

# Bifurcated Estuary Models

## Core Components and Initialization

The `BifurcatedEstuaryModels` class, implemented in C#, defines a bifurcated estuarine grid with a main channel and two branches, enabling simulation of spatially varying dynamics in estuarine circulation models. It integrates with the `AsymmTidalMix` class to provide grid initialization and branch-specific adjustments for tidal velocity and turbulent mixing. The class is initialized with the following parameters:

- Number of cells in main channel:  $n_{\text{main}} = 30$
- Number of cells per branch:  $n_{\text{branch}} = 20$
- Estuary length:  $L = 10000$  m
- Bifurcation point:  $x_b = 0.6L$  (fraction of estuary length)
- Depth scaling for branch 1:  $s_{d1} = 0.8$  (shallower)
- Depth scaling for branch 2:  $s_{d2} = 1.2$  (deeper)
- Tidal velocity scaling for branch 1:  $s_{t1} = 0.7$  (reduced tidal influence)
- Tidal velocity scaling for branch 2:  $s_{t2} = 1.3$  (enhanced tidal influence)

The constructor sets these parameters, which control the geometry and dynamics of the bifurcated estuary.

## Functioning Logic

The class provides three main methods to support estuarine modeling:

1. `InitializeBifurcatedGrid`: Creates an unstructured grid with cells for the main channel and two branches, assigning coordinates, depths, areas, and neighbor connectivity.
2. `GetBranchIndex`: Identifies whether a cell belongs to the main channel or one of the branches.
3. `AdjustMixingDynamics`: Modifies turbulent viscosity and turbulent kinetic energy (TKE) production based on branch-specific depth and tidal scaling.
4. `AdjustTidalVelocity`: Scales tidal velocity at branch ends to reflect differing tidal influences.

## Grid Initialization

The `InitializeBifurcatedGrid` method constructs a grid with  $n_{\text{main}} + 2 \cdot n_{\text{branch}} = 70$  cells:

- **Main Channel:**  $n_{\text{main}} = 30$  cells from  $x = 0$  (ocean) to  $x = x_b = 0.6L$ .

- Cell width:  $\Delta x_{\text{main}} = \frac{x_b}{n_{\text{main}}}$
- X-coordinate:  $x_i = i\Delta x_{\text{main}} + \text{rand} \cdot 0.5\Delta x_{\text{main}}$ , where  $\text{rand} \in [0, 1)$ .
- Depth:  $h_i = 5.0 + 5.0 \sin\left(\frac{\pi x_i}{L}\right)$  m
- Area:  $A_i = \Delta x_{\text{main}} \cdot 100 \cdot (0.8 + \text{rand} \cdot 0.4)$  m<sup>2</sup>
- **Branch 1:**  $n_{\text{branch}} = 20$  cells from  $x = x_b$  to  $x = L$ .
  - Cell width:  $\Delta x_{\text{branch}} = \frac{L-x_b}{n_{\text{branch}}}$
  - X-coordinate:  $x_i = x_b + i\Delta x_{\text{branch}} + \text{rand} \cdot 0.5\Delta x_{\text{branch}}$
  - Depth:  $h_i = \left[5.0 + 5.0 \sin\left(\frac{\pi x_i}{L}\right)\right] \cdot s_{d1}$  m
  - Area:  $A_i = \Delta x_{\text{branch}} \cdot 100 \cdot (0.8 + \text{rand} \cdot 0.4)$  m<sup>2</sup>
- **Branch 2:**  $n_{\text{branch}} = 20$  cells from  $x = x_b$  to  $x = L$ .
  - Same cell width and x-coordinate as Branch 1
  - Depth:  $h_i = \left[5.0 + 5.0 \sin\left(\frac{\pi x_i}{L}\right)\right] \cdot s_{d2}$  m
  - Area: Same as Branch 1
- **Neighbor Connectivity:**
  - Main channel: Linear (cell  $i$  connects to  $i - 1$  and  $i + 1$ ), with the last cell ( $i = n_{\text{main}} - 1$ ) connecting to the first cells of both branches.
  - Branch 1: Linear (cell  $i$  connects to  $i - 1$ ,  $i + 1$ ), with the first cell connecting to the bifurcation point.
  - Branch 2: Similar to Branch 1.

A fixed random seed ensures reproducibility.

## Branch Identification

The GetBranchIndex method assigns a cell to a region:

$$\text{Branch Index} = \begin{cases} 0 & \text{if } i < n_{\text{main}} \text{ (main channel)} \\ 1 & \text{if } n_{\text{main}} \leq i < n_{\text{main}} + n_{\text{branch}} \text{ (branch 1)} \\ 2 & \text{if } i \geq n_{\text{main}} + n_{\text{branch}} \text{ (branch 2)} \end{cases} \quad (1)$$

## Mixing Dynamics Adjustment

The AdjustMixingDynamics method modifies turbulent viscosity ( $\nu_T$ ) and TKE production ( $P_k$ ) based on the branch:

- **Mixing Modifier:**

$$m = \begin{cases} 1.0 & \text{(main channel)} \\ s_{d1} = 0.8 & \text{(branch 1, reduced mixing)} \\ s_{d2} = 1.2 & \text{(branch 2, enhanced mixing)} \end{cases} \quad (2)$$

- **Turbulent Viscosity:**  $\nu_T \leftarrow \nu_T \cdot m$
- **TKE Production:**

$$P_k \leftarrow P_k + \nu_T \cdot \text{Shear}_k^2 \cdot m \cdot \frac{u_t \cdot s_t}{\max(0.01, |u_t|)} \quad (3)$$

where  $u_t$  is the tidal velocity,  $s_t = s_{t1}$  or  $s_{t2}$  for branches 1 and 2, respectively, and  $P_k \in [0, 10^{-3}] \text{ m}^2/\text{s}^3$ .

## Tidal Velocity Adjustment

The AdjustTidalVelocity method scales the tidal velocity at branch ends:

$$u'_t = \begin{cases} u_t \cdot s_{t1} & \text{(branch 1, cell } i = n_{\text{main}} + n_{\text{branch}} - 1) \\ u_t \cdot s_{t2} & \text{(branch 2, cell } i = n_{\text{main}} + 2 \cdot n_{\text{branch}} - 1) \\ u_t & \text{(main channel)} \end{cases} \quad (4)$$

## Physical and Mathematical Models

The BifurcatedEstuaryModels class employs the following models:

- **Grid Geometry:**

$$x_i = i\Delta x + \text{rand} \cdot 0.5\Delta x \quad (5)$$

$$h_i = \left( 5.0 + 5.0 \sin\left(\frac{\pi x_i}{L}\right) \right) \cdot s_d, \quad s_d = \begin{cases} 1.0 & \text{(main channel)} \\ s_{d1} & \text{(branch 1)} \\ s_{d2} & \text{(branch 2)} \end{cases} \quad (6)$$

$$A_i = \Delta x \cdot 100 \cdot (0.8 + \text{rand} \cdot 0.4) \quad (7)$$

- **Turbulent Viscosity Adjustment:**

$$\nu_T \leftarrow \nu_T \cdot m, \quad m = \begin{cases} 1.0 & \text{(main channel)} \\ 0.8 & \text{(branch 1)} \\ 1.2 & \text{(branch 2)} \end{cases} \quad (8)$$

- **TKE Production Adjustment:**

$$P_k \leftarrow P_k + \nu_T \cdot \text{Shear}_k^2 \cdot m \cdot \frac{u_t \cdot s_t}{\max(0.01, |u_t|)}, \quad P_k \in [0, 10^{-3}] \quad (9)$$

- **Tidal Velocity Scaling:**

$$u'_t = u_t \cdot s_t, \quad s_t = \begin{cases} 0.7 & \text{(branch 1)} \\ 1.3 & \text{(branch 2)} \\ 1.0 & \text{(main channel)} \end{cases} \quad (10)$$

These models capture the geometric and dynamic variations in a bifurcated estuary, enabling realistic simulation of tidal and mixing processes in distinct channels.