Adaptive Mesh Refinement for Estuarine Modeling

The `AdaptiveMeshRef` class, implemented in C# within the `EstuarineCirculationModelin` namespace, simulates estuarine circulation with adaptive mesh refinement (AMR) for salinity, temperature, sediment, and velocity fields. It dynamically refines a 2D grid (400×100) based on gradients in key variables, improving resolution in regions of high variability (e.g., estuarine turbidity maximum, fronts) while modeling tidal, wind, riverine, and Coriolis effects.

Simulation Logic

The class operates on a base grid ($N_x = 400$, $N_z = 100$) over an estuary of specified length and depth, with refinement applied to cells exhibiting high gradients. Key features include:

- **Initialization**: Stores input profiles for salinity, temperature, sediment, horizontal (u) and vertical (w) velocities, and bedload, along with physical parameters (e.g., viscosity, diffusivities, settling velocity, tidal and wind forcing).
- **Data Structures**:
  - Base grid arrays ($[400, 100]$) for salinity, temperature, sediment, and velocities.
  - `bedloadProfile` ($[400]$) for bottom sediment transport.
  - `refineFlags` ($[400, 100]$) to mark cells for 2×2 refinement.
  - Dictionaries (`fineSalinityGrids`, etc.) for refined 2×2 subgrids.
- **Refinement**: The `RefineGrid` method flags cells with gradient magnitudes in the top 10% (threshold factor 0.9) and initializes fine grids using bilinear interpolation.
- **Update**: The `UpdateFields` method advances fields using smaller timesteps ($\Delta t/4$) in refined regions, applying boundary conditions and updating coarse grid values by averaging fine grid results.

Simulation steps:

1. **Gradient-Based Refinement**: Computes gradients for salinity, temperature, and u-velocity, flagging cells where the combined gradient magnitude exceeds a threshold.
2. **Field Updates**: Updates refined regions with substeps, then updates coarse grid and bedload.
3. **Access Methods**: Provides `GetSalinity`, `GetTemperature`, `GetSediment`, `GetUVelocity`, and `GetWVelocity` to retrieve values from fine or coarse grids.

Physical and Mathematical Models

Grid Refinement

The base grid has spacings $\Delta x = L/(N_x - 1)$, $\Delta z = H/(N_z - 1)$, where L is estuary length and H is depth. Refined cells use $\Delta x_{\text{fine}} = \Delta x/2$, $\Delta z_{\text{fine}} = \Delta z/2$, and timestep $\Delta t_{\text{fine}} = \Delta t/4$. Gradient magnitude for cell (i, j) is:

$$G = \sqrt{\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2 + \left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2} \tag{1}$$

where $\frac{\partial S}{\partial x} \approx \frac{S_{i+1,j} - S_{i-1,j}}{2\Delta x}$, etc. Cells with G in the top 10% are refined.

Velocity Update

Horizontal velocity u in refined cells is updated using:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - w\frac{\partial u}{\partial z} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - fw + \frac{u_{\text{tidal}} - u}{T_{\text{tidal}}} + u_{\text{wind}} + \xi \tag{2}$$

where ν is kinematic viscosity, f is the Coriolis parameter, $u_{\text{tidal}} = A\cos(2\pi t/T_{\text{tidal}})$, T_{tidal} is tidal period, A is tidal amplitude, $u_{\text{wind}} = \frac{\tau_{wx}}{\rho_0 \nu} e^{-z/\delta}$ for surface cells ($\delta = \sqrt{2\nu/|f|}$), and $\xi = (k_w \tau_w (1 - z/H) + I) A (r - 0.5)$ adds wind mixing and turbulence ($r \sim \text{Uniform}[0, 1]$). Vertical velocity w is updated similarly:

$$\frac{\partial w}{\partial t} = -u\frac{\partial w}{\partial x} - w\frac{\partial w}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + fu + w_{\text{wind}} + \xi \tag{3}$$

Boundary conditions:

- River ($x = 0$): $u = Q(t)$, $w = 0$.
- Ocean ($x = L$): $u = u_{\text{tidal}}$, $w = 0$.
- Bottom ($z = 0$): $u = w = 0$.
- Surface ($z = H$): $u = u_{\text{tidal}} + \tau_{wx}/(\rho_0 \nu)$, $w = \tau_{wz}/(\rho_0 \nu)$.

Wind stress is $\tau_w = \rho_a C_d U_w^2$, with $U_w = U_{\text{wind}}|\sin(2\pi t/T_{\text{wind}})|$, $\rho_a = 1.225\,\text{kg/m}^3$, $C_d = 0.0012$.

Salinity and Temperature Update

Salinity S is updated using:

$$\frac{\partial S}{\partial t} = -u\frac{\partial S}{\partial x} - w\frac{\partial S}{\partial z} + \nu\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial z^2}\right) \tag{4}$$

Temperature T uses thermal diffusivity κ:

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - w\frac{\partial T}{\partial z} + \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

Boundary conditions:

- River: $S = 10(1 - Q(t)/Q_{\text{max}})$, $T = 14 - 4Q(t)/Q_{\text{max}}$.
- Ocean: $S = 35$, $T = 15$.
- Bottom: $\partial S/\partial z = 0$, $\partial T/\partial z = 0$.
- Surface: $\partial S/\partial z = 0$, $T = 15$.

River discharge is $Q(t) = 0.1 + A_r \sin(2\pi t/T_r)$, A_r is river discharge amplitude, T_r is river period.

Sediment Transport

Suspended sediment concentration C is updated with settling:

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} - (w + w_s)\frac{\partial C}{\partial z} + D_s\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2}\right) \tag{6}$$

where w_s is settling velocity, D_s is sediment diffusivity, and $C \geq 0$. Boundary conditions:

- River: $C = 200Q(t)/Q_{\text{max}}$.
- Ocean: $C = 0$.
- Bottom: $\partial C/\partial z = 0$.
- Surface: $\partial C/\partial z = 0$.

Bedload Transport

Bedload q_b at $z = 0$ is updated using:

$$\frac{\partial q_b}{\partial t} = E - D - \frac{\partial q_b}{\partial x} \tag{7}$$

where erosion rate $E = \alpha_e u_*^2$, $u_* = \sqrt{|\tau_b|/\rho_0}$, $\tau_b = \rho_0 \nu (\partial u/\partial z)|_{z=0}$, deposition rate $D = -w_s C(x, z = 1)$, and bedload flux:

$$q_b = \begin{cases} 8\left(\theta - \theta_c\right)^{1.5}\sqrt{\left(\frac{\rho_s}{\rho_0} - 1\right) g d^3}, & \theta > \theta_c \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

where $\theta = \tau_b/((\rho_s - \rho_0) g d)$, $\theta_c = 0.047$, $\rho_s = 2650\,\text{kg/m}^3$, $d = 0.001\,\text{m}$, $\alpha_e = 0.0001$.

Notes

- The model uses central differencing for spatial derivatives and explicit time integration.
- Fine grids are updated with 2×2 substeps to maintain stability (CFL condition).

- Turbulence and wind mixing introduce stochastic perturbations proportional to tidal amplitude.
- No GUI is implemented, but fields can be accessed for external visualization.