Baroclinic Flow

Core Components and Initialization

The `BaroclinicFlow` class, implemented in C#, models density-driven flows in an estuary, focusing on salinity and temperature transport influenced by baroclinic pressure gradients. The estuary spans a length L and depth H, discretized into n grid points with spatial step $\Delta x = L/n$. Key parameters include:

- Gravitational acceleration: $g = 9.81\,\mathrm{m/s^2}$
- Reference density: $\rho_0 = 1000\,\mathrm{kg/m^3}$
- Base horizontal diffusivity: $K_x = 0.1\,\mathrm{m^2/s}$
- Base vertical diffusivity: $K_z = 0.01\,\mathrm{m^2/s}$
- Mixing rate: $\gamma = 0.005\,\mathrm{s^{-1}}$
- Turbulent length scale: $l = 10\,\mathrm{m}$
- Critical Richardson number: $Ri_c = 0.25$
- Specific heat capacity of water: $C_p = 4184\,\mathrm{J/kg\cdot K}$
- Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2\cdot K^4}$
- Surface albedo: $\alpha = 0.06$
- Emissivity: $\epsilon = 0.97$
- Latent heat of vaporization: $L_v = 2.45 \times 10^6\,\mathrm{J/kg}$
- Bowen ratio: $B = 0.61$
- Wind speed: $U = 5\,\mathrm{m/s}$
- Cloud cover fraction: $C_c = 0.5$
- Atmospheric temperature: $T_a = 15\,°\mathrm{C}$

The class initializes arrays for salinity, temperature, density, and baroclinic pressure gradients, with computations driven by the equation of state for density.

Functioning Logic

The `BaroclinicFlow` class computes:

1. Density using the equation of state via `EqOfState.ComputeDensity`.
2. Baroclinic pressure gradient based on density gradients.
3. Eddy diffusivities (K_x, K_z) modulated by velocity shear and stratification.
4. Surface heat flux incorporating shortwave, longwave, sensible, and latent heat.
5. Salinity and temperature transport using a semi-implicit scheme with Van Leer flux limiter.

The model supports external eddy viscosity inputs for compatibility with turbulence models.

Baroclinic Pressure Gradient

The `ComputeBaroclinicGradient` method calculates the baroclinic pressure gradient:

$$\frac{\partial p_b}{\partial x} \approx g\rho h \frac{\partial \rho/\partial x}{\rho_0} \tag{1}$$

where:

- $\rho = \text{ComputeDensity}(S, T, P)$, with pressure $P = \rho_0 g h / 10^4$ dbar
- $\partial \rho/\partial x = (\rho_{i+1} - \rho_{i-1})/(2\Delta x)$

Boundary conditions set $\partial p_b/\partial x$ at endpoints equal to adjacent interior points.

Eddy Diffusivities

The `ComputeEddyDiffusivities` method computes:

- Horizontal diffusivity:

$$K_x = \nu_t + l\sqrt{\left|\frac{\partial u}{\partial x}\right|}, \quad \frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} \tag{2}$$

- Vertical diffusivity:

$$K_z = \frac{\nu_t + K_z^{\text{base}}}{1 + \max(0, Ri/Ri_c)}, \quad Ri = \frac{g(\partial \rho/\partial x)/\rho_0}{(\partial u/\partial x)^2} \tag{3}$$

where ν_t is the eddy viscosity (default $K_x^{\text{base}} = 0.1\,\text{m}^2/\text{s}$).

Surface Heat Flux

The `ComputeSurfaceHeatFlux` method calculates the total heat flux:

$$Q_{\text{surface}} = Q_{\text{sw}} - Q_{\text{lw}} - Q_{\text{sensible}} - Q_{\text{latent}} \tag{4}$$

where:

- Shortwave: $Q_{\text{sw}} = Q_{\text{solar}}(1-\alpha)(1-C_c)\max(0, \cos(2\pi t/T_d))$, $T_d = 24 \cdot 3600\,\text{s}$
- Longwave: $Q_{\text{lw}} = \epsilon\sigma(T_w^4 - T_a^4)$, with temperatures in Kelvin
- Sensible: $Q_{\text{sensible}} = \rho_{\text{air}} C_{p,\text{air}} C_h U (T_w - T_a) B$
- Latent: $Q_{\text{latent}} = \rho_{\text{air}} L_v C_e U (q_s - q_a)/100$, with q_s, q_a from Tetens formula

The flux is scaled by velocity gradient and converted to a temperature source term:

$$Q_t = \frac{Q_{\text{surface}}(1 + |\partial u/\partial x|)}{\rho_0 C_p h} \tag{5}$$

Transport Equations

Salinity and temperature are updated using:

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (u\phi) = \nabla \cdot (K_x \frac{\partial \phi}{\partial x} + K_z \frac{\partial \phi}{\partial z}) + Q_\phi \tag{6}$$

where ϕ is S or T, and Q_ϕ includes source/sink terms:

- Salinity: $Q_s = -\gamma|\partial u/\partial x|S$ (river, $x < x_s$) or $\gamma|\partial u/\partial x|(S_{\text{ocean}} - S)$ (ocean, $x \geq x_s$)
- Temperature: Q_t from surface heat flux, plus vertical mixing $\alpha_z(T_{\text{ocean}} - T)$

Advection uses the Van Leer limiter:

$$\phi_{\text{interface}} = \phi_i + 0.5\phi_{\text{VL}}(\phi_{i+1} - \phi_i), \quad \phi_{\text{VL}} = \max(0, \min(2r/(1+r), 2)), \quad r = \frac{\phi_i - \phi_{i-1}}{\phi_{i+1} - \phi_i + 10^{-10}} \tag{7}$$

Diffusion is solved semi-implicitly (Crank-Nicolson) using a tridiagonal system:

$$a_i = -\alpha_x, \quad b_i = 1 + 2\alpha_x, \quad c_i = -\alpha_x, \quad \alpha_x = \frac{\kappa_x \Delta t}{2\Delta x^2} \tag{8}$$

$$d_i = \phi_i + \Delta t \left(-\frac{f_{i+1} - f_i}{\Delta x} + Q_\phi + \alpha_z(\phi_{\text{boundary}} - \phi_i) \right) + \alpha_x(\phi_{i+1} - 2\phi_i + \phi_{i-1}) \tag{9}$$

Boundary conditions: $S = 0$, $T = 15\,°\text{C}$ at $x = 0$; $S = S_{\text{ocean}}$, $T = T_{\text{ocean}}$ at $x = L$.

Physical and Mathematical Models

The `BaroclinicFlow` class simulates density-driven estuarine flows using the following models:

- **Density Calculation**: Equation of state:

$$\rho = \text{ComputeDensity}(S, T, P), \quad P = \frac{\rho_0 g h}{10^4}\,\text{dbar} \tag{10}$$

- **Baroclinic Pressure Gradient**:

$$\frac{\partial p_b}{\partial x} = g\rho h \frac{\rho_{i+1} - \rho_{i-1}}{2\rho_0 \Delta x} \tag{11}$$

- **Eddy Diffusivities**:

$$K_x = \nu_t + l\sqrt{\left|\frac{u_{i+1} - u_{i-1}}{2\Delta x}\right|} \tag{12}$$

$$K_z = \frac{\nu_t + 0.01}{1 + \max(0, Ri/0.25)}, \quad Ri = \frac{g(\rho_{i+1} - \rho_{i-1})/(2\rho_0 \Delta x)}{(u_{i+1} - u_{i-1})^2/(2\Delta x)^2} \tag{13}$$

- **Surface Heat Flux**:

$$
\begin{aligned}
Q_{\text{sw}} &= 1000(1-0.06)(1-0.5)\,\text{max}(0, \text{cos}(2\pi t/86400)) && (14)\\
Q_{\text{lw}} &= 0.97 \cdot 5.67 \times 10^{-8}(T_w^4 - T_a^4) && (15)\\
Q_{\text{sensible}} &= 1.225 \cdot 1005 \cdot 0.0012 \cdot 5(T_w - 15) \cdot 0.61 && (16)\\
Q_{\text{latent}} &= 1.225 \cdot 2.45 \times 10^6 \cdot 0.0015 \cdot 5(q_s - q_a)/100 && (17)\\
Q_t &= \frac{Q_{\text{sw}} - Q_{\text{lw}} - Q_{\text{sensible}} - Q_{\text{latent}}(1 + |\partial u/\partial x|)}{\rho_0 C_p h} && (18)
\end{aligned}
$$

  where $q_s = 6.1078 \cdot 10^{7.5T_w/(T_w+237.3)}$, $q_a = 0.8 \cdot 6.1078 \cdot 10^{7.5T_a/(T_a+237.3)}$.

- **Transport Equations**: For salinity and temperature:

$$
\frac{\partial \phi}{\partial t} + \frac{\partial (u\phi)}{\partial x} = \frac{\partial}{\partial x}\left(K_x \frac{\partial \phi}{\partial x}\right) + \frac{K_z(\phi_{\text{boundary}} - \phi)}{h^2} + Q_\phi \tag{19}
$$

  with $Q_s = \gamma|\partial u/\partial x|(-S$ or $S_{\text{ocean}} - S)$, Q_t from heat flux, and Van Leer flux limiter for advection.

- **Tridiagonal Solver**: Thomas algorithm for semi-implicit diffusion:

$$
\begin{aligned}
c_i' &= \frac{c_i}{b_i - a_i c_{i-1}'}, \quad d_i' = \frac{d_i - a_i d_{i-1}'}{b_i - a_i c_{i-1}'} && (20)\\
\phi_i &= d_i' - c_i' \phi_{i+1} && (21)
\end{aligned}
$$

These models capture baroclinic dynamics, heat exchange, and turbulent mixing, ensuring numerical stability through clamping and flux limiters.