Asymmetric Tidal Mixing

Core Components and Initialization

The `AsymmTidalMix` class, implemented in C#, simulates asymmetric tidal mixing in a bifurcated estuarine system using a 3D hydrodynamic solver on an unstructured grid with sigma coordinates. It integrates physical processes such as tidal forcing, Stokes drift, internal tides, tidal straining, and turbulence using a $k - \epsilon$ model. The class is initialized with the following parameters:

- Estuary length: $L = 10000\,\mathrm{m}$
- Number of cells: $n_{\text{cells}} = 70$ (30 main channel, 20 branch 1, 20 branch 2)
- Number of sigma layers: $n_\sigma = 10$
- Time step: $\Delta t = 100\,\mathrm{s}$
- Kinematic viscosity: $\nu = 10^{-6}\,\mathrm{m}^2/\mathrm{s}$
- Gravitational acceleration: $g = 9.81\,\mathrm{m}/\mathrm{s}^2$
- Reference density: $\rho_0 = 1000\,\mathrm{kg}/\mathrm{m}^3$
- Coriolis parameter: $f = 10^{-4}\,\mathrm{s}^{-1}$
- Tidal amplitude: $A_t = 1.0\,\mathrm{m}/\mathrm{s}$
- Default bed shear stress: $\tau_b = 0.1\,\mathrm{N}/\mathrm{m}^2$
- Default tidal period: $T_t = 43200\,\mathrm{s}$ (12 hours)

The class uses a Windows Forms interface for user interaction and visualization, incorporating models for mixing (`RichardsonNumDepAndSSIMix`), tidal effects (`SimpsonHunterMechanismParam`), and bifurcated estuary geometry (`BifurcatedEstuaryM`
Each cell in the unstructured grid is defined by the `Cell` class, storing:

- X-coordinate: X (m)
- Depth: h (m)
- Volume: $V = Ah$ (m³, where A is the cell area)
- Arrays per sigma layer: turbulent kinetic energy (K), dissipation rate (ϵ), velocities (u, v, w), pressure (p), salinity (S), turbidity, shear
- Neighbor indices for connectivity

Initial conditions include a linear salinity gradient ($S = 35(1 - x/L)\,\mathrm{PSU}$), $K = 10^{-4}\,\mathrm{m}^2/\mathrm{s}^2$, and $\epsilon = 10^{-6}\,\mathrm{m}^2/\mathrm{s}^3$.

Functioning Logic

The `AsymmTidalMix` class manages:

1. **User Interface**: A Windows Forms window with controls for bed shear stress (τ_b), tidal period (T_t), visualization options (velocity vectors, streamlines, salinity isosurface, density slice, turbidity fronts, shear layer), and simulation controls (start, pause, reset).
2. **Grid Initialization**: Uses `BifurcatedEstuaryModels` to create an unstructured grid with 70 cells, representing a main channel and two branches.
3. **Simulation Update**: Advances the simulation using a 3D Navier-Stokes solver with Boussinesq approximation, incorporating:
   - Tidal forcing with asymmetry (flood: 1.2, ebb: 0.8)
   - Stokes drift and internal tide effects via `SimpsonHunterMechanismParam`
   - Turbulent viscosity via `RichardsonNumDepAndSSIMix`
   - Bifurcated dynamics via `BifurcatedEstuaryModels`
4. **Turbulence Modeling**: Updates K and ϵ using a $k-\epsilon$ model with contributions from shear, internal tides, and bifurcated mixing.
5. **Visualization**: Displays profiles, isosurfaces, and hodographs in two panels.
6. **Output Console**: Reports average TKE, velocity, salinity, turbidity, shear, Richardson number, and tidal phase.

Simulation Update

The `UpdateSimulation` method advances the simulation by $\Delta t = 100\,\text{s}$, updating the tidal phase:

$$\phi_t = \frac{2\pi t}{T_t} \tag{1}$$

The simulation follows these steps:

1. **Intermediate Velocities**: Solve Navier-Stokes equations:

$$\frac{u^* - u^n}{\Delta t} = -u\frac{\partial u}{\partial x} + (\nu + \nu_T)\nabla^2 u + fv - \frac{\tau_b}{\rho_0}\delta_{k=0}(u + u_s) \tag{2}$$

$$\frac{v^* - v^n}{\Delta t} = -u\frac{\partial v}{\partial x} + (\nu + \nu_T)\nabla^2 v - f(u + u_s) - \frac{\tau_b}{\rho_0}\delta_{k=0}v \tag{3}$$

$$\frac{w^* - w^n}{\Delta t} = -w\frac{\partial w}{\partial z} + (\nu + \nu_T)\nabla^2 w - \frac{g(\rho - \rho_0)}{\rho_0} + w_{\text{tide}} \tag{4}$$

   where u_s is Stokes drift, w_{tide} is internal tide vertical velocity, ν_T is turbulent viscosity, and $\rho = \rho_0 + 0.8S$. Boundary conditions:

   - Ocean ($i = 0$): $u = A_t \sin(\phi_t)(1-\sigma)$, $v = w = 0$, $S = 35\,\text{PSU}$
   - Branch ends ($i = 49, 69$): $u = 0.1(1-\sigma) + A_t'(1-\sigma)$, $v = w = 0$, $S = 0\,\text{PSU}$
   - Bed ($k = 0$): No-slip ($u = v = w = 0$)

   Velocities are capped: $u, v, w \in [-2, 2]\,\text{m/s}$.

2. **Pressure Poisson Equation**: Solve for pressure to enforce incompressibility:

$$\nabla^2 p = \frac{\rho_0}{\Delta t} \nabla \cdot \mathbf{u}^* \tag{5}$$

using successive over-relaxation (SOR) with 20 iterations. Pressure is capped: $p \in [-10^5, 10^5]$ Pa.

3. **Velocity Correction**:

$$u^{n+1} = u^* - \frac{\Delta t}{\rho_0} \frac{\partial p}{\partial x} \tag{6}$$

$$v^{n+1} = v^* \tag{7}$$

$$w^{n+1} = w^* - \frac{\Delta t}{\rho_0} \frac{\partial p}{\partial z} \tag{8}$$

4. **Turbulence Update**: Solve $k - \epsilon$ equations:

$$\frac{k^{n+1} - k^n}{\Delta t} = -u \frac{\partial k}{\partial x} + \frac{\partial}{\partial x} \left(\frac{\nu + \nu_T}{\sigma_k} \frac{\partial k}{\partial x} \right) + P_k + P_{\text{it}} - \epsilon \tag{9}$$

$$\frac{\epsilon^{n+1} - \epsilon^n}{\Delta t} = -u \frac{\partial \epsilon}{\partial x} + \frac{\partial}{\partial x} \left(\frac{\nu + \nu_T}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x} \right) + C_{1\epsilon} \frac{\epsilon}{k} P_k - C_{2\epsilon} \frac{\epsilon^2}{k} \tag{10}$$

where P_k is shear production, P_{it} is internal tide TKE production, $\sigma_k = 1.0$, $\sigma_\epsilon = 1.3$, $C_{1\epsilon} = 1.44$, $C_{2\epsilon} = 1.92$. $k \in [10^{-6}, 10^{-3}]$ m^2/s^2, $\epsilon \in [10^{-8}, 10^{-5}]$ m^2/s^3.

5. **Salinity and Turbidity**: Update using advection-diffusion with tidal straining:

$$\frac{S^{n+1} - S^n}{\Delta t} = -u \frac{\partial S}{\partial x} + \frac{\partial}{\partial z} \left(\frac{\nu + \nu_T}{\sigma_S} \frac{\partial S}{\partial z} \right) + S_{\text{strain}} \tag{11}$$

$$\frac{T^{n+1} - T^n}{\Delta t} = -u \frac{\partial T}{\partial x} + \frac{\partial}{\partial z} \left(\frac{\nu + \nu_T}{\sigma_T} \frac{\partial T}{\partial z} \right) + P_T - \frac{T}{1000} \tag{12}$$

where S_{strain} is the tidal straining term, $P_T = 0.5k$ at the bed, $\sigma_S = \sigma_T = 1.0$. Salinity is capped: $S \in [0, 35]$ PSU; turbidity: $T \in [0, 100]$.

6. **Shear Calculation**:

$$\text{Shear} = \sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}, \quad \text{Shear} \in [0, 10] \, \text{s}^{-1} \tag{13}$$

Visualization

The `VisualizationPanel_Paint` method renders:

- **Density Slice**: Color-coded density ($\rho = \rho_0 + 0.8S$) across cells and sigma layers (blue gradient).
- **Salinity Isosurface**: Lines at $S = 15$ PSU (green, thicker during flood).
- **Shear Layer**: Lines scaled by shear magnitude (orange, thicker during flood).

- **Turbidity Fronts**: Lines at 50% of maximum turbidity (brown, thicker during flood).
- **Velocity Vectors**: Arrows showing u, w at selected cells and layers (black, thicker during flood).
- **Streamlines**: Paths following average u velocity (purple, thicker during flood).
- **Turbulence and Velocity Profiles**: Blue (TKE) and red (velocity magnitude) lines along the estuary.

Values are scaled to panel dimensions, with checks for NaN/Infinity and bounds enforcement.

The `HodographPanel_Paint` method plots hodographs for u, w velocities at three sample points (main channel, branch 1, branch 2) in blue, green, and red, respectively, updated every $T_t/12$.

Physical and Mathematical Models

The `AsymmTidalMix` class employs:

- **Navier-Stokes with Boussinesq**:
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho_0}\nabla p + (\nu + \nu_T)\nabla^2 \mathbf{u} + \mathbf{f} \times \mathbf{u} - \frac{g}{\rho_0}\nabla \rho + \mathbf{F} \tag{14}$$
$$\nabla \cdot \mathbf{u} = 0 \tag{15}$$
where **F** includes bed friction, Stokes drift, and internal tides.
- **Turbulence ($k - \epsilon$)**:
$$\frac{\partial k}{\partial t} + u\frac{\partial k}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\nu + \nu_T}{\sigma_k}\frac{\partial k}{\partial x}\right) + P_k + P_{\text{it}} - \epsilon \tag{16}$$
$$\frac{\partial \epsilon}{\partial t} + u\frac{\partial \epsilon}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\nu + \nu_T}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x}\right) + C_{1\epsilon}\frac{\epsilon}{k}P_k - C_{2\epsilon}\frac{\epsilon^2}{k} \tag{17}$$
- **Salinity and Turbidity**:
$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} = \frac{\partial}{\partial z}\left(\frac{\nu + \nu_T}{\sigma_S}\frac{\partial S}{\partial z}\right) + S_{\text{strain}} \tag{18}$$
$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{\partial}{\partial z}\left(\frac{\nu + \nu_T}{\sigma_T}\frac{\partial T}{\partial z}\right) + P_T - \frac{T}{1000} \tag{19}$$
- **Boundary Conditions**:
  - Ocean: Tidal velocity and salinity
  - Branch ends: Freshwater inflow and scaled tidal velocity
  - Bed: No-slip and bed friction

The model integrates tidal asymmetry, turbulence, and estuarine geometry to simulate realistic circulation and mixing dynamics, with interactive visualization and user-configurable parameters.