# Total Variation Diminishing

## Core Components and Initialization

The `TotalVariationDiminishing` class, implemented in C#, simulates estuarine circulation in a two-dimensional (x-z) domain using a total variation diminishing (TVD) numerical scheme. It supports both structured and unstructured grids, with visualization capabilities for salinity, temperature, and velocity fields. The estuary has a length $L = 1000$ m and depth $H = 10$ m, discretized into a structured grid of $200 \times 100$ points ($\Delta x = L/200$, $\Delta z = H/100$) or an unstructured triangular mesh. Key parameters include:

- Gravitational acceleration: $g = 9.81$ m/s$^2$
- Reference density: $\rho_0 = 1000$ kg/m$^3$
- Thermal expansion coefficient: $\alpha = 2 \times 10^{-4}$ °C$^{-1}$
- Saline contraction coefficient: $\beta_s = 8 \times 10^{-4}$ PSU$^{-1}$
- Reference temperature: $T_0 = 20$ °C
- Reference salinity: $S_0 = 35$ PSU
- Tidal amplitude: $A = 0.3$ m
- Tidal period: $T = 60$ s
- Horizontal diffusion coefficient: $\kappa_h = 0.01$ m$^2$/s

The model initializes fields for salinity ($S$), temperature ($T$), density ($\rho$), velocity components ($u$, $w$), turbulent kinetic energy ($k$), dissipation rate ($\epsilon$) or specific dissipation rate ($\omega$), eddy viscosity ($\nu_t$), and eddy diffusivity ($\kappa_t$). Initial conditions for the structured grid are:

$$S(x,z) = 17.5\left(1 + \tanh\left(\frac{x-500}{50}\right)\right)\left(1 - 0.5\frac{z}{H}\right) \tag{1}$$

$$T(x,z) = \max\left(15, 20 + 2\left(1 - \tanh\left(\frac{x-500}{50}\right)\right) - 2\frac{z}{H}\right) \tag{2}$$

$$\rho(x,z) = \rho_0\left(1 - \alpha(T - T_0) + \beta_s(S - S_0)\right) \tag{3}$$

with $u = A(1 - z/H)$, $w = -0.02(z/H)$, $k = 10^{-4}$ m$^2$/s$^2$, $\epsilon = C_\mu k^{1.5}/0.1$, $\omega = k/(0.09 \cdot 0.1)$, $\nu_t = C_\mu k^2/\epsilon$, and $\kappa_t = \nu_t/0.7$. Unstructured grids use a similar setup with bathymetry.

## Functioning Logic

The `UpdateSimulation` method advances the simulation by a fixed time step $\Delta t = 0.025$ s, updating fields via:

1. Velocity updates using tidal and buoyancy forcing.
2. Advection-diffusion of salinity, temperature, and turbulence quantities using a TVD scheme with HLL flux limiter.

3. Turbulence modeling with either $k-\epsilon$ or $k-\omega$ models.
4. Visualization updates for plan view, cross-section, contour, or quiver plots.

The simulation supports user interaction via a GUI, allowing control of visualization modes, grid type, depth, x-position, tidal period, and turbulence model.

## Velocity Update

The velocity field is updated with tidal and buoyancy effects:

$$u(x,z,t) = A\sin\left(\frac{2\pi t}{T}\right)\left(1-\frac{z}{H}\right) \tag{4}$$

$$\frac{\partial w}{\partial t} = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial x} + \nu_t\frac{\partial^2 w}{\partial z^2} \tag{5}$$

For unstructured grids, bathymetry adjusts the depth: $H(x) = H(0.5+0.5\,\text{min}(1, x/200))$. The vertical velocity $w$ is constrained: $-0.5 \leq w \leq 0.5\,\text{m/s}$.

## TVD Advection-Diffusion

Salinity, temperature, and turbulence quantities are updated using a TVD scheme with HLL flux limiter:

$$\frac{\partial \phi}{\partial t} + \frac{\partial (u\phi)}{\partial x} + \frac{\partial (w\phi)}{\partial z} = \kappa_h\frac{\partial^2 \phi}{\partial x^2} + \kappa_t\frac{\partial^2 \phi}{\partial z^2} + S_\phi \tag{6}$$

where $\phi$ is $S, T, k, \epsilon$, or $\omega$, and $S_\phi$ is a source term (e.g., river/ocean salinity/temperature sources). The HLL flux is:

$$F_{\text{HLL}} = \frac{s_R v\phi_L - s_L v\phi_R + s_L s_R(\phi_R - \phi_L)}{s_R - s_L}, \quad s_L = v - |v|, \quad s_R = v + |v| \tag{7}$$

A limiter ensures stability:

$$\phi = \text{max}(0, \text{min}(2r, \text{min}(1, (1+r)/2))), \quad r = \frac{\phi - \phi_{\text{upwind}}}{\phi_{\text{downwind}} - \phi + 10^{-10}} \tag{8}$$

## Turbulence Models

The solver supports $k-\epsilon$ and $k-\omega$ models:

$$\frac{\partial k}{\partial t} + u\frac{\partial k}{\partial x} + w\frac{\partial k}{\partial z} = \frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial z}\right) + P - \epsilon \quad (\text{k-}\epsilon) \tag{9}$$

$$\frac{\partial \epsilon}{\partial t} + u\frac{\partial \epsilon}{\partial x} + w\frac{\partial \epsilon}{\partial z} = \frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial z}\right) + C_{1\epsilon}\frac{\epsilon}{k}P - C_{2\epsilon}\frac{\epsilon^2}{k} \tag{10}$$

$$\frac{\partial k}{\partial t} + u\frac{\partial k}{\partial x} + w\frac{\partial k}{\partial z} = \frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial z}\right) + P - \beta^* k\omega \quad (\text{k-}\omega) \tag{11}$$

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + w\frac{\partial \omega}{\partial z} = \frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_\omega}\frac{\partial \omega}{\partial z}\right) + \alpha_\omega\frac{\omega}{k}P - \beta\omega^2 \tag{12}$$

where $P = P_{\text{shear}} + P_{\text{buoy}}$, with $P_{\text{shear}} = \nu_t \left(\frac{\partial u}{\partial z}\right)^2$, $P_{\text{buoy}} = -g\kappa_t \frac{\partial \rho}{\partial z}/\rho_0$. Constants are $C_\mu = 0.09$, $\sigma_k = 1.0$, $\sigma_\epsilon = 1.3$, $C_{1\epsilon} = 1.44$, $C_{2\epsilon} = 1.92$, $\sigma_{k,\omega} = 0.5$, $\sigma_\omega = 0.5$, $\beta^* = 0.09$, $\beta = 0.075$, $\alpha_\omega = 0.52$. Eddy viscosity is:

$$\nu_t = \begin{cases} C_\mu \frac{k^2}{\epsilon} & (\text{k-}\epsilon) \\ \frac{k}{\omega} & (\text{k-}\omega) \end{cases} \tag{13}$$

Eddy diffusivity is $\kappa_t = \nu_t/0.7$.

## Visualization

The GUI provides four visualization modes:

- **Plan View**: Plots salinity, temperature, and velocity magnitude at a selected depth.
- **Cross-Section**: Plots profiles at a selected x-position.
- **Contour Plot**: Displays salinity and temperature as color maps.
- **Quiver Plot**: Shows velocity vectors.

For unstructured grids, fields are interpolated over triangles. The output textbox displays maximum salinity, temperature, and velocity.