Shallow Water Equations 2D Solver

Core Components and Initialization

The `ShallowWaterEq2D` class, implemented in C#, simulates two-dimensional estuarine circulation using the shallow water equations, incorporating tidal forcing, river discharge, wind stress, storm surges, and salinity transport. The estuary is defined with length $L = 10,000\,\text{m}$, width W, and depth $H = 10\,\text{m}$, discretized into a grid of $n_x \times n_y$ points with spatial steps $\Delta x = L/(n_x - 1)$ and $\Delta y = W/(n_y - 1)$. Key parameters include:

- Gravitational acceleration: $g = 9.81\,\text{m/s}^2$
- Freshwater density: $\rho_0 = 1000\,\text{kg/m}^3$
- Ocean water density: $\rho_{\text{ocean}} = 1025\,\text{kg/m}^3$
- Kinematic viscosity: $\nu = 10^{-6}\,\text{m}^2/\text{s}$
- Salinity diffusion coefficient: $\kappa = 10^{-4}\,\text{m}^2/\text{s}$
- Eddy viscosity: $\nu_t = 0.01\,\text{m}^2/\text{s}$
- Coriolis parameter: $f = 10^{-4}\,\text{s}^{-1}$
- Bottom friction coefficient: $C_f = 0.0025$
- Atmospheric pressure gradient: $\nabla P_{\text{atm}} = 0.0001\,\text{Pa/m}$
- Seasonal salinity amplitude: $S_a = 2.0\,\text{PSU}$

The model initializes 2D arrays for velocity components (U, V), water surface elevation (η), and salinity (S). Initial conditions set $U = Q_r/(WH)$, $V = 0$, $\eta = A\sin(2\pi x/L)$, and $S = S_g x$, where Q_r is river discharge, A is tidal amplitude, and S_g is the salinity gradient.

Functioning Logic

The `Update` method advances the simulation by a time step determined by the CFL condition:

$$\Delta t = C \min\left(\frac{\Delta x}{|u_{\max}| + \sqrt{gH}}, \frac{\Delta y}{|v_{\max}| + \sqrt{gH}}, \frac{\Delta x^2}{2(\nu + \nu_t)}, \frac{\Delta y^2}{2(\nu + \nu_t)}\right) \tag{1}$$

where $C = 0.4$ is the Courant number. The method:

1. Computes tidal forcing: $\eta_t = A\sin(2\pi t/T)$, $u_t = A(2\pi/T)\cos(2\pi t/T)$.
2. Applies wind stress (τ_x, τ_y) via `WindForcing`.
3. Includes storm surge: $\eta_s = A_s e^{-t/86400}$.
4. Accounts for tide-surge interaction and wave effects.
5. Updates interior points using the shallow water equations and salinity transport.

6. Applies wetting and drying if enabled via `WetAndDryAlgo`.
7. Checks for numerical stability.

Shallow Water Equations

The model solves the 2D shallow water equations for momentum and continuity:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g\frac{\partial \eta}{\partial x} - \frac{g}{\rho_0}\frac{\partial \rho}{\partial x} + (\nu + \nu_t)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + fv + \frac{\tau_x}{\rho_0 H} - C_f\frac{|u|u}{H} + \frac{\partial P_{\text{atm}}}{\partial x} + u_w \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial \eta}{\partial y} - \frac{g}{\rho_0}\frac{\partial \rho}{\partial y} + (\nu + \nu_t)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - fu + \frac{\tau_y}{\rho_0 H} - C_f\frac{|v|v}{H} \tag{3}$$

$$\frac{\partial \eta}{\partial t} = -(H + \eta)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{4}$$

where $u_w = 0.5k_w\sqrt{gH}$ is wave-induced velocity, $k_w = 2\pi/(L/10)$. Advection terms use a minmod flux limiter:

$$\phi = \max(0, \min(1, r)), \quad r = \frac{q - q_{\text{upwind}}}{q_{\text{downwind}} - q + 10^{-10}} \tag{5}$$

Salinity Transport

Salinity evolves via an advection-diffusion equation:

$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} + v\frac{\partial S}{\partial y} = (\kappa + \nu_t)\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2}\right) + 0.1|\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}|S \tag{6}$$

Discretized with flux limiters for advection and central differences for diffusion, salinity is constrained: $0 \leq S \leq 35$ PSU.

Boundary Conditions

Boundary conditions are:

- At $x = 0$: $u = Q_r/(WH)$, $v = 0$, $\eta = 0$, $S = 0$.
- At $x = L$: $u = u_t$, $v = 0$, $\eta = \eta_t + \eta_s + 0.05AA_s \sin(2\pi t/T)$, $S = 35 + S_a \sin(2\pi t/(365 \cdot 86400))$.
- At $y = 0, W$: Reflective conditions for u, η, and S; $v = 0$.

Wetting and Drying

If enabled, the `WetAndDryAlgo` sets $u = v = 0$ and maintains η and S in dry cells (depth below `minDepth`), preventing unphysical flow.

Key Outputs

The model provides:

- Velocity fields: $U(x, y)$, $V(x, y)$.
- Water surface elevation: $\eta(x, y)$.
- Salinity field: $S(x, y)$.