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# Structures of cognitive and metacognitive reading strategy use for reading comprehension of geometry proof

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**Abstract** In this study, we explored the structural relationship between the students' perceived use of cognitive and metacognitive reading strategies (CMRS) and their reading comprehension of geometry proof (RCGP), and we also examined the differences in students' perceived use of reading strategies among the poor, moderate and good comprehenders. A sample of ninth graders ($N$=533) completed a RCGP test and then the CMRS questionnaire. In the exploratory factor analysis with one subsample ($n$=150), principal component analysis was used to extract factors of CMRS use for improving the CMRS instrument. Another subsample of students ($n$=370) participated in the study on the confirmatory factor analysis with structural equation modelling method. Results revealed that the use of metacognitive reading strategies exerts an executive function over that of cognitive reading strategies, which directly influenced students' RCGP. Our interesting findings were that good comprehenders tended to employ more metacognitive reading strategies for planning and monitoring comprehension and more cognitive reading strategies for elaborating proof compared with the moderate comprehenders, who in turn employed these strategies more often compared with the poor comprehenders.

**Keywords** Geometry proof · Reading comprehension · Reading strategy

## 1 Introduction

### 1.1 Difficulty in understanding mathematical proof

There has been extensive research on college students' difficulties in understanding mathematical proof. Some of those difficulties are related to reading comprehension. For example, Selden and Selden (2003) found that undergraduates tended to focus on surface features of arguments and that their ability to validate arguments was very limited. These

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difficulties may result from their lack of proof techniques, analysing proof structures (Solow, 2002) and deductive proof schemes (Harel & Sowder, 2007).

Regarding high school students' difficulties, we can find more studies that investigated their difficulties in conjecturing and constructing proof than those that investigated their difficulties in understanding mathematical proof. Some studies that explored high school students' understanding of proof found that they may view evidence as valid arguments (Chazan, 1993), incorrectly evaluate the generality of algebraic arguments (Healy & Hoyles, 2000), and perform poorly on RCGP (Yang & Lin, 2008). Few studies justified cognitive sources of difficulties in high school students' understanding of proofs.

### 1.2 Reading strategy as an approach to learning mathematics

Researchers who study language learning have emphasized that reading strategy use is an approach to improving reading comprehension. Moreover, many empirical studies showed that successful learners differ from less successful ones in both the quantity and quality of CMRS use (Phakiti, 2003). Reading to learn is essential not only to language learning but also to academic learning in all subject areas and to lifelong learning (Dreyer & Nel, 2003; Pritchard, Romeo & Muller, 1999).

Although the number of studies on reading strategy in mathematics education is less than that in language arts, an increasing number of studies have shown the significance of applying reading strategies to learning mathematics. For example, transactional reading strategies can lead secondary school students to discussion and sense-making in mathematics classrooms (Borasi, Siegel, Fonzi & Smith, 1998). Reading strategies used in reciprocal teaching method were adopted to design tasks of reading mathematics proofs in order to trigger or structure students' schemas (Yang & Lin, 2009). Nonetheless, there are still relatively few studies that explore how students read mathematics proof.

### 1.3 Aims of this study

We have pointed out in section 1.1 and 1.2 that students have difficulties in understanding mathematical proof, and that reading strategy is one approach to learning mathematics. On the other hand, deep comprehension arises from an effortful attempt to construct a coherent understanding of what a text is about by engaging in specific reading strategies (Graesser, Singer & Trabasso, 1994; Magliano, Todaro, Millis, Wiemer-Hastings, Kim & McNamara, 2005). Accordingly, more research is anticipated on reading strategies related to understanding mathematical proof.

Thus, this study was designed to explore structural relationships of the perceived metacognitive and cognitive reading strategy use to RCGP for junior high school students. If the contribution of the two categories of perceived reading strategy use to RCGP is determined, then the possible relationships between the two categories become an issue. In addition, the differences among successful, moderate and poor comprehenders in their perceived metacognitive and cognitive reading strategy use are important for further understanding why RCGP is difficult for some students.

In sum, two principal research questions of this study emerged:

1. What is the relationship between perceived metacognitive and cognitive reading strategy use, and how are the two categories of perceived reading strategy use related to RCGP?

2. Are there any differences among successful, moderate and poor comprehenders in their perceived use of metacognitive and cognitive reading strategies for RCGP?

In the current study, proof is reduced to correct deductive organization presented in junior high school textbooks, as in Appendix A.

## 2 Theoretical background

### 2.1 Reading comprehension of geometry proof

Because proofs are viewed as a genre of mathematical text, one way to capture understanding of proofs is through the approach of reading comprehension. Reading comprehension of proofs means understanding proofs from the essential elements of knowing how a proof operates and why a proof is right, in addition to knowing what a proof can prove (Yang & Lin, 2008). The process of reading proofs and the resulting comprehension are difficult to observe because not all of it is conscious; some comprehension occurs at the subconscious level. Therefore, Yang and Lin formulated the operational definitions of five facets—basic knowledge, logical status, integration or summarisation, generality and application—to develop an instrument for measuring RCGP of ninth-grade students.

The facet of basic knowledge requires one to understand mathematical terms, figures and symbols in a proposition and its proof. The facet of logical status requires one to correctly recognize the status of arguments which may be premises, conclusions or applied properties in proof. The facet of summarisation requires one to identify the core of the proof or the critical proof idea. The facet of generality requires one to correctly recognize accuracy of a proposition and to identify what is validated by the proof. The facet of application requires one to appropriately apply the proposition in another situation. The operational definitions were formulated according to learning goals for ninth graders. Some contents, like improving a valid proof method or formulating extended questions, were not included.

### 2.2 Research on reading strategy in language

Reading comprehension involves conscious and unconscious use of various strategies (Johnston, 1983). Reading strategies have been defined as goal–directed actions which are undertaken by readers for planning and monitoring their efforts in order to decode text, understand words and construct meaning of texts (Alexander & Judy, 1988; Pereira-Laird & Deane, 1997). Besides, reading strategies differ from reading skills. Reading strategies are reflective, and reading skills are automatic actions that result in decoding and comprehension with speed, efficiency, and fluency and usually occur without control involved (Afflerbach, Pearson, & Paris, 2008).

Reading strategies are usually classified into three categories depending on the level or type of thinking process involved: *cognitive*, *metacognitive* and *social* affective strategies (Anastasiou & Griva, 2009). In this study, we focused on the CMRS. Pereira-Laird and Deane (1997) explained these strategies as follows:

> Metacognitive strategies involve planning, monitoring, and regulation activities that take place before, during, and after any thinking act such as reading. In contrast, cognitive strategies refer to integrating new material with prior knowledge. In

> contrast, cognitive strategies refer to integrating new material with prior knowledge. Cognitive strategies that students use to acquire, learn, remember, retrieve and understand the material while reading include rehearsal, elaboration, and organizational strategies. (p. 190)

Accordingly, cognitive reading strategies are the actions readers take while interacting directly with the text whereas metacognitive reading strategies are intentional, planned tactics by which learners monitor, identify and remediate their reading (Anastasiou & Griva, 2009; Sheorey & Mokhtari, 2001).

Readers have been found to employ a wide range of reading strategies to comprehend texts (Paris, Wasik & Turner, 1991). Good comprehenders are more able than poor comprehenders to distinguish between surface-structure and deep-structure information, predict or question while reading, notice inconsistencies in a text and use strategies to make these inconsistencies understandable (Garner, 1980; Olson, Duffy & Mack, 1984). Furthermore, Purpura (1999) surprisingly found that CMRS use would not both exert a direct and positive impact on second language test performance. It was confirmed in Purpura's study that the metacognitive process as a latent trait showed no direct impact on performance; however, it produced significant, indirect effects on performance by means of the cognitive process. This suggests a strong need for investigating the relationships between CMRS use and their impacts on RCGP.

### 2.3 Research on reading mathematical proof

Research related to reading mathematical proofs showed that undergraduates may check proofs step by step, follow arguments logically, generate examples and make sure the ideas in a proof make sense while validating proofs (Selden & Selden, 2003). In a study of undergraduate mathematics majors, Weber, Brophy and Lin (2008) found that successful mathematics majors were more likely than non-successful majors to use reading strategies, such as reformulating definitions and reflecting on statements before the statements were proved.

In a study of secondary students, Yang, Lin and Wang (2007) found that students with different levels of reading comprehension might employ the same reading strategies at different time points (before, while or after reading), and students with similar knowledge background might use different reading strategies. Furthermore, Wang (2008) found that successful students used more reading strategies in predicting, finding key points, representing transformations, clarifying and summarising.

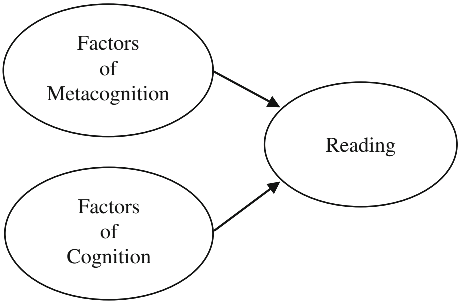
Aside from these findings, there is no study on CMRS for RCGP, and it is still questionable whether more strategy use predicts better RCGP.

## 3 Methods

According to Purpura's (1997) hypothesis, we postulated that there is a strong relationship between CMRS use, whereby the integration of metacognition and cognition would result in better performance (the non-mediated model). This model (see Fig. 1) represents one possible relationship, that is, the two types of reading strategy use have the same position for RCGP.

However, the empirical study (Purpura, 1997) showed that students' metacognitive strategy use exerts an executive function over their cognitive strategy use, which directly

**Fig. 1** Non-mediated model




influenced their performance (the mediated model). This model (see Fig. 2) represents another possible relationship, that is, the two constructs have different orders of a hierarchical structure for RCGP.

To assess RCGP and students' perceived reading strategy use, we firstly developed two instruments, which would are described in Sections 3.2 and 3.3, respectively. Then, the structural relationships among cognitive strategies, metacognitive strategies and RCGP were confirmed by structural equation modelling (SEM). SEM is a technique used for specifying and estimating models of linear relationships among variables including both measured variables and latent variables (Byrne, 2001).

## 3.1 Participants

In Taiwan, the grades 1–9 curriculum is compulsory in schools, and the content of the learning materials does not vary substantially from school to school. A total of 533 grade 9 students participated in this study; all of the participants came from 17 classes at the same public junior high school in Taipei (when students enrolled in the school, they were randomly assigned to classes where the students' mean scores on an intelligence test were not significantly different statistically). The public junior high school was selected by convenience sampling because the school's mathematics teachers were willing to collaborate with the researchers in this study. In order to encourage students to respond to the questionnaire seriously, they were informed that their score on the RCGP test would be part of their mathematics semester grade and their responses to the instrument of reading strategy use would be taken into account when designing instruction for teaching mathematical proof in the future.

The participating students had been taught some properties of triangles (including congruent and similar triangles) by means of geometric calculation or manipulative reasoning in the second semester of grade 8. Moreover, they had just been taught how to construct a deductive proof with two or more steps in the first semester of grade 9 before this study. The geometry proof problems are in the form of "Given X, show Y." or "Given X, is Y correct?" with a figure (X,Y) in which the figural meanings of X and Y are

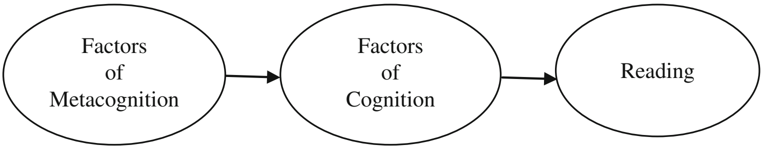



**Fig. 2** Mediated model

embedded, and students are asked to construct the intermediate condition to bridge X and Y by forward or backward inference (ref., Heinze, Cheng, Ufer, Lin & Reiss, 2008).

After discarding 21 cases because of missing data in one of the two instruments, we had 268 male and 265 female students left in the sample. Therefore, data from 96.2% of the total student cohort were used. Prior to analysing the data, the participants were randomly assigned to one of two groups with different analytical methods. The data were analysed in two ways: (1) the first sample of ninth grader ($n$=150) participated in the study on exploratory analyses mainly using principal component analysis in order to extract interpretable scales from the Likert-type responses and to obtain internal-consistency reliability estimates, and (2) the second sample of ninth graders ($n$=370) participated in the study on confirmatory factor analysis with SEM, and 13 cases were discarded due to incomplete data.

### 3.2 Instruments for measuring RCGP

Two sets of research instruments were employed in the study: (1) RCGP test and (2) CMRS use questionnaire or the CMRS instrument. The geometrical content of the RCGP test was adopted from a standard Taiwanese grade 9 mathematics textbook (see Appendix A). The instrument of the RCGP test was designed according to the previously described operational definitions of facets of RCGP (Yang & Lin, 2008) (see the outline in Table 1). Sixteen questions of RCGP were included in the test (see Table 1 and Appendix A), which took about 30 min for students to complete.

Except for questions (1) and (2), all questions were in the multiple choice format. Some questions contained multiple correct choices, so partial scores of 1, 2 or 3 were given. For example, question (7) asked which properties should be applied in a given proof. There were three correct answers among the four choices. If students selected only three correct answers, they would get a score of 3. If students selected the three correct answers and the wrong choice, they would get a score of 2. If students selected one of the three correct answers and the wrong choice, they would get zero score.

Quantitative measures regarding the five facets were derived from the 16 corresponding question items: basic knowledge comprehension, logical status comprehension, summarisation comprehension, generality comprehension, and application comprehension (see Table 1). The Cronbach's alpha reliability coefficient for this instrument was 0.783 for the grade 9 participants in this study.

### 3.3 Instrument for measuring reading strategy use

The CMRS instrument measures two broad categories of reading strategy use, namely, cognitive strategy use and metacognitive strategy use. Cognitive strategies are ongoing mental activities used by readers to utilize their knowledge and inference to decode the given text, and metacognitive strategies are directive and regulative actions used by readers to monitor and evaluate their cognition. A brief description of each category and the number of items within each category are given below (Anastasiou & Griva, 2009).

1. Cognitive strategies are composed of direct interaction with the text and contribute to the initiation of comprehension. Such strategies include reading the proposition first, underlining, guessing from the attached figure, extracting, numbering proof steps and so forth (11 items).

**Table 1** Structure of reading comprehension of geometry proof test

| Facet | Object of comprehension | Operational definition | Item number | Score[a] |
|---|---|---|---|---|
| Basic knowledge | Content of premise or conclusion | Recognizing the meaning of a symbol in figure | 1 | 0, 1 |
| | | Explaining the meaning of a property | 2 | 0, 1, 2 |
| | | Recognizing the meaning of a property | 3 | 0, 1 |
| Total score of basic knowledge | | | | 4 |
| Logical status | Status of premise | Recognizing a condition applied directly | 4 | 0, 1 |
| | Logical relation between premise and conclusion | Judging the logical order of statements | 5 | 0, 1, 2 |
| | | | 6 | 0, 1, 2 |
| | Property applied to derive conclusion from premise | Recognizing which properties are applied | 7 | 0, 1, 2, 3 |
| Total score of logical status | | | | 8 |
| Summary | Multiple arguments and critical ideas | Identifying critical procedures, premises, or conclusions | 8-1 | 0, 1, 2 |
| | | | 8-2 | 0, 1 |
| | | Identifying critical ideas of a proof | 10-1 | 0, 1, 2 |
| | | | 10-2 | 0, 1, 2 |
| Total score of summary | | | | 7 |
| Generality | Proposition or proof | Judging the correctness | 11, 12 | 0, 1; 0, 1 |
| | All arguments and attached figure | Identifying what is validated by this proof | 9, 13-1, 13-2 | 0, 1; 0, 1 |
| | | | | 0, 1 |
| Total score of generality | | | | 5 |
| Application | Knowing to apply in other situations | Application in the same premises | 14, 15 | 0, 1; 0, 1, 2, 3 |
| | | Identifying the different premises | 16 | 0, 1 |
| Total score of application | | | | 5 |

[a] Some questions require multiple right responses or choices, so partial scores of 1, 2 or 3 are given

2. Metacognitive strategies require intentional reading, careful monitoring and regulation. Such strategies include having a purpose in mind, identifying key elements of proof, monitoring one's own understanding or misunderstanding and thinking about the proposition and so forth (20 items).

The CRMS items were derived from the studies on students' reading strategies used for RCGP (Wang, 2008; Yang, Lin & Wang, 2007; Yang & Wang, 2008). The CMRS initially consisted of 31 items, each of which used a five-point Likert scale ranging from 1 ("I never or almost never do this") to 5 ("I always or almost always do this"). Thus, the higher the score, the more frequent the perceived use of the reading strategy.

Because the correlation coefficient between item 5 ("I consider what the arguments can prove.") and the whole instrument was less than 0.4, we deleted this item for exploring the number of factors by the exploratory factor analysis with a sample of 150 participants. The principal component analysis method was used to form uncorrelated linear combinations of the observed variables. Decisions about the number of components to retain were based on eigenvalues greater than one. The initial component solution was followed by varimax rotation with Kaiser Normalization, which is an orthogonal rotation method that simplifies the interpretation of the components (Tabachnick & Fidell, 1996).

### *3.3.1 Exploratory factor analysis for cognitive reading strategy use*

The analysis of the 11 items for cognitive strategies yielded two factors which explained 55.45% of the total variance. In order to increase the explanatory power of the total variance, two items of cognitive strategies (items 24 and 25) were deleted. Nine items of cognitive strategies were re-analysed in the exploratory factor analyses. The two factors, containing five and four items respectively, explained 59.34% of the total variance and represented two sets of reading strategies. The rotated factor patterns are shown in Table 2. The two factors emerging from the analysis were interpreted and named as *elaborating* and *indicating* proof.

The cognitive strategies used in elaborating a proof are related to constructing a situation model, which incorporates the textual information with a reader's pre-existing knowledge structures (Magliano, Radvansky & Copeland, 2007), and are helpful for bridging either the proof text and figures or the proof and one's own knowledge. For instance, students may visualise part of proof text in the corresponding part of a figure to enrich their comprehension. The cognitive strategies for indicating proof are related to highlighting important information, a strategy frequently employed by fluent readers (Cioffi, 1986). Highlighting also helps a reader to identify specific pieces of information for further processing (O'Donnell, 1993). For example, the proof text is underlined according to students' perceived importance and the information is kept in mind for later processing.

### *3.3.2 Exploratory factor analysis for metacognitive reading strategy use*

The analysis of the 19 items for metacognitive strategies yielded two factors which explained 58.37% of the total variance. In order to increase the explanatory power of the total variance, three items of metacognitive strategies (items 6, 7 and 23) were deleted. Sixteen items of metacognitive strategies were re-analysed in the exploratory factor analysis. The three factors, containing eight, five and three items respectively, explained 61.31% of the total variance and represented three sets of reading strategies. The rotated factor patterns are shown in Table 3. The three factors emerging from the analysis were interpreted and named as *planning*, *monitoring* and *regulating*.

**Table 2** Exploratory factor analysis loadings of cognitive strategies

| | Inventory item of cognitive strategies | Factor[a] | |
|---|---|---|---|
| | | 1 | 2 |
| 1 | I read the proposition of the proof first | 0.756 | |
| 4 | I try to guess and draw an inference from figures | 0.722 | |
| 12 | I try to keep reading to see how the steps are going | 0.565 | |
| 26 | I draw a figure to enhance my understanding | 0.764 | |
| 27 | I label the figure according to the proof text | 0.722 | |
| 10 | I identify the key steps from the proof text | | 0.544 |
| 20 | I underline (or circle) the property or definition in the text | | 0.873 |
| 21 | I underline (or circle) the key steps in the proof | | 0.904 |
| 22 | I number proof steps to organize the proof process | | 0.613 |

[a] Loadings of greater than 0.4 were outputted

**Table 3** Exploratory factor analysis loadings of metacognitive strategies

| | Inventory item of metacognitive strategies | Factor | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| 3 | I think about how to prove after reading the proposition | 0.734 | | |
| 11 | I try to look back to enhance my understanding | 0.648 | | |
| 13 | I try to think over what I don't understand | 0.603 | | |
| 14 | I skim through the text and then read carefully | 0.696 | | |
| 15 | I skim through the properties of the proof | 0.708 | | |
| 16 | I identify the given before reading the whole proof | 0.710 | | |
| 18 | I identify the important steps in the text | 0.654 | | |
| 25 | I would read the text step by step | 0.614 | | |
| 17 | I identify the claim before reading the whole proof | | 0.587 | |
| 19 | I consider the relation between steps | | 0.652 | |
| 29 | I ask myself how the proof started and came to the conclusion | | 0.633 | |
| 30 | I ask myself which part I don't understand | | 0.700 | |
| 31 | I ask myself the relation between figures and proof process | | 0.759 | |
| 2 | I consider the correctness of the proposition | | | 0.705 |
| 8 | I think about the application of the proposition | | | 0.658 |
| 9 | I think about how to modify or alter the proposition | | | 0.757 |

The planning strategies are related to the formulation of specific goals (e.g. to prove or to find out) and the reading process (e.g. to skim through or to read step by step). The monitoring strategies are related to items about proof structure (e.g. to think about how the proof starts and comes to the conclusion) and focus on logical chains of proof steps (e.g. to consider the relation between steps). The regulating strategies are related to the application of the proposition. The three factors are compared with Pintrich's (1999) definition of planning, monitoring and regulating strategies, which respectively help readers plan the comprehension process, notice the comprehension situations, and repair the comprehension performance. However, the meanings of planning, monitoring and regulating strategies in geometry proof reading are quite different from those in language reading. Specifically, these three strategies—planning what to find rather than planning how to find, monitoring logical relations in addition to the status of understanding and regulating propositions to enhance comprehension instead of repairing comprehension—are respectively the characteristics of the three factors.

One may ask why Item 16 ("I identify the given before reading the whole proof") and Item 17 ("I identify the claim before reading the whole proof") were included in planning and monitoring factors, respectively. Conceptually, these two items appear to be very much related. One possible reason is that different types of inferences may follow, that is, the identification of the given is followed by forward inference to predict how to prove, which is item 3 of the planning factor ("I think about how to prove after reading the proposition."). In contrast, to identify the claim, one has to infer backward to explain the logics between steps, which is item 19 of the monitoring factor.

The final version of the CMRS instrument consisted of nine items on cognitive reading strategy use (see Table 2) and 16 items on metacognitive reading strategy use (see Table 3).

This instrument would be used to measure the frequencies of perceived use of cognitive and metacognitive strategies by students while reading geometry proof.

### 3.4 Analysis

Measurements and structural models were analysed with AMOS 18 (Arbuckle, 2009). Because some of the observed variables were nonsymmetrical, unweighted least squares factor extraction was used to estimate the parameters (Joreskog & Sorbom, 1984). In order to standardize each item with the same total score, each student's score on each facet of RCGP was divided by the total score of each facet and then multiplied by five because CMRS used a five-point Likert scale.

Based on the scores of RCGP, the top or bottom 27% of the 533 participants were identified as good or poor comprehenders, and the others were identified as moderate comprehenders. To compare the reading strategies used among good ($n$=141), moderate ($n$=250) and poor comprehenders ($n$=142), MANOVA/ANOVA test and post hoc analysis were conducted to examine the differences in the use of cognitive and metacognitive strategies among the three groups. Besides the statistical significance, partial eta-square ($\eta^2$) as a measure of effect size was also reported. For easy interpretation, 0.01 is small effect size, 0.06 is medium and 0.14 is large (Cohen, 1973).

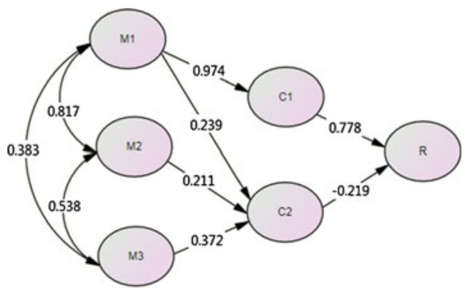
## 4 Results

### 4.1 Relationships among cognitive and metacognitive strategies and RCGP

Of all analyses reported, all matrices of parameter estimates were positively definite, and the iterative estimation procedure converged. In order to ensure that no parameter estimates were out of range and models were both substantial and simple, the modified mediated model, as shown in Fig. 3, was used. The following symbols were used to designate latent variables: (1) M1=metacognitive reading strategy use for planning factor; (2) M2=metacognitive reading strategy use for monitoring factor; (3) M3= metacognitive reading strategy use for regulating factor; (4) C1=cognitive reading strategy use for elaborating proof factor; (5) C2=cognitive reading strategy use for indicating proof factor; (6) $R$=RCGP.

Because the Chi-square test was sensitive to sample size, we also utilized the goodness-of-fit index (GFI), the adjusted goodness-of-fit index (AGFI), the normed fit index (NFI), the root mean square residual (RMR), the parsimony goodness-of-fit index (PGFI) and the parsimonious normed fit index (PNFI) to compensate for the sensitivity. By looking at the

**Fig. 3** Modified mediated model



variances and covariances accounted for by the model, GFI showed how closely the model replicated the observed covariance matrix. GFI was not sensitive to sample size, and AGFI was adjusted to take into account the degrees of freedom in the model. NFI assessed the model by comparing the $\chi^2$ value of the model to the $\chi^2$ value of the null model. RMR was an index of the degree of discrepancy between elements in the sample and the hypothesized covariance matrix. PGFI and PNFI were based on GFI and NFI, respectively, after adjusting for degrees of freedom.

A rule of thumb for GFI, AGFI and NFI is that a value of 0.95 is indicative of good fit. The RMR values (values range from 0 to 1.00) as low as 0.08 are deemed acceptable (Hu & Bentler, 1999). The goodness-of-fit indices shown in Table 4 indicated that the non-mediated model did not provide a good fit for the data in this study since the goodness-of-fit indices were all well below 0.90. On the contrary, the modified mediated model provided a good fit for the data. PGFI of 0.829 and PNFI of 0.877 also indicated good model parsimony.

The actual factor loadings of metacognitive strategy use, cognitive strategy use and RCGP items on the hypothesized latent factors are shown in Table 5. Factor loadings generated by the covariance matrix exceeded 0.50 in all instances except the application facet of RCGP.

The estimated parameters of the modified mediated model are displayed in Fig. 2. The metacognitive reading strategy factors were found to be strongly intercorrelated, and this was particularly true for the correlation between M1 and M2, $r$=0.817. However, it was still less than 0.9 so converging to one factor was not necessary. The effect coefficients (total effects) for the modified mediated model are included in Table 6. The results indicated that two factors had the largest impacts on RCGP: M1 and C1, with total effects 0.706 and 0.778, respectively. The impact of M2 on RCGP was not mediated through C1 but was decreasingly mediated through C2. Moreover, M1 had a unique effect (0.974) on C1 and M3 had the largest total effect (0.372) on C2.

The C1 factor (elaborating proof) represents a group of strategies used by students to directly deal with the proof and to initiate comprehension, and the C2 factor (indicating proof) represents a group of strategies to keep track of key steps. The M1 factor (planning) represents strategies that direct reading goals and manage reading process, the M2 factor (monitoring) represents strategies that students use to evaluate their comprehension by concentrating on the meaning of relations between proof steps and figures, and lastly, the M3 factor (regulating) represents students' adaptive strategies concerning the application of the proposition.

The SEM results showed that metacognitive reading strategy use was a higher-order construct, which was mediated by cognitive reading strategy use to influence RCGP. Specifically, the use of cognitive strategies for Elaborating proof could indeed facilitate RCGP, whereas the use of cognitive strategies for Indicating proof was the most inferior factor of reading strategy use that impeded RCGP. Furthermore, planning, mediated through cognitive strategies for elaborating proof, was the second

**Table 4** Comparison of models with the measures of goodness of fit

| | $\chi^2$ | *df* | *p* | GFI | AGFI | NFI | RMR | PGFI | PNFI |
|---|---|---|---|---|---|---|---|---|---|
| Independence | 32,228 | 435 | <0.01 | 0.268 | 0.217 | 0 | 0.433 | 0.250 | 0 |
| Mediated | 1,182.6 | 396 | 0.08 | 0.973 | 0.968 | 0.963 | 0.083 | 0.829 | 0.877 |
| Non-mediated | 11,302 | 396 | <0.01 | 0.743 | 0.698 | 0.649 | 0.257 | 0.633 | 0.591 |

**Table 5** Confirmatory factor analysis loadings

| | Item | Loading |
|---|---|---|
| Metacognitive strategy use | | |
| M1 | 3 | 0.744 |
| | 11 | 0.729 |
| | 13 | 0.719 |
| | 14 | 0.637 |
| | 15 | 0.696 |
| | 16 | 0.657 |
| | 18 | 0.730 |
| | 25 | 0.606 |
| M2 | 17 | 0.741 |
| | 19 | 0.739 |
| | 29 | 0.703 |
| | 30 | 0.613 |
| | 31 | 0.697 |
| M3 | 2 | 0.689 |
| | 8 | 0.726 |
| | 9 | 0.781 |
| Cognitive strategy use | | |
| C1 | 1 | 0.605 |
| | 4 | 0.726 |
| | 12 | 0.775 |
| | 26 | 0.665 |
| | 27 | 0.643 |
| C2 | 10 | 0.708 |
| | 20 | 0.694 |
| | 21 | 0.745 |
| | 22 | 0.615 |
| RCGP | | |
| R | B | 0.623 |
| | L | 0.660 |
| | S | 0.701 |
| | G | 0.638 |
| | A | 0.454 |

important factor of reading strategy use to enhance RCGP. The intercorrelation among planning, monitoring and regulating strategies showed the complexity and interconnectivity of these metacognitive processes. These factors intertwined with each other and then directed the deployment of appropriate cognitive strategy use for elaborating instead of Indicating proof.

## 4.2 Comparison among good, moderate and poor comprehenders

Table 7 presents the descriptive statistics of students' RCGP scores and strategy use categorized by good, moderate and poor performance on the RCGP test. The scores of

**Table 6** Effect coefficients of latent variables

| Variable | Direct effect | Indirect effect | Total effect |
|---|---|---|---|
| R | | | |
| C1 | 0.778 | –[a] | 0.778 |
| C2 | −0.219 | –[a] | −0.219 |
| M1 | –[a] | 0.706 | 0.706 |
| M2 | –[a] | −0.046 | −0.046 |
| M3 | –[a] | −0.081 | −0.081 |
| C1 | | | |
| M1 | 0.974 | –[a] | 0.974 |
| C2 | | | |
| M1 | 0.239 | –[a] | 0.239 |
| M2 | 0.211 | –[a] | 0.211 |
| M3 | 0.372 | –[a] | 0.372 |

[a] No direct or indirect paths hypothesized

performance on the RCGP test increased from the poor comprehenders to the moderate comprehenders and then to the good comprehenders. According to their mean scores on RCGP, the good comprehenders correctly answered most of the 16 questions, the poor comprehenders failed to comprehend the facets of *summary*, *generality* and *application*, and moderate comprehenders performed better on the facet of summary than the poor comprehenders did.

The RCGP mean scores of the good, moderate and poor comprehenders were significantly different ($F_{(2,\ 530)}$=1,331.328, $p$<0.01, $\eta^2$=0.834). Regarding all factors of strategy use except C2, the good comprehenders responded with more perceived use of

**Table 7** Descriptive statistics by levels of RCGP

| | RCGP | Mean | SD |
|---|---|---|---|
| Reading comprehension of geometry proof (RCGP) | Good | 4.22 | 0.32 |
| | Moderate | 3.15 | 0.38 |
| | Poor | 1.77 | 0.50 |
| Cognitive reading strategy use for *Elaborating* proof (C1) | Good | 4.49 | 0.58 |
| | Moderate | 4.24 | 0.66 |
| | Poor | 3.65 | 0.89 |
| Cognitive reading strategy use for *Indicating* proof (C2) | Good | 3.15 | 0.97 |
| | Moderate | 3.20 | 0.95 |
| | Poor | 2.80 | 0.99 |
| Metacognitive reading strategy use for *Planning* comprehension (M1) | Good | 4.37 | 0.60 |
| | Moderate | 4.12 | 0.66 |
| | Poor | 3.52 | 0.86 |
| Metacognitive reading strategy use for *Monitoring* comprehension (M2) | Good | 3.80 | 0.82 |
| | Moderate | 3.37 | 0.80 |
| | Poor | 2.93 | 0.90 |
| Metacogntive reading strategy use for *Regulating* comprehension (M3) | Good | 2.68 | 0.89 |
| | Moderate | 2.57 | 0.92 |
| | Poor | 2.34 | 0.90 |

Good comprehender ($n$=141), moderate comprehenders ($n$=250) and poor comprehenders ($n$=142)

**Table 8** MANOVA results for good, moderate and poor comprehenders

| Dependent variables | $F$ | $df$ | $p$ | $\eta^2$ |
|---|---|---|---|---|
| C1 | 56.72 | 2 | <0.01 | 0.176 |
| C2 | 8.51 | 2 | <0.01 | 0.031 |
| M1 | 58.69 | 2 | <0.01 | 0.181 |
| M2 | 41.12 | 2 | <0.01 | 0.134 |
| M3 | 5.76 | 2 | <0.01 | 0.021 |

reading strategies compared to the moderate comprehenders, and the moderate comprehenders responded with more perceived use of reading strategies compared to the poor comprehenders.

Table 8 shows the results of MANOVA, and there was a statistically significant difference in all five factors of CMRS among the good, moderate and poor comprehenders. The level of RCGP had small effect sizes to predict cognitive strategy use for indicating proof factor (C2) and regulating strategy use factor (M3) but had large effect sizes to predict cognitive strategy use for elaborating proof factor (C1), planning (M1) and monitoring (M2) strategy use factors.

The Dunnett's T3 post hoc tests (see Table 9) were conducted to examine which differences were statistically significant. There was no statistically significant difference in either C2 or M3 between the good and moderate comprehenders. Moreover, no statistically significant difference was observed in M3 between moderate and poor comprehenders. In general, the good comprehenders perceived significantly higher use of metacognitive reading strategy for *planning* and *monitoring* comprehension and significantly higher use of cognitive reading strategy use for *elaborating* proof compared to the moderately successful comprehenders who in turn perceived significantly higher use of these strategies compared with the poor comprehenders.

**Table 9** Dunnett's T3 post hoc tests of differences across three groups

| Dependent variable | I | J | Mean differences (I–J) | Standard error | $p$ |
|---|---|---|---|---|---|
| C1 | Good | Moderate | 0.250 | 0.075 | <0.01 |
| | Moderate | Poor | 0.593 | 0.075 | <0.01 |
| C2 | Good | Moderate | −0.054 | 0.102 | 0.867 |
| | Moderate | Poor | 0.400 | 0.101 | <0.01 |
| | Good | Poor | 0.346 | 0.113 | <0.01 |
| M1 | Good | Moderate | 0.251 | 0.744 | <0.01 |
| | Moderate | Poor | 0.600 | 0.074 | <0.01 |
| M2 | Good | Moderate | 0.430 | 0.088 | <0.01 |
| | Moderate | Poor | 0.440 | 0.087 | <0.01 |
| M3 | Good | Moderate | 0.108 | 0.095 | 0.522 |
| | Moderate | Poor | 0.234 | 0.094 | 0.041 |

## 5 Discussion

Researchers have given relatively little attention to how students read mathematical proofs. In response to this lack of attention, this study set out to justify theoretically the structural relationship of CMRS use to RCGP. Practically, this study validated an instrument to measure students' perceived use of CMRS for RCGP. This study provided evidence for a model of good comprehenders who were in control of the reading process and actively engaged in *planning* what to identify and *monitoring* comprehension for logical coherence. Furthermore, the metacognitive reading strategies directed the deployment of appropriate cognitive strategies for *elaborating* proof—in the same way as the orchestra conductor directs the players in creating a harmonious performance (cf. Vandergrift, 2003)—to interact with the proof and achieve the final goal of RCGP. It should be noted that cognitive reading strategy use for *indicating* proof might impede RCGP.

Moreover, the perceived use of cognitive and metacognitive strategies across the good, moderate and poor comprehenders differed quantitatively. The good comprehenders perceived significantly higher use of metacognitive reading strategies in planning and monitoring comprehension and cognitive reading strategies in elaborating proof compared to the moderate comprehenders who in turn perceived significantly higher use of these strategies compared to the unsuccessful comprehenders.

When the two factors of cognitive reading strategy use are considered, most of the C1 items (elaborating proof) could be viewed as strategies for understanding deep structure of proof whereas most of the C2 items (indicating proof) could be viewed as strategies for underlining superficial structure of proof. As for the three factors of metacognitive reading strategy use, most of the M1 items (planning comprehension) could be viewed as element-oriented strategies which direct readers to focus on important elements of a proof, most of the M2 items (monitoring comprehension) could be viewed as relation-oriented strategies which direct readers to focus on relations between proof steps, and all of the M3 items (regulating comprehension) could be viewed as proposition-oriented strategies which encourage students to apply and modify the proposition. In particular, successful comprehenders in this study used element-oriented, relation-oriented and deep-structure strategies to enhance their RCGP. The results seemed to be compatible with Soranastaporn and Chuedoung's finding (1999) that successful comprehenders did more planning than unsuccessful comprehenders did and Brenna's finding (1995) that good comprehenders used text aids (e.g. figures) to predict or guess the writer's ideas.

Language research has shown that successful comprehenders encountering an easy task might report low use of some strategies because the task was simply so easy that strategies were not needed (Phakiti, 2003). On the contrary, unsuccessful comprehenders might use strategies inappropriately even though the strategies are available to them (Vann & Abraham, 1990). Besides, the unsuccessful test-takers showed more frequent use of metacognitive strategies in retrieving information from the long-term memory compared with the successful test-takers who undertook metacognitive strategies to help them understand and remember for secondary language reading (Purpura, 1999). Similarly, the quality of perceived use of reading strategies may differ between good and poor groups of RCGP. To incorporate reading strategies into mathematics instruction, it is necessary to investigate the quality of perceived use of reading strategies. This may shed light on how to design element-oriented and relation-oriented activities for planning and monitoring comprehension and deep-structure activities for elaborating proof to develop a transformational proof scheme (Harel, 2001).

This instrument does have a few limitations. First of all, one cannot tell with absolute certainty from the instrument alone whether students actually engage in the strategies they perceive using. To better understand the psychology of the learners, future research will be needed to investigate the relationship between their perceived strategy use in general and their actual strategy use in a specific situation. Next, teachers and researchers should consider the students' responses to this instrument as only one source of information about their perceived reading strategy use and these responses must be analysed in conjunction with data from other measures such as think-aloud protocols or retrospective interviews. Finally, strategies to enhance understanding or to resolve misunderstanding, for example, generating examples (Dahlberg & Housman, 1997) and formulating competing hypothesis (Yang, 2010), were not considered in designing this instrument for RCGP.

Given the nature of the cognitive and metacognitive constructs involved, it needs to be acknowledged that the relationships of CMRS use to RCGP could have been far more complicated than those that have been found or implied. Besides, the structural relationship of CMRS use to reading comprehension of mathematical proof may change with proof methods, contents and formats. Moreover, this study was not comprehensive enough because other factors such as affect and beliefs (Aarnoutse & Schellings, 2003; Law, 2009; Paris & Oka, 1986; Weinberg & Wiesner, 2010)—that are related to both RCGP and perceived use of CMRS—were excluded in this model.

Nevertheless, the CMRS instrument has been validated to measure the perceived use of cognitive and metacognitive strategies in this study, and the modified mediated model provides basic information for further exploration of the relationships between reading strategy use and RCGP. Not only will this help advance the study of how students understand proofs, but this will also reveal students' perceived use of strategies through their questionnaire responses. More important, the instrument of CMRS would allow teachers to assess to what extent students with poor RCGP perceived their use of reading strategies, and this modified mediated model would further suggest which reading strategies were crucial for students' RCGP. In addition, this study has provided evidence that besides knowledge and logical reasoning (cf. Lin & Yang, 2007), CMRS can also explain the variation in their RCGP.

Although there is a need for advancing students' reading comprehension of mathematical proof, only a few interventions have so far been designed to fulfil this need. For example, Alcock (2009) recognized the need to teach students to think more carefully about what has been read and to design a set of resources to demonstrate how one might go about understanding proofs by examining their internal logical relationships and overall structures. If learning how to sustain the process of reading comprehension is necessary, CMRS should be considered in future design of interventions to facilitate students' understanding of mathematical proof. Yang and Lin (submitted) have described and evaluated a teaching intervention designed with reading strategies for RCGP elsewhere.

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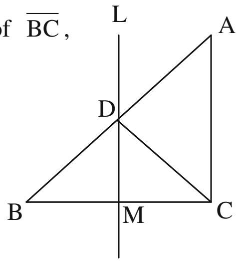
## Appendix A

**Problem**：

As shown in the figure, L, the perpendicular bisector of $\overline{BC}$, intersects $\overline{AB}$ at D, and intersects $\overline{BC}$ at M; and $\overline{DA}=\overline{DB}$; must $\angle$ DCA and $\angle$ DAC be equal?




**To this problem**, **Tom gives the following proof:**

As shown in the figure,

since L, the perpendicular bisector of $\overline{BC}$, intersects $\overline{BC}$ at M, …….. Line 1

$\angle$ BMD $=\angle$ CMD $= 90^\circ$ and $\overline{BM}=\overline{CM}$. ……………….……….…… Line 2

And $\overline{DM}=\overline{DM}$, ……………………..………………………………….. Line 3

$\therefore$ $\triangle$ BMD $\cong$ $\triangle$CMD (SAS). ……………..……………………..…… Line 4

$\therefore$ $\overline{DB}=\overline{DC}$ (corresponding sides). …………..………..……………… Line 5

$\because$ $\overline{DA}=\overline{DB}$, ……………..…………………..……………..………………… Line 6

From Line 5 and Line 6 $\Rightarrow$ $\overline{DA}=\overline{DC}$. ……………………..……….…… Line 7

Because $\overline{DA}=\overline{DC}$, $\angle$ DCA $=\angle$DAC. ………………..………………… Line 8

Answer the following on the basis of this question and the proof process:

1. Do you agree that $\overline{BM}=\overline{CM}$? Explain why or why not.
2. Label ∠BMD in this figure as 1 and ∠CMD as 2.
3. If △BMD and △CMD are congruent, what is the corresponding side of $\overline{DB}$?
4. Besides the known conditions (the perpendicular bisector of $\overline{BC}$ intersects $\overline{AB}$ at D, and intersects $\overline{BC}$ at M, and $\overline{DA}=\overline{DB}$), which conditions can be directly applied without any explanation?
5. If someone suggests that the proof process of lines 1, 2, 4, 3, 5, 6, 7 and 8 is correct, after lines 3 and 4 are interchanged, would you agree with his or her opinion?
6. If someone suggests that the proof process of lines 6, 1, 2, 3, 4, 5, 7 and 8 is correct, after the position of line 6 has been changed, would you agree with his or her opinion?
7. Which properties are applied in this proof?
8. On the basis of the question and the proof,

   (8-1) Which premises are necessarily required?

   (8-2) What final conclusion is derived from these premises?
9. Which statements can be validated from this proof?
10. In this proof process, an important result is first derived from the condition that L, the perpendicular bisector of $\overline{BC}$, intersects $\overline{BC}$ at M and other conditions.

(10-1) What is this important result?

(10-2) According to this important result in (10–1) and $\overline{DA}=\overline{DB}$, one condition can be derived to confirm ∠DCA=∠DAC. What is this condition?

11. Choose the correct statements.
12. Do you agree that this proof process is correct?
13. Statement A: If L, the perpendicular bisector of $\overline{BC}$ of △ABC, intersects $\overline{AB}$ at D, and intersects $\overline{BC}$ at M, and $\overline{DA}=\overline{DB}$; then ∠DCA and ∠DAC must be equal.

(13-1) Do you agree that this proof process can prove that Statement A is always correct?

(13-2) Do you agree that this proof process can prove that Statement A is sometimes correct and sometimes incorrect?

Answer the following questions on the basis of what you know.

14. There is a circle with centre point P, radius $\overline{PS}$ and $\overline{PQ}$. If T is the midpoint of $\overline{PQ}$, $\overline{ST}\perp\overline{PQ}$, and S is the midpoint of $\overline{PR}$, is △RSQ an isosceles triangle?
15. There are three points P, Q and R. If S is the midpoint of $\overline{PQ}$ and $\overline{ST}\perp\overline{PQ}$, what conclusions can be derived?
16. If D is the midpoints of $\overline{AE}$, and $\overline{BD}$ and $\overline{AE}$ are perpendicular to each other, and $\overline{AB}=\overline{BC}$, then ∠AEC=90°. Is this correct?

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