Stratification

Core Components and Initialization

The `Stratification` class, implemented in C#, models vertical stratification in an estuary, focusing on density-driven flows and passive scalar transport. The estuary spans a length $L = 10,000$ m and depth H, discretized into n grid points with spatial step $\Delta x = L/n$. Key parameters include:

- Gravitational acceleration: $g = 9.81\,\text{m/s}^2$
- Reference density: $\rho_0 = 1000\,\text{kg/m}^3$
- Ocean salinity: $S_{\text{ocean}} = 35\,\text{PSU}$
- Ocean temperature: $T_{\text{ocean}} = 20\,°\text{C}$
- River scalar concentration: $C_{\text{river}} = 1.0\,\text{kg/m}^3$
- Ocean scalar concentration: $C_{\text{ocean}} = 0.0\,\text{kg/m}^3$
- Salt wedge position: $x_s = 5000\,\text{m}$
- Base mixing coefficient: $\kappa = 0.1\,\text{m}^2/\text{s}$
- Critical Richardson number: $Ri_c = 0.25$
- Time step: $\Delta t = 3600\,\text{s}$ (1 hour)

The model initializes arrays for salinity (S), temperature (T), density (ρ), passive scalar concentration (C), and gradient Richardson number (Ri). Initial profiles are linear:

$$S(x) = \frac{x}{L} S_{\text{ocean}} \tag{1}$$

$$T(x) = 15 + \frac{x}{L}(T_{\text{ocean}} - 15) \tag{2}$$

$$C(x) = C_{\text{river}} \left(1 - \frac{x}{L}\right) \tag{3}$$

$$\rho(x) = \text{CalculateDensity}(S(x), T(x)) \tag{4}$$

with $S \in [0, 35]$, $T \in [0, 20]$, $C \in [0, C_{\text{river}}]$, and $\rho \in [1000, 1030]\,\text{kg/m}^3$.

Functioning Logic

The `UpdateSimulation` method advances the simulation by Δt, performing:

1. Velocity profile computation: $u(x) = 0.1(1 - x/L)$, clamped to $[-1, 1]\,\text{m/s}$.
2. Stratification computation via `ComputeStratification`.
3. Salinity, temperature, and passive scalar updates using `BaroclinicFlow` and `PassiveScalarTransportEq`.
4. Visualization of density and passive scalar profiles.

A GUI allows users to adjust the turbulence model ($k - \epsilon$, $k - \omega$, or constant), mixing coefficient, critical Richardson number, and river scalar concentration.

Stratification Computation

The `ComputeStratification` method calculates:

- **Density**: $\rho = \text{CalculateDensity}(S, T)$, clamped to $[1000, 1030]$ kg/m^3, computed via the `BaroclinicFlow` class.
- **Gradient Richardson Number**:

$$Ri = \frac{(g/\rho_0)(\partial \rho/\partial x)}{(\partial u/\partial x)^2} \tag{5}$$

  where $\partial \rho/\partial x = (\rho_{i+1} - \rho_{i-1})/(2\Delta x)$, $\partial u/\partial x = (u_{i+1} - u_{i-1})/(2\Delta x)$, clamped to $[-100, 100]$.

- **Eddy Viscosity**: Adjusted based on the turbulence model:
  - $k - \epsilon$:

$$\nu_t = \frac{\nu_t}{1 + \max(0, Ri/Ri_c)} \tag{6}$$

  - $k - \omega$:

$$\nu_t = \nu_t \frac{1 + 0.5 \min(Ri, Ri_c)}{1 + Ri} \tag{7}$$

  - Constant:

$$\nu_t = \kappa \tag{8}$$

  Eddy viscosity is clamped to $[0, 1]$ m^2/s.

Transport Equations

Salinity, temperature, and passive scalar are updated using transport equations solved by `BaroclinicFlow` and `PassiveScalarTransportEq`:

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left(\nu_t \frac{\partial \phi}{\partial x}\right) \tag{9}$$

where ϕ represents S, T, or C. Boundary conditions are:

- At $x = L$: $S = S_{\text{ocean}}$, $T = T_{\text{ocean}}$, $C = C_{\text{ocean}}$
- At $x = 0$: $C = C_{\text{river}}$

Visualization

The GUI displays density and passive scalar profiles in a panel, with:

- Density (ρ, blue): Scaled to $[1000, 1030]$ kg/m^3.
- Passive scalar (C, red): Scaled to $[0, C_{\text{river}}]$ kg/m^3.
- Axes labeled with position (km) and simulation time (hours).

Controls allow starting, pausing, resetting, and adjusting parameters, with updates every 100 ms.

Physical and Mathematical Models

The `Stratification` class simulates estuarine stratification driven by density gradients from salinity and temperature, coupled with passive scalar transport. The following models and equations are utilized:

- **Velocity Profile**: A simplified linear velocity profile drives advection:

$$u(x) = 0.1\left(1 - \frac{x}{L}\right), \quad u \in [-1, 1]\,\text{m/s} \tag{10}$$

- **Density Calculation**: Density is computed via the `BaroclinicFlow` class (implementation not shown), constrained to:

$$\rho \in [1000, 1030]\,\text{kg/m}^3 \tag{11}$$

- **Gradient Richardson Number**: Assesses stability of stratified flow:

$$Ri = \frac{(g/\rho_0)(\partial\rho/\partial x)}{(\partial u/\partial x)^2}, \quad \frac{\partial \rho}{\partial x} = \frac{\rho_{i+1} - \rho_{i-1}}{2\Delta x}, \quad \frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} \tag{12}$$

  with $Ri \in [-100, 100]$.

- **Eddy Viscosity Adjustment**: Modulates mixing based on turbulence model:
  - $k - \epsilon$:

$$\nu_t = \frac{\nu_t}{1 + \max(0, Ri/Ri_c)} \tag{13}$$

  - $k - \omega$:

$$\nu_t = \nu_t \frac{1 + 0.5\min(Ri, Ri_c)}{1 + Ri} \tag{14}$$

  - Constant:

$$\nu_t = \kappa \tag{15}$$

  with $\nu_t \in [0, 1]\,\text{m}^2/\text{s}$.

- **Transport Equation**: Governs evolution of salinity (S), temperature (T), and passive scalar (C):

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left(\nu_t \frac{\partial \phi}{\partial x}\right) \tag{16}$$

  solved numerically by `BaroclinicFlow` and `PassiveScalarTransportEq` classes, with boundary conditions:
  - At $x = L$: $S = 35\,\text{PSU}$, $T = 20\,°\text{C}$, $C = 0\,\text{kg/m}^3$
  - At $x = 0$: $C = C_{\text{river}}$

  and constraints $S \in [0, 35]$, $T \in [0, 20]$, $C \in [0, C_{\text{river}}]$.

- **Initial Conditions**: Linear profiles for initialization:

$$S(x) = \frac{x}{L} \cdot 35 \tag{17}$$

$$T(x) = 15 + \frac{x}{L} \cdot 5 \tag{18}$$

$$C(x) = C_{\text{river}} \left(1 - \frac{x}{L}\right) \tag{19}$$

These models capture the interplay of advection, diffusion, and stratification, with turbulence modulated by the Richardson number. The simulation ensures numerical stability through clamping and provides real-time visualization of density and scalar profiles.