

Homeork1

Kaymon McCain

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1. Question:

- (a) $\frac{35}{5} = 7$
 $T(n) = C_{op}C(n)$
 $C(n) = \frac{1}{3}n^3$
 $\frac{T(7n)}{T(n)} \approx \frac{C_{op}(7n)}{C_{op}C(n)} \approx \frac{\frac{1}{3}(7n)^3}{\frac{1}{3}n^3} \approx 343 \text{ times}$
- (b) $T_{old}(n_1) \approx \frac{1}{3}n_1^3$
 $T_{new}(n_2) \approx \frac{\frac{1}{3}n_2^3}{25}$
 $\sqrt{25} \approx \frac{n_2}{n_1}$

2. Question:

- (a) they are equal growth $n^2 + n = 2000n^2$
(b) $100n^2 < 0.01n^3$
(c) $\log_2(n) = \ln(n)$
(d) $\log_2^2(n) > \log_2(n)^2$
(e) $2^{n-1} = 2^n$
(f) $(n-1)! = n!$

3. Question:

- (a) $x(n-1) + 3 = x(n)$
 $x(n-2) + 3 + 3\ldots$
 $x(n-k) + 3k$
 $n-k=1 \quad k=n-1$
 $3(n-1) + 3 = x(n)$
 $O(n)$
- (b) $4x(n-1) + 7 = x(n)$
 $3[3x(n-2)] + 7 + 7\ldots$
 $3^k x(n-k) + 7k$
 $n-k=0 \quad n=k$
 $3^n 8 + 7n$
 $O(3^n)$

$$\begin{aligned}
(c) \quad & x(n-1) + n^2 = x(n) \\
& x(x-2) + n^2 + (n-1)^2 \\
& x(x-3) + n^2 + (n-2) + (n-1)^2 \dots \\
& x(n-k) + \sum_{j=0}^{k-1} (n-j)^2 \\
& n-k=0 \quad n=k \\
& 9 + n * n^2 \\
& O(n^3)
\end{aligned}$$

$$\begin{aligned}
(d) \quad & x(n) = x\left(\frac{n}{3}\right) + n \\
& x(n) = x\left(\frac{n}{3^2}\right) + \frac{n}{3} + n \\
& x(n) = x\left(\frac{n}{3^3}\right) + \frac{n}{3^2} + \frac{n}{3} + n \dots \\
& x(n) = x\left(\frac{n}{3^k}\right) + \sum_{j=0}^{k-1} \left(\frac{n}{3^{k-j}}\right) (3^k) \\
& x(n) = 8 + 3^{k+1} \\
& x(n) = 8 + 3(3)^k \\
& x(n) = 8 + 3n \\
& O(n)
\end{aligned}$$

$$\begin{aligned}
(e) \quad & x(n) = x\left(\frac{n}{7}\right) + 5 \\
& x(n) = x\left(\frac{n}{7^2}\right) + 5 + 5 \\
& x(n) = x\left(\frac{n}{7^3}\right) + 5 + 5 + 5 \dots \\
& x(n) = x\left(\frac{n}{7^k}\right) + 5k \\
& x(n) = 9 + 5 \log_3(n) \\
& O(\log_3(n))
\end{aligned}$$

4. Question:

- (a) This algorithm starts with an empty array called count[] and my A[] = 60,35,81,98,14,47. This algorithm compares on the if statement $A[i] < A[j]$. By using this kind of if statement we will have counts first pass through the loop as count[i] = 3,0,1,1,0,0. Basically count is only storing the index which the sorted array needs so each integer will be placed where it belongs. In the end of the loop count will be count[] = 3,1,4,5,0,2. Finally, the sorted array S[] will be sorted based on count[] list of index.
- (b) No it is not stable. If we had integers that equal it wouldn't work.
- (c) No because there are two arrays.

5. Question:

$$5 \lg(n+100)^{10}, \ln^2(n), \sqrt[3]{n}, 0.001n^4 + 3n^3 + 1, 3^n, 2^{2n}, (n-2)!$$

6. Question:

- (a) 2 for the first question, and 12 for the second question.
- (b) It will be $\frac{1}{9}$ for the worst case because $\binom{10}{2} = 45$ and you have 5 best case. For the average case $\frac{3}{9}$