Homework 9 411021348 AM

Question 1

$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$, let $f(x) = \frac{1}{x(\ln x)^3}$

$\int \frac{1}{x(\ln x)^3}\,dx$ let $u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \int \frac{1}{u^3}\,du = -\frac{1}{2u^2}$ by power rule $= \frac{-1}{2\ln^2 x} + C$

$\therefore \int_2^{\infty} \frac{1}{x(\ln x)^3} = \lim_{b\to\infty} \left[\frac{-1}{2\ln^2(x)}\right]_2^b$

$\Rightarrow \lim_{b\to\infty} \left[\frac{-1}{2\ln^2(b)} + \frac{1}{2\ln^2(2)}\right] = 0 + \frac{1}{2\ln^2(2)} = \frac{1}{2\ln^2(2)}$

$\therefore \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$ converges

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Question 2 -

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$$ let $f(x) = \dfrac{1}{x\ln(x)}$ continous, positive, decreasing $[2,\infty]$

$$\int_{2}^{\infty} \frac{1}{n\ln(x)}\,dx \quad , \text{let } u = \ln(x), \quad du = \frac{1}{x}\,dx \quad , \quad x=2,\ u=\ln 2 \quad ; \quad x=b,\ u=\ln b$$

$$= \lim_{b\to\infty} \int_{2}^{b} \frac{1}{x\ln(x)}\,dx = \lim_{b\to\infty} \int_{\ln 2}^{\ln b} \frac{1}{u}\,du$$

$$= \lim_{b\to\infty} \Big[\ln(u)\Big]_{\ln(2)}^{\ln(b)} = \lim_{b\to\infty} \Big[\ln|\ln b| - \ln|\ln 2|\Big] = \infty$$

$\therefore$ Integral test $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ diverges.

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Question 3

a) $\sum_{n=2}^{\infty} \frac{n^2+n}{n^3-2}$ $\sum_{n=2}^{\infty} \frac{1}{n}$ for $[2, \infty)$

$a_n \left\{ \frac{6}{6} + \frac{12}{25} + \frac{20}{62} \cdots + \frac{n^2+n}{n^3-2} \right\}$

$b_n \left\{ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \cdots + \frac{1}{n} \right\}$

So $\frac{n^2+n}{n^3-2} > \frac{1}{n}$ for $[2, \infty)$

$a_n > b_n > 0$ for $[2, \infty)$

$\sum_{n=2}^{\infty} \frac{1}{n}$ is a harmonic series that diverges.

By comparison theorem, if $\sum b_n$ diverges then $\sum a_n$ diverges.

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b) $\sum_{n=2}^{\infty} \frac{n^2-n}{n^3+2}$, $\sum_{n=2}^{\infty} \frac{1}{n}$

Using limit comparison Test: $L = \lim_{n\to\infty} \frac{a_n}{b_n}$

$= \lim_{n\to\infty} \left[\frac{\frac{n^2-n}{n^3+2}}{\frac{1}{n}} \right] = \lim_{n\to\infty} \frac{n^3-n^2}{n^3+2} = 1$ ~~by hi~~

Since $0 < L < \infty$ and $\sum b_k$ is a harmonous series that diverges, then a_k diverges.

diverges #

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Question 4

$$\sum_{n=1}^{\infty} \frac{n^2 + \cos^2 n}{n^3}, \quad -1 \leq \cos(n) \leq 1, \quad 0 \leq \cos^2(n) \leq 1$$

So, $\frac{n^2 + \cos^2 n}{n^3} \geqslant \frac{n^2 + 0}{n^3} = \frac{n^2}{n^3} = \frac{1}{n}$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

a_n $\qquad\qquad\qquad\qquad$ b_n

$a_n \geqslant b_n > 0$

$\sum b_n$ is divergent as $\sum_{n=1}^{\infty} \frac{1}{n}$ is a harmonic series

It then follows that $\sum a_n$ is divergent.