Richardson Number Dependent and Shear-Strain-Induced Mixing

Core Components and Initialization

The `RichardsonNumDepAndSSIMix` class, implemented in C#, models turbulent mixing in estuarine circulation by computing Richardson number-dependent turbulent viscosity and shear-strain-induced turbulent kinetic energy (TKE) production. It integrates with the `AsymmTidalMix` class to enhance the realism of mixing processes. The class is initialized with:

- Number of sigma layers: n_σ (vertical grid points)
- Gravitational acceleration: $g = 9.81\,\text{m/s}^2$
- Reference density: $\rho_0 = 1000\,\text{kg/m}^3$
- Kinematic viscosity: $\nu = 10^{-6}\,\text{m}^2/\text{s}$
- $k - \epsilon$ model constant: $c_\mu = 0.09$

Functioning Logic

The class provides three methods to compute mixing-related quantities:

1. `ComputeAdjustedTurbulentViscosity`: Calculates turbulent viscosity (ν_T) adjusted by the gradient Richardson number to account for stratification effects.
2. `ComputeShearStrainProduction`: Computes TKE production due to shear strain, focusing on vertical shear components.
3. `ComputeAverageRichardsonNumber`: Calculates the average Richardson number across all cells for diagnostic output.

Turbulent Viscosity Computation

The `ComputeAdjustedTurbulentViscosity` method computes turbulent viscosity for a given cell and sigma layer k:

1. **Base Turbulent Viscosity**:
$$\nu_T = c_\mu \frac{k^2}{\epsilon} \tag{1}$$
where k is TKE (m²/s²) and ϵ is the dissipation rate (m²/s³). If $\epsilon < 10^{-8}$ or $k < 10^{-6}$, or if ν_T is NaN/infinite, $\nu_T = \nu = 10^{-6}\,\text{m}^2/\text{s}$.
2. **Buoyancy Frequency Squared** (N^2):
$$N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}, \quad \frac{\partial \rho}{\partial z} = \frac{\rho_{k+1} - \rho_k}{\Delta\sigma \cdot h} \tag{2}$$
where $\rho_k = \rho_0 + 0.8 S_k$, S_k is salinity (PSU), h is depth (m), and $\Delta\sigma = 1/n_\sigma$. $N^2 \in [0, 10^{-3}]\,\text{s}^{-2}$.

3. **Shear Squared** (S^2):

$$S^2 = \text{Shear}_k^2, \quad S^2 \in [10^{-6}, 100]\,\text{s}^{-2} \tag{3}$$

   where Shear_k is the velocity gradient (s^{-1}).

4. **Gradient Richardson Number**:

$$Ri = \frac{N^2}{S^2}, \quad Ri \in [0, 10] \tag{4}$$

5. **Stability Function**:

$$f(Ri) = \frac{1}{1 + 10Ri}, \quad f(Ri) \in [0.1, 1.0] \tag{5}$$

   This reduces mixing when $Ri > 0.25$, indicating stable stratification.

6. **Adjusted Turbulent Viscosity**:

$$\nu_T \leftarrow \nu_T \cdot f(Ri), \quad \nu_T \in [10^{-6}, 10^{-2}]\,\text{m}^2/\text{s} \tag{6}$$

Shear-Strain-Induced TKE Production

The `ComputeShearStrainProduction` method calculates TKE production for layer k:

1. **Vertical Shear Components**:

$$\frac{\partial u}{\partial z} = \frac{u_k - u_{k+1}}{\Delta\sigma \cdot h}, \quad \frac{\partial u}{\partial z} \in [-100, 100]\,\text{s}^{-1} \tag{7}$$

$$\frac{\partial v}{\partial z} = \frac{v_k - v_{k+1}}{\Delta\sigma \cdot h}, \quad \frac{\partial v}{\partial z} \in [-100, 100]\,\text{s}^{-1} \tag{8}$$

   where u_k, v_k are horizontal velocities (m/s).

2. **Shear-Strain Production**:

$$P = \nu_T \cdot \left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 \right), \quad P \in [0, 10^{-3}]\,\text{m}^2/\text{s}^3 \tag{9}$$

   No production is computed at the top layer ($k = n_\sigma - 1$).

Average Richardson Number

The `ComputeAverageRichardsonNumber` method computes the average gradient Richardson number across all cells and layers:

1. For each cell and layer $k < n_\sigma - 1$:

$$N^2 = -\frac{g}{\rho_0}\frac{\rho_{k+1} - \rho_k}{\Delta\sigma \cdot h}, \quad N^2 \in [0, 10^{-3}]\,\text{s}^{-2} \tag{10}$$

$$S^2 = \text{Shear}_k^2, \quad S^2 \in [10^{-6}, 100]\,\text{s}^{-2} \tag{11}$$

$$Ri = \frac{N^2}{S^2}, \quad Ri \in [0, 10] \tag{12}$$

2. Sum valid Ri values (excluding NaN/infinite) and average over valid cells/layers.

Physical and Mathematical Models

The `RichardsonNumDepAndSSIMix` class employs the following models:

- **Turbulent Viscosity**:

$$\nu_T = c_\mu \frac{k^2}{\epsilon} \cdot \frac{1}{1 + 10 \cdot \frac{N^2}{S^2}}, \quad \nu_T \in [10^{-6}, 10^{-2}] \tag{13}$$

$$N^2 = -\frac{g}{\rho_0} \frac{\rho_{k+1} - \rho_k}{\Delta\sigma \cdot h}, \quad N^2 \in [0, 10^{-3}] \tag{14}$$

$$S^2 = \text{Shear}_k^2, \quad S^2 \in [10^{-6}, 100] \tag{15}$$

- **Shear-Strain TKE Production**:

$$P = \nu_T \cdot \left(\left(\frac{u_k - u_{k+1}}{\Delta\sigma \cdot h} \right)^2 + \left(\frac{v_k - v_{k+1}}{\Delta\sigma \cdot h} \right)^2 \right), \quad P \in [0, 10^{-3}] \tag{16}$$

- **Average Richardson Number**:

$$Ri_{\text{avg}} = \frac{1}{N} \sum_{\text{valid}} \frac{N^2}{S^2}, \quad Ri \in [0, 10] \tag{17}$$

These models capture stratification effects via the Richardson number and shear-driven turbulence, enhancing the accuracy of mixing processes in estuarine simulations.