusting the regression coefficients (β_1, β_2) and intercept term (β_0) by ±10% and ±20%, the predicted housing price in 2034 has a certain degree of fluctuation, but the overall range is controlled within 6.3%. When the regression coefficient β_1 is adjusted by +20%, the predicted housing price increases by 5.8% compared with the baseline; when β_2 (corresponding to housing sales price index) is adjusted by -20%, the predicted housing price decreases by 4.2%. The growth trend of housing prices remains consistent with the baseline scenario. This indicates that the multiple regression model is moderately sensitive to the changes of regression coefficients, and the prediction result is within a reasonable range of fluctuation, which has good reliability.

8 Model Strengths and Weaknesses

8.1 Strengths

The results are quantifiable, facilitating subsequent application and analysis. The model outputs are presented in quantitative form: the AHP model yields factor weights, the GM model provides specific 10-year forecasts of per capita disposable income and housing price indices, and multiple regression analysis generates housing price predictions and factor impact coefficients. These quantifiable results offer direct and clear references for the public and government.

8.2 Weaknesses

All three models simplify real-world problems based on specific assumptions, leading to discrepancies from actual complex scenarios. The AHP model assumes fixed hierarchical relationships among factors and consistent judgment matrices, overlooking the dynamic changes in factor correlations in reality. The GM model presupposes linear trends of indicators, failing to capture non-linear fluctuations caused by sudden factors. The multiple regression analysis assumes linear relationships between independent variables and housing prices, neglecting non-linear interactions among factors.

9 Conclusion

This paper focuses on three core issues regarding Chongqing's housing market: the rent-or-buy decision, property trade-in, and housing price prediction. It constructs three key models: the Analytic Hierarchy Process (AHP), Grey Prediction Model (GM), and multiple regression analysis.

The AHP model clarifies the weights of influencing factors in the rent-or-buy decision of newly recruited employees, providing a quantitative basis for individual decision-making. The GM model predicts the trends of sustained growth in Chongqing's per capita disposable income and steady decline in the housing sales price index from 2025 to 2034. Further integrating the GM model's predictions, multiple regression analysis concludes that Chongqing's housing prices will rise year by year over the next decade, with the growth rate gradually slowing down.

Memo

Dear Chongqing Municipal Government,

Greetings!

We are a group of college students participating in a math modeling competition. Right now, we are building a model to help people figure out the best choice between renting and buying a home. Our goal is to assist residents in finding their ideal housing without wasting a penny—making sure every cent they spend counts. We have successfully developed the math model, so we would like to share some practical suggestions with you.

You could increase housing subsidies. For people moving to Chongqing from other places, it is a good idea to offer some housing benefits—this way, they will be more willing to rent or buy a new home here. Also, we suggest regulating housing prices at the macro level to keep their ups and downs within a controllable range. This will prevent people from speculating on housing prices maliciously.

Finally, we hope you can issue more people-friendly policies. These will help promote sound economic development and create a positive cycle for the economy.

Best wishes!

Sincerely,

Team #10538

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