Spectral Analysis Tool

The `SpectralAnalyzer` class, implemented in C# within the `EstuarineCirculationModeli` namespace, provides a graphical user interface (GUI) for spectral and Empirical Orthogonal Function (EOF)/Proper Orthogonal Decomposition (POD) analysis of estuarine circulation data. It processes time series (e.g., Richardson number, velocity, water level) and spatial-temporal data (e.g., salinity, velocity profiles) to analyze tidal dynamics, turbulence, and mixing events.

Simulation Logic

The class simulates data collection and analysis through a timer-driven loop (`UpdateSimulation`), with a default interval of 100 ms. The simulation progresses until the specified duration ($T = 86400$ s, one day) is reached, collecting data at a user-defined sampling rate ($\Delta t = 3600$ s, one sample per hour). The GUI allows users to:

- Select analysis type: Spectral Analysis or EOF/POD Analysis.
- Choose variables: Richardson number, velocity, water level (for spectral analysis); salinity, velocity profile (for EOF/POD).
- Set plot scale (linear or log-log), window type (Hanning, Hamming, Blackman, Rectangular), and EOF/POD mode (1, 2, or 3).
- Control simulation via Analyze, Pause, Reset, and Clear buttons.

Data is stored in:

- `timeSeriesData`: List of scalar values for spectral analysis.
- `salinityData`: List of 2D arrays $[100, 1]$ for salinity (10x10 grid flattened).
- `velocityData`: List of 2D arrays $[10, 1]$ for velocity profiles (10 depth levels).
- `mixingEvents`: List of detected mixing events with time and value.

The simulation logic:

1. **Initialization**: Clears data, sets `currentTime = 0`, and starts the timer (`isAnalyzing = true`).
2. **Data Generation**: At each timestep ($t \leftarrow t + \Delta t$):
   - For spectral analysis, generates synthetic data for the selected variable (e.g., Richardson number) using `GenerateSyntheticData`.
   - For EOF/POD, generates salinity (`GenerateSyntheticSalinityData`) or velocity (`GenerateSyntheticVelocityData`).
   - Detects mixing events by computing the standard deviation over a window (size 5) and comparing against thresholds (0.2 for Richardson number, 0.03 for velocity, 0.5 for water level).

3. **Analysis**: When `currentTime` reaches `duration`, the timer stops, and `PerformAnalysi`
   triggers either `PerformSpectralAnalysis` or `PerformEOFAnalysis`.
4. **Visualization**: Updates two panels:
   - `visualizationPanel`: Plots power spectral density (PSD) or time series (spectral analysis), or spatial modes (EOF/POD).
   - `heatmapPanel`: Plots time-frequency heatmap (spectral analysis) or temporal coefficients (EOF/POD).

Physical and Mathematical Models

Synthetic Data Generation

Synthetic data mimics estuarine dynamics with tidal and inertial influences:

- **Richardson Number**: Models stratification stability:

$$Ri(t) = 0.5 + 0.3\sin\left(\frac{2\pi t}{T_{\text{tidal}}}\right) + 0.2\sin\left(\frac{2\pi t}{T_{\text{inertial}}}\right) + \text{noise} \tag{1}$$

  where $T_{\text{tidal}} = 43200\,\text{s}$ (12-hour semi-diurnal tide), $T_{\text{inertial}} = 17 \times 3600\,\text{s}$ (inertial period at 45° latitude), and noise is a random jump (~ 0.5 with 2% probability).

- **Velocity**: Represents tidal-driven flow:

$$u(t) = 0.1 + 0.05\sin\left(\frac{2\pi t}{T_{\text{tidal}}}\right) + 0.02\sin\left(\frac{2\pi t}{T_{\text{tidal}}/2}\right) + \text{noise} \tag{2}$$

  where noise is random (~ 0.01).

- **Water Level**: Combines tidal constituents:

$$\eta(t) = 1.0\sin\left(\frac{2\pi t}{T_{\text{M2}}}\right) + 0.5\sin\left(\frac{2\pi t}{T_{\text{K1}}}\right) \tag{3}$$

  where $T_{\text{M2}} = 44712\,\text{s}$ (M2 tide), $T_{\text{K1}} = 86148\,\text{s}$ (K1 tide).

- **Salinity**: Models spatial-temporal variations:

$$S(x, y, t) = \left[30.0 + 2.0\sin\left(\frac{2\pi t}{T_{\text{M2}}}\right)\right](1.0 - 0.1(x + y)) + \text{noise} \tag{4}$$

  on a 10x10 grid, with noise (~ 0.5).

- **Velocity Profile**: Depth-dependent tidal flow:

$$u(z, t) = \left[0.1\sin\left(\frac{2\pi t}{T_{\text{M2}}}\right)\right](1.0 - 0.1z) + \text{noise} \tag{5}$$

  for 10 depth levels, with noise (~ 0.01).

Spectral Analysis

Spectral analysis uses Welch's method to estimate the power spectral density (PSD):

1. **Segmentation**: Divides the time series into overlapping segments (length $N = 2^{\lfloor \log_2 \min(256, |\text{data}|) \rfloor}$, overlap $N/2$).
2. **Windowing**: Applies a window function:
   - Hanning: $w(n) = 0.5\left(1 - \cos\left(\frac{2\pi n}{N-1}\right)\right)$
   - Hamming: $w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$
   - Blackman: $w(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$
   - Rectangular: $w(n) = 1.0$

   Window power: $P_w = \frac{1}{N}\sum_{n=0}^{N-1} w(n)^2$.
3. **FFT**: Computes the Fast Fourier Transform (FFT) for each segment using the Cooley-Tukey algorithm:

$$X(k) = \sum_{n=0}^{N-1} x(n)w(n)e^{-i\frac{2\pi kn}{N}}, \quad k = 0, \ldots, \frac{N}{2} \tag{6}$$

4. **PSD**: Calculates power spectrum:

$$\text{PSD}(k) = \frac{1}{N_s P_w \Delta t} \sum_{\text{segments}} |X(k)|^2 \tag{7}$$

   where N_s is the number of segments, and frequencies are $f_k = \frac{k}{N\Delta t}$.
5. **Short-Time Fourier Transform (STFT)**: Computes time-frequency heatmap:

$$\text{STFT}(k, s) = \left| \sum_{n=0}^{N-1} x(n + s(N - \text{overlap}))w(n)e^{-i\frac{2\pi kn}{N}} \right|^2 \frac{1}{P_w \Delta t} \tag{8}$$

   for segment s.
6. **Turbulence Detection**: Checks for a $-5/3$ spectral slope in the log-log PSD (first 10 frequencies, $f > 10^{-4}$ Hz):

$$\text{slope} = \frac{\sum(\log f_i - \bar{\log f})(\log \text{PSD}_i - \bar{\log \text{PSD}})}{\sum(\log f_i - \bar{\log f})^2} \tag{9}$$

   Turbulence is detected if $|\text{slope} + \frac{5}{3}| < 0.5$.
7. **Tidal Constituents**: Identifies M2 ($f_{\text{M2}} = \frac{1}{44712}$ Hz) and K1 ($f_{\text{K1}} = \frac{1}{86148}$ Hz) frequencies within $2\Delta f = \frac{2}{N\Delta t}$.

EOF/POD Analysis

EOF/POD decomposes spatial-temporal data into orthogonal modes:

1. **Data Matrix**: For salinity ($100 \times T$) or velocity ($10 \times T$), where T is the number of time steps. Mean subtraction per spatial point:

$$X(s,t) \leftarrow X(s,t) - \frac{1}{T} \sum_{t=1}^{T} X(s,t) \tag{10}$$

2. **Singular Value Decomposition (SVD)**: Approximated via power iteration for the top 3 modes:
   - Initialize random vector $\mathbf{v}$, normalize: $\mathbf{v} \leftarrow \frac{\mathbf{v}}{\|\mathbf{v}\|}$.
   - Iterate (20 times):

$$\mathbf{u} = X\mathbf{v}, \quad \mathbf{u} \leftarrow \frac{\mathbf{u}}{\|\mathbf{u}\|} \tag{11}$$

$$\mathbf{v} \leftarrow X^T\mathbf{u}, \quad \mathbf{v} \leftarrow \frac{\mathbf{v}}{\|\mathbf{v}\|} \tag{12}$$

   - Compute singular value: $\sigma = \|X\mathbf{v}\|$, spatial mode: $\mathbf{u} = \frac{X\mathbf{v}}{\sigma}$, temporal coefficients: $\mathbf{v}$.
   - Deflate: $X \leftarrow X - \sigma\mathbf{u}\mathbf{v}^T$.

3. **Explained Variance**: For singular values σ_i:

$$\text{Variance}_i = \frac{\sigma_i^2}{\sum \sigma_j^2} \times 100\% \tag{13}$$

Mixing Events

Mixing events are detected by computing the standard deviation over a window of 5 samples:

$$\sigma = \sqrt{\frac{1}{5} \sum_{i=1}^{5} (x_i - \bar{x})^2} \tag{14}$$

A mixing event is flagged if σ exceeds the threshold (0.2 for Richardson number, 0.03 for velocity, 0.5 for water level).

Visualization

- **Spectral Analysis**:
  - `visualizationPanel`: Plots PSD (linear or log-log) or time series with mixing events (red dots). M2 and K1 frequencies are marked.
  - `heatmapPanel`: Displays STFT heatmap, with power scaled as $\log_{10}(\text{STFT})$ (log-log) or linear, using red-blue color gradient.

- **EOF/POD Analysis**:
  - `visualizationPanel`: For salinity, plots spatial mode on a 10x10 grid (red-blue gradient); for velocity, plots mode vs. depth.
  - `heatmapPanel`: Plots temporal coefficients vs. time.