Spring 2019

Exercise 5 | EKF Simultaneous Localization And Mapping (SLAM)

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Goal …

- Putting everything from exercise 3 and 4 together
  - Forward integrate state with wheel odometry (EKF Prediction)
  - Update state with measurements (EKF measurement update)
    - Detect lines in lidar scan
    - Associate lines with previous detections
    - Evaluate innovation term for measurements

- **New: No ground truth map of environment → map is in EKF state**

EKF recap

Prediction

State Propagation

$$\hat{\boldsymbol{x}}_t = f\left(\boldsymbol{x}_{t-1}, \boldsymbol{u}_t\right)$$

Covariance Propagation

$$\hat{\mathbf{P}}_t = \mathbf{F}_x \mathbf{P}_{t-1} \mathbf{F}_x^\top + \mathbf{F}_u \mathbf{Q} \mathbf{F}_u^\top$$

Update

Measurement

$$\hat{\mathbf{z}}^i = \begin{bmatrix} \hat{\alpha}^i \\ \hat{r}^i \end{bmatrix} = h(\hat{\mathbf{x}}_t, i)$$

Innovation

$$\hat{\mathbf{y}}_t = \mathbf{z}_t - \hat{\mathbf{z}}_t$$

Optimal gain

$$\mathbf{K} = \hat{\mathbf{P}}_t \cdot \mathbf{H}^\top \cdot (\mathbf{R} + \mathbf{H} \cdot \hat{\mathbf{P}}_\mathbf{t} \cdot \mathbf{H}^\top)^{-1}$$

Update

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}\mathbf{H})\hat{\mathbf{P}}_t$$
$$\mathbf{x}_t = \hat{\mathbf{x}}_t + \mathbf{K} \cdot \hat{\mathbf{y}}_t$$

Assumptions

- Landmarks are static:

$$\dot{\alpha} = 0 \qquad \dot{r} = 0$$

- Known number of landmarks
  - Highly simplifies implementation as no bookkeeping necessary
  - $$\mathbf{x} = \begin{bmatrix} x & y & \theta & \alpha^1 & r^1 & \dots & \alpha^i & r^i \end{bmatrix}^T$$
- First two landmarks/lines remain fixed (global coordinate system)

Task 1

Prediction

State Propagation

$$\hat{x}_t = f\left(x_{t-1}, u_t\right)$$

Covariance Propagation

$$\hat{\mathbf{P}}_t = \mathbf{F}_x \mathbf{P}_{t-1} \mathbf{F}_x^\top + \mathbf{F}_u \mathbf{Q} \mathbf{F}_u^\top$$

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$$\hat{\mathbf{z}}^i = \begin{bmatrix} \hat{\alpha}^i \\ \hat{r}^i \end{bmatrix} = h(\hat{\mathbf{x}}_t, i)$$

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Update

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}\mathbf{H})\hat{\mathbf{P}}_t$$
$$\mathbf{x}_t = \hat{\mathbf{x}}_t + \mathbf{K} \cdot \hat{\mathbf{y}}_t$$

What is new since Exercise 4?

Suggestion:
Practice derivation of Jacobians by hand!

Task 1 - Solution

- $$\hat{\mathbf{x}}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_t) = \begin{bmatrix} x_{t-1} + (\Delta s_l + \Delta s_r)/2 \cdot cos(\theta_{t-1} + (\Delta s_l - \Delta s_r)/2b) \\ y_{t-1} + (\Delta s_l + \Delta s_r)/2 \cdot sin(\theta_{t-1} + (\Delta s_l - \Delta s_r)/2b) \\ \theta_{t-1} + (\Delta s_r - \Delta s_l)/b \\ \alpha^1_{t-1} \\ r^1_{t-1} \\ \vdots \\ \alpha^i_{t-1} \\ r^i_{t-1} \end{bmatrix}$$

- f = x;
  f(1) = f(1) + (u(1)+(u(2)))/2 * cos(x(3) + (u(2)-u(1))/(2*b));
  f(2) = f(2) + (u(1)+(u(2)))/2 * sin(x(3) + (u(2)-u(1))/(2*b));
  f(3) = f(3) + (u(2)-u(1))/(b);

Task 1 - Solution

$$\mathbf{F}_x = \frac{\partial f(\mathbf{x}_{t-1}, \mathbf{u}_t)}{\partial \mathbf{x}} \qquad \mathbf{F}_u = \frac{\partial f(\mathbf{x}_{t-1}, \mathbf{u}_t)}{\partial \mathbf{u}}$$

- $$\mathbf{F}_x = \begin{bmatrix} 1 & 0 & -sin(\theta - \frac{\Delta s_l - \Delta s_r}{2b}) \cdot \frac{\Delta s_l + \Delta s_r}{2} & 0 & \dots & 0 \\ 0 & 1 & cos(\theta - \frac{\Delta s_l - \Delta s_r}{2b}) \cdot \frac{\Delta s_l + \Delta s_r}{2} & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

- ```
F_x(1:3,1:3) = [...
   1, 0, -sin(x(3) - (u(1) - u(2))/(2*b))*(u(1)/2 + u(2)/2)
   0, 1, cos(x(3) - (u(1) - u(2))/(2*b))*(u(1)/2 + u(2)/2)
   0, 0, 1
   ];
```
```

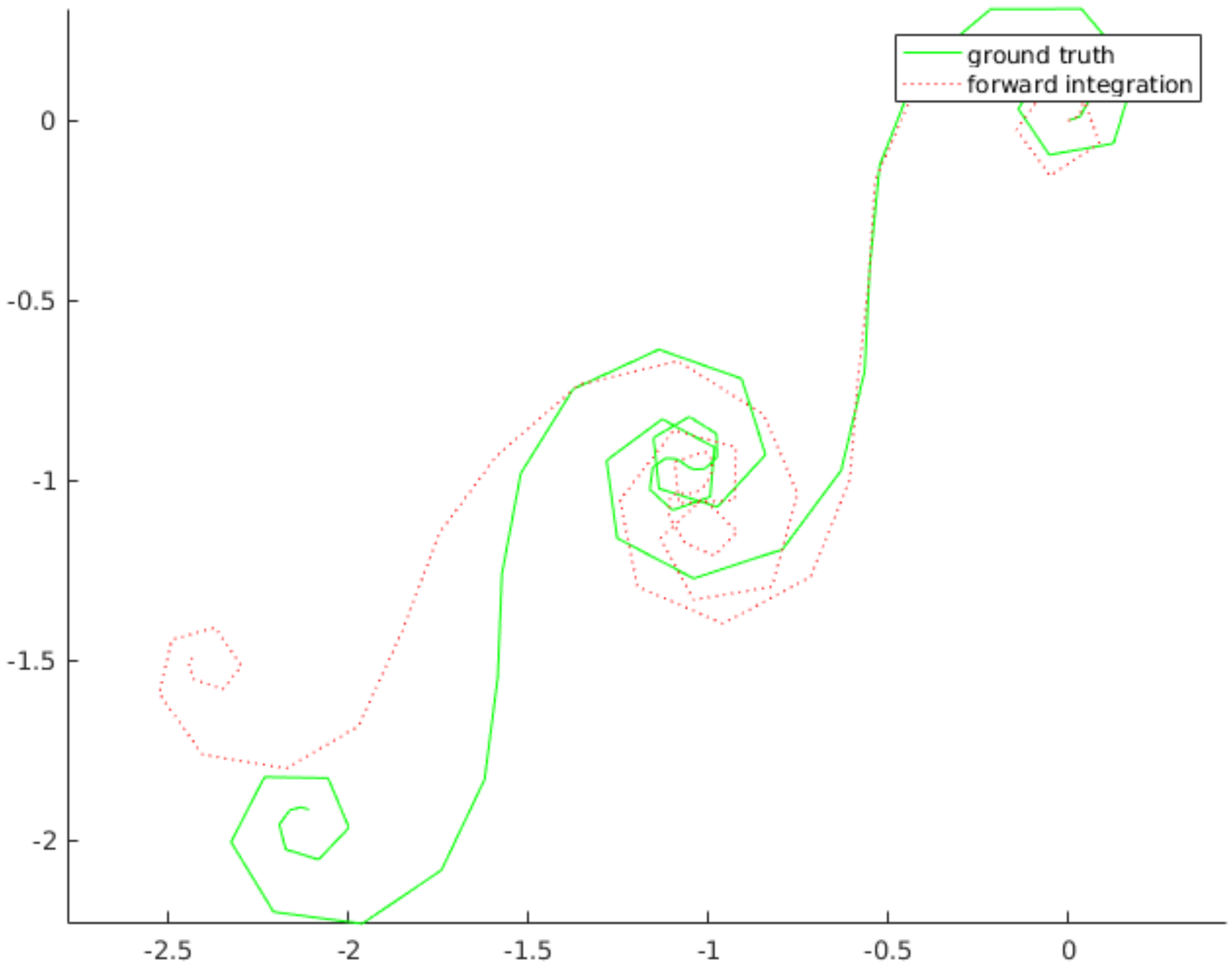
Task 1 - Solution

- $$\mathbf{F}_x = \frac{\partial f\left(\mathbf{x}_{t-1}, \mathbf{u}_t\right)}{\partial \mathbf{x}} \qquad \mathbf{F}_u = \frac{\partial f\left(\mathbf{x}_{t-1}, \mathbf{u}_t\right)}{\partial \mathbf{u}}$$

$$\mathbf{F}_u = \begin{bmatrix} \frac{cos(\theta - \frac{\Delta s_l - \Delta s_r}{2b})}{2} + sin(\theta - \frac{\Delta s_l - \Delta s_r}{2b}) \cdot \frac{\Delta s_l/2 + \Delta s_r/2)}{2b} & \frac{cos(\theta - \frac{\Delta s_l - \Delta s_r}{2b})}{2} - sin(\theta - \frac{\Delta s_l - \Delta s_r}{2b}) \cdot \frac{\Delta s_l/2 + \Delta s_r/2)}{2b} \\ \frac{sin(\theta - \frac{\Delta s_l - \Delta s_r}{2b})}{2} - cos(\theta - \frac{\Delta s_l - \Delta s_r}{2b}) \cdot \frac{\Delta s_l/2 + \Delta s_r/2)}{2b} & \frac{sin(\theta - \frac{\Delta s_l - \Delta s_r}{2b})}{2} + cos(\theta - \frac{\Delta s_l - \Delta s_r}{2b}) \cdot \frac{\Delta s_l/2 + \Delta s_r/2)}{2b} \\ -1/b & 1/b \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}$$

- F_u(1:3,1:2) = [...
  cos(x(3) - (u(1) - u(2))/(2*b))/2 + (sin(x(3) - (u(1) - u(2))/(2*b))*(u(1)/2 + u(2)/2))/(2*b), cos(x(3) - (u(1) - u(2))/(2*b))/2 - (sin(x(3) - (u(1) - u(2))/(2*b))*(u(1)/2 + u(2)/2))/(2*b)
  sin(x(3) - (u(1) - u(2))/(2*b))/2 - (cos(x(3) - (u(1) - u(2))/(2*b))*(u(1)/2 + u(2)/2))/(2*b), sin(x(3) - (u(1) - u(2))/(2*b))/2 + (cos(x(3) - (u(1) - u(2))/(2*b))*(u(1)/2 + u(2)/2))/(2*b)
  -1/b, 1/b
  ];

Task 1 - Solution



Task 2

Prediction

State Propagation

$$\hat{x}_t = f\left(x_{t-1}, u_t\right)$$

Covariance Propagation

$$\hat{\mathbf{P}}_t = \mathbf{F}_x \mathbf{P}_{t-1} \mathbf{F}_x^\top + \mathbf{F}_u \mathbf{Q} \mathbf{F}_u^\top$$

Update

Measurement

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Update

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}\mathbf{H})\hat{\mathbf{P}}_t$$

$$\mathbf{x}_t = \hat{\mathbf{x}}_t + \mathbf{K} \cdot \hat{\mathbf{y}}_t$$

- How to reuse measurement function of Ex4?
- Derive Jacobian by hand! $\mathbf{H} = \frac{\partial h(\hat{\mathbf{x}}_t, i)}{\partial \mathbf{x}}$
- First two landmarks are fixed → don't update

Task 2 - Solution

- $$h(\hat{\mathbf{x}}_t, i) = \begin{bmatrix} \alpha^i - \hat{\theta}_t \\ r^i - (\hat{x}_t \cdot cos\alpha^i + \hat{y}_t \cdot sin\alpha^i) \end{bmatrix}$$

- ```
  h = [...
      m(1) - x(3)
      m(2) - (x(1)*cos(m(1)) + x(2)*sin(m(1)))
      ];
  ```
  ```

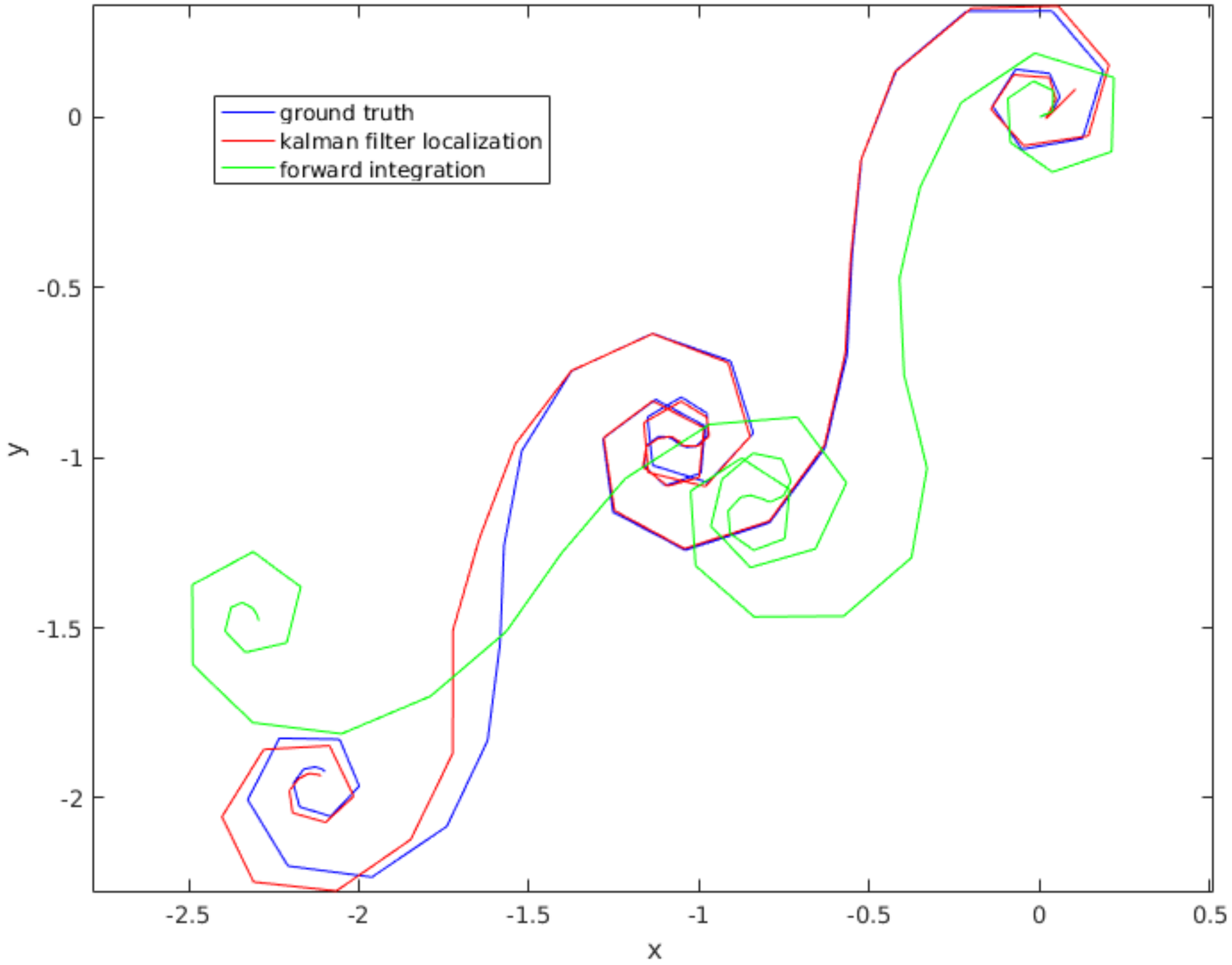
Task 2 - Solution

$$\mathbf{H} = \frac{\partial h(\hat{\mathbf{x}}_t, i)}{\partial \mathbf{x}}$$

- $$\mathbf{H} = \begin{bmatrix} 0 & 0 & -1 & \dots & 1 & 0 & \dots \\ -cos\alpha^i & -sin\alpha^i & 0 & \dots & \hat{x}_t \cdot sin\alpha^i - \hat{y}_t \cdot cos\alpha^i & 1 & \dots \end{bmatrix}$$

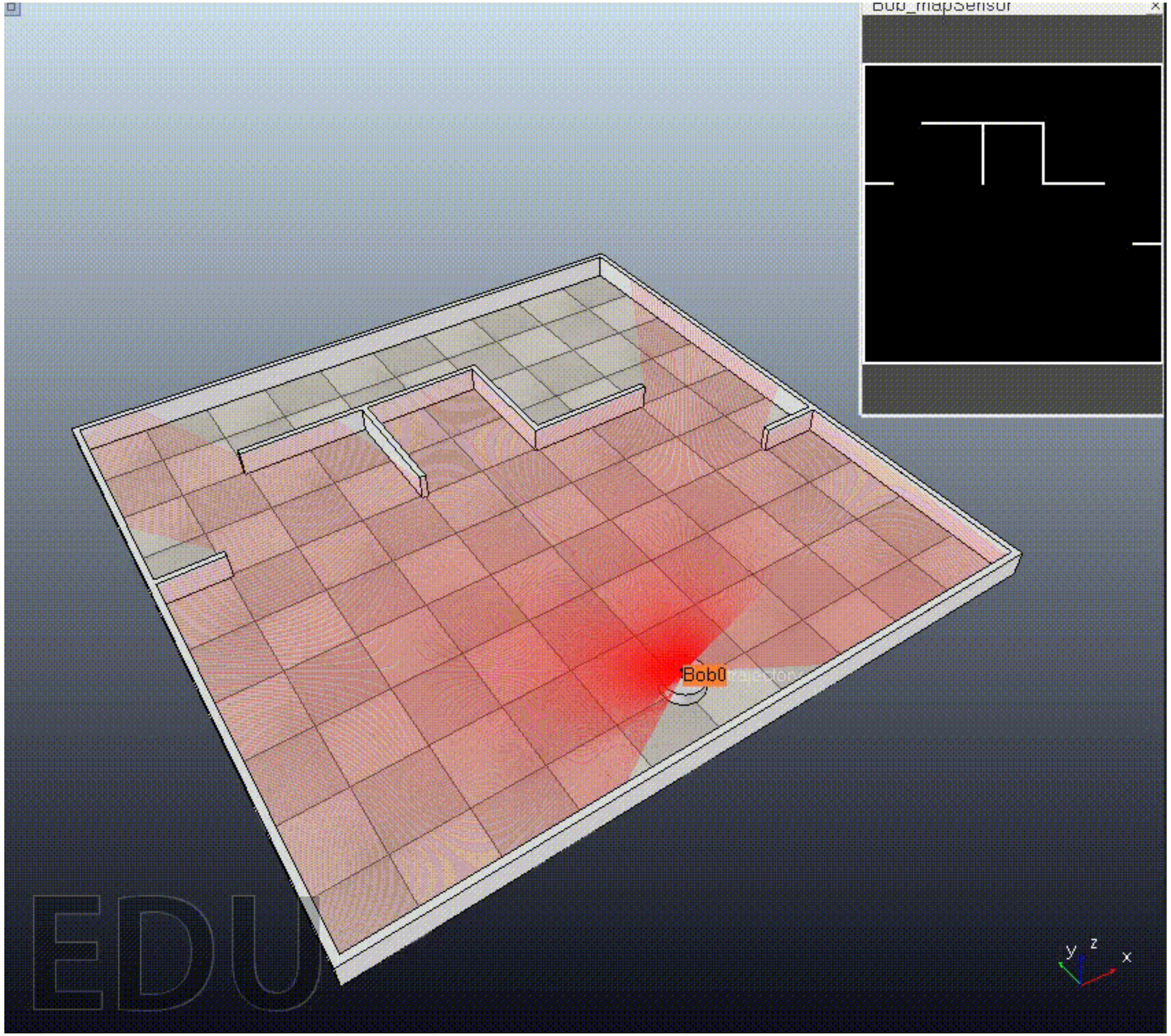
-
```
H_x = zeros(2,length(x));
H_x(1:2,1:3) = [...
   0,          0,          -1
   -cos(m(1)), -sin(m(1)),  0
   ];

%Do not correct first two landmarks as they remain fixed
if (idxLandmark>2)
   H_x(1,3 + (idxLandmark-1)*2+1) = 1;
   H_x(2,3 + (idxLandmark-1)*2+1) = x(1)*sin(m(1)) - x(2)*cos(m(1));

   H_x(2,3 + (idxLandmark)*2) = 1;
end
```

Task 2 - Solution



Task 3

- Try it out in V-REP!



Real application