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Homework 10 Part B

No 1

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^4}{4^n}$$

$$|a_n| = \left| (-1)^n \frac{n^4}{4^n} \right| = \frac{n^4}{4n}, \; |a_{n+1}| = \left| (-1) \frac{(n+1)^4}{4^{n+1}} \right.$$

$$= \frac{(n+1)^4}{4^{n+1}}$$

$$\lim_{n\to\infty} \frac{(n+1)^4}{n^4} \cdot \frac{4^n}{4^{n+1}} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^4 \cdot \frac{1}{4}$$

$$= \frac{1}{4} \cdot \left(\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)\right) = \frac{1}{4} \cdot (1+0) = \frac{1}{4} < 0$$

2) By ratio test, the series is Asbolutely convergent.

No 2.

$$\sum_{n=1}^{\infty} n^2 e^{n^3} = \sum_{n=1}^{\infty} \frac{n^2}{e^{n^3}}, \quad |a_n| = \left|\frac{n^2}{e^{n^3}}\right| \text{ and } |a_{n+1}| = \frac{(n+1)^2}{e^{(n+1)^3}}$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^2}{e^{(n+1)^3}} \cdot \frac{e^{n^3}}{n^2} \right| = \lim_{n\to\infty} \frac{(n+1)^2 \cdot e^{n^3-(n+1)^3}}{n^2}$$

$$= \lim_{n\to\infty} \left(\frac{(n^2+2n+1)\, e^{n^3-(n^3+3n^2+3n+1)}}{n^2} \right) = \lim_{n\to\infty} \left(\frac{(n^2+2n+1)e^{-3n^2-3n-1}}{n^2} \right)$$

$$= \lim_{n\to\infty} \left(\frac{\cancel{n^2} e^{-3n^2-3n-1}}{\cancel{n^2}} \right) + \lim_{n\to\infty} \left(\frac{2n e^{-3n^2-3n-1}}{n^2} \right) + \lim_{n\to\infty} \left(\frac{e^{-3n^2-3n-1}}{n^2} \right)$$

$0 + 0 + 0 \therefore L < 0$ Converges by Ratio test.

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Question 3:

$$\sum_{K=1}^{\infty} \frac{1}{K\sqrt{K^2+1}}$$, Apply limit comparison test.

$$L = \lim_{K \to \infty} \left(\frac{\frac{1}{K\sqrt{K^2+1}}}{\frac{1}{K^2}} \right) = \lim_{K \to \infty} \frac{K^2}{K\sqrt{K^2+1}} = \lim_{K \to \infty} \frac{K}{\sqrt{K^2+1}}$$

Factor out K:

$$\therefore \lim_{K \to \infty} \frac{K}{K\sqrt{1+K^2}} = \lim_{K \to \infty} \frac{1}{\sqrt{1+\frac{1}{K^2}}} = \frac{1}{\sqrt{1+1}} = 1$$

L is finite and greater than 0

$\Rightarrow$ Converges

Question 4

$$\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

Apply Series Ratio test: $\left| \dfrac{\frac{3^{(n+1)}(n+1)^2}{(n+1)!}}{\frac{3^n n^2}{n!}} \right| = \left| \dfrac{3^{(n+1)}(n+1)^2 n!}{3^n n^2 n!(n+1)} \right|$

$$= \left| \frac{3(n+1)}{n^2} \right|$$

$$\lim_{n\to\infty} \left| \frac{3(n+1)}{n^2} \right| = 3 \cdot \lim_{n\to\infty} \frac{(n+1)}{n^2} = 3 \cdot \lim_{n\to\infty} \frac{n}{n^2} + \lim_{n\to\infty} \frac{1}{n^2}$$

$= 3 \cdot (0+0) = 0 \;\therefore\; L < 1$ by Ratior test

converges

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Question 5

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}} = \text{Root test.}$$

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(n!)^n}{n^{4n}}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{(n!)^n}{n^{4n}}} = \lim_{n\to\infty} \frac{n!}{n^4} = \infty$$

$$\left(\dots + a_7 + a_8 + a_9 + \dots \right.$$

$$\left. \dots + \frac{5040}{2401} + \frac{40320}{4096} + \frac{362880}{6561} + \dots \right)$$

Diverges by Root test