**ECE1008 (SENSORS & CONTROL SYSTEMS) EXPERIMENT – 8**

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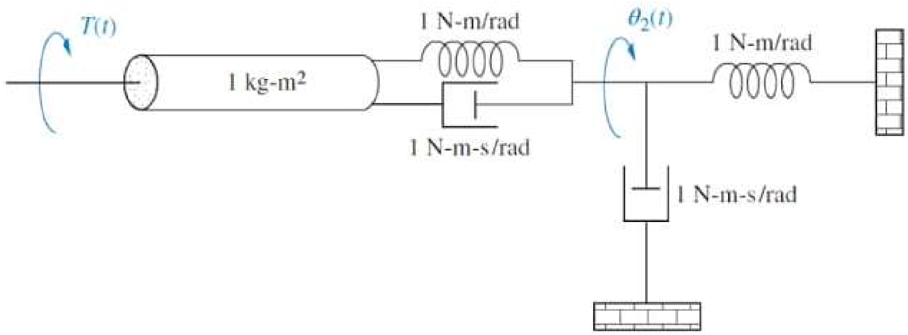
AIM –
Evaluation of time-domain performance indices of given rotational mechanical system




Derivation of mathematical model (transfer function) of the given rotational mechanical system considering the given specifications :–

using D'Alembert's principle for $J_1$:-

$$T(t) = T_{J_1}(t) + T_{B_1}(t) + T_{K_1}(t)$$

$$T(t) = J\frac{\partial^2\theta(t)}{\partial t^2} + B_1\left[\frac{\partial\theta(t)}{\partial t} - \frac{\partial\theta_1(t)}{\partial t}\right] + K_1\left[\theta(t)\right]$$

$$T(t) = \frac{J\partial^2\theta(t)}{\partial t} + B\left[\frac{\partial\theta(t)}{\partial t} - \frac{\partial\theta_2(t)}{\partial\theta}\right] + K\left[\theta_1(t) - \theta_2(t)\right]$$

We have,
$B_1 = 1$ Nms/rad.
$B_2 = 1$ Nms/rad.
$J = 1$ Kgm$^2$; $K_1 = 1$ Nm/rad.

$$T(t) = \frac{\partial^2\theta(t)}{\partial t} + \left[\frac{\partial\theta(t)}{\partial t} - \frac{\partial\theta_2(t)}{\partial t}\right] + \theta_1(t) - \theta_2(t)$$

using Laplace Transformation :-

$$T(s) = s^2\theta(s) + s\theta(s) - s\theta_2(s) + \theta_1(s) - \theta_2(s)$$

$$T(s) = (s^2+s+1)\theta(s) - (s+1)\theta_2(s)$$

For $\theta = T_{K_1}(t) + T_{B_1}(t) + T_{B_2}(t) + T_{K_2}(t)$.

$$= K_1\left[\theta_2(t) - \theta(t)\right] + B_1\left[\frac{\partial\theta_2(t)}{\partial t} - \frac{\partial\theta(t)}{\partial t}\right] + B_2\frac{\partial\theta_2(t)}{\partial t} + K_2\theta_2(t).$$

$$= \theta_2(s) - \theta(s) + s\theta_2(s) - s\theta(s) + \theta_2(s) + s\theta_2(s)$$

$$= 2(s+1)\theta_2(s) - (s+1)\theta(s)$$

Writing in matrix form:-

$$\begin{bmatrix} s^2+s+1 & -(s+1) \\ -(s+1) & 2(s+1) \end{bmatrix} \begin{bmatrix} \theta_1(s) \\ \theta_2(s) \end{bmatrix} = \begin{bmatrix} T(s) \\ 0 \end{bmatrix}$$

$$\Delta = (2s^2+s+1)(s+1)$$

$$\Delta_2 = \begin{bmatrix} s^2+s+1 & T(s) \\ -(s+1) & 0 \end{bmatrix} = T(s)(s+1).$$

$$\theta_2(s) = \frac{\Delta_2}{\Delta} = \frac{T(s)\cancel{(s+1)}}{\cancel{(s+1)}(2s^2+s+1)} = \frac{T(s)}{2s^2+s+1}$$

$$\Rightarrow \boxed{T.F. = \frac{1}{2s^2+s+1}}$$

Calculation of time domain specifications (delay time, rise time, peak time, peak overshoot, settling time, steady state error) –

Compare the TF $\frac{0.5}{s^2+0.5s+0.5}$ with standard 2nd order system $TF = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

Standard form of transfer function

Therefore,

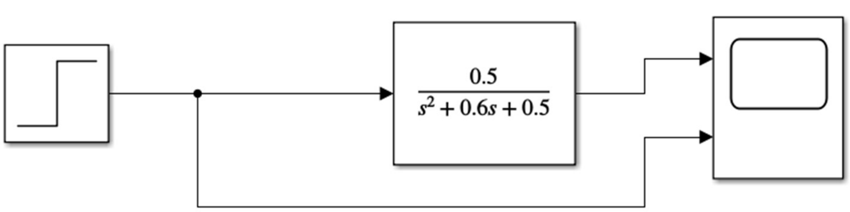
$$\omega_n = \sqrt{0.5} = 0.7071 \quad \text{and} \quad 2\zeta\omega_n = 0.5 \Rightarrow \zeta = \frac{0.5}{2\sqrt{0.5}} = 0.3535$$

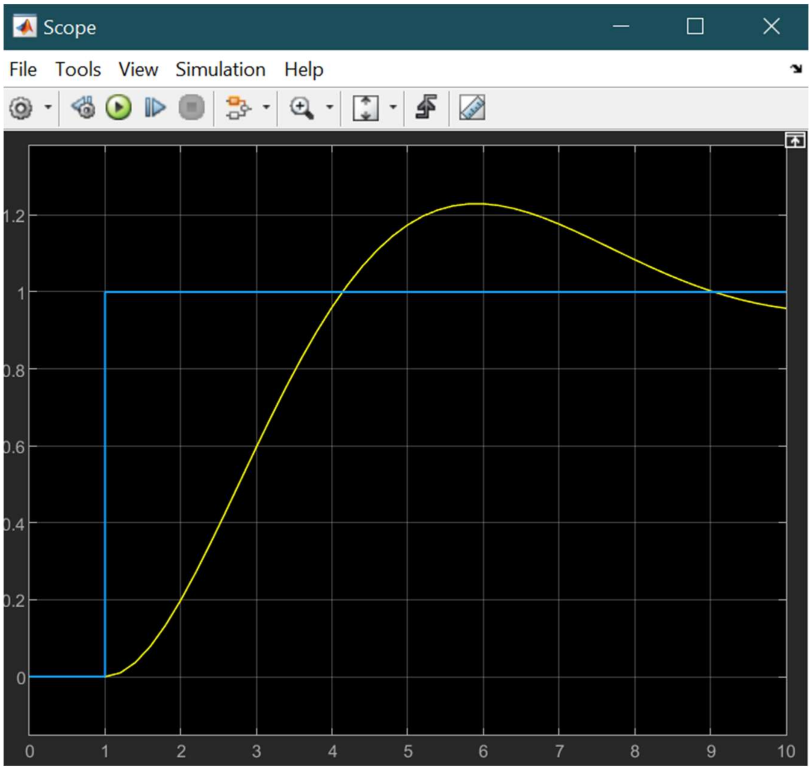
**Delay Time ($t_d$):** $$t_d = \frac{1+0.7\zeta}{\omega_n} = \frac{1+(0.7\times0.3535)}{0.7071} = 1.7641\,\text{sec}$$

**Rise Time ($t_r$):** $$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}(\zeta)}{\omega_n\sqrt{1-\zeta^2}}$$

$$= \frac{3.14-1.2094}{0.7071\sqrt{1-0.12496225}} = \frac{1.9306}{0.6614} = 2.918\,\text{sec}$$

BLOCK DIAGRAM USING SIMULINK –






OBSERVATION TABLE –

| *Parameter* | *Theoretical Value* | *Practical Value* |
|---|---|---|
| Delay Time | 1.8342 secs | 2.7 |
| Rise Time | 3.138 secs | 3.8 |
| Peak Time | 4.903 secs | 6.0 |
| Peak Overshoot | 22.9% | 4.13% |
| Setting Time | At 5% = 10 | 15.9 |
| | At 2% = 13.33 | 22 |
| Steady State Error | 0 | 0 |