MAT1002 Lab Exp. No.1 **24/02/2021**

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Problem using MATLAB : Creating stress matrix and its eigenvalues for simply supported beam.

Aim: Calculating and visualizing the Eigen values of stress matrix for simply supported beam

Problem statement: Find principal stresses for a twodimensional simply supported beam by finding the eigenvalues of the stress matrix with variable components.

Mathematical Background: The principal stresses are the eigenvalues of the stress matrix.
The principal stress at the point will be the eigenvalues of the stress matrix S.

Definitions:
A nonzero vector x is an eigenvector (or characteristic vector) of a square matrix A if there exists a scalar λ such that Ax = λx. Then λ is an eigenvalue (or characteristic value) of A.

Properties of Eigenvalues and Eigenvectors

1. The sum of the eigenvalues of a matrix equals the trace of the matrix.
2. A matrix is singular if and only if it has a zero eigenvalue.
3. The eigenvalues of an upper (or lower) triangular matrix are the elements on the main diagonal.
4. If λ is an eigenvalue of A and A is invertible, then 1/λ is an eigenvalue of matrix inverse(A)
5. If λ is an eigenvalue of A then kλ is an eigenvalue of kA where k is any arbitrary scalar.
6. If λ is an eigenvalue of A then λ k is an eigenvalue of Ak for any

positive integer k
7. If λ is an eigenvalue of A then λ is an eigenvalue of A

Eigenvectors :

To each distinct eigenvalue of a matrix A there will correspond at least one eigenvector which can be found by solving the appropriate set of homogenous equations. If λi is an eigenvalue then the corresponding eigenvector xi is the solution of (A – λi I)Xi = 0

Eigenvalues :

Let x be an eigenvector of the matrix A. Then there must exist an eigenvalue λ such that Ax = λx or, equivalently, Ax - λx = 0 or (A – λI)x = 0 If we define a new matrix B = A – λI, then Bx = 0 If B has an inverse then x = B-10 = 0. But an eigenvector cannot be zero. Thus, it follows that x will be an eigenvector of A if and only if B does not have an inverse, or equivalently det(B)=0, or det(A – λI) = 0 This is called the characteristic equation of A. Its roots determine the eigenvalues of A.

MATLAB syntax

p =poly(A) : where A is an nxn matrix returns an n+1 element row vector whose elements are the coefficients of the characteristic polynomial, det(λI - A), which are stored in p

r =roots(p) : Returns a column vector r whose elements are the roots of the polynomial p.

[V,D] = eig(A) : D=diagonal matrix with eigenvalues on its diagonal; V = modal matrix whose columns are the corresponding eigenvectors.

eye(n) : Returns an n x n identity matrix

INPUT :

```
%Code for calculating eigenvalues of matrix
clc
clear all
A=[3 0 -1; 0 1 0; 2 0 0];
Eigenvalues of A=eig(A);
[X,D]= eig(A);
p=poly(A); % Coefficients of the characteristic polynomial
r = roots(p); % To find the roots of the characteristic
sum of eigenvalues = sum(r);
Trace of A=trace(A); %Note that sum of eigenvalues is Trace
of A
Product of eigenvalues of A = prod(r);
Determinant of A = det(A);
Inverse of A = inv(A);
Eigenvalues of invA= eig(inv(A));
Eigenvalues of Transpose of A= eig(A');
% Note that eigenvalues of A and Transpose of A are same
Eigenvalues of B= eig(A^2+3*A+2*eye(3));
% Understand the property it is satisfying
```

OUTPUT :

```
>>
A =
     3     0    -1
     0     1     0
     2     0     0

Eigenvalues_of_A =

     2
     1
     1
```

```
X =
    0.7071    0.4472         0
         0         0    1.0000
    0.7071    0.8944         0

D =
     2     0     0
     0     1     0
     0     0     1

p =

     1    -4     5    -2
```

```
r =

   2.0000
   1.0000 + 0.0000i
   1.0000 - 0.0000i

sum_of_eigenvalues =     4.0000

Trace_of_A =     4

product_of_eigenvalues_A =     2.0000

Determinant_of_A =   2

Inverse_A =

         0         0    0.5000
         0    1.0000         0
   -1.0000         0    1.5000
```

```
Eigenvalues_of_invA =

    0.5000
    1.0000
    1.0000

Eigenvalues_Transpose_of_A =

     2
     1
     1

Eigenvalues_of_B =

    12
     6
     6
```

STRESS ANALYSIS

• The number of components and some other transformation properties, the stress can be expressed as a 3 x 3 matrix

• Since the shearing stresses have the equalities , the stress matrix is symmetric.

• If we changed the orientation of a particular plane the normal stress component will vary

• There exists a special orientation where the normal stress will be a maximum, and these are called principal planes and the normal stresses acting on them are called the principal stresses

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

• The general three-dimensional case, the theory to determine principal stresses and the planes on which they act is formulated by the eigenvalue problem $[\sigma]\{n\} = \lambda\{n\}$

$$\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{zx} = \tau_{xz}$$

• where σ is the stress matrix, {n} is the principal direction vector and λ (the eigenvalue) is the principal stress. Thus solving the eigenvalue problem will determine up to three distinct principal stresses and the corresponding three principal directions. It turns out for this application (3x3, symmetric real matrix) the principal directions are mutually orthogonal.

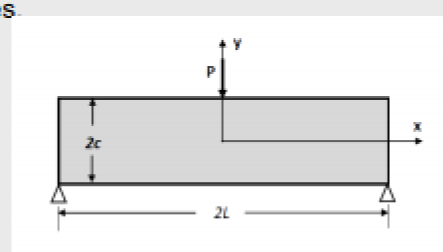
• The shear stress components will vanish on these three principal planes and so for a coordinate system that is aligned with the principal directions the stress matrix takes on the simplified diagonal form

INPUT(Practice Problem) :

Q If the individual stresses in the beam at any point (x,y) are given by

$$\sigma_x = -\frac{3P}{4c^3}(L-x)y, \sigma_y = 0, \tau_{xy} = -\frac{3P}{4c^3}\left(c^2 - y^2\right)$$

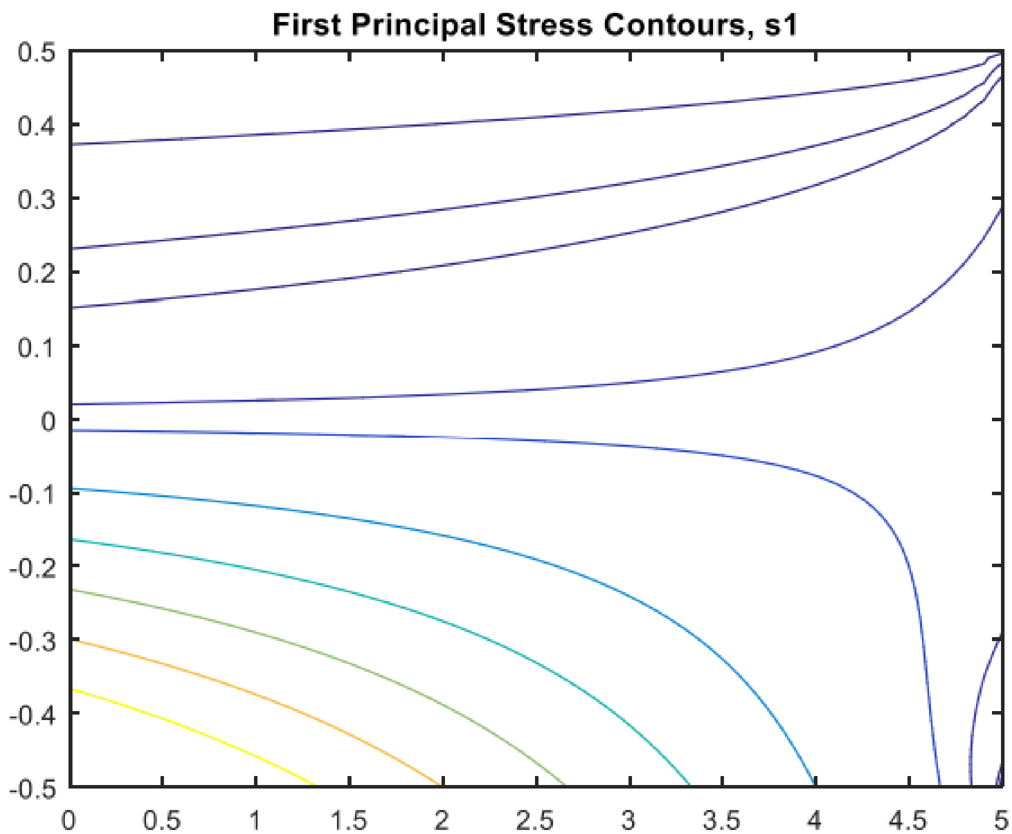
Where P denotes the force 2c denotes the height and 2L denotes the length of the beam. Write the stress matrix and hence calculate the principal stresses.



Draw the stress distribution in the beam using contour plot

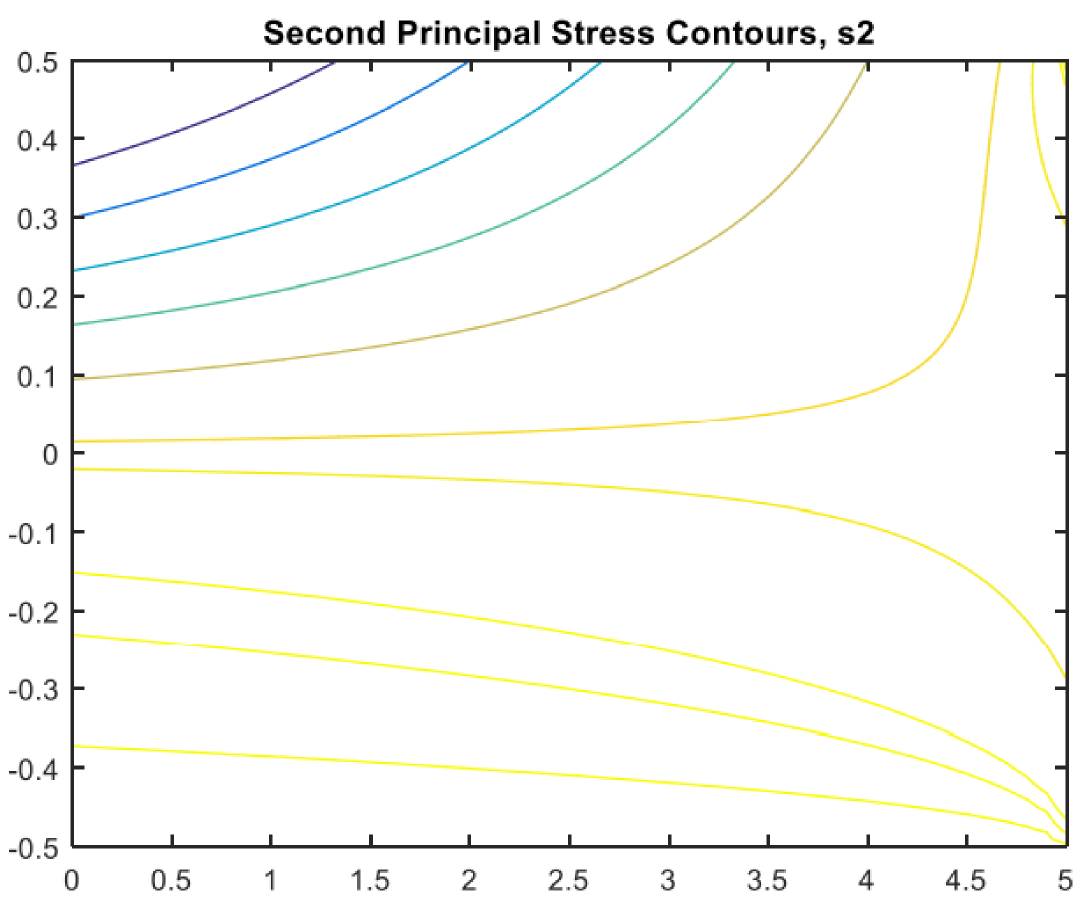
```
clc

clear all
clf
c=0.5;P=1;L=5;
x=0:0.1:L;
y=-c:0.01:c;
[X,Y]=meshgrid(x,y);
sx=-(3/(4*c^3))*(L-x).*Y;
sy=zeros(length(y),length(x));
txy=-(3/(c^3))*(c^2-Y.^2);
%Create Stress Matrix
for i=1:length(y)
    for j=1:length(x)
        s=[sx(i,j),txy(i,j),txy(i,j),sy(i,j)];
        p=eig(s);
        s1(i,j)=p(2);
        s2(i,j)=p(1);
    end
end
%Plot distributions of principle stresses s1 and s2
figure(1)
contour(X,Y,s1,[0.01,0.05,0.1,0.5,1,3,5,7,9,11])
title('First Principle Stress Contours, s1')
axis tight
figure(2)
contour(X,Y,s1,[-0.01,-0.05,-0.1,-0.5,-1,-3,-5,-7,-9,-11])
axis tight
title('Second Principle Stress Contours, s2')
```

OUTPUT :



First Principal Stress Contours, s1
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

:



Second Principal Stress Contours, s2
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

INPUT (Practice Problem)

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$ Find

```
%Code for calculating eigenvalues of matrix
clc
A=[1 2 1; 6 -1 0; -1 -2 -1];
Eigenvalues of A=eig(A);
[X,D]= eig(A);
p=poly(A); % Coefficients of the characteristic polynomial
r = roots(p); % To find the roots of the characteristic
sum of eigenvalues = sum(r);
Trace of A=trace(A); %Note that sum of eigenvalues is Trace
of A
product of eigenvalues of A=prod(r);
Determinant of A =det(A);
Inverse of A = inv(A);
Eigenvalues of invA= eig(inv(A));
Eigenvalues Transpose of A = eig(A);
% Note that eigenvalues of A and Transpose of A are same
Eigenvalues of B= eig(A^2+3*A+2*eye(3));
% Understand the property it is satisfying
```

OUTPUT

```
>> A =[1 2 1; 6 -1 0; -1 -2 -1]

A =

     1     2     1
     6    -1     0
    -1    -2    -1
```

```
[V,D]=eigs(A)

V =

    0.4082   -0.4851   -0.0697
   -0.8165   -0.7276   -0.4180
   -0.4082    0.4851    0.9058


D =

   -4.0000         0         0
         0    3.0000         0
         0         0    0.0000


  Sum of eigenvalues = -1.000


  Product of eigenvalues = -12.000
```