Hydrodynamic Solver

Core Components and Initialization

The `HydrodynamicSolver` class, implemented in C#, models the hydrodynamics of estuarine flow, incorporating momentum, pressure, and turbulence effects. The model discretizes an estuary of length $L = 10,000\,\mathrm{m}$ and depth $H = 10\,\mathrm{m}$ into $N = 100$ grid points, with spatial step $\Delta x = L/N$ and time step $\Delta t = 10\,\mathrm{s}$. Key parameters include:

- Kinematic viscosity: $\nu = 10^{-6}\,\mathrm{m}^2/\mathrm{s}$
- Coriolis parameter: $f = 10^{-4}\,\mathrm{s}^{-1}$
- Reference density: $\rho = 1000\,\mathrm{kg}/\mathrm{m}^3$
- Gravitational acceleration: $g = 9.81\,\mathrm{m}/\mathrm{s}^2$
- Resolution threshold: $\Delta x \leq 200\,\mathrm{m}$ for non-hydrostatic mode

The solver initializes arrays for velocity (u), pressure (p), turbulent kinetic energy (k), dissipation rate (ϵ), and eddy viscosity (ν_t). Initial conditions set velocity to zero, pressure to hydrostatic ($p = \rho g H$), $k = 10^{-3}\,\mathrm{m}^2/\mathrm{s}^2$, $\epsilon = 10^{-6}\,\mathrm{m}^2/\mathrm{s}^3$, and $\nu_t = 0.01\,\mathrm{m}^2/\mathrm{s}$. A `BaroclinicFlow` object computes density-driven pressure gradients.

Functioning Logic

The `Solve` method updates the flow field each time step:

1. Determines non-hydrostatic mode: enabled if $\Delta x \leq 200\,\mathrm{m}$ or overridden by `UseNonHydrostaticOverride`.
2. Computes baroclinic pressure gradient using salinity and temperature profiles.
3. Solves the momentum equation for velocity.
4. Updates turbulent quantities using the $k - \epsilon$ model.
5. Updates pressure, either non-hydrostatically or hydrostatically based on water level.

Momentum Equation

The momentum equation includes advection, pressure gradient, baroclinic, diffusion, and Coriolis terms:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{1}{\rho}\frac{\partial p_b}{\partial x} + (\nu + \nu_t)\frac{\partial^2 u}{\partial x^2} - fu \tag{1}$$

where p_b is the baroclinic pressure. Discretized:

$$\text{Advection: } u_i \frac{u_{i+1} - u_{i-1}}{2\Delta x} \tag{2}$$

$$\text{Pressure gradient: } -\frac{p_{i+1} - p_{i-1}}{2\rho\Delta x} \tag{3}$$

$$\text{Diffusion: } (\nu + \nu_{t,i}) \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} \tag{4}$$

Boundary conditions set $u_0 = Q_r/H$ (river inflow) and $u_{N-1} = u_t$ (tidal velocity).

Turbulence Model

The $k - \epsilon$ model governs turbulence:

$$\frac{\partial k}{\partial t} = -u\frac{\partial k}{\partial x} + P - \epsilon + \frac{\partial}{\partial x}\left(\frac{\nu + \nu_t}{\sigma_k}\frac{\partial k}{\partial x}\right) \tag{5}$$

$$\frac{\partial \epsilon}{\partial t} = -u\frac{\partial \epsilon}{\partial x} + C_{1\epsilon}\frac{\epsilon}{k}P - C_{2\epsilon}\frac{\epsilon^2}{k} + \frac{\partial}{\partial x}\left(\frac{\nu + \nu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x}\right) \tag{6}$$

where $P = \nu_t \left(\frac{\partial u}{\partial x}\right)^2$ is shear production, and constants are $C_\mu = 0.09$, $\sigma_k = 1.0$, $\sigma_\epsilon = 1.3$, $C_{1\epsilon} = 1.44$, $C_{2\epsilon} = 1.92$. Eddy viscosity is:

$$\nu_t = C_\mu \frac{k^2}{\epsilon} \tag{7}$$

Boundary conditions set k and ϵ at the domain edges equal to adjacent interior points.

Pressure Update

Pressure is updated based on the mode:

- Non-hydrostatic:

$$p_i = p_i - \rho\frac{u_{i+1} - u_{i-1}}{2\Delta x}\frac{\Delta x^2}{\Delta t} \tag{8}$$

- Hydrostatic:

$$p_i = \rho g\left(H + \eta\frac{i\Delta x}{L}\right) \tag{9}$$

where η is the water level.

Key Outputs

The solver provides:

- Velocity at a point: $u(x)$, interpolated from the profile.
- Velocity profile: u_i.
- Eddy viscosity at a point: $\nu_t(x)$.
- Non-hydrostatic status.