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The influence of self-efficacy and metacognitive prompting on math problem-solving efficiency

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Abstract

A regression design was used to test the unique and interactive effects of self-efficacy beliefs and metacognitive prompting on solving mental multiplication problems while controlling for mathematical background knowledge and problem complexity. Problem-solving accuracy, response time, and efficiency (i.e. the ratio of problems solved correctly to time) were measured. Students completed a mathematical background inventory and then assessed their self-efficacy for mental multiplication accuracy. Before solving a series of multiplication problems, participants were randomly assigned to either a prompting or control group. We tested the *motivational efficiency hypothesis*, which predicted that motivational beliefs, such as self-efficacy and attributions to metacognitive strategy use are related to more efficient problem solving. Findings suggested that self-efficacy and metacognitive prompting increased problem-solving performance and efficiency separately through activation of reflection and strategy knowledge. Educational implications and future research are suggested.

Keywords: Self-efficacy; Self-regulation; Metacognitive prompting; Math problem-solving; Strategy

1. Introduction

Solving dual-digit multiplication problems mentally, without calculation aids, is a cognitively complex task requiring focused attention, skill, sustained effort, and strategic awareness (Ashcraft, 1992). A variety of individual difference factors including back-

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ground knowledge (Alexander, 1992; Schoenfeld, 1987), working memory (Campbell & Xue, 2001; DeStefano & LeFevre, 2004) and beliefs, such as self-efficacy (Pajares & Kranzler, 1995; Pajares & Miller, 1994) have been found to boost math problem-solving performance. Collectively, these attributes influence a "mathematical disposition" (DeCorte, Verschaffel, & Op 'T Eynde, 2000, p. 689), which combined with self-regulatory skill, is conducive to optimal problem-solving success.

Superior problem-solving accuracy is partially influenced by an individual's ability and willingness to use strategies to monitor, adjust, and reflect upon the problem-solving process (Zimmerman, 1989). Broadly defined as self-regulation and situated within social cognitive theory (Pintrich & De Groot, 1990; Zimmerman, 2000), these strategies include providing cues or prompts (i.e. asking an individual "What steps are you using to solve the problem?" or "What is the best method to solve the problem?"). Prompting stimulates awareness of task characteristics, performance strategies, and evaluation of outcomes (Butler & Winne, 1995; Winne, 1998).

Although prior studies have shown prompting results in superior mathematical performance (Kramarski and Gutman; 2006; Kramarski & Zeichner, 2001; Schoenfeld, 1987) and self-efficacy beliefs mediate problem-solving success (Bandura, 1986; Pajares, 2003; Pajares & Miller, 1994), few studies have investigated the interaction of these factors, which is important when creating optimal instructional environments. Additionally, studies that investigated these factors individually, measured performance through problem-solving accuracy (number of correct responses), but not from the perspective of problem-solving efficiency (the ratio of accuracy over time).

Conventional studies have examined self-efficacy beliefs as an antecedent variable that exerts influence upon achievement outcomes. This singular measurement of self-efficacy outcomes employs a broad holistic view that diminishes the critical influence of context upon self-efficacy beliefs (Bandura, 1997). The lack of a fine-grained distinction within the current self-efficacy literature is surprising considering the domain specificity of self-efficacy (Bandura, 1997) and the need to broaden the self-efficacy construct (Labone, 2004). Problem solving is context bound (Schoenfeld, 1987) and situational constraints, such as limited instruction time and individual student differences warrant efficiency considerations. Additionally, investigation of dual outcomes such as accuracy and efficiency support conceptual models that emphasize the multi-dimensional nature of constructs, similar to the model of teacher efficacy proposed by Woolfolk Hoy, Davis, and Pape (2006) or the evolving conceptions of the self described by Martin (2007).

The goal of our study was to test the separate and interactive effects of self-efficacy beliefs and metacognitive prompting on solving mental multiplication problems while controlling for mathematical background knowledge and problem complexity. This question is important for both empirical and practical reasons. Empirically, the differential impact of self-efficacy beliefs and metacognitive prompting upon efficiency are unknown. From a practical perspective, understanding how individual differences are regulated to achieve optimal problem solving success is critical for both classroom (Zimmerman, 2000) and computer-based learning environments (Azevedo, 2005).

1.1. Self-efficacy and self-regulatory strategies

One of the most powerful beliefs related to problem-solving success is an individual's domain-specific self-efficacy. Self-efficacy, the conviction in one's ability to successfully organize and execute courses of action to meet desired outcomes (Bandura, 1986), is a reliable predictor of mathematical performance (Pajares, 1996). Self-efficacy is related to extended effort (Lopez, Lent, Brown, & Gore, 1997), persistence (Bouffard-Bouchard, 1990; Pajares & Kranzler, 1995; Pajares & Miller, 1994), and adaptive cognitive competencies including goal setting (Bandura, 1997; Schunk, 1990) and productive goal orientations (Schunk & Ertmer, 1999; Wolters, Yu, & Pintrich, 1996). Self-efficacy accounts for variance in mathematical performance accuracy beyond background knowledge (Pajares, 2003; Pajares & Miller, 1994), and was recently linked to the efficiency of mathematical problem solving (Hoffman & Schraw, 2007).

A direct relationship exists between learners high in self-efficacy and the use of metacognitive strategy to generate successful performance outcomes (Braten, Samuelstuen, & Stromso, 2004; Kitsantas, 2000; Pintrich & De Groot, 1990), and self-efficacy has been found to influence all phases of the self-regulation process (Schunk & Ertmer, 2000). For example, Zimmerman, Bandura, and Martinez-Pons (1992) studied the relationship between perceived self-efficacy for academic achievement and setting of academic goals. Results indicated self-efficacy and strategy use accounted for 31% of the variance in students' academic attainment. Self-regulatory factors were found to mediate the influence of prior achievement, along with the establishment and attainment of goals.

Horn, Bruning, Schraw, and Curry (1993) measured students' general academic ability, domain knowledge of human development, self-efficacy for reading and writing, and self-reported use of study strategies. Positive relationships existed between performance and both general ability and domain knowledge, while self-efficacy was found to have a positive influence concerning the number, type, and implementation of strategy use. The authors surmised enhancement of self-efficacy lead to automaticity of strategy and was compensatory for achievement; however, knowledge, self-efficacy, and strategy were "separable components of learning" (p. 476).

Connectivity among motivational constructs like self-efficacy, intrinsic value, and perceived task value are linked to the use of self-regulatory strategies. In a correlational study (Pintrich & De Groot, 1990), middle school students completed self-report instruments measuring cognitive strategy use and assessments such as tests, essays, and seatwork measured achievement. Self-efficacy was the best predictor of cognitive engagement and academic performance. Students with the expectation of task success reported the greatest degree of metacognitive strategy use and a willingness to persist at tasks described as uninteresting or difficult.

The current study employed mental multiplication as a problem-solving task. Mathematical problem-solving success is a combination of ability, estimation of task success, and beliefs about the subject and nature of mathematical tasks linked with the efficient use of strategies (Garofalo & Lester, 1985; Seitz & Schumann-Hengsteler, 2000). Previous studies have indicated a moderately challenging task in conjunction with achievement relative to goals, combined with moderate and reasonable expectation of task success, and a willingness to apply strategies results in problem-solving accuracy (Butler & Winne, 1995).

However, most studies that investigate the interplay of strategy and self-efficacy have only examined the predictive ability on performance accuracy. Research that investigates efficiency is important because, due to time considerations, differences in problem-solving accuracy are not always identical to differences in problem-solving efficiency (Paas, Tuovinen, Tabbers, & Van Gerven, 2003). Identifying factors that contribute to problem solving under instructional constraints constitutes an important step in defining overall problem solving efficiency. It is unknown whether self-efficacy increases problem-solving efficiency, even though it is acknowledged that self-efficacy is related to mathematical problem-solving accuracy (Lopez et al. 1997; Pajares & Kranzler, 1995; Pajares & Miller, 1994).

1.2. What is metacognitive prompting?

Metacognitive prompting (MP) is an externally generated stimulus that activates reflective cognition, or evokes strategy use with the objective of enhancing a learning or problem-solving outcome. Alternatively, MP has been referred to as metacognitive cueing (Veenman, Kerseboom, & Imthorn, 2000), reflective prompting (Davis, 2003), self-metacognitive questioning (Kramarski & Gutman, 2006), guided cooperative questioning (King, 1994), and self-generated inferences (Wittrock, 1990). The reflective cognition evoked by MP is related to the use of strategies such as self-monitoring (Kauffman, 2004), self-reflection (Lin & Lehman, 1997), and self-explanations (Chi, Hutchinson, & Robin, 1989). Some of these operationalized definitions are the roots for prescriptive models of self-regulation (see Boekaerts, Pintrich, & Zeidner, 2000; Butler & Winne, 1995; Wittrock, 1990, for complete reviews).

The commonality in these descriptions is the availability of thought provoking information designed to stimulate and facilitate the problem-solving process. Unlike feedback, which provides knowledge of results (Butler & Winne, 1995; Mory, 2004), or supplies corrective enrichment information (Mevarech & Kramarski, 1997) like a heuristic, MP stimulates problem-solving reflection. MP is helpful in promoting metacognitive monitoring, a well supported component of metacognition whereby the individual judges and evaluates ongoing activities. This reflective process of inducing mental imagery and creating an association to existing ideas has been a pedagogical technique used since ancient times (Wittrock, 1990).

MP is important, as it has been linked to improved performance across domains and contexts (Schoenfeld, 1987). For example, during the writing process, idea-prompting statements were provided to sixth graders as a means to generate self-questioning during a think aloud protocol (Scardamalia, Bereiter, & Steinbach, 1984). During "points of stuckness" (p. 179), suggestive cues such as "An important point I haven't considered is" were incorporated into the protocol. The reflective introjections contributed to posttest essay gains in topical exposition, elaboration of goals, and central ideas.

Methods such as "IMPROVE" (Kramarski & Gutman, 2006; Kramarski & Zeichner, 2001; Mevarech & Kramarski, 1997) used self-metacognitive questioning, as illustrated by "What is the problem all about?" (2006, p. 25), to help students bootstrap self-regulation strategies during math problem solving. Focusing upon content reflection and use of differential strategies, the IMPROVE method, which is four-step questioning strategy designed to evoke mathematical reasoning, resulted in superior math achievement. Questioning guided students in the process of identifying problem structure, created connections with prior knowledge, and served as a catalyst for strategy use which enhanced mathematical reasoning (Mevarech & Kramarski, 1997).

Davis (2003) contrasted two types of reflection prompts with middle school students: generic prompts designed to activate awareness of conceptual ideas, and directed prompts that included specific project solving hints. Generic prompts, exemplified by "What are we thinking about now?" (p. 100), resulted in greater coherence and productivity of science projects. A control group was not part of the design; however, results supported the role of prompting as a preferential scaffolding technique to enhance understanding of science.

In sum, this research supports the contention that prompting can be a catalyst to evoke the use of self-regulation strategies, such as understanding the nature of a problem, selecting and monitoring strategy, evaluating outcomes, and revising and sometimes abandoning strategies if deemed unsuccessful (Garofalo & Lester, 1985). However, mere knowledge of the strategies induced by reflective prompting is not sufficient to guarantee usage, as metacognition is a combination of both strategies and belief (Schoenfeld, 1987; DeCorte, Verschaffel, & Op 'T Eynde, 2000).

The effectiveness of prompting requires awareness of strategic application, perception of a need, motivational endorsement, and a belief in the function of using strategies. An individual may possess a strategy but not use it, may detect a need and ignore it, or may not have the necessary resources to execute the strategy. As Bandura (1997) mused, "knowing what to do is only part of the story" (p. 223).

Several scenarios may render the effects of prompting questionable. Availability deficiencies occur when individuals do not have metacognitive skills at their disposal, and a production deficiency may result when available strategies are deemed irrelevant to the task at hand (Veenman et al., 2000). Similarly, if a learner "doesn't register a cue's presence, then a cue has no value" (Butler & Winne, 1995, p. 251). Beliefs may even distort the intended message (Butler & Winne, 1995). In some cases, the problem-solver may find prompting intrusive (Salomon & Globerson, 1987) and blatantly reject the prompting message.

Problem solvers that deem the prompt as unnecessary or superfluous to the task may result in mindlessness, a situation whereby potentially helpful cues are ignored. Salomon and Globerson (1987) described the necessity of volitional, metacognitively guided employment of non-automatic, effortful processes to be personally and situationally relevant to facilitate "mindfulness" (p. 624), which is the active engagement and receptivity towards a prompting message. Thus, prompting perceived as functionally unnecessary or personally irrelevant, superfluous, or redundant, may prove counterproductive to the problem-solving process and result in lower performance. In order for MP to be effective the problem solver must be aware and motivated to use the prompt, and the strategy activated by the prompt should be relevant and deemed effective to solve the problem.

1.3. The influence of background knowledge

A belief in task mastery devoid of ability can lead to debilitating performances (Bandura, 1997; McCombs & Marzano, 1990). In problem-solving situations, learners filter information through a network of cognitive representations including beliefs about the self, background knowledge, strategies, and task engagement (Butler & Winne, 1995; Schoenfeld, 1987). Ultimately, success is a function of ability, motivation, and regulation of strategy in the face of difficulty.

A prerequisite to the awareness and use of strategy is domain knowledge (Alexander, 1992; Alexander & Judy, 1988; Murphy & Alexander, 2002). As an individual increases domain expertise, the availability and production of associated strategy use is increased (Pressley & Afflerbach, 1995). The evocation of strategy is aligned with the evolving degree of problem-solving expertise; as knowledge increases, the efficiency of strategic application follows (Alexander & Judy, 1988).

The model of domain learning (Murphy & Alexander, 2002) specifies the multidimensionality of knowledge, interest, and strategy. In an empirical test of the model, as an individual's "interactive knowledge" (p. 199), (i.e. the combination of domain expertise in tandem with strategic processing) improved, the subsequent frequency of deep-level processing strategies also increased. A similar model, the "good strategy user model for mathematics" (Pressley, 1986), dictates metastrategic knowledge, like interactive knowledge, allows the problem-solver to exert effort towards problem resolution that might otherwise be devoted to knowledge acquisition.

In models of this nature, tasks such as problem representation become automated (Alexander & Judy, 1988). Similarly, the facilitative hypothesis (Nietfeld & Schraw, 2002) explains how background knowledge promotes the use of self-regulation strategy due to the availability of cognitive resources and knowledge serving as a basis for evaluation of ongoing performance (Nietfeld & Schraw, 2002). For example, King (1994) investigated differences among the use of various integration, comprehension, and factual questions to determine the influence on the knowledge construction process in 4th and 5th grade students. Results concluded questions, which promote connections to prior knowledge, and connection of ideas promoted learning.

A problem solver must have the motivation to use strategy in conjunction with background knowledge. McCombs and Marzano (1990) posited the need for both agency and ability and the integrated influence upon self-efficacy. Impairment occurs when cognitive systems are isolated from metacognitive components and self-system development. Skill instruction is critical, but cannot be thought of as distinct from self-assessment and the use of metacognitive strategy. Knowledge must be created in harmony with self-systems that, in turn, generate greater self-efficacy.

In sum, existing research leads to three main conclusions. First, MP promotes problem-solving success, but the differential impact is predicated upon both the availability and willingness to use strategies (DeCorte, Verschaffel, & Op 'T Eynde, 2000; Kramarski & Gutman, 2006; Mevarech & Kramarski, 1997; Schoenfeld, 1987; Veenman, Prins, & Elshout, 2002). Secondly, ability and ability beliefs influence the receptivity to prompting (Bandura, 1997; Braten et al., 2004; Butler & Winne, 1995; Kitsantas, 2000; Pintrich & De Groot, 1990). Suggestions prompted by MP may be perceived as unnecessary or intruding if a problem-solver is high in expertise, or is unwilling or unmotivated to devote resources towards activating strategy (Salomon & Globerson, 1987).

Finally, precise determination as to the role of self-efficacy and strategy provoked by MP necessitates controlling for background knowledge (Alexander, 1992; Alexander & Judy, 1988; Murphy & Alexander, 2002). Individuals high in self-efficacy should have more resources to devote to the strategies induced by MP, although MP should result in more time and effort devoted to monitoring, and impact efficiency (Butler & Winne, 1995).

Thus, our goal was to examine whether self-efficacy beliefs and MP enhanced problem-solving efficiency, when controlling for mathematical problem-solving ability and problem complexity. Prompting may have an interactive effect upon math problem-solving efficiency. It is possible problem solvers high in self-efficacy may find prompting intrusive and unnecessary and impedes both performance and efficiency. Conversely, it is possible high self-efficacy may promote mindfulness (Salomon & Globerson, 1987) since individuals with greater beliefs in their self-efficacy use greater effort, persistence, and self-regulation strategies in the pursuit of reaching goals (Pintrich & De Groot, 1990; Zimmerman et al., 1992). These factors have important implications for instructional approaches, especially under constraints of classroom learning, such as solving difficult problems or having limited time.

An interaction between self-efficacy and prompting will support the assumption that MP can externalize strategy use and work in conjunction with the expectation of problem-solving success. Differences among complexity and either self-efficacy or MP will support the contention of a collective or singular compensatory influence upon problem-solving efficiency in the face of difficulty. Lastly, performance differences between MP and a control group should lend support to the presumption that MP can compensate for overall deficits when controlling for ability.

1.4. The present study

The present study investigated the separate and interactive influence of self-efficacy beliefs and MP upon problem-solving accuracy, problem-solving time, and problem-solving efficiency under unrestricted time conditions. We controlled for the effect of background knowledge to better examine the unique variation shared by self-efficacy and MP. Participants completed a mathematical background inventory, and then assessed their self-efficacy for mental multiplication accuracy, subsequently participants were randomly assigned to either a MP or control group, before solving a series of multiplication problems. Problem complexity was manipulated using two levels of math problems with each participant solving an equal number of problems at each complexity level.

Hoffman & Schraw (2007) proposed the *motivational efficiency hypothesis* which states that positive motivational beliefs such as self-efficacy (Bandura, 1986; Bandura, 1997; Butler & Winne, 1995; Pintrich, 2000; Pintrich & De Groot, 1990), personal goal orientations (Ames & Archer, 1988; Stone, 2000), intrinsic motivation (Zimmerman, 1989), engagement and attributions to metacognitive strategy use (Butler & Winne, 1995; Linnenbrink & Pintrich, 2003) are related to more efficient problem solving. We predicted individuals with powerful self-efficacy beliefs judiciously dedicate resources and strategic knowledge towards the problem-solving process resulting in problem-solving efficiency.

Our primary goal was to investigate the interaction between self-efficacy and MP with regards to both accuracy and efficiency outcomes. We believe that higher degrees of self-efficacy may interact with MP to produce superior accuracy, but reduced efficiency due to time considerations, for three inter-related reasons. One reason is that high self-efficacy individuals use the most appropriate problem-solving strategy (Siegler; 1988; Walczyk, 1994), but exhibit more effort (Chen, 2002) resulting in increased problem solving time. A second reason is individuals high in self-efficacy exhibit mindfulness as a result of dedicating more attention, exhibit a greater frequency of self-regulated learning, and respond favorably to MP. A third reason is that high self-efficacy individuals experience fewer distractions due to anxiety and handicapping (Ashcraft & Kirk, 2001; Eysenck & Calvo, 1992; Hopko, Hunt, & Armento, 2005).

We made several predictions regarding the relationship between self-efficacy, MP, and problem complexity. First, we expected individuals with higher self-efficacy to be responsive to MP, be more accurate solving math problems, and more accurate overall. A second prediction was that MP would interact with self-efficacy. We expected MP to play a stronger role on accuracy and efficiency when self-efficacy was high and when problems were more complex. Lastly, we expected MP to result in greater problem-solving accuracy.

2. Methods

2.1. Participants and design

Participants were undergraduate students enrolled in educational psychology courses at a large southwestern university ($N = 81$, $18 = \text{M}$, $63 = \text{F}$) who volunteered for the study to fulfill course requirements. The mean grade point average for the students was 3.26. No differences based upon GPA were observed for any of the outcome measures.

Three dependent measures by complexity level were recorded: number of correct responses to each of 42 mental multiplication problems, aggregate time in milliseconds to solve each problem, and problem-solving efficiency (the aggregate number of correct responses per difficulty level divided by response time). The study employed a regression design in order to assess the relationships among self-efficacy, MP, and background knowledge and allowed for the determination of unique variance contributed by each independent variable. Each participant solved the same problems at each of the two levels of problem complexity.

2.2. Materials and procedures

Standardized written and oral instructions were presented to all participants for all tasks. The first task consisted of an informed consent form. After reading and signing the consent form, the participants completed an inventory of 15 math problems (French, Ekstrom, & Price, 1962) designed to measure basic ability and background knowledge in multiplication and addition. The math inventory was a 20-minute timed test that consisted of multiple-choice items requiring paper and pencil calculations (i.e. How many candy mints can you buy for 50 cents at the rate of 2 for 5 cents?). The test was scored for accuracy of responses to the math problems. Credit was given only for a completely correct response. Partial credit was not possible. The number of accurate responses was used to control for participants' overall mathematical ability.

The second task was completed on the computer and consisted of two steps. The first step was to complete a self-efficacy assessment of mental multiplication ability. Participants indicated their degree of confidence in mentally (without any calculation aids) solving eight different mental multiplication problems similar in format and complexity to the criterial task. This method of measuring self-efficacy was equivalent to Lopez et al. (1997). Students were told to rate their ability to solve the problems in a "reasonable" amount of time and were informed that unlimited time could not be a factor in compiling their self-efficacy assessments. The problems for the self-efficacy assessment were listed on the computer screen and were rated on an eleven-point scale ranging from no confidence at all (0), to total confidence (100) in solving the multiplication problems.

In the next task, participants solved 42 mental multiplication problems. Each participant solved (20_{easy}) 2-digit $\times$ 1-digit problems with three-digit solutions (i.e. $49 \times 9 = 441$) and (20_{hard}) 2-digit $\times$ 2-digit problems with three-digit solutions (i.e. $45 \times 12 = 540$). The first two problems were designed to acclimate participants to the problem-solving process and were not included in the statistical analysis. Mental multiplication problems were chosen for two main reasons. First, adults already have experience with mental multiplication and they can also make accurate estimates of own ability to solve (Chen, 2002; Stone, 2000). Second, from a design perspective, we had control over the complexity of the problems, which proves important to account for the problem size effect (Campbell & Xue, 2001), which indicates multiple-digit problems are more complex than those with single digits.

The computer keyboard was used to input problem answers in an on-screen dialog box. Each problem was presented individually on the computer screen. The participant had to provide an answer to one problem before being able to move to the next one. Problems were randomized but presented in the same sequence to each student to control for order effects (Seitz & Schumann-Hengsteler, 2000). Participants were instructed to solve the problems as quickly and accurately as possible without making mistakes. The task itself was not timed, but participants were aware of the recording of accuracy and solving time for each individual problem. Participants were monitored during all tasks to ensure the problems were solved mentally without any calculation aids, finger counting, or recording of partial answers in the onscreen answer box.

Participants were randomly assigned to either a metacognitive prompt group (i.e. Have you considered other problem solving strategies?) or a control group that did not receive any prompting. The MPs were developed with the purpose of connection making and integration of strategy with background knowledge (King, 1994; Kramarski & Gutman, 2006; Mevarech & Kramarski, 1997). The MPs did not supply any information related to the answer accuracy like regular feedback (Butler & Winne, 1995). The prompts were targeting strategy, connection, and metacognitive thought (i.e. Can you make the problem less difficult to solve?). A larger pool of questions was initially developed. Through discussions, the authors independently agreed (inter-rater reliability of .90) upon 10 questions to be used in the prompting condition (see Appendix A for the complete list of prompts).

The ten different prompts, one prompt per approximately every fourth math problems appeared randomly on the computer monitor between math problems. When a prompt appeared on the screen the participants responded to the prompt by inputting a response to the prompting question. Prompting time was segregated from overall responses time in order to avoid latency confounds. The computer was programmed to deliver the prompts after a variable number of problem responses. Thus, participants could not anticipate when a prompt would appear on the computer monitor. All participants in the prompt condition received the same number of prompts.

A 2-minute break was included after the first 20 problems to avoid cognitive fatigue. Once finished, the participants had to click a "submit" button on the computer screen. The computer recorded both accuracy and latency of response for each problem. After the computer tabulated results, the participants received a completion message via the computer, and then were verbally debriefed by the researcher.

3. Results

Based upon a number of significant interactions among general mathematical ability and both self-efficacy and MP, general ability was not used as a covariate. Subsequently, an ATI regression analysis was performed (Pedhazur, 1997) for each dependent variable at each level of complexity (labeled easy, hard). General mathematical ability, self-efficacy, and MP were the independent variables, with vector coding used for the categorical variable of MP. The analysis was performed using SPSS 14.0 for Windows. Results were screened for violation of assumptions and outliers, resulting in nine cases being removed from the statistical analysis due to excessive problem-solving time. Based upon the research hypothesis, an interaction term was created between the independent variables of self-efficacy and MP, and mean centering was used to control for multicollinearity. A separate model was created excluding the interaction term from the model when the interaction term was not significant (Aiken & West, 1991).

A preliminary comparison of males and females was performed using problem-solving accuracy and problem-solving response time. Both analyses indicated that gender did not affect results on the dependent measures, $t_{(79)} = .610$, $p = .543$ and $t_{(79)} = -1.789$, $p = .077$, respectively. Reliability coefficients using Cronbach's alpha were calculated to determine the reliability of the eight-item self-efficacy measure. The measure was highly reliable, $\alpha = .958$.

3.1. Descriptive statistics and correlations

Means and standard deviations for each dependent measure and for general mathematical ability, self-efficacy, and MP variables are presented in Table 1. Zero-order correlations were conducted and are presented in Table 2. Highlights of the correlation table revealed a significant positive relationship between general mathematical ability, self-efficacy, and GPA as well as problem-solving accuracy and total efficiency. Additionally, a positive relationship between self-efficacy and both problem-solving accuracy and problem-solving efficiency was found. MP was correlated with problem-solving accuracy and efficiency, but only at the more complex level, suggesting a differential impact of prompting upon performance. As expected, strong relationships were observed among the problem-solving accuracy and problem-solving efficiency outcome variables.

3.2. Problem-solving accuracy

The total number of problems answered correctly, at each complexity level, determined problem-solving accuracy. As can be seen in Table 3, no significant interactions between the self-efficacy variable and the MP variable were observed for either easy or hard problems.

Table 1
Descriptive statistics

Variable	Mean	*SD*	Observed range	Possible range
Accuracy (number correct)				
Easy	15.94	3.36	6.00–20.00	.00–20.00
Hard	8.85	5.42	1.00–20.00	.00–20.00
Response time (milliseconds × 1000)				
Easy	311.12	98.92	137.66–626.75	.001–∞
Hard	780.27	233.97	329.23–1319.51	.001–∞
Efficiency (accuracy/time × 1000)				
Easy	57.70	24.07	14.36–130.72	.001–∞
Hard	12.37	8.98	1.085–35.42	.001–∞
Self-efficacy	71.37	24.61	11.25–100.00	.00–100.00
General math ability	7.28	3.16	.00–14.00	.00–15.00

Table 2
Correlations

Study variable	1	2	3	4	5	6	7	8	9	10	11	12
1. GPA	—	.203	.408[a]	.101	.424[a]	−069	−.055	.108	.458	.341[a]	−.163	.401[a]
2. Self-efficacy		—	.309[a]	.276[b]	.453[a]	−.345[a]	−.003	.346[a]	.402[a]	.437[a]	−.072	.323[a]
3. Math test			—	.253[b]	.643[a]	−.060	−.150	.137	.636[a]	.559[a]	−.182	.506[a]
4. Accuracy easy				—	.530[a]	−.452[a]	−.121	.737[a]	.489[a]	.805[a]	−.265[b]	.610[a]
5. Accuracy hard					—	−.331[a]	−.049	.491[a]	.912[a]	.930[a]	−.235[b]	.610[a]
6. Time easy						—	.398[a]	−.856[a]	−.453[a]	−.427[a]	.654[a]	−.622[a]
7. Time hard							—	−.369[a]	−.386[a]	−.087	.891[a]	−.529[a]
8. Efficiency easy								—	.595[a]	.663[a]	−.593[a]	.762[a]
9. Efficiency hard									—	.851[a]	−.532[a]	.915[a]
10. Total accuracy										—	−.279[b]	.796[a]
11. Prompt time removed											—	−.732
12. Total efficiency												—

[a] Correlation is significant at the 0.01 level (2-tailed).
[b] Correlation is significant at the 0.05 level (2-tailed).

After removing the interaction term from the regression model, at the first level of problem complexity, the overall model was significant, $F_{(3,77)} = 3.125$, $MSR = 10.474$, $p = .05$, $R^2 = .109$. However, when testing for differences in slope between variables, no significant differences were observed, but the self-efficacy variable was in the direction of statistical significant, $\beta = .226$, $SE = .010$, $t = 1.966$, $p < .06$.

At the second level of complexity, the overall model was also significant, $F_{(3,77)} = 27.799$, $MSR = 14.690$, $p = .05$, $R^2 = .520$, however, regression of problem-solving accuracy on self-efficacy and MP was not statistically significant. Individually, general mathematical ability, self-efficacy and MP were all predictive of problem-solving accuracy, respectively, $\beta = 509$, $SE = .147$, $t = 5.965$, $p < 001$, $\beta = .316$, $SE = 0.19$, $t = 3.749$, $p < .001$ and $\beta = .194$, $SE = .440$, $t = 2.378$, $p < .025$. Examination of squared semipartial correlations between self-efficacy and problem solving accuracy revealed that 47.1% of the variation in problem-solving accuracy was accounted for by general mathematical ability, 29.6 % by self-efficacy and 18.8% by MP.

3.3. Problem-solving time

Problem-solving time was determined by the latency of response for each of the 40 problems of mental multiplication. Response time was recorded in milliseconds, and converted to seconds for ease of analysis. Results were aggregated to provide a total problem-solving time for each level of problem complexity. Results were screened for extreme scores prior to analysis.

No significant interactions were observed for problem-solving time for easy problems. After removing the interaction term from the regression model, at the first level of complexity, the regression of problem-solving time on general mathematical ability, self-efficacy and MP were statistically significant, $F_{(3,77)} = 4.501$, $MSR = 8650.667$, $p < .01$, $R^2 = .149$. Self-efficacy was a statistically significant predictor of problem-solving time, $\beta = -.392$, $SE = .451$, $t = -3.490$, $p = .001$. Squared semipartial correlations between self-efficacy and problem-solving time revealed that 36.7% of the variation in

Table 3
Results of multiple regression analysis by predictor and complexity level, interactions analyzed in separate model

Predictor variable	Self-efficacy × MP[a]				Math ability				Self-efficacy				MP			
	β	*SE*	*T*	*P*=	β	*SE*	*T*	*P*<	β	*SE*	*T*	*P*<	β	*SE*	*T*	*P*<
Accuracy																
Easy	−195	.015	−1.73	.087	.176	.124	1.511	.14	.226	.016	1.966	.06	.038	.372	.346	.74
Hard	−.116	.018	−1.39	.168	.509	.147	5.965	.001	.316	.019	3.749	.001	.194	.440	2.378	.025
Time																
Easy	−.043	.452	−.383	.703	.093	3.561	.822	.42	−.392	.451	−3.490	.005	−.172	10.68	−1.59	.12
Hard	−.296	1.099	−2.57	.02[b]	−.164	9.018	−1.35	.181	.048	1.142	.397	.693	−.004	27.04	−.034	.974
Efficiency																
Easy	−.118	.110	−1.05	.296	.005	.875	.040	.968	.357	.111	3.152	.005	.117	2.622	1.070	.29
Hard	−.019	.032	−.220	.826	.514	.250	5.862	.001	.265	.032	3.059	.005	.214	.749	2.554	.025

[a] β reflects use of centered means.
[b] Overall model was not significant.

problem-solving time was accounted for by self-efficacy. No other significant findings were observed.

3.4. Problem-solving efficiency

Problem-solving efficiency was determined by computing the ratio between the number of problems solved accurately and problem-solving time for each of the 40 problems of mental multiplication. Results were summed to provide a total problem-solving efficiency score for each level of problem complexity. The ratio of problem-solving performance to problem-solving time was multiplied by 1000 for ease of comparison.

No significant interactions were observed for problem-solving efficiency at either complexity level. After removing the interaction term from the regression model, at the first level of complexity, the regression of problem-solving efficiency on general mathematical ability, self-efficacy and MP was statistically significant, $F_{(3,77)} = 3.962$, $MSR = 521.680$, $p < .025$, $R^2 = .134$. Self-efficacy was a statistically significant predictor of problem-solving efficiency, $\beta = .357$, $SE = .111$, $t = 3.152$, $p < .005$. Squared semipartial correlations between self-efficacy and problem-solving efficiency revealed 33.4% of the variation in problem-solving efficiency was accounted for by self-efficacy.

Similar results were observed at the second level of problem complexity, as the regression of problem-solving efficiency on general mathematical ability, self-efficacy and MP, after removing the interaction term, revealed statistical significance, $F_{(3,77)} = 24.993$, $MSR = 42.502$, $p < .001$, $R^2 = .493$. The consistent finding of self-efficacy as a predictor of problem-solving efficiency was again supported, $\beta = .265$, $SE = .032$, $t = 3.059$, $p < .005$. Additionally, both general mathematical ability and MP were significant predictors of problem-solving efficiency, $\beta = .514$, $SE = .250$, $t = 5.862$, $p < .001$ and $\beta = .214$, $SE = .749$, $t = 2.554$, $p < .025$. Examination of squared semipartial correlations between problem solving efficiency and the variables revealed 47.6% of the variation in problem-solving efficiency was accounted for by general mathematical knowledge, 14.2% by self-efficacy, and 20.7% by MP.

A composite efficiency variable was created by taking the sum of all accurate responses at both complexity levels and dividing by the total problem-solving time and subtracting prompt completion time. This analysis was done to determine if the influence of MP on efficiency was a function of problem-solving performance or problem-solving time. After removing prompting time, interactions were not observed, however upon removal of the interaction term the overall model was significant, $F_{(3,77)} = 15.692$, $MSR = 122.781$, $p < .001$, $R^2 = .379$. Unlike when MP time was included in the model, the findings indicated a significant effect for MP, $\beta = .315$, $SE = 1.272$, $t = 3.392$, $p = .001$.

4. Discussion

The main study focus was to determine the influence of self-efficacy and metacognitive prompting upon mathematical problem-solving accuracy and efficiency when controlling for mathematical background knowledge. Results supported the prediction that self-efficacy influences accuracy above and beyond mathematical background knowledge, and is consistent with previous findings (Lopez et al., 1997; Pajares & Kranzler, 1995; Pajares & Miller, 1994). Our results add to the literature by providing support for mediating influence of self-efficacy upon outcomes that require efficiency considerations.

MP influenced both problem-solving accuracy and problem-solving efficiency. The findings suggest that under conditions of increasing complexity, metacognitive prompting may induce greater cognitive awareness and the utilization of typically unmindful problem-solving strategies. Cognitive arithmetic involves the mental representation of processes (Ashcraft, 1992), and the use of memory representations, such as stored associations and procedural processing (Campbell & Graham, 1985; DeStefano & LeFevre, 2004; Siegler, 1988). To solve an arithmetic problem, the solver must encode the presented information, perform the calculation, and then provide a response (DeStefano & LeFevre, 2004; Logie, Gilooly & Wynn, 1994). The awareness of alternative strategies may promote additional scrutiny of problem representation and resolution thus leading to greater problem-solving accuracy, albeit at the expense of longer problem-solving time.

The facilitating effects of metacognitive prompting were not observed on the time measure and were inconsistent for the efficiency measure. This result is due in part to the increased time necessary to assimilate prompts and provide a response to the prompting question. When the time devoted to recording prompting requests by participants was eliminated from the analysis, significant efficiency outcomes were observed. This finding suggested MP could promote efficiency if the strategies induced by MP can be automatized and is consistent with previous research that indicated problem-solvers invoke compensatory processes to overcome processing obstacles (Walczyk & Griffith-Ross, 2006). Future studies should control for prompting latency to accurately assess the influence of prompting. Providing a choice for prompting may also be important in order to avoid the mindlessness paradigm in which prompts are seen as intrusive and potential consumers of precious mental resources.

Main effects for both self-efficacy and prompting in the absence of interactive effects suggest either variable may boost problem-solving success separately. Participants higher in self-efficacy solved more problems accurately and efficiently regardless of prompting. This is consistent with previous findings indicating that high-self efficacy learners self-monitor (Zimmerman, 1989), implying prompting may be superfluous for these individuals (Salomon & Globerson, 1987). Individuals high in self-efficacy may not need prompting because they already monitor (Butler & Winne, 1995). Additionally, time effects for self-efficacy were observed which provided additional evidence that individuals higher in self-efficacy may discount the messages provided by MP and instead rely upon automatic strategies, while likely devoting time consuming resources to problem solving.

MP was only a significant influence upon accuracy and efficiency for more complex problems, suggesting the benefits of increased metacognitive awareness may be unnecessary when individuals perceive simplicity and do not acknowledge the necessity for using advanced problem-solving strategies. Veenman et al. (2002) suggested differences exist between the availability versus the production of metacognitive skill usage. Likely, in the current study, strategies used for complex problems remained dormant for easier problems, yet were evoked in the face of difficulty, despite the increased time required. College students apparently possess and use strategic capability, discounting the production deficiency explanation in this study.

The current results supported the *motivational efficiency hypothesis*, which states that adaptive motivational beliefs such as self-efficacy (Bandura, 1986; Bandura, 1997; Butler & Winne, 1995; Pintrich, 2000; Pintrich & De Groot, 1990), and attributions to metacognitive strategy use (Butler & Winne, 1995; Linnenbrink & Pintrich, 2003) are related to more efficient problem solving and contribute unique variance above and beyond background knowledge. The consistent self-efficacy effects suggest the importance of both strategy and beliefs in mathematical problem-solving efficiency presumably by motivating the problem solver in positive ways. Prior research has indicated "The adequate use of cognitive and volitional self-regulatory skills in a specific task and instructional context depends on the subjective perception and interpretation of that dynamically changing context based on the knowledge, skills, beliefs and strategies one possess; this is, the different components of a mathematical disposition." (DeCorte et al., 2000, p. 692). Future research should identify additional factors, such as epistemology and affect, which may also explain unique variance associated with optimal motivation and extend findings to dispositions in other domains.

Self-efficacy for mathematical problem solving may be a multi-faceted concept akin to Woolfolk's (2006) notion of teacher efficacy, which includes the dimensions of student engagement, instructional strategies, and classroom management, or Martin's (2007) underlying conception of views of the self, which are posited to include an expressive, managerial, and communal self. Conceptual models of this nature advocate the influence of context upon the appraisal of internal beliefs. The current research supports a similar paradigm that deconstructs self-efficacy into fine-grained distinctions with multiple integrative and weighted factors influencing self-efficacy assessments contingent upon outcomes of accuracy or efficiency.

An interpretation of this nature is consistent with Bandura's (1997) description of self-efficacy as a construct that includes task contingent subskills that vary in importance based upon task demands. Further clarity is warranted to decipher the precise distinction between self-efficacy for performance compared to self-efficacy for efficiency. The influence of self-efficacy upon problem-solving time was only found for easier problems under unrestricted conditions; however, different outcomes may result in restricted conditions or as the problem-solving context changes (Walczyk & Griffith-Ross, 2006).

4.1. Educational implications and future directions

Social cognitive theory advocates reciprocity among environmental, personal, and behavioral factors (Bandura, 1997). The current results emphasize the triadic nature of motivational efficiency and have at least two educational implications: the influence of self-efficacy on efficiency outcomes, and judicious use of reflective hints to facilitate problem-solving success. Given the typical constraints encountered in the classroom environment, such as lack of engaged time (Marks, 2000), educators should adapt methods to change both student self-perceptions and implement strategies to overcome problem-solving limitations.

Our findings have broader relevance to the self-efficacy literature, as this is the first study that we know of to demonstrate the effect of self-efficacy on problem-solving efficiency when controlling for background knowledge. The consistent effects on efficiency have important implications for problem solving and instruction. When faced with instructional constraints educators may need to be more efficient.

Our results suggest that for problems of lower complexity, self-efficacy was related to quicker problem solving and may improve performance without increasing problem-solving time. We encourage others to replicate and extend our findings to other outcome measures, which incorporate control of problem complexity. Future research should measure both self-efficacy for accuracy and self-efficacy for efficiency to determine the precise additive, integrative, and conjoint impact of multiple self-efficacy assessments.

Previous research indicated monitoring is an informative influence promoting intrinsic regulation relative to goals (Butler & Winne, 1995). Educators should consider infusing MP into instruction as a means to foster self-reflective awareness. Typically, elementary and secondary classrooms are saturated with overt reminders designed to stimulate strategic thought and domain awareness. The current study provides evidence that explicit presentation of prompting comes at the expense of time; automatization of the reflective messages can achieve similar problem-solving benefits, more efficiently.

It is unknown if the advantages of increased self-efficacy and MP will be observed in other problem-solving situations. Mental multiplication is a familiar task to many college students and may not be reflective of novel problem-solving situations. The generalization of these results to other domains may not be warranted. The utilization of a similar methodology for more complex problem-solving tasks is encouraged.

Metacognitive prompting is not a panacea. Students must be willing to invest both the necessary time and effort to reap the rewards of strategic advantage. Halpern (1998) advocated, "Metacognitive monitoring skills need to be made explicit and public so that they can be examined and feedback can be given about how well they are functioning." (p. 454). Educators should strive towards creating a delicate balance between building confidence in a student's ability to leverage both the availability and production of strategy (Veenman et al., 2002), while concurrently helping students automatize the process. More research is needed to fully understanding the relationship among domain specific knowledge, self-efficacy, and strategy use to completely identify the variables that influence motivational efficiency.

1. Have you solved similar problems before?
2. What strategy can you use to solve these problems?
3. What steps are you using to solve the problem?
4. Can your answer be checked for accuracy?
5. Are you sure your answers are correct?
6. Can the problem be solved in steps?
7. What strategy are you using to solve the problems?
8. Is there a faster method to solve the problem?
9. Are these problems similar to addition in any way?
10. What is the best method to solve the problem?

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