MAT1002 Experiment **20/03/2021**

Name: Khan Mohd. Owais Raza
Registration No. : 20BCD7138

Aim: To find the power series solution and visualize it for the second order initial value problem with ordinary point.

Problem using MATLAB: Series Solutions for ODE

Question :

Find the series solution of the second order ode

$y''+y=0, y(0)=1, y'(0)=1$

Assume $y=\sum_{n=0}^{\infty} a_n x^n$ then $y(0)=1=a_0$,

$y'(0)=1=a_1$ and visualize it with exact solution.

Recurrence relation

$$a_n = -\frac{a_{n-2}}{n(n-1)}; n = 2,3,4,...$$

Exact solution is

$$y_{ex} = \cos(x) + \sin(x)$$

MATLAB code :

```
clc
clear all
syms x
n=input('Enter the number of terms');
for i=3:n
    a(1)=1;
    a(2)=1;
    a(i)=-a(i-2)/(i)*(i-1);
end
y_s=poly2sym(fliplr(a),x);
y_ex=dsolve('D2y+y=0', 'y(0)=1', 'Dy(0)=1','x');
figure(1)
h1=ezplot(y_s);
hold on
h2=ezplot(y_ex);
hold off
set(h1, 'color', 'g', 'linestyle', '—')
legend('Series  sol.','Exact sol.');
```

OUTPUT :

```
Enter the number of terms

100
y_s=poly2sym(fliplr(a),x)


y_s =


- (5735008957433605*x^99)/36028797018963968 -
(4526842849830837*x^98)/36028797018963968 +
(5792938340842025*x^97)/36028797018963968 +
(4573035123808703*x^96)/36028797018963968 -
(2926329677332569*x^95)/18014398509481984 -
(4620670906348377*x^94)/36028797018963968 +
(5914266295240561*x^93)/36028797018963968 +
(583728372477521*x^92)/4503599627370496 -
(5977860556479707*x^91)/36028797018963968 -
(4720585968731257*x^90)/36028797018963968 +
(3021775665912819*x^89)/18014398509481984 +
```

(2386518461969691*x^88)/18014398509481984 -
(3055728201484873*x^87)/18014398509481984 -
(4827275979893239*x^86)/36028797018963968 +
(6181703028291237*x^85)/36028797018963968 +
(2441703547969255*x^84)/18014398509481984 -
(3127214473135567*x^83)/18014398509481984 -
(4941542894699683*x^82)/36028797018963968 +
(3164891755944429*x^81)/18014398509481984 +
(312612850807983*x^80)/2251799813685248 -
(1601982246836069*x^79)/9007199254740992 -
(1266082045772331*x^78)/9007199254740992 +
(1622260503125133*x^77)/9007199254740992 +
(5129255467487905*x^76)/36028797018963968 -
(6573315285390149*x^75)/36028797018963968 -
(5196745671007483*x^74)/36028797018963968 +
(6660959489195351*x^73)/36028797018963968 +
(658371495482367*x^72)/4503599627370496 -
(6752205509595287*x^71)/36028797018963968 -
(2670062176122933*x^70)/18014398509481984 +
(6847306995645925*x^69)/36028797018963968 +
(1354102960748059*x^68)/9007199254740992 -
(1736635832229039*x^67)/9007199254740992 -
(2748032479165179*x^66)/18014398509481984 +
(7050223080094009*x^65)/36028797018963968 +
(1394834667455053*x^64)/9007199254740992 -
(3579344025278497*x^63)/18014398509481984 -
(5666515836536153*x^62)/36028797018963968 +
(3636159009806727*x^61)/18014398509481984 +
(2878955626627239*x^60)/18014398509481984 -
(7391536347803839*x^59)/36028797018963968 -
(5853876440808719*x^58)/36028797018963968 +
(1879204156221315*x^57)/9007199254740992 +
(2977402672480297*x^56)/18014398509481984 -
(956086325095055*x^55)/4503599627370496 -
(6061141154692033*x^54)/36028797018963968 +
(121683714103007*x^53)/562949953421312 + (771673063676069*x^52)/4503599627370496
- (61989816618513*x^51)/281474976710656 -
(6292103442281793*x^50)/36028797018963968 + (15801325804719*x^49)/70368744177664
+ (6417945511127429*x^48)/36028797018963968 - (8061900920775*x^47)/35184372088832
- (6551652709275917*x^46)/36028797018963968 + (514589420475*x^45)/2199023255552 +
(6694079942086263*x^44)/36028797018963968 - (263012370465*x^43)/1099511627776 -

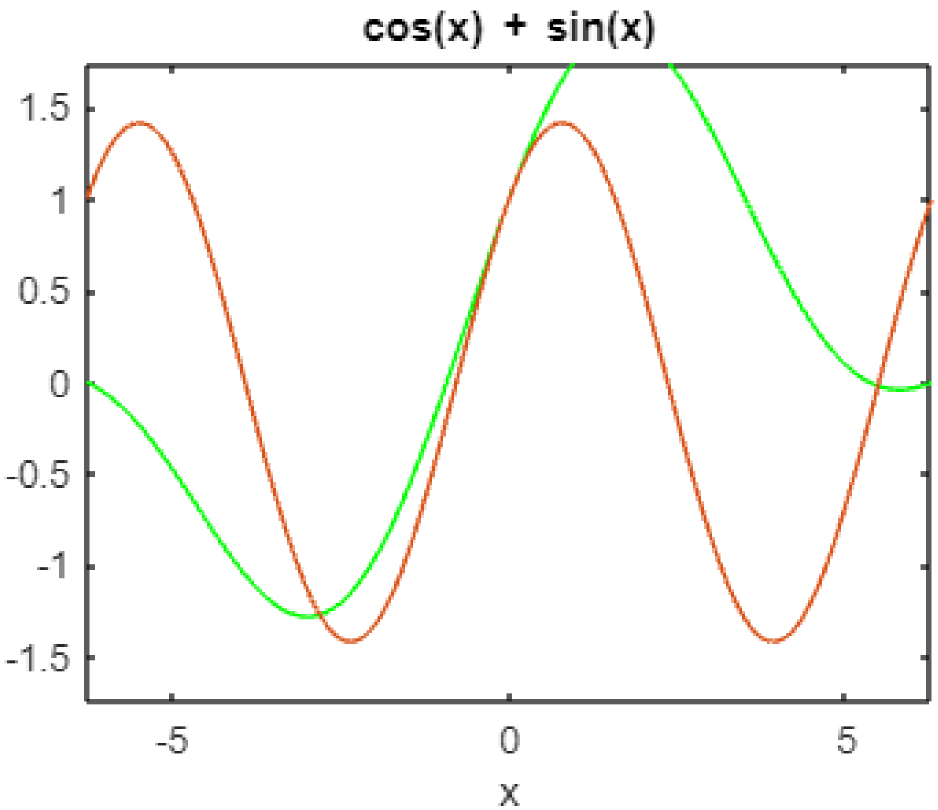
```
(106972158165441*x^42)/562949953421312 + (67282234305*x^41)/274877906944 +
(3504611657991591*x^40)/18014398509481984 - (34461632205*x^39)/137438953472 -
(3592226949441381*x^38)/18014398509481984 + (4418157975*x^37)/17179869184 +
(3686759237584575*x^36)/18014398509481984 - (2268783825*x^35)/8589934592 -
(7578338432812737*x^34)/36028797018963968 + (583401555*x^33)/2147483648 +
(3900615369830085*x^32)/18014398509481984 - (300540195*x^31)/1073741824 -
(4022509600137275*x^30)/18014398509481984 + (9694845*x^29)/33554432 +
(8313186506950369*x^28)/36028797018963968 - (5014575*x^27)/16777216 -
(8610086025055739*x^26)/36028797018963968 + (1300075*x^25)/4194304 +
(1117655397483197*x^24)/4503599627370496 - (676039*x^23)/2097152 -
(4656897489513321*x^22)/18014398509481984 + (88179*x^21)/262144 +
(4868574648127563*x^20)/18014398509481984 - (46189*x^19)/131072 -
(5112003380533941*x^18)/18014398509481984 + (12155*x^17)/32768 +
(5396003568341383*x^16)/18014398509481984 - (6435*x^15)/16384 - (2048*x^14)/6435
+ (429*x^13)/1024 + (1024*x^12)/3003 - (231*x^11)/512 - (256*x^10)/693 +
(63*x^9)/128 + (128*x^8)/315 - (35*x^7)/64 - (16*x^6)/35 + (5*x^5)/8 + (8*x^4)/15
- (3*x^3)/4 - (2*x^2)/3 + x + 1

y_ex=dsolve('D2y+y=0', 'y(0)=1', 'Dy(0)=1','x')
y_ex =

cos(x) + sin(x)


h1=ezplot(y_s)


h1 =


  Line with properties:


              Color: [0 0.4470 0.7410]
          LineStyle: '-'
          LineWidth: 0.5000
             Marker: 'none'
         MarkerSize: 6
    MarkerFaceColor: 'none'
              XData: [1×308 double]
              YData: [1×308 double]
              ZData: [1×0 double]
```

```
AlignVertexCenters: off
        Annotation: [1×1 matlab.graphics.eventdata.Annotation]
      BeingDeleted: off
        BusyAction: 'queue'
     ButtonDownFcn: ''
          Children: [0×0 GraphicsPlaceholder]
          Clipping: on
             Color: [0 0.4470 0.7410]
         ColorMode: 'auto'
       ContextMenu: [0×0 GraphicsPlaceholder]
         CreateFcn: ''
    DataTipTemplate: [1×1 matlab.graphics.datatip.DataTipTemplate]
         DeleteFcn: ''
       DisplayName: ''
  HandleVisibility: 'on'
           HitTest: on
     Interruptible: on
          LineJoin: 'round'
         LineStyle: '-'
     LineStyleMode: 'auto'
         LineWidth: 0.5000
            Marker: 'none'
   MarkerEdgeColor: 'auto'
   MarkerFaceColor: 'none'
     MarkerIndices: [1×308 uint64]
        MarkerMode: 'auto'
        MarkerSize: 6
            Parent: [1×1 Axes]
     PickableParts: 'visible'
          Selected: off
 SelectionHighlight: on
       SeriesIndex: 1
               Tag: ''
              Type: 'line'
          UserData: []
           Visible: on
             XData: [1×308 double]
         XDataMode: 'manual'
       XDataSource: ''
             YData: [1×308 double]
       YDataSource: ''
```

```
ZData: [1×0 double]
ZDataSource: ''
```

OUTPUT :

For n=100



Question :

Find the series solution

$y''+xy=0, y(0)=1, y'(0)=1$

Recurrence relation is

$$a_{n+2} = -\frac{a_{n-1}}{(n+2)(n+1)}, a_0 = y(0) = 1, a_1 = y'(0) = 1$$

Also $a_2 = 0$

INPUT(MATLAB Code):

```
clc
clear all
syms x
   n=input('Enter the number of terms');
   for i=4:n
          a(1)=1;
          a(2)=1;
          a(3)=0;
          a(i)=-(a(i-3)/((i)*(i-1)));
     end
      y_s=poly2sym(fliplr(a),x);
```

OUTPUT:

```
Enter the number of terms
25
 y_s=poly2sym(fliplr(a),x)

y_s =

(2015632321182053*x^24)/32451855365842672678315602057 6256 -
(540486010400375*x^22)/633825300114114700748351602688 -
(4724138252770437*x^21)/1267650600228229401496703205376 +
(4273217519727965*x^19)/990352031428304219919299 3792 +
(1065699156630831*x^18)/618970019642690137449562112 -
(3171528627923099*x^16)/193428131138340667952988 16 -
(5694829868246003*x^15)/967140655691703339764940 8 +
(6739498334336585*x^13)/151115727451828646838272 +
(1334725750370157*x^12)/944473296573929042739 2 -
(598920262133427*x^10)/73786976294838206464 - x^9/45360 + x^7/1120 + x^6/504 -
x^4/20 - x^3/12 + x + 1
```

```
a(i)=-(a(i-3)/((i)*(i-1)))

a =

  Columns 1 through 20

    1.0000    1.0000         0   -0.0833   -0.0500         0    0.0020    0.0009
0   -0.0000   -0.0000         0    0.0000    0.0000         0   -0.0000   -0.0000
0    0.0000    0.0000

  Columns 21 through 25

         0   -0.0000   -0.0000         0    0.0000

a(1)=1

a =

  Columns 1 through 20

    1.0000    1.0000         0   -0.0833   -0.0500         0    0.0020    0.0009
0   -0.0000   -0.0000         0    0.0000    0.0000         0   -0.0000   -0.0000
0    0.0000    0.0000

  Columns 21 through 25

         0   -0.0000   -0.0000         0    0.0000

a(2)=1

a =

  Columns 1 through 20

    1.0000    1.0000         0   -0.0833   -0.0500         0    0.0020    0.0009
0   -0.0000   -0.0000         0    0.0000    0.0000         0   -0.0000   -0.0000
0    0.0000    0.0000
```

```
  Columns 21 through 25

         0   -0.0000   -0.0000         0    0.0000

a(3)=0

a =

  Columns 1 through 20

    1.0000    1.0000         0   -0.0833   -0.0500         0    0.0020    0.0009
0   -0.0000   -0.0000         0    0.0000    0.0000         0   -0.0000   -0.0000
0    0.0000    0.0000

  Columns 21 through 25

         0   -0.0000   -0.0000         0    0.0000
```

Aim: To visualize Legendre polynomials up to order 6

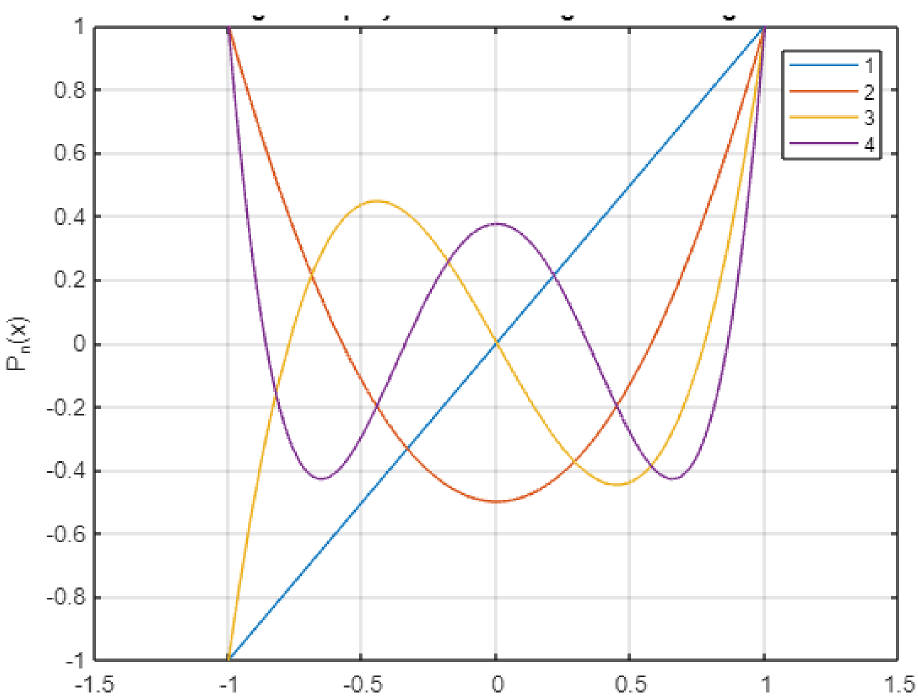
Bonnet's Recurrence relation

$P_0(x) = 1, P_1(x) = x$

$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$

INPUT (MATLAB Code):

```
clc
clear all
X=linspace(-1,1,500);
X=X(:);
N=6;
Y=zeros(numel(X),N);
Y(:,1)=1;
Y(:,2)=X;
for n=1:(N-1)
c(n)=(2*n+1)/(n+1);
d(n)=n/(n+1);
Y(:,n+2)=c(n)*(X.*Y(:,n+1))-d(n)*Y(:,n);
end
figure(1)
plot(X,Y(:,1:6), 'linewidth',1.5)
legend('P_{0}','P_{1}','P_{2}','P_{3}','P_{4}','P_{5}',
'location', 'best')

clc
clear all
syms x y
fplot(legendreP(1:4, x))
axis([-1.5 1.5 -1 1])
grid on
ylabel('P_n(x)')
title('Legendre polynomials of degrees 1 through 4')
legend('1','2','3','4','5','6','Location','best')
```



Aim: Visualize the Bessel function of first kind up to order 5

Bessel function of first kind are the solutions of the differential equation

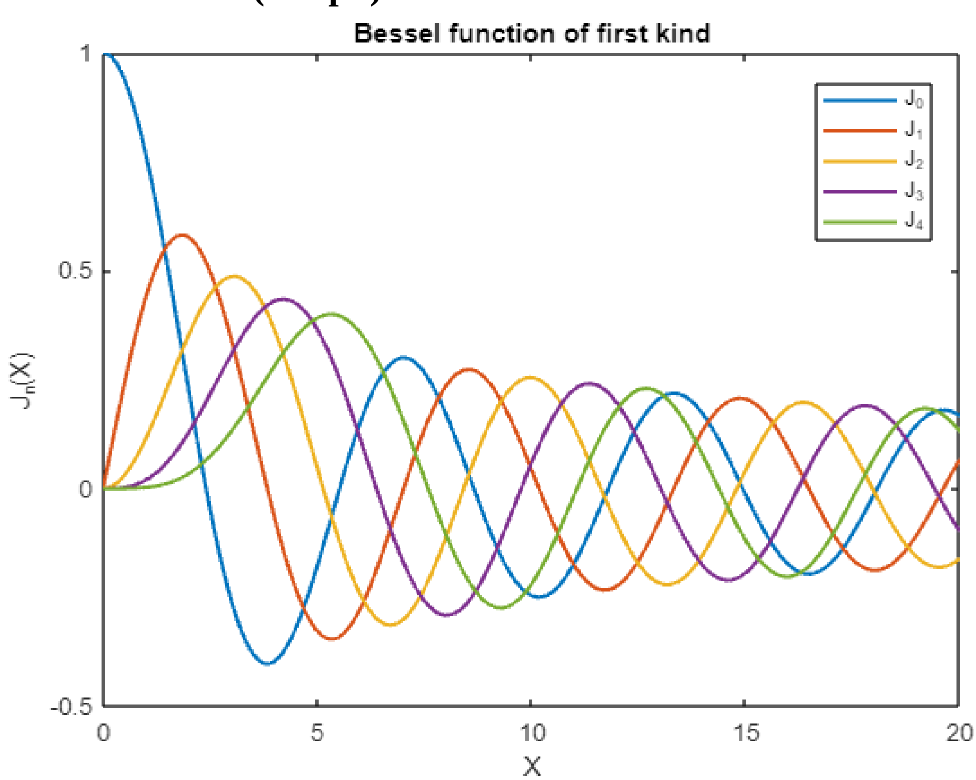
$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right)y = 0$$

where the solutions are finite at the origin

INPUT (MATLAB Code):

```
clc
clear all
X=(0:0.1:20);
J=zeros(length(X),5);
for i=0:4
J(:,i+1)=besselj(i,X);
end
figure (1)
plot(X, J(:,1:5), 'linewidth',1.5)
title('Bessel function of first kind');
xlabel('X');
ylabel('J_{n}(X)');
legend('J_{0}','J_{1}','J_{2}','J_{3}','J_{4}','locatio
n', 'best')
```

OUTPUT (Graph):



Aim : Visualize Bessel function of second kind up to order 5

Bessel function of second kind $Y_n(x)$ are the solutions of the second order ODE

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right)y = 0$$

where the solutions are singular at the origin.

INPUT (MATLAB Code):

```
clc
clear all
X=(0.1 :0.1:10);
Y=zeros(length(X),5);
for i=0:4
Y(:,i+1)=bessely(i,X);
end
plot(X,Y(:,1:5),'linewidth',1.5);
xlabel('X');
ylabel('Y_{n}(x)')
title('Bessel function of second kind');
legend('Y_{0}','Y_{1}','Y_{2}','Y_{3}','Y_{4}','locatio
n', 'best');
axis([0.1 10 -2 2])
```

OUTPUT (Graph):