**MAT1002 Lab Exp. No.3 06/03/2021**

**Name :** Khan Mohd. Owais Raza

**Registration No. :** 20BCD7138

**Problem using MATLAB:** Sprung Mass Displacement in a Quarter Car (One Wheel) Model

**Aim :**
• To write MATLAB code for system of second order differential equations of the form X"+ AX = 0 using Diagonalization
• Determining the sprung mass displacement in a car suspension for one wheel. Solving a coupled system of ordinary differential equations derived from the mathematical model.

**Mathematical form:**
Using the transformation Y = P X, the given system of differential equations can be reduced to uncoupled system as X" + AX = 0 , where D is the diagonal matrix with eigen values of A as diagonal elements and Q is the modal matrix corresponding to A.

**MATLAB syntax used :**
• eig:eig(A) returns a vector of the eigenvalues of matrix A
• solve:solve(eq, x) returns the set of all complex solutions of an equation or inequality eq with respect to x.

**Problem:**

Solve the following system of equations

$$y_1'' = -5y_1 - 2y_1$$

$$y_2'' = -2y_1 - 2y_2$$

**Solution by MATLAB code :**

```
clc
clear all
close all
syms x1(t) x2(t)
A=Input('Enter the coefficient matrix A')
lambda=eig(A)
fprintf('eigen values of A are %f%f\n\n',lambda)
for i=1:length(lambda)
    temp=null(A-lambda(i)*eye(size(A)),'r');
    P(:,i)=temp./min(temp);l
end
disp('the model matrix is:\n')
disp(P)
D=inv(P)*A*P;
Sol1=dsolve(diff(x1,2)+D(1)*x1==0);
Sol2=dsolve(diff(x2,2)+D(4)*x2==0);
Y=P*[Sol1;Sol2]
```

**Output:**

```
Y =
2*C5*cos(6^(1/2)*t) + 2*C6*sin(6^(1/2)*t) + C2*cos(t) +
C3*sin(t)
C5*cos(6^(1/2)*t) + C6*sin(6^(1/2)*t) - 2*C2*cos(t) +
2*C3*sin(t)
```

**Exercise:**

Solve the following system of equations

1.

$$x_1'' = -2x_1 - 5x_1$$

$$x_2'' = -5x_1 - 2x_2$$

2.

$$x_1'' = -5x_1 - 2x_1$$

$$x_2'' = +2x_1 - 2x_2$$

**Q.1)**

$$x_1'' = -2x_1 - 5x_1$$

$$x_2'' = -5x_1 - 2x_2$$

**INPUT:**

```
clc
clear all
syms x1(t) x2(t)
A=input('Enter the coefficient matrix A');
lambda=eig(A);
%fprintf('eigen values of A are %f%f\n\n',lambda)
for i=1:length(lambda)
    temp=null(A-lambda(i)*eye(size(A)),'r');
    P(:,i)=temp./min(temp);
end
disp('the model matrix is:\n')
disp(P)
D=inv(P)*A*P;
X=[x1;x2]
Sol1=dsolve(diff(x1,2)+D(1)*x1==0);
Sol2=dsolve(diff(x2,2)+D(4)*x2==0);
Y=P*[Sol1;Sol2]
```

**OUTPUT :**

```
Enter the coefficient matrix A
[-2 -5;-5 -2]
the model matrix is:\n
     1     1
     1    -1


X(t) =

x1(t)
x2(t)


Y =
```

```
C1*cos(3^(1/2)*t) + C1*exp(7^(1/2)*t) + C2*exp(-7^(1/2)*t) -
C2*sin(3^(1/2)*t)
C1*exp(7^(1/2)*t) - C1*cos(3^(1/2)*t) + C2*exp(-7^(1/2)*t) +
C2*sin(3^(1/2)*t)

disp(P)
     1     1
     1    -1

D=inv(P)*A*P

D =


    -7     0
     0     3

Sol1=dsolve(diff(x1,2)+D(1)*x1==0)

Sol1 =

C1*exp(7^(1/2)*t) + C2*exp(-7^(1/2)*t)

Sol2=dsolve(diff(x2,2)+D(4)*x2==0)

Sol2 =

C1*cos(3^(1/2)*t) - C2*sin(3^(1/2)*t)

lambda = eig(A)

lambda =

    -7
     3
```

## Q.2)

$$x_1'' = -5x_1 - 2x_1$$

$$x_2'' = +2x_1 - 2x_2$$

**INPUT:**

```
clc
clear all
syms x1(t) x2(t)
A=input('Enter the coefficient matrix A');
lambda=eig(A);
%fprintf('eigen values of A are %f%f\n\n',lambda)
for i=1:length(lambda)
    temp=null(A-lambda(i)*eye(size(A)),'r');
    P(:,i)=temp./min(temp);
end
disp('the model matrix is:\n')
disp(P)
D=inv(P)*A*P;
X=[x1;x2]
Sol1=dsolve(diff(x1,2)+D(1)*x1==0);
Sol2=dsolve(diff(x2,2)+D(4)*x2==0);
Y=P*[Sol1;Sol2]
```

**OUTPUT :**

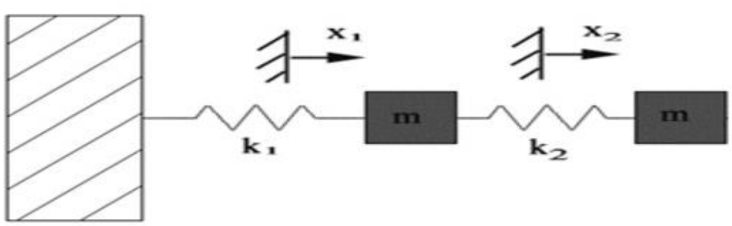
```
Enter the coefficient matrix A
[-5 -2;2 -2]
the model matrix is:\n
   1.0000 + 0.0000i   1.0000 + 0.0000i
  -0.7500 - 0.6614i  -0.7500 + 0.6614i


X(t) =

x1(t)
x2(t)
```

```
Y =

                                                  C1*exp(-
(2^(1/2)*t*(7 - 7^(1/2)*1i)^(1/2))/2) + C1*exp(-(2^(1/2)*t*(7 +
7^(1/2)*1i)^(1/2))/2) + C2*exp((2^(1/2)*t*(7 - 7^(1/2)*1i)^(1/2))/2) +
C2*exp((2^(1/2)*t*(7 + 7^(1/2)*1i)^(1/2))/2)
((7^(1/2)*1i)/4 - 3/4)*(C1*exp(-(2^(1/2)*t*(7 + 7^(1/2)*1i)^(1/2))/2)
+ C2*exp((2^(1/2)*t*(7 + 7^(1/2)*1i)^(1/2))/2)) - ((7^(1/2)*1i)/4 +
3/4)*(C1*exp(-(2^(1/2)*t*(7 - 7^(1/2)*1i)^(1/2))/2) +
C2*exp((2^(1/2)*t*(7 - 7^(1/2)*1i)^(1/2))/2))

disp(P)
   1.0000 + 0.0000i   1.0000 + 0.0000i
  -0.7500 - 0.6614i  -0.7500 + 0.6614i

D=inv(P)*A*P

D =

  -3.5000 + 1.3229i   0.0000 - 0.0000i
   0.0000 + 0.0000i  -3.5000 - 1.3229i

X=[x1;x2]

X(t) =

x1(t)
x2(t)

Sol1=dsolve(diff(x1,2)+D(1)*x1==0)

Sol1 =

C1*exp(-(2^(1/2)*t*(7 - 7^(1/2)*1i)^(1/2))/2) + C2*exp((2^(1/2)*t*(7 -
7^(1/2)*1i)^(1/2))/2)
```

```
Sol2=dsolve(diff(x2,2)+D(4)*x2==0)

Sol2 =

C1*exp(-(2^(1/2)*t*(7 + 7^(1/2)*1i)^(1/2))/2) + C2*exp((2^(1/2)*t*(7 + 7^(1/2)*1i)^(1/2))/2)

Y=P*[Sol1;Sol2]



Y =

                                                                C1*exp(-
(2^(1/2)*t*(7 - 7^(1/2)*1i)^(1/2))/2) + C1*exp(-(2^(1/2)*t*(7 + 
7^(1/2)*1i)^(1/2))/2) + C2*exp((2^(1/2)*t*(7 - 7^(1/2)*1i)^(1/2))/2) + 
C2*exp((2^(1/2)*t*(7 + 7^(1/2)*1i)^(1/2))/2)
((7^(1/2)*1i)/4 - 3/4)*(C1*exp(-(2^(1/2)*t*(7 + 7^(1/2)*1i)^(1/2))/2) 
+ C2*exp((2^(1/2)*t*(7 + 7^(1/2)*1i)^(1/2))/2)) - ((7^(1/2)*1i)/4 + 
3/4)*(C1*exp(-(2^(1/2)*t*(7 - 7^(1/2)*1i)^(1/2))/2) + 
C2*exp((2^(1/2)*t*(7 - 7^(1/2)*1i)^(1/2))/2))
```

**Practice Problem:**

Consider the system below with 2 masses and 2 springs.




Assume the masses $m = 10kg$ *and* spring constants $k_1 = k_2 = 1N/m$. Find the displacements $x_1$ and $x_2$ in the system using diagonalization method.

**INPUT (MATLAB code):**

```
clc
clear all
syms x1(t) x2(t)
A=input('Enter the coefficient matrix A');
lambda=eig(A);
%fprintf('eigen values of A are %f%f\n\n',lambda)
for i=1:length(lambda)
    temp=null(A-lambda(i)*eye(size(A)),'r');
    P(:,i)=temp./min(temp);
end
disp('the model matrix is;\n')
disp(P)
D=inv(P)*A*P;
X=[x1;x2];
Sol1=dsolve(diff(x1,2)+D(1)*x1==0);
Sol2=dsolve(diff(x2,2)+D(4)*x2==0);
Y=P*[Sol1;Sol2];
```

**OUTPUT :**

```
Enter the coefficient matrix A
[0.2 -0.1;-0.1 0.1]
the model matrix is;\n
    1.0000    1.0000
    1.6180   -0.6180

>> lambda=eig(A)

lambda =

    0.0382
    0.2618

>> disp(P)
    1.0000    1.0000
    1.6180   -0.6180

>> D=inv(P)*A*P

D =

    0.0382    0.0000
    0.0000    0.2618
```

```
>> X=[x1;x2]

X(t) =

x1(t)
x2(t)

>> Sol1=dsolve(diff(x1,2)+D(1)*x1==0)

Sol1 =

C1*cos((11009420709978678^(1/2)*t)/536870912) - C2*sin((11009420709978678^(1/2)*t)/53687091

>> Sol2=dsolve(diff(x2,2)+D(4)*x2==0)

Sol2 =

C1*cos((294764422404433^(1/2)*t)/33554432) - C2*sin((294764422404433^(1/2)*t)/33554432)
```

```
>> Y=P*[Sol1;Sol2]
C1*cos((294764422404433^(1/2)*t)/33554432) -
C2*sin((294764422404433^(1/2)*t)/33554432) +
C1*cos((11009420709978678^(1/2)*t)/536870912) -
C2*sin((11009420709978678^(1/2)*t)/536870912)
(347922205179541*C2*sin((294764422404433^(1/2)*t)/33554432))/562949953421312
-
(347922205179541*C1*cos((294764422404433^(1/2)*t)/33554432))/562949953421312
+
(7286977268806823*C1*cos((11009420709978678^(1/2)*t)/536870912))/450359962737
0496 -
(7286977268806823*C2*sin((11009420709978678^(1/2)*t)/536870912))/450359962737
0496
```