Simpson-Hunter Mechanism Parameterization

Core Components and Initialization

The `SimpsonHunterMechanismParam` class, implemented in C#, models physical processes related to tidal straining and internal tides in estuarine circulation, focusing on wave-induced Stokes drift, internal tide effects, and tidal straining (Simpson-Hunter mechanism). The class is initialized with the following parameters:

- Number of sigma layers: n_σ (vertical grid points)
- Wave amplitude: $a = 0.5\,\text{m}$ (default)
- Wave period: $T_w = 10.0\,\text{s}$ (default)
- Internal tide amplitude: $A_{\text{it}} = 0.05\,\text{m/s}$ (default)
- Simpson-Hunter coefficient: $C_{\text{sh}} = 0.1$ (default, for tidal straining)
- Gravitational acceleration: $g = 9.81\,\text{m/s}^2$
- Reference density: $\rho_0 = 1000.0\,\text{kg/m}^3$

The constructor sets these parameters, allowing customization of wave, internal tide, and tidal straining effects.

Functioning Logic

The class provides three methods to compute contributions to estuarine dynamics:

1. `ComputeStokesDrift`: Calculates the x-direction Stokes drift velocity due to surface waves.
2. `ComputeInternalTideEffect`: Computes vertical velocity perturbations and turbulent kinetic energy (TKE) production from internal tides.
3. `ComputeTidalStraining`: Evaluates the tidal straining effect (Simpson-Hunter mechanism) on the salinity gradient.

Stokes Drift Computation

The `ComputeStokesDrift` method calculates the Stokes drift velocity in the x-direction using a simplified shallow-water approximation:

- Wave number: $k = \frac{2\pi}{T_w\sqrt{gh}}$, where h is the water depth.
- Depth coordinate: $z = \sigma h$, where σ is the sigma coordinate (0 at bottom, 1 at surface).
- Stokes drift velocity:

$$u_s = a \cdot \frac{2\pi}{T_w} \cdot e^{-2kz} \cdot \sin(\phi_t) \tag{1}$$

where a is the wave amplitude, ϕ_t is the tidal phase, and the velocity is capped: $u_s \in [-0.5, 0.5]$ m/s for numerical stability.

Internal Tide Effect

The `ComputeInternalTideEffect` method computes vertical velocity perturbations and TKE production due to internal tides:

- **Buoyancy frequency squared**:

$$N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}, \quad \frac{\partial \rho}{\partial z} = \frac{\rho_2 - \rho_1}{\Delta\sigma \cdot h} \tag{2}$$

  where $\rho_k = \rho_0 + 0.8 S_k$, and S_k is the salinity at layer k. N^2 is capped: $N^2 \in [0, 10^{-3}]\,\mathrm{s}^{-2}$.

- **Vertical velocity perturbation**:

$$w_{\text{tide}} = A_{\text{it}} \sin(\phi_t) \sqrt{N^2} \cos\left(\frac{2\pi z}{h}\right) \tag{3}$$

  where $w_{\text{tide}} \in [-0.2, 0.2]$ m/s.

- **TKE production**:

$$\text{TKE} = 0.1 A_{\text{it}} \left(\frac{\partial w}{\partial z}\right)^2, \quad \frac{\partial w}{\partial z} = \frac{w_k - w_{k+1}}{\Delta\sigma \cdot h} \tag{4}$$

  TKE is capped: $\text{TKE} \in [0, 10^{-4}]\,\mathrm{m}^2/\mathrm{s}^3$. No effect is applied at the top layer ($k = n_\sigma - 1$).

Tidal Straining Effect

The `ComputeTidalStraining` method models the Simpson-Hunter mechanism, which describes how tidal currents modify salinity gradients:

- **Salinity gradient**:

$$\frac{\partial S}{\partial x} = \begin{cases} \frac{S_{i+1,k} - S_{i-1,k}}{2\Delta x} & \text{if } 0 < i < n_{\text{cells}} - 1 \\ \frac{S_{i+1,k} - S_{i,k}}{\Delta x} & \text{if } i = 0 \\ \frac{S_{i,k} - S_{i-1,k}}{\Delta x} & \text{if } i = n_{\text{cells}} - 1 \end{cases} \tag{5}$$

  where Δx is the average cell width, and $\frac{\partial S}{\partial x} \in [-10, 10]$ PSU/m.

- **Tidal straining term**:

$$\text{Straining} = -C_{\text{sh}} u_t \frac{\partial S}{\partial x} \tag{6}$$

  where u_t is the tidal velocity, and the term is capped: $\text{Straining} \in [-1, 1]$ PSU/s. No effect is applied at the top layer or boundary cells.

Physical and Mathematical Models

The `SimpsonHunterMechanismParam` class employs the following models:

- **Stokes Drift**:

$$u_s = a \cdot \frac{2\pi}{T_w} \cdot e^{-2kz} \cdot \sin(\phi_t), \quad u_s \in [-0.5, 0.5] \tag{7}$$

- **Internal Tide**:

$$N^2 = -\frac{g}{\rho_0} \frac{\rho_2 - \rho_1}{\Delta\sigma \cdot h}, \quad N^2 \in [0, 10^{-3}] \tag{8}$$

$$w_{\text{tide}} = A_{\text{it}} \sin(\phi_t) \sqrt{N^2} \cos\left(\frac{2\pi z}{h}\right), \quad w_{\text{tide}} \in [-0.2, 0.2] \tag{9}$$

$$\text{TKE} = 0.1 A_{\text{it}} \left(\frac{w_k - w_{k+1}}{\Delta\sigma \cdot h}\right)^2, \quad \text{TKE} \in [0, 10^{-4}] \tag{10}$$

- **Tidal Straining**:

$$\text{Straining} = -C_{\text{sh}} u_t \frac{\partial S}{\partial x}, \quad \frac{\partial S}{\partial x} \in [-10, 10], \quad \text{Straining} \in [-1, 1] \tag{11}$$

These models capture wave-induced transport, internal tide-driven vertical mixing, and tidal straining effects on stratification, enhancing the realism of estuarine circulation simulations.