Linear Programming Model for Portfolio Optimization

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1 Introduction

Linear Programming can be used as a tool for optimization. Provided suitable constraints, it can be used to maximize or minimize objectives such as *profits, returns, cost of transportation,etc.* In this paper, the same is done for optimizing the returns as a result of investing in various investment options to create an optimized portfolio.

2 Data Collection

Information related to return rate of stocks and risk was calculated from the Opening and Closing Price. The return rate for Gold ETF and risk score are calculated similarly. The data of the Mutual Fund option was gathered directly from online sources.

3 Linear Programming

In a linear programming model, we have a objective function to be maximized or minimized subject to constraints. The equations are made in terms of the decision variables, x_i's. In general, we have an objective;

$$\sum_{i=1}^{n} c_i x_i$$

Subject to:

$$\sum_{j=1}^{n} a_{i_j} x_j \leq b_i;\ 1 \leq i \leq p$$

$$\sum_{j=1}^{m} a_{i_j} x_j \leq b_i;\ p+1 \leq i \leq k$$

$$\sum_{j=1}^{n} a_{i_j} x_j \leq b_i;\ k+1 \leq i \leq m$$

$$x_j \geq 0$$

4 Formulation of Linear Programming Model

In our portfolio optimization problem, we have an initial upper bound of 10L i.e., maximum of 10L can be invested in the portfolio. On the other hand, we also have the following constraints:

1. Not more than 2L investment in any option. In each option, not more than 2L can be invested. This is done to diversify investment.
2. Investment in stocks cannot exceed 25% of the total invested sum.
3. Atleast 15% investment in Stock B.
4. The total risk should not exceed 2.0. This is calculated by using the weighed average of the individual risks and setting it below 2.

We have the following investment options: Stock A, Stock B, Gold ETF, Mutual funds and Govt. Bonds. Accordingly the decision variables:

i. x_1 denotes the investment made in Stock A.

ii. x_2 denotes the investment made in Stock B.

iii. x_3 denotes the investment made in Gold ETF.

iv. x_4 denotes the investment made in Mutual Funds.

v. x_5 denotes the investment made in Govt. Bonds.

The following table contains the return rate and risk scores of these options:

Investment Options	Return Rate(%)	Risk Score
Stock A	6.75	0.92
Stock B	0.42	0.11
Gold ETF	0.42	3.12
Mutual Fund	1.54	10.94
Bonds	7.03	0

Table 1: Data

Thus we have the objective function:

$$Z = 0.0675x_1 + 0.0042x_2 + 0.0042x_3 + 0.0154x_4 + 0.0703x_5$$

subject to:

I. $x_1 + x_2 + x_3 + x_4 + x_5 \leq 1000000$

II. $0.92x_1 + 0.11x_2 + 3.12x_3 + 10.94x_4 \leq 2(x_1 + x_2 + x_3 + x_4 + x_5)$
i.e. $1.08x_1 + 1.89_x2 - 1.12x_3 - 8.94x_4 + 2_x5 \geq 0$

III. $x_1 + x_2 \leq 0.25(x_1 + x_2 + x_3 + x_4 + x_5)$
i.e. $3x_1 + 3x_2 - x_3 - x_4 - x_5 \leq 0$

IV. $x_2 \geq 0.15(x_1 + x2_+x_3 + x_4 + x_5)$
i.e. $3x_1 - 17x_2 + 3x_3 + 3x_4 + 3x_5 \leq 0$

V. $x_1, x_2, x_3, x_4, x_5 \leq 200000$

VI. $x_1, x_2, x_3, x_4, x_5 \geq 0$

The last constraint imposes the fact that the decision variables are positive.

5 Results

The problem is solved using the Google OR Tools. The code is provided here. On solving, we get the following investment values:

Investment Option	Sum to be invested	Return
Stock A	59,428.21	4,011.4
Stock B	89,142.34	374.5
Gold ETF	2,00,000	84000
Mutual Fund	45,711.57	703.96
Bonds	2,00,000	14,060

Table 2: Return on investment

Note that the returns of total 19,989.76 are monthly. The individual returns are mentioned in 2.