

CHAPTER-2CONCEPT LEARNING & THE GENERAL-TO-SPECIFIC ORDERING

concept learning ~~task~~ means inferring a boolean valued func. from training examples of its i/p & o/p.

of concept learning Task:

- Consider an example where a friend enjoys water sport.

concept learning task is "Enjoy Sport".

example	Sky	Air Temp	Humidity	Wind	Water	Forecast	Enjoy Sport
1.	sunny	warm	normal	strong	warm	same	yes
2.	sunny	warm	high	strong	warm	same	yes
3.	rainy	cold	high	strong	warm	change	no
4.	sunny	warm	high	strong	cool	change	yes

Table: 2.1

- The "Attribute" - Enjoy sport rep. whether the person enjoys water sport that day or not.
- Task: To learn to predict "enjoysport" of an arbitrary day based on other values of attributes.
- Hypothesis repn: - Conjunction of constraints on the instance attributes.
- So Hypothesis is a vector having 6 constraints here, i.e., sky, Airtemp, humidity, wind, water & forecast.
 - For each attribute, hypothesis will be:
 - (i) "?" = Any value is acceptable for this attribute ex:- warm.
 - (ii) Specify a single reqd. value ex:- ?
 - (iii) "∅" - No value is acceptable.

- Positive Example: - If an instance x satisfies all constraints of hypothesis 'h' then 'h' classifies x as +ve example $\Rightarrow h(x) = 1$.
- ex: ① (?, cold, High, ?, ?, ?) \Rightarrow on cold day with high humidity
- ② (?, ?, ?, ?, ?, ?) \Rightarrow enjoy sports every day \Rightarrow every day +ve
- ③ (∅, ∅, ∅, ∅, ∅, ∅) \Rightarrow No day is a positive example

→ The "Enjoy Sport" Concept Learning Task:-

- Given:-
Instances X : possible days, each described by attributes
 1. Sky (sunny, cloudy & rainy)
 2. Air Temp (warm & cold)
 3. Humidity (Normal & High)
 4. Wind (strong & weak)
 5. Water (warm & cool)
 6. Forecast (Same & Change)

Hypothesis H : Each hypothesis is described by constraint
on attributes.

"?" → any value acceptable

"x" → no value acceptable.
specific value

Target concept C : Enjoy Sport: $X \rightarrow \{0, 1\}$

Training Examples D : +ve & -ve examples of Target func.

Determine:-

A hypothesis "h" in H such that

$$h(x) = C(x) \quad \forall x \in X$$

→ Notation:-

- Instances: set of items over which the concept is defined
is called set of instances, X .

- Target Concept: The concept or func. to be learned is
called Target Concept, C .
It can be any boolean valued func.
defined over the instances X .

$$C: X \rightarrow \{0, 1\}$$

Ex:- In "enjoy sport" concept, if
 $C(x)=1 \Rightarrow$ if Enjoy Sport = Yes
 & $C(x)=0 \quad \text{if} \quad \text{No.}$

- Training examples :- when learning a target concept a learner is provided a set of training examples. They consist of an instance x from X , along with its target concept value $c(x)$.
 - If $c(x) = 1 \Rightarrow$ Positive Training example,
 - If $c(x) = 0 \Rightarrow$ Negative "
 - It is written in ordered pairs $(x, c(x))$ for defining a training example.
Here, x = instance & $c(x)$ = Target Concept Value
 - The set of available training set examples are denoted by "D".

- All possible Hypotheses :- - Denoted by H
 - Given a target concept "c"
 - Using a set of Training examples, the learner must hypothesize or estimate "c".
 - So, the set of all possible hypotheses is H in order to identify of target concept.
 - Each hypothesis in set " H " is denoted by " h ". which is a boolean value defined over X .
 $\Rightarrow h: X \rightarrow \{0, 1\}$

- The goal of the learner is to find a hypothesis ' h' such that
$$h(x) = c(x) \forall x \in X$$

→ The Inductive Learning Hypothesis :-

It is a hypothesis which is found to approximate target func. well, over a sufficiently large set of training examples & it will also approximate the target func. well, over other unobserved examples.

Concept learning as search:

(15)

- Concept learning is like a task of searching through a large space of hypotheses (defined by hypothesis repn)
 - Here, goal is to find hypothesis, which fits the best to the training examples.
- ex: - In "Enjoy Sport" learning task, H = Hypotheses set
 X = Instances set

The Attribute Sky has 3 possible values & Remaining Air Temp, Humidity, Wind, Water & forecast has 2 possible values.

Thus there are $3 \times 2 \times 2 \times 2 \times 2 \times 2 = 96$ instances in X .

- coming to Hypotheses there are $5 \times 4 \times 4 \times 4 \times 4 \times 4 = 5120$

"Syntactically distinct Hypotheses" in H .

- Every hypotheses with one or more \emptyset symbol rep. empty set of instance \Rightarrow instance is -ve.
- Thus now "Semantically distinct Hypotheses" are

$$\text{only } 1 + (4 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = 973$$

General-to-Specific Ordering of Hypotheses:

Def: - Let h_j and h_k be boolean valued func., defined over X . Then ' h_j ' is "more general than- or equal" to h_k ($h_j \geq_g h_k$) iff

$$(\forall x \in X) [(h_k(x) = 1) \rightarrow (h_j(x) = 1)]$$

ex: - $h_1 = \langle \text{sunny}, ?, ?, \text{strong}, ?, ? \rangle$

$h_2 = \langle \text{sunny}, ?, ?, ?, ?, ?, ? \rangle$

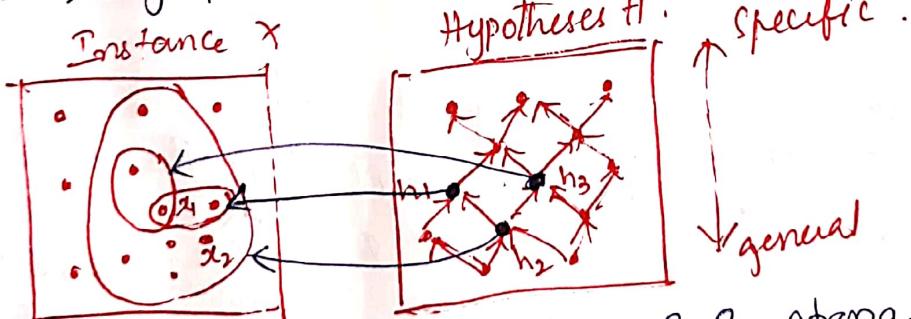
Consider set of instances classified as +ve by h_1 & h_2

As seen, any instance classified by h_1 will also be classified +ve by h_2 (B)
 $\Rightarrow h_2$ is more general than h_1
 $(\because h_2$ imposes fewer constraints on instance, also, it classifies more instances to be +ve)

- In general, for any instance "x" in the set of instances X and any hypothesis "h" in set of hypotheses H , we say, x satisfies h iff $h(x) = 1$.
- Given 2 hypotheses h_k & h_j , if any ^{instance} satisfying h_k also satisfies h_j then h_j is more general or equal to h_k .
- strictly, If $(h_j \geq g h_k) \wedge (h_k \not\geq g h_j)$ then h_j is more general than h_k . (or) ~~but~~
 h_k is more specific than h_j . (Inverse)

→ Example, Assume 3 hypotheses h_1 , h_2 and h_3 from "enjoy sport" example.

~~Also $h_2 \geq g h_1$ since, every instance~~



$$x_1 = \langle \text{sunny, warm, high, strong, cool, same} \rangle \quad h_1 = \langle \text{sunny, ?, ?, ?, strong, ?, ?} \rangle$$

$$x_2 = \langle \text{sunny, warm, high, light, warm, same} \rangle \quad h_2 = \langle \text{sunny, ?, ?, ?, ?, ?, ?} \rangle$$

$$h_3 = \langle \text{sunny, ?, ?, ?, cool, ?} \rangle$$

- Here we see that, $h_2 \geq g h_1$, because every instance that satisfies h_1 also satisfies h_2 ; $h_2 \not\geq g h_3$
 $\Rightarrow \geq g$ relⁿ depends only on instances & not on the classifying them.

- A \Rightarrow_{Rel} defines a partial order on hypothesis space (Rel is Reflexive, Antisymmetric & Transitive).
- The structure is partial order \Rightarrow there may be pairs of hypotheses such as $h_1 \& h_2$ such that $h_1 \not\geq h_2$ and $h_2 \not\geq h_1$.

Fund-S: finding a Maximally Specific Hypothesis:

- It is an appl'n of more general than partial order.
- Here, we begin with most specific possible hypothesis in H & generalize it; when it fails to cover an observed +ve Training example (classifies +ve).

Fund-S Algorithm: -

1. Initialise h to most specific hypothesis in H .
2. For each +ve Training instance x .
 - For each attribute constraint a_i in h , if constraint a_i is satisfied by x then do nothing
 - else Replace a_i in h with next more general constraint satisfied by?
3. O/p hypothesis h .

Example: -

- consider "Enjoy Sport task".

- (i) Initialise h to most specific hypothesis in H .

$$h \leftarrow \{\emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset\}.$$

- (ii) From table 2.1, none of constraints are satisfied by this so it is replaced by next more general constraint that fits this example.

$$h \leftarrow \{\text{Sunny, Warm, Normal, Strong, Warm, Same}\}$$

[It's less specific b'coz all instances are -ve except one +ve Training example, (See Table 2.1 to relate)]

→ The second training example forces algo. to further generalise h , which can be done by substituting any attribute value in ' h ' by a '?'.

$$h \leftarrow \langle \text{sunny}, \text{warm}, ?, \text{Strong}, \text{warm}, \text{Same} \rangle$$

→ Upon encountering a -ve Training example (3rd one) find s algo simply ignores it. So no revision of ' h ' is needed.

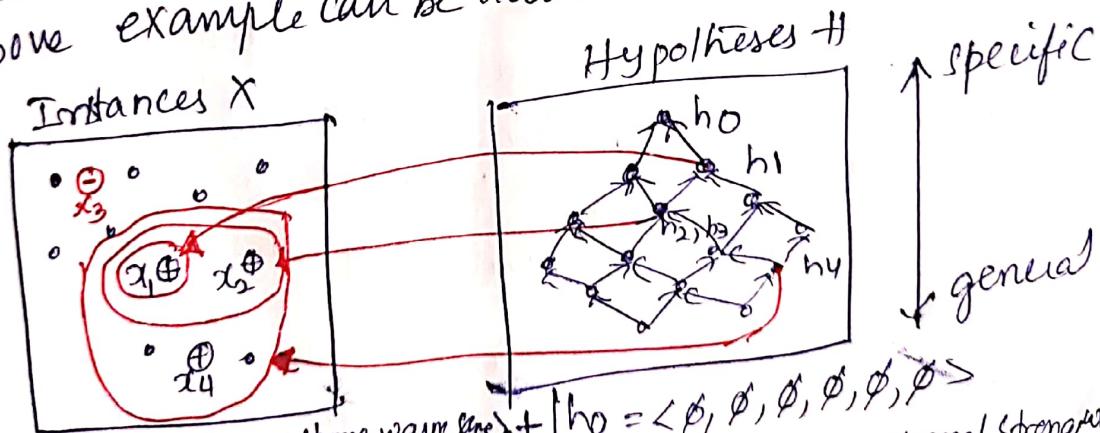
→ the 4th example (+ve) leads to further generalisation of ' h '.

$$h \leftarrow \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ? \rangle$$

→ In this Algo., "More general than" partial ordering is thus used to organise the search for an acceptable hypothesis.

→ At each step, the hypothesis is generalised as far as necessary to cover the new +ve examples thus making the hypothesis more consistent with Training examples. Hence it is named as "Ford-S" Algorithm.

→ Above example can be illustrated further with foll. fig.



$$x_1 = \langle \text{sunny}, \text{warm}, \text{normal}, \text{Strong}, \text{warm}, \text{Same} \rangle + h_0 = \langle ?, ?, ?, ?, ?, ? \rangle$$

$$x_2 = \langle \text{sunny}, \text{warm}, \text{high}, \text{Strong}, \text{warm}, \text{Same} \rangle + h_1 = \langle \text{sunny}, \text{warm}, \text{Normal}, \text{Strong}, \text{warm}, \text{Same} \rangle$$

$$x_3 = \langle \text{rainy}, \text{cold}, \text{high}, \text{Strong}, \text{warm}, \text{cha } \rangle - h_2 = \langle \text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{Same} \rangle$$

$$x_4 = \langle \text{sunny}, \text{warm}, \text{high}, \text{Strong}, \text{cool}, \text{cha } \rangle + h_3 = \langle \text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{Same} \rangle$$

$$x_4 = \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle + h_4 = \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle$$

- "Find-S" is guaranteed to o/p most specific hypothesis within H that is consistent with +ve training examples. Finally, it is also consistent with -ve training examples provided the correct target concept is contained in H & also provided the training examples are correct. (9)
- few voids in this Algo. in the form of Questions are:
 - * Has learner converged to correct target concept?
 - * why prefer most specific hypothesis?
 - * Are Training examples consistent?
 - * what if there are several maximally specific consistent hypotheses?

Version spaces and "The Candidate_Elimination Algorithm."

- Helps to address limitations of Find-S Algo.
- "Find-S" Algo gives as an o/p, only single hypothesis from H that is consistent with Training examples.
- "Candidate_Elimination" Algo on other hand, o/p's a set of "all hypotheses" that are consistent with Training examples.
- It also uses "More_general_than" partial ordering, but produces o/p without explicitly enumerating all of its members. Hence it is more compact rep'n of set of consistent Hypotheses.
- Both "Find-S" & "Candidate_Elimination" Algos. perform poorly when noisy Training data is given.
- Appl'n of "Candidate_Elimination" Algo:
 - Learning regularities in chemical mass Spectroscopy
 - Learning Control rules for heuristic search

→ Representation:

- Consistent:

A hypothesis, h , is said to be consistent with a set of training examples ' D ' iff $h(x) = c(x)$ for each example $\langle x, c(x) \rangle$ in D .

$$\text{consistent } (h, D) \equiv (\forall \langle x, c(x) \rangle \in D) h(x) = c(x)$$

NOTE :

An example ' x ' satisfies a hypothesis ' h ' when $h(x) = 1$ irrespective of x is +ve or -ve example of Target Concept vs. consistent.
 x is consistent with h is not only dependent on target concept but also its compulsory that $h(x) = c(x)$.

- Version Space:

It is denoted as $VS_{H, D}$ w.r.t hypothesis space H & training examples D .

It is a subset of Hypotheses from H consistent with Training examples in D .

→ The "List-Then-Eliminate" Algorithm:

It is used to rep. the Version space by simply listing all of its members.

Algo:-

1. Version space \leftarrow a list containing every hypothesis in H

2. For each Training example, $\langle x, c(x) \rangle$

- remove from "version space" any hypotheses ' h ' for which $h(x) \neq c(x)$

3. O/p. of the list of hypotheses in Version space

→ Limitation:-

- * It can be applied on finite Hypotheses space H ,
* Exhaustive enumeration of all ' h ' in H is reqd.

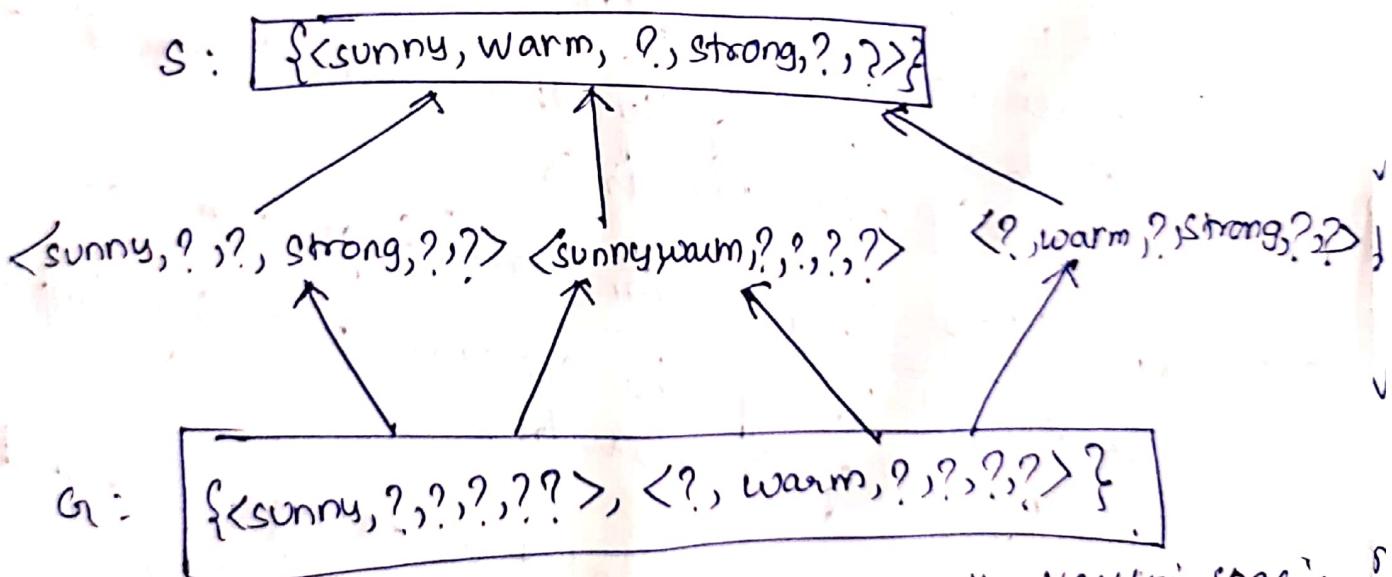
→ Advantages:-

- * guarantees to P all hypotheses consistent
with training data.

→ More compact representation for Version spaces:-

- Here, Version space is rep. by most general & least general members, thus setting boundaries for version space. They form general boundary set & specific boundary set.
- For this, we use example of "enjoy sport task" & its o/p of "Find-S" algorithm, which is:

$$h = \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle$$



- In "candidate & elimination" Algo., the version space is rep. by storing only its most general members, " G " & and most specific members " S ".

- General Boundary " G ": - G w.r.t Hypotheses space H & training data D is set of maximally general members of H consistent with D

$$G = \{g \in H \mid \text{consistent}(g, D) \wedge (\exists g' \in H) [(g' \supseteq g) \wedge \text{consistent}(g', D)]\}$$

- specific boundary's:
 S w.r.t H & D is set of minimally general members of H
 Consistent with D

- Version space rep" theorem:-

Euclidean Space

Let X be arbitrary set of instances,
 H be set of boolean valued hypotheses defined
 on X .
 Arbitrary Target concept defined

Let H be set of a boolean. Let H be an arbitrary Target concept defined over X . $\{0, 1\}$ be an arbitrary concept.

Let H be an arbitrary hypothesis class.
 Let $c: X - \{0, 1\} \rightarrow \{0, 1\}$ be an arbitrary function.
 Let $C: X - \{0, 1\} \rightarrow \{0, 1\}$ be an arbitrary function.
 Let $c: X - \{0, 1\} \rightarrow \{0, 1\}$ be an arbitrary function.

Let $C: X \rightarrow \{0, 1\}^S$ be an arbitrary function. Consider a set of training examples $\{(x, c(x))\}$.
 i.e. c is well defined,

Let D be an arbitrary set such that $S \& G$ are well defined,

$\nexists x, H, C \& D$ such that $\{ \forall c \in S \} (\exists g \in G) (g \geq_g h \geq_g s) \}$,

$$VS_{H,D} = \{ h \in H \mid \exists s \in S \text{ such that } h = s \}$$

proof: - satisfying RHS is in V_{S+D} .
 satisfies RHS

* Every h satisfying RHS is a member of $V\mathcal{S}_{H,D}$ satisfies RHS.

- * Every h in $S_{H,D}$ satisfies κ 's
- * Every member of $V_{S_{H,D}}$ satisfies κ 's
must be proved.

* Eve
This must be proved,
Let 'g' is an arbitrary member of G
 $"$ " H

" b ", that $g \geq g^h \geq g^s$

such that $g \geq g^h \geq g^s$ must be satisfied by all +ve examples in D (as per definiteness) $\rightarrow s$ should also be satisfied by all in D.

$\rightarrow \delta$ must be satisfied by
it should also be satisfied by
samples in D)

$\therefore h \geq g^k \Rightarrow$ it shows +ve examples n, i.e. it is satisfied by a -

$\therefore h \geq g \Rightarrow a \dots$ +ve example
As per defⁿg G₂, g can't be satisfied by a -ve example in
 $\therefore b$ also " " " " member of V_{S_H, D}

i defⁿ g G, g can't be same
if $g > g^h \Rightarrow h$ also " " "
with $D \supseteq$ his mem

$\Rightarrow h$ is consistent with $D \Rightarrow$ his membership of S_H, D

→ "Candidate Elimination" Learning Algorithm:-

(23)

1. Initialise G_0 to set of maximally general Hypotheses w.t.o t
2. " " S " " " " specific " "
3. For each training example ' d ', do :
 - If d is positive example:
 - (i) Remove from ' G ' any hypothesis inconsistent with d .
 - (ii) For each hypothesis ' h ' in S that is inconsistent with d :
 - (a) Remove ' h ' from ' S '.
 - (b) Add to ' S ' all minimal generalizations ' h' of ' h ' such that:
 - (i) h' is consistent with d .
 - (ii) some member $g \in G$ is more general than h .
 - (c) Remove from S , any hypothesis i.e. more general than another hypothesis in S .
 - If d is Negative example:
 - (i) Remove from ' S ' any hypothesis inconsistent with d .
 - (ii) For each hypothesis ' g ' in G i.e. not consistent with d :
 - (a) Remove ' g ' from ' G '.
 - (b) Add to ' G ' all minimal generalizations ' h ' of ' g ' such that:
 - (i) h is consistent with d .
 - (ii) some member $s \in S$ is more specific than h .
 - (c) Remove from G , any hypothesis i.e. less general than another hypothesis in G .

Example :-

→ Initialise G & S

$$G_0 \leftarrow \{<?, ?, ?, ?, ?, ?, ?>\} \text{ Most general hypothesis}$$

$$S_0 \leftarrow \{\emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset\} \text{ " specific " , " " }$$

→ Candidate Elimination Trace :-

Training Examples :

1. $\langle \text{sunny}, \text{warm}, \text{Normal}, \text{strong}, \text{warm}, \text{same}, \text{Enjoy sport} = \text{yes} \rangle$.

2. $\langle \text{sunny}, \text{warm}, \text{high}, \text{strong}, \text{warm}, \text{same}, \text{Enjoy sport} = \text{yes} \rangle$.

- S_0 & G_0 are initial boundary sets.
- Training Examples 1 & 2 force S boundary to become more general, as in "Find S " Algo.
- They don't have any effect on G boundary.

$$S_0: \boxed{\{\emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset\}}$$

$$S_1: \boxed{\{\text{sunny}, \text{warm}, \text{Normal}, \text{strong}, \text{warm}, \text{same}\}}$$

$$S_2: \boxed{\{\text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{same}\}}$$

$$G_0, G_1, G_2: \boxed{\{<?, ?, ?, ?, ?, ?, ?>\}}$$

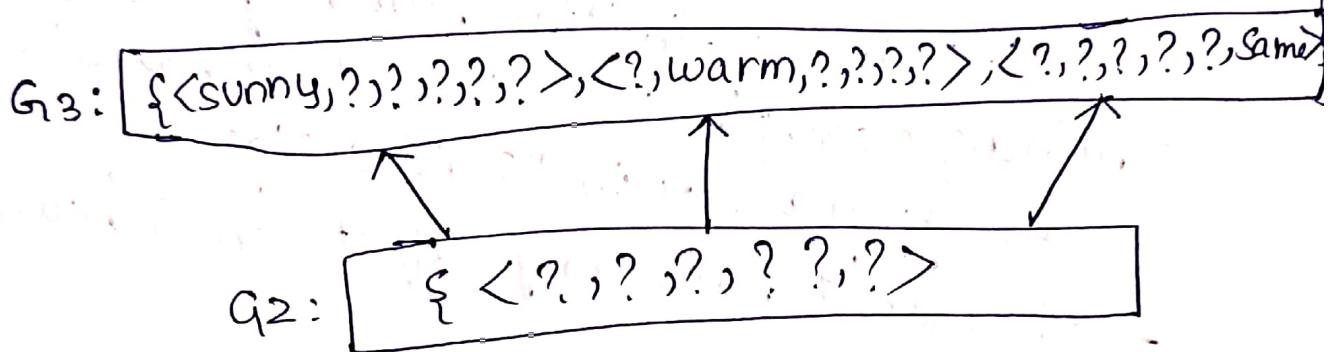
→ candidate-elimination Trace 2:-

Training Example:

3. $\langle \text{Rainy}, \text{cold}, \text{High}, \text{strong}, \text{warm}, \text{change} \rangle$,
 Enjoy sport = No

∴ it's a -ve example, it will now force G_2 boundary to be specialised to G_3 . & there will be several alternative maximally general hypotheses that are included.

$S_2, S_3 : \{\langle \text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{same} \rangle\}$



→ candidate-elimination Trace 3:-

Training Example:

4. $\langle \text{sunny}, \text{warm}, \text{high}, \text{strong}, \text{cool}, \text{change} \rangle$,
 Enjoy sport = Yes.

∴ +ve Training example, it generalizes 'S' boundary from S_3 to S_4 .

Also one member of G_3 should be deleted since

it is no longer more general than S_4

boundary.

S3: $\{ \langle \text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{same} \rangle \}$

S4: $\{ \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle \}$

G4: $\{ \langle \text{sunny}, ?, ?, ?, ?, ? \rangle \langle ?, \text{warm}, ?, ?, ?, ? \rangle \}$

G3: $\{ \langle \text{sunny}, ?, ?, ?, ?, ? \rangle \langle ?, \text{warm}, ?, ?, ?, ? \rangle \langle ?, ?, ?, ?, ?, \text{same} \rangle \}$

Final version :-

- Set S4 & G4 delimit version space of all hypotheses consistent with set of incrementally observed training examples.
- The foll. learned version space is independent of the seq. in which training examples present.
- S & G boundaries move monotonically closer, delimiting smaller & smaller version space of candidate hypotheses.

S4: $\{ \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle \}$

$\langle \text{sunny}, ?, ?, \text{strong}, ?, ?, ? \rangle \langle \text{sunny}, \text{warm}, ?, ?, ?, ? \rangle \langle ?, \text{warm}, ?, \text{strong}, ?, ? \rangle$

G4: $\{ \langle \text{sunny}, ?, ?, ?, ?, ? \rangle \langle ?, \text{warm}, ?, ?, ?, ? \rangle \}$

Remarks On Version space & "Candidate-Elimination":

- "Version space" learned by the Candidate-Elimination algorithm will converge towards the hypothesis that correctly describes the target concept, provided
 - (i) There are no errors in the training examples.
 - (ii) There is some hypothesis in H that correctly describes the target concept.
- Also if training data contains errors, Algo. will remove correct target concept from Version space & may create an empty version space.
- If the learner is allowed to conduct experiments in which it chooses next instance (a.k.a. Query) then it obtains correct classification for this instance from an external oracle (ex: Nature or Teacher).

Example:-

For "Enjoy sport" task, the "Query strategy" would be to choose an instance that would be classified +ve by some of these hypotheses, but negative by others such as:

(sunny, warm, Normal, light, warm, same)
- With optimal query strategy, size of Version space is reduced by half with each new example. The correct target concept can therefore be found with only $\lceil \log_2 |V_{S1}| \rceil$ experiments

- It is possible to classify certain examples, with same degree of confidence as if the target concept had been uniquely identified, even if it uses partially learned concept.
- Consider an instance A, which was not amongst the training examples, the learner will classify it as +ve. since, condition will be met iff instance satisfies every member of s. (with definition of more general than).
- ^{why} For instance B, it will be classified Negative by every hypothesis in version space; given a partially learned concept. (Just check if instance satisfies none of members of G).
- For instance C, Half of version space hypotheses classify it +ve & $\frac{1}{2}$ as -ve. Hence Learner can't classify this instance confidently.
- In case of instance D, it is classified +ve by 2 of version space hypotheses & -ve by other 4 \Rightarrow Negative classification it is.

<u>Instance</u>	<u>Sky</u>	<u>AirTemp</u>	<u>Humidity</u>	<u>Wind</u>	<u>Water</u>	<u>Forecast</u>	<u>Enjoy Sport</u>
A	sunny	warm	normal	strong	cool	change	?
B	rainy	cold	"	light	warm	same	?
C	sunny	warm	"	"	"	"	?
D	sunny	cold	"	strong	"	"	?

Inductive Bias:-

- "candidate elimination" Algo converges iff. accurate training examples & its initial hypothesis space contains target concept.
- A Biased Hypothesis space:
 - Assume that hypothesis space contains unknown target concept.
 - In "Enjoy Sport" example, hypothesis space was restricted to include only conjunctions of attribute values.
 - Becoz of this restriction, hypothesis space is unable to rep. even simple disjunctive target concepts such as "Sky = sunny or Sky = cloudy".

Ex	sky	Air Temp	Humidity	Wind	Water	Forecast	Enjoy sport
1	sunny	warm	Normal	strong	cool	change	Yes
2	cloudy	warm	"	"	"	"	Yes
3	Rainy	warm	"	"	"	"	No

There are no hypothesis which are consistent with these 3 examples above.

- The maximally specific hypothesis from H is S_2 : (? , Warm, Normal, strong, cool, change)

- problem:- The learner is biased to consider only Conjunction hypotheses.

An Unbiased Learner:-

Powerset:- A hypothesis space capable of representing every "Teachable concept" i.e. Capable of rep. every possible subset of instances of X .
 \Rightarrow Set of all subsets of X is called powerset of X .

Ex: - X for "Enjoy sports" is 9^6 .

(30)

$$\text{So, Distinct subsets} = 2^{1 \times 1} = 2^6 = 10^{28}.$$

- ④ Reformulate "Enjoy sports" Task in unbiased way,
lets define new hypothesis space H' .
 H' rep. every subset of instances \Rightarrow corresponds to power set of X .
lets allow arbitrary disjunctions, conjunctions & negations.
 \Rightarrow "sky=sunny" or "sky=cloudy" can be defined as
 $\langle \text{sunny}, ?, ?, ?, ?, ? \rangle \vee \langle \text{cloudy}, ?, ?, ?, ?, ? \rangle$
- ④ - If a "Candidate-Elimination" Algo is used then
S boundary is disjunction of observed +ve example whereas
G boundary will be negated disjunction of -ve examples
so, we have to present every single instance in X
as Training example, instead of S & G.

→ Futility of Bias-Free learning:

* Fundamental Property of Inductive Inference

- A learner that makes no a priori assumptions regarding the identity of the target concept has no rational basis for classifying any unseen instances.
- Inductive learning requires some form of prior assumptions or Inductive bias to classify new instances.
- Consider an arbitrary learning algo. 'L'. & an arbitrary set of Training data $D_C = \{ \langle x, c(x) \rangle \}$
Here c = Some arbitrary Target concept.
- After Training, 'L' classifies a new instance x_i .
- Let $L(x_i, D_C)$ denotes that classification (+ve or -ve).
- Inductive Inference step performed by L is:
$$(D_C \wedge x_i) \rightarrow L(x_i, D_C)$$

where \Rightarrow symbol means Inductively inferred,
 $\Rightarrow y \Rightarrow z \Rightarrow x$ is inductively inferred from y .

- Assume $L = \text{candidate_Elimination Algo}$
 $D_c = \text{Training data of "enjoy sport" Task}$.
 Here we should also define inductive bias of L .

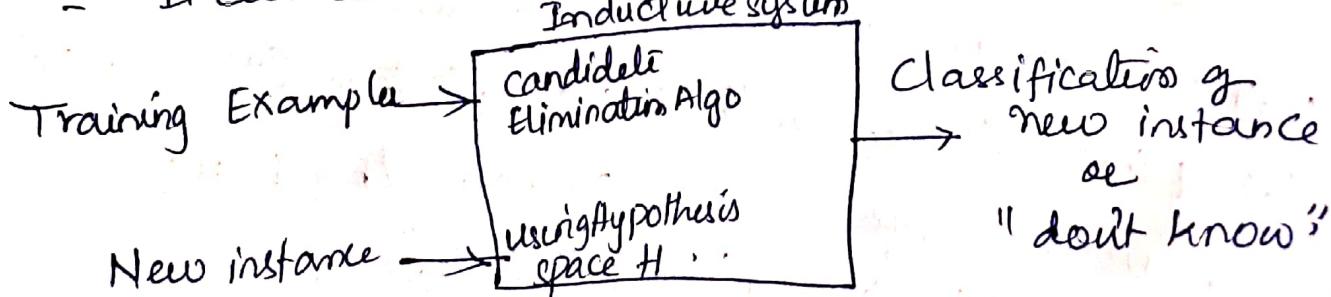
Definition:-

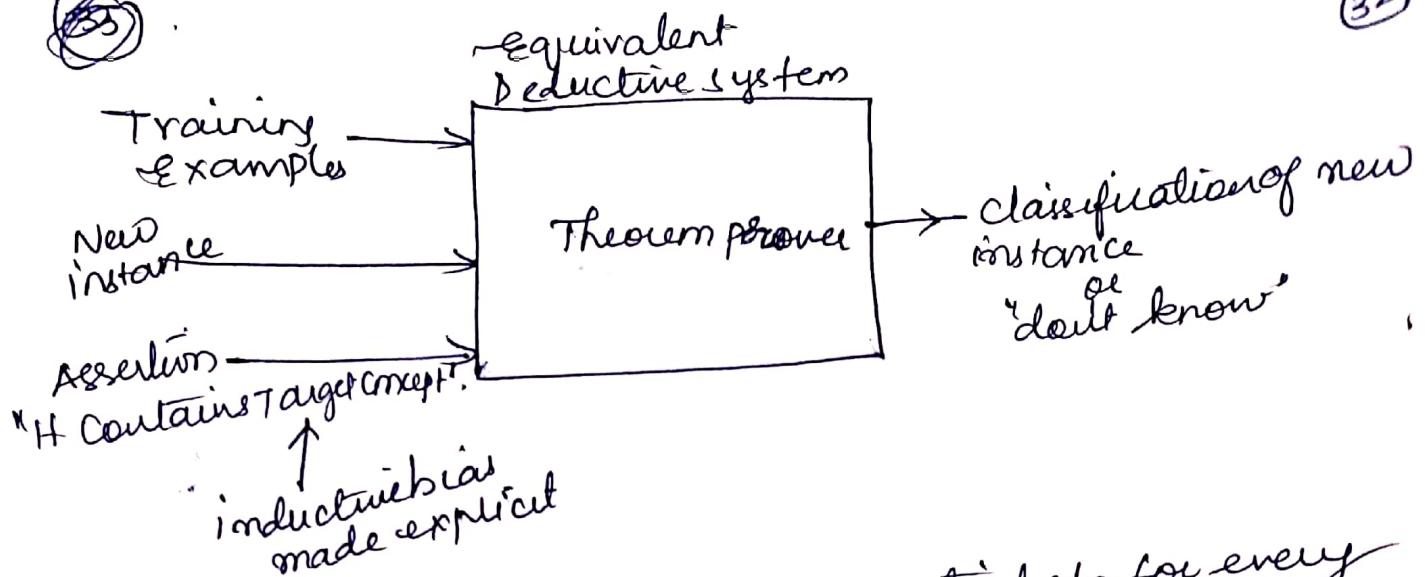
inductive bias of L is any minimal set of Assertions 'B' such that for any target concept $C \in$ corresp. Training examples D_c

$$(\forall x_i \in X) [(B \wedge D_c \wedge x_i) \vdash L(x_i; D_c)]$$

[Note :- $y \vdash z \Rightarrow z$ follows deductively from y
 i.e. z is provable from y]

- Given D_c , use candidate_elimination Algo.
- compute Version Space VS_{H, D_c}
- classify new instance x_i (+ve or -ve)
- Inductive bias : Assumption that $C \in H$.
- The classification $L(x_i; D_c)$ follows deductively from $B = \{C \in H\}$ along with Data D_c & description instance x_i
- so, if we assume $C \in H$ then it follows deductively that $C \in VS_{H, D_c}$. Also, classification $L(x_i; D_c)$ is to be unanimous vote of all hypothesis in VS_{H, D_c}
- $L(x_i) = L(x_i; D_c)$
- It can be described by foll. fig:





These 2 sys. will produce identical o/p for every possible i/p set of train set & new instances set X.

→ Inductive Bias Advantages:-

- (i) provides non-procedural means of characterizing their policy for generalizing beyond observed data
- (ii) Allows comparison of diff. learners accdg to strength of inductive bias

→ The 3 Algo. Examples from Weakest to strongest bias are:-

(i) Rule-Learner :-

- Simply storing each observe Training example in memory
- It has no inductive bias
- Instances are classified by looking them up in memory. If found, the stored classification is returned else sys. refuses to classify the new instance.

(ii) Candidate-Elimination :-

- New instances are classified only when all members of the current version space agree on the classification
- It refuses to classify new instance. It has strong inductive bias

(iii) Find-S :-

- Finds most specific hypotheses consistent with the training examples. Then it uses this hypothesis to classify the subsequent instance. It has a stronger inductive bias