MAT1002 Lab Experiment No. 6 **10/04/2021**

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Aim: Finding the vertical deflection in a cantilever beam subjected to variable load and material properties and visualization of it.

Methodology: Solving governing equation of vertical deflection in swimming pool diving board using Laplace transform.

Laplace Transform : The Laplace Transform of a function f(t),defined for all real numbers t > 0 is the function F(s) defined by,

$$L[f(t)] = \int_0^\infty e^{-st} f(t)dt = F(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Inverse Laplace Transform : The inverse Laplace Transform of F(s) is defined by

$$f(t) = L^{-1}[F(s)] = \int_0^\infty e^{-st} F(s)dt \quad \ldots\ldots\ldots\ldots\ldots(2)$$

List of MATLAB Commands used :
1.] *laplace(f)* : Returns the Laplace transform of f using the default independent variable t and the default transformation variable s.
2.] *laplace(f, transVar)* : Uses the specified transformation variable transVar
3.] *laplace(f, var, transVar***)** : Uses the specified independent variable var and transformation variable transVar instead of t and s respectively
4.] *ilaplace(F)* : Returns the inverse Laplace transform of F using the default independent variable s for the default transformation variable t. If F does not contain s, ilaplace uses symvar.
5.] *ilaplace(F, transVar)* : Uses the specified transformation variable transVar
6.] *ilaplace(F, var, transVar)* : Uses the specified independent variable var and transformation variable transVar instead of s and t respectively.
7.] *heaviside(t − a)* : To input the heaviside's unit step function H(t − a) .
8.] *dirac(t − a)* : To input the dirac delta function δ(t − a).
9.] *laplace(diff(f(t), t), t, s)* : s*laplace(f(t), t, s) - f(0)

Example 1

Write the MATLAB code which computes the Laplace Transform of

$$f(t)=\begin{cases} t^2, & t<2, \\ t-1, & 2<t<3 \\ 7, t>3. \end{cases}$$

MATLAB code:

```
clear all
clc
syms t
f=input('Enter the function in terms of t:');
F=laplace(f );
F=simplify(F);
```

OUTPUT:

```
Enter the function in terms of t:

t^2*(heaviside(t-0)-heaviside(t-2))+(t-1)*(heaviside(t-2)-heaviside(t-3)) +
7*(heaviside(t-3))

Y = 5/(s*exp(3*s))-3/(s*exp(2*s))-3/((s^2)*exp(2*s))-1/((s^2)*exp(3*s))-
2/((s^3*exp(2*s)) + 2/s^3
```

Example 2:

Solve $y'' + 2y' + 10y = 1 + 5(t-5), y(0) = 1, y'(0) = 2$

MATLAB Code :

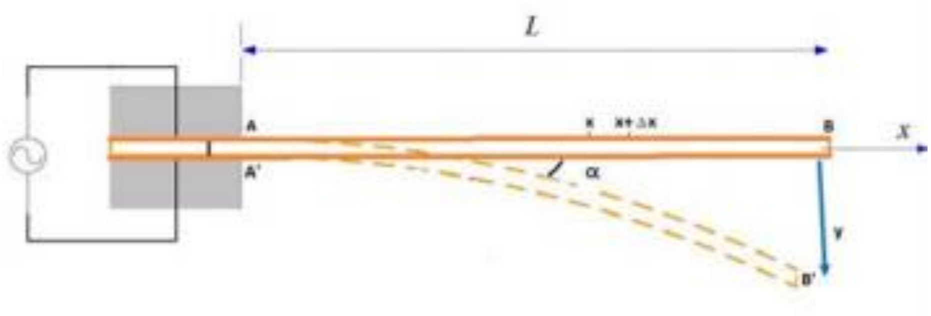
```
clc
clear all
syms t s Y
y2=diff(sym('y(t)'),2);
y1=diff(sym('y(t)'),1);
y0=sym('y(t)');
a = input('The Coefficient of D2y = ');
b = input('The Coefficient of Dy = ');
c = input('The Coefficient of y = ');
nh = input('Enter the non homogenous part = ');
eqn=a*y2+b*y1+c*y0-nh;
LTY=laplace(eqn,t,s);
if (a==0)
d = input('The initial value at 0 is ');
LTY=subs(LTY,'laplace(y(t), t, s)','y(0)',Y,d)
else
```

```
d = input('The initial value at 0 is ');
e = input('The initial value at 0 is ');
LTY=subs(LTY,'laplace(y(t), t, s)','y(0)','D(y)(0)',Y,d,e)
end
eq=collect(LTY,Y);
Y=simplify(solve(eq,Y));
y=simplify(ilaplace(Y,s,t));
```

OUTPUT:

```
The Coefficient of D2y = 1
The Coefficient of Dy = 2
The Coefficient of y = 10

Enter the non homogenous part = 1 + 5*dirac(t-5)

The initial value at 0 is 1

The initial value at 0 is 2

LTY = 10 ∗ Y − s − 5/exp(5 ∗ s) + 2 ∗ Y ∗ s + Y ∗ s 2 − 1/s − 4 y =
(cos(3 ∗ t) − sin(3 ∗ t)/3)/exp(t) − (cos(3 ∗ t) + sin(3 ∗ t)/3)/(10 ∗
exp(t)) + (4 ∗ sin(3 ∗ t))/(3 ∗ exp(t)) + (5 ∗ heaviside(t − 5) ∗ exp(5
− t) ∗ sin(3 ∗ t − 15))/3 + 1/10
```

Mathematical Modelling:



1.] Stress is proportional to strain.Thus, the equation is valid only for beams that are not stressed beyond the elastic limit.
2.] The curvature is always small.
3.] Any deflection resulting from the shear deformation of the material or shear stresses is neglected.
4.] For the deflected shape of the beam, the slope α at any point is defined as tan α = dy/dx . Assuming tan α = α we can write α = dy/dx .

5.] The curvature of a plane curve at a point can be expressed as

$$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left(1 + \frac{dy}{dx}^2\right)^{\frac{3}{2}}}$$

6.] In the elastic curve of beam dy/dx is very small so we can neglect its higher order terms. $\frac{1}{\rho} = \frac{d^2y}{dx^2}$

7.] From the theory of elasticity, if x is the distance of the section from the left end of the beam then $1/\rho = M(x)/EI$ where M=Bending moment, E=Modulus of Elasticity & I=Moment of inertia of the cross section.

8.] The governing equation for an elastic curve: $\frac{d^2y}{dx^2} = \frac{-M(x)}{EI}$.

9.] When a beam supports a distributive load w(x) then dM/dx = V(Shear force) and dV/dx = −w. Therefore,

$$\frac{d^4y}{dx^4} = \frac{w(x)}{EI}$$

Vertical deflection in a swimming pool diving board subjected to a distributed load can be seen as the deflection in a cantilever beam of length L subjected to a distributed load w(x) which is the solution of the differential equation:

$$\frac{d^4y}{dx^4} = \frac{w(x)}{EI}$$

subjected to the boundary conditions y(0) = y'(0) = y''(L) = y'''(L) = 0.
Note : y''(L) = 0 because there is no bending moment and y'''(L) = 0 because there is no shear at that point.

Problem:

Find the deflection in a cantilever beam subjected to the following conditions $y(0) = y'(0) = y''(L) = y'''(L) = 0$ by taking $L = 3$, $E = 2.1 * 10^{11} N/mm^2$, $I = 4.5 * 10^{-11} mm^4$ and $w(x) = x$.

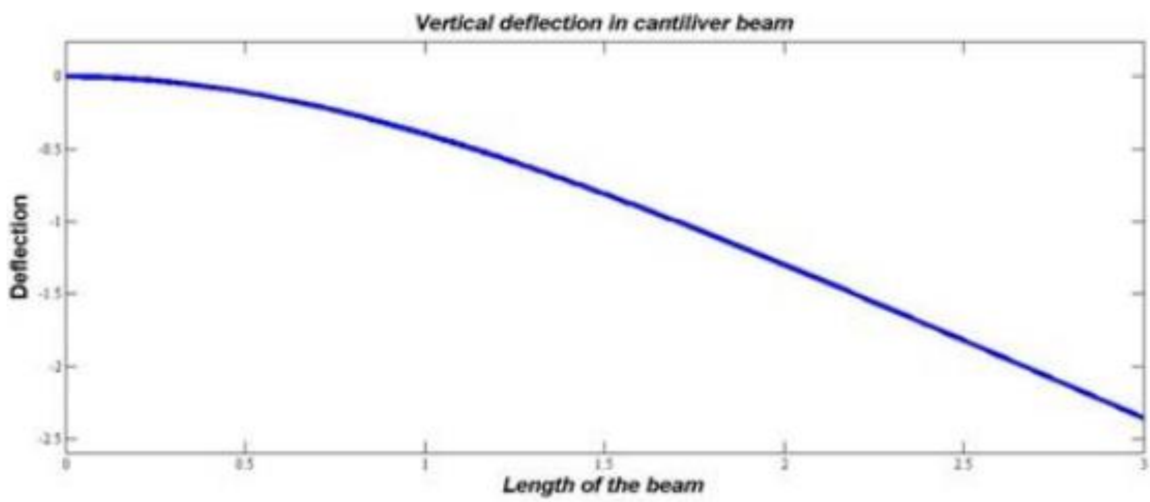
MATLAB Code:

```
clc
clear all
syms x s C D Y
y4=diff(sym('y(x)'),4);
y0=sym('y(x)');
L=input('Enter the length of the beam:');
E=input('Enter Modulus of elasticity:');
I=input('Enter Moment of inertia of the cross section:');
w=input('Enter distributive load w(x):');
eqn=E*I*y4-w;
LTY=laplace(eqn,x,s);
a=input('Enter y(0):');
b=input('Enter Dy(0):');
c=input('Enter D2y(L):');
d=input('Enter D3y(L):');
LTY=subs(LTY,{'laplace(y(x),x,s)','y(0)','D(y)(0)','D(D((y)))(0)','D(D
(D((y))))(0)'},{Y, a, b, C, D});
eq=collect(LTY,Y);
Y=simplify(solve(eq,Y));
y=simplify(ilaplace(Y,s,x));
eq1=subs(diff(y,x,2),x,L);
eq2=subs(diff(y,x,3),x,L);
[C, D]=solve(eq1, eq2);
gen=subs(y);
def=subs(def,heaviside(x-L),0);
fzplot(-def,[0,L])
title('Vertical deflection in cantilever beam')
xlabel('Length of the beam')
ylabel('Deflection')
```

OUTPUT :

```
Enter the length of the beam: 3
                Length of the beam = 3

Enter Modulus of elasticity: 2.1 * 10^(-11)
                Modulus of elasticity = 2.1 * 10^(-11)

Enter Moment of inertia of the cross section: 4.5 * 10^(-11)
                Moment of inertia = 4.5 * 10^(-11)

Enter distributive load w(x): x*(heaviside(x-L)-heaviside(x))
                Distributive load = x*(heaviside(x-L)-heaviside(x))

Enter y(0):0
```

```
                y(0) = 0

Enter Dy(0):0
                Dy(0) = 0

Enter D2y(L):0
                D2y(L) = 0

Enter D3y(L):0
                D3y(L) = 0

LTY =-(189/20)∗D-(189/20)∗s∗C+(1-(3∗s+1)∗exp(-3∗s))/s^2+ 189/20∗s^4∗Y
C =-20/21
D =10/21
def =(5/63)*x^3-(10/21)*x^2-(1/1134)*x^5
```



Exercise:

Find the deflection in a cantilever beam subjected to the following conditions $y(0) = y'(0) = y''(L) = y'''(L) = 0$ by taking $L = 2$, $E = 2.1 * 10^{11} N/mm^2$, $I = 4.5 * 10^{-11} mm^4$ and

$$w(x) = \begin{cases} x, & x < 1, \\ x - L, & 1 < x < L. \end{cases}$$

INPUT (MATLAB Code) :

```
clc
clear all
syms x s C D Y
y4=diff(sym('y(x)'),4);
y0=sym('y(x)');
L=input('Enter the length of the beam:');
E=input('Enter Modulus of elasticity:');
I=input('Enter Moment of inertia of the cross section:');
w=input('Enter distributive load w(x):');
eqn=E*I*y4-w;
LTY=laplace(eqn,x,s);
a=input('Enter y(0):');
b=input('Enter Dy(0):');
c =input('Enter D2y(L):');
d = input('Enter D3y(L):');
LTY=subs(LTY,{'laplace(y(x),x,s)','y(0)','D(y)(0)','D(D((y)))(0)','D(D
(D((y))))(0)'},{Y, a, b, C, D});
eq=collect(LTY,Y);
Y=simplify(solve(eq,Y));
y=simplify(ilaplace(Y,s,x));
eq1=subs(diff(y,x,2),x,L);
eq2=subs(diff(y,x,3),x,L);
[C, D]=solve(eq1, eq2);
gen=subs(y);
def=subs(def,heaviside(x-L),0);
fzplot(-def,[0,L])
title('Vertical deflection in cantilever beam')
xlabel('Length of the beam')
ylabel('Deflection')
```

OUTPUT :

```
Enter the length of the beam: 2
               Length of the beam = 2

Enter Modulus of elasticity: 2.1 * 10^(-11)
               Modulus of elasticity = 2.1 * 10^(-11)

Enter Moment of inertia of the cross section: 4.5 * 10^(-11)
               Moment of inertia = 4.5 * 10^(-11)

Enter distributive load w(x): x*(heaviside(x-L)-heaviside(x))
               Distributive load = x*(heaviside(x-L)-heaviside(x))

Enter y(0):0
               y(0) = 0

Enter Dy(0):0
               Dy(0) = 0

Enter D2y(L):0
               D2y(L) = 0

Enter D3y(L):0
               D3y(L) = 0

LTY =-(189/20)*D-(189/20)*s*C+(1-(3*s+1)*exp(-3*s))/s^2+ 189/20*s^4*Y
C =-20/21
D =10/21
def =(5/63)*x^3-(10/21)*x^2-(1/1134)*x^5
```

Exercise:

Solve the differential equation

$$y''+2y'+10y=1+5(t-5),\ y(0)=1,\ y'(0)=2$$

MATLAB Code :

```
clc
clear all
syms x s C D Y
y4=diff(sym('y(x)'),4);
y0=sym('y(x)');
L=input('Enter the length of the beam:');
E=input('Enter Modulus of elasticity:');
I=input('Enter Moment of inertia of the cross section:');
w=input('Enter distributive load w(x):');
eqn=E*I*y4-w;
LTY=laplace(eqn,x,s);
a=input('Enter y(0):');
b=input('Enter Dy(0):');
c =input('Enter D2y(L):');
d = input('Enter D3y(L):');
LTY=subs(LTY,{'laplace(y(x),x,s)','y(0)','D(y)(0)','D(D((y)))(0)','D(D(D((y))
))(0)'},{Y, a, b, C, D});
eq=collect(LTY,Y);
Y=simplify(solve(eq,Y));
y=simplify(ilaplace(Y,s,x));
eq1=subs(diff(y,x,2),x,L);
eq2=subs(diff(y,x,3),x,L);
[C, D]=solve(eq1, eq2);
gen=subs(y);
def=subs(def,heaviside(x-L),0);
fzplot(-def,[0,L])
title('Vertical deflection in cantilever beam')
xlabel('Length of the beam')
ylabel('Deflection')
```

Output :

```
Enter the length of the beam: 2

      L = 2

Enter Modulus of elasticity: 2.1 * 10^(-11)
      E = 2.1000e-11
```

```
I=input('Enter Moment of inertia of the cross section:')
Enter Moment of inertia of the cross section: 4.5 * 10^(-11)

        I =  4.5000e-11

w=input('Enter distributive load w(x):')
Enter distributive load w(x): x*(heaviside(x-L)-heaviside(x))
    w = x*(heaviside(x - 2) - heaviside(x))

a=input('Enter y(0):')
Enter y(0): 1

  a = 1

b=input('Enter Dy(0):')
Enter Dy(0): 2

  b = 2

y=simplify(ilaplace(Y,s,x))
 y =Y*dirac(x)

eq1=subs(diff(y,x,2),x,L)
 eq1 =0

eq2=subs(diff(y,x,3),x,L)
 eq2 =0

gen=subs(Y)
 gen = Y
```