

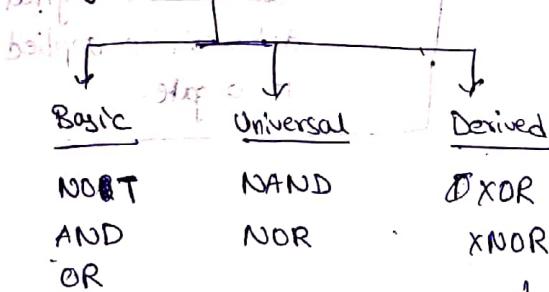
Digital Logic & Design (GATE)

1. Boolean Algebra

→ Distributive law: $x+yz = (x+y)(x+z)$

→ De Morgan's law: $\overline{A+B} = \bar{A} \cdot \bar{B}$, $\overline{A+B+C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$, $\overline{AB} = \bar{A} + \bar{B}$, $\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$

Logic Gates



→ AND, OR, NAND, NOR - single i/p signal

(from minimum) 9.2.6 controlled gates

(thus not suitable for staircase operation)

bottom, not included by

Suitable for staircase operation

→ Propagation delay: time taken by a gate to produce o/p from i/p.

→ Bubbled NAND = OR, bubbled NOR = AND

→ Bubbled AND = NOR, bubbled OR = NAND

→ Single bubbled XOR = X-NOR, single bubbled X-NOR = X-OR

Min no of NAND
to get n i/p NAND
 $= 2n-3$
say for NOR

→ Bubbled XOR = X-OR, Bubbled X-NOR = X-NOR

→ X-OR gate: o/p is high for odd no of 1's (coincidence gate)

→ X-NOR gate: o/p is high for even no of 1's (non-coincidence gate)

XOR

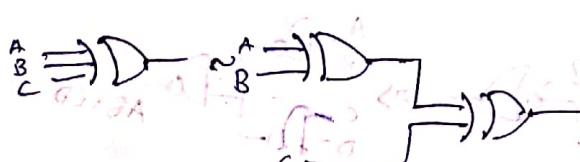
odd, bottom or top is 1 output

X-NOR

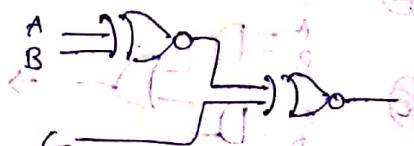
$A \oplus B = \bar{A}\bar{B} + A\bar{B}$

$\Rightarrow \bar{D} = A \oplus B = \bar{A}\bar{B} + A\bar{B}$ at level out

3-i/p EX-OR

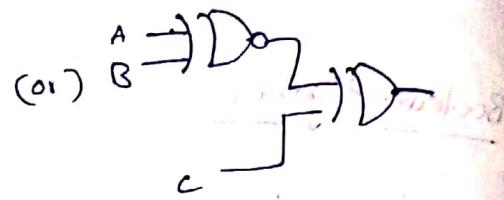
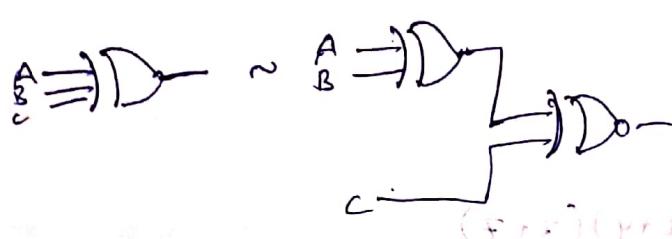


$$(A+B+C) \oplus (A+B+C) = A+B+C$$



3-i/p X-NOR

(ABCD) \rightarrow output logic level



→ Venn diagrams for logic gates

$$S = \overline{A} \cdot \overline{B} \cdot \overline{C} = \overline{A + B + C}$$

Obtaining expression from given truth table

Four methods:

long method (Minterm)

short method (i) S.O.P (minterm i.e., 1)

(ii) P.O.S (maxterm i.e., 0)

(iii) K-Map

(iv) Tabulation method

fan-in: no. of i/p to a gate

fan-out: no. of o/p connected to a gate.

operating speed: The max frequency at which digital data can be applied to a gate.

AB	CD	ABCD
00	00	0000
00	11	0011
11	00	1100
11	11	1111

i/p	I/P	Term Designation			
		Σ S.O.P	Σ M.O.P	Σ T.L.P.O.S	Σ O.M.T
0	00	$\bar{x}\bar{y}$	M ₀	$x\bar{y}$	M ₀
1	01	$\bar{x}y$	M ₁	$x\bar{y}$	M ₁
2	10	$x\bar{y}$	M ₂	$x\bar{y}$	M ₂
3	11	xy	M ₃	$\bar{x}\bar{y}$	M ₃

→ Sum of all minterms is '1'.

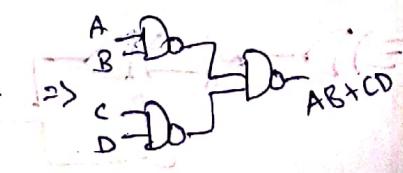
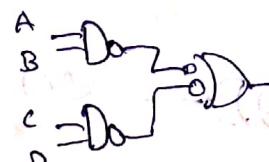
→ Product of all maxterms is '0'.

→ Redundant term: It is a repeated term whose removal does not

disturb original canonical form.

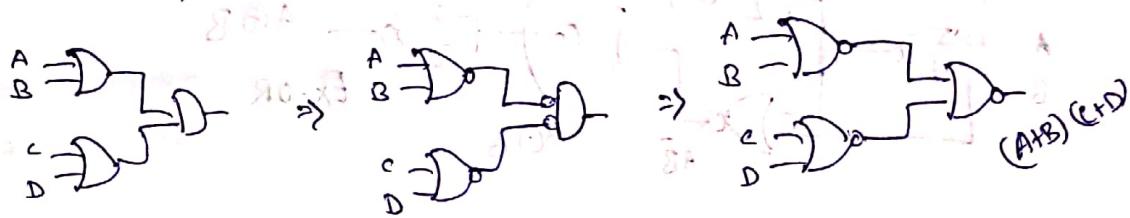
→ Two level SOP expression can be implemented 2-level NAND gates

Ex: Consider AB+CD



→ Two level P.O.S expression can be represented with 2 level NOR gates

Eg: consider $(A+B)(C+D)$



Dual Function (FD)

→ Dual function for a given function is generated by replacing \oplus with \ominus and \ominus with \oplus . and \otimes with \mid and \mid with \otimes

→ If all minterms / Maxterms produced by the i/p func and its dual function are same then the i/p func is called a self Dual Function (SDF)

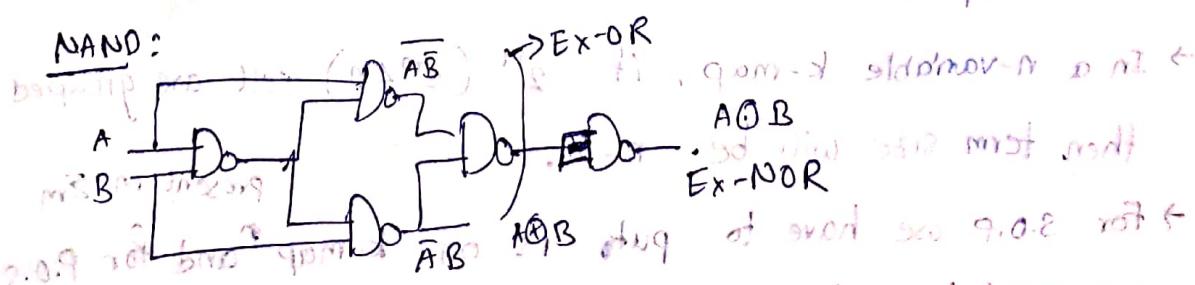
→ Max no of SDFs that can be implemented with n binary variables = $2^{2^{n-1}}$

→ Max no of normal boolean func that can be implemented with n binary variables = 2^{2^n}

→ Universal Gates

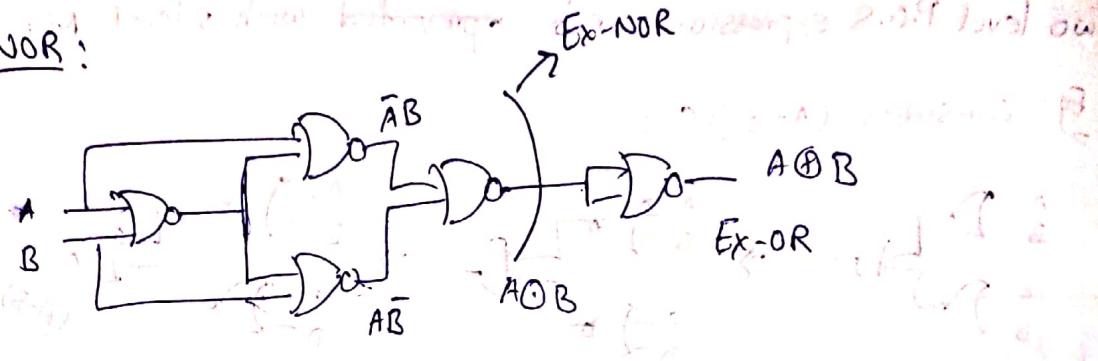
→ Check implementation of NOT, OR, AND, NAND, NOR with NAND & NOR

→ Exclusive gates with NAND/NOR gates:



$$(A, B, C)_{\text{NOT}} = (0, 0, 1) \quad (A, B, C)_{\text{OR}} = (0, 1, 1) \quad (A, B, C)_{\text{AND}} = (1, 0, 0) \quad (A, B, C)_{\text{NAND}} = (1, 0, 1) \quad (A, B, C)_{\text{NOR}} = (0, 1, 0)$$

NOR:



→ ~~4~~ NAND gates for Ex-OR: 4

NAND gates for Ex-NOR = 5

~~5~~ NOR gates for Ex-OR: 5

NOR gates for Ex-NOR: 4

∴ Min 2 Nos. of basic gates required to implement Ex-OR.

2. k-Maps:

→ In n-variable k-map, each cell has chances for grouping with other cells. (7/2) without loss

(→ std method) of k-mapping is don't care to OR with

i) get all PIs (Prime implicants: All possible groupings)

ii) get EPIs (Essential PI: PIs which are grouped only once)

iii) if EPIs are not sufficient get selective PIs and n

→ k-Maps give minimized expression in most of the cases, but not in all the cases.

→ Remember to take 0 as \bar{A} and 1 as A while determining P.O.S expression.

→ In a n-variable k-map, if 2^m ($2^m \leq n$) cell are grouped then term size will be $n-m$. present in Σ_m

→ For S.O.P we have to put 1's on k-map and for P.O.S we need to put 0's on k-map present in Π_M .

$$\text{i.e., } F(ABC) = \sum_m (0, 1, 5, 7) \Rightarrow F(ABC) = \Pi_M (2, 3, 4, 6) \quad (\text{P.O.S})$$

3. Number System:

→ Conversion b/w base r and r^n system:

* Direct conversion is possible by replacing each symbol in r^n system with n bits.

$$\text{Ex: } (24)_8 = (\underline{\underline{010100}})_2$$

$$(\underline{\underline{01101101}})_2 = (\underline{\underline{001101101}})_2 = (155)_8$$

$$\hookrightarrow (6B)_{16}$$

* While converting from r to r^n if bits can be partitioned into n groups, then add zeroes to start of integer part, but for decimal part add at the end.

$$\text{Ex: } (11010 \cdot 1101)_2 = (011010 \cdot 110100)_2 = (32 \cdot 64)_8$$

Fractional data Conversions:

$$(26.8125)_{10} = (?)_2$$

$$(\underline{\underline{11010 \cdot 1101}})_2 = (?)_{10}$$

$$(26)_{10} = (11010)_2$$

for decimal part

$$\begin{array}{r} 0.8125 \times 2 \\ \downarrow \\ 1 \quad 1.6250 \times 2 \\ \downarrow \\ 1 \quad 1.25 \times 2 \\ \downarrow \\ 0 \quad 0.25 \times 2 \\ \downarrow \\ 1 \quad 0.5 \times 2 \\ \downarrow \\ 1 \quad 1.0 \end{array}$$

* In decimal part power starts from -1^{st} .

$$\Rightarrow (26.8125)_{10} = (11010.1101)_2$$

Example: To convert 0.8125₁₀ to binary, we have to multiply it by 2 repeatedly until we get 0.

$$\therefore 0.8125 \times 2 = 1.6250 \rightarrow 11010 \cdot 1101$$

∴ 0.8125₁₀ = 0.1101₂ { 1st part is 0 because 0.8125 is less than 1.

Signed Data Representation:

- In any representation the data is represented normally.
- Result for given data is same if 1's or 2's complement is performed twice.

(i) Signed Magnitude Notation:

0000 } to
0111 }
 { +7

Range: (-2^{n-1}) to $(2^{n-1} - 1)$

1000 } -8
1111 } -7
 { -1

→ It contains an invalid notation i.e., -0

(ii) Signed 1's Complement Notation:

0000 } to
0111 }
 { +7

Range: $(-2^{n-1} - 1)$ to $(2^{n-1} - 1)$

1000 } -8
1111 } -0
 { -1

→ It also contains -0.

(iii) Signed 2's Complement Notation:

0000 } to
0111 }
 { +7

Range: (-2^{n-1}) to $(2^{n-1} - 1)$

1000 } -8
1111 } -1
 { -1

→ It permits 8421 weighted code considering M.S.B (sign bit) as -ve weight.
i.e., $1011 \rightarrow {}^8421 1011 \Rightarrow -8 + 2 + 1 = -5$

→ permits sign bit extension for converting smaller size signed data to larger size

i.e., $+1011 = 11011 = 111011 = 1111011 = \dots$

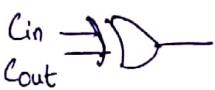
These padded bits are known as dummy bits / Guard bits.

Overflow(OVFL) in signed 2's complement Notation:

→ OVFL does not occur on adding one +ve & one -ve data.

→ OVFL inverts the target sign bit. i.e., if target is '1', result sign bit is '0'.

Overflow detection:

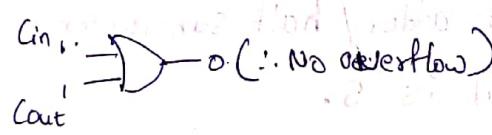


Cin: Obtained by adding data.

Cout: Obtained by adding sign bits.

Eg: $\begin{array}{r} \text{Cin } 1 \\ \text{1111 } (-1) \end{array}$

$\begin{array}{r} \text{Cout } 1 \\ \text{1111 } (-1) \\ \hline \text{Cout } 1 \\ \text{1110 } (+1) \\ \hline \text{Answer} \end{array}$



If res is -ve
we'll obtain it in
2's comp form.

If it is +ve we'll
get in normal form

→ Check examples in notes for better understanding

4. Combinational Circuits (Better to refer notes for perfection)

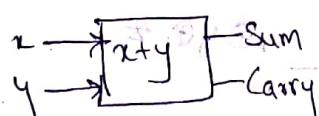
→ faster, no memory, no clock

magnitude comparator

→ Eg: Adders, subtractors, Mux, Demux, Encoder, Decoder etc.

→ Mux is universal combinational circuit.

Half Adder:



$$\text{Sum} = x \oplus y$$

$$\text{Carry} = xy$$



implementation:

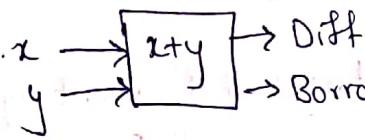
→ 5 NAND gates

→ 5 NOR gates

↳ difficult

stop using the adder now go to next slide

Half Subtractor:



$$\text{Diff} = x \oplus y$$

$$\text{Borrow} = \bar{x}y$$

Implementation:

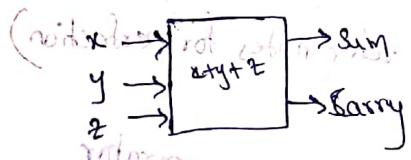
→ 5 NAND gates

→ 5 NOR gates

Note:

* ~~Min no of~~ For a full adder/half adder/half subtractor min no of NAND/NOR gates required is 5.

Full Adder:



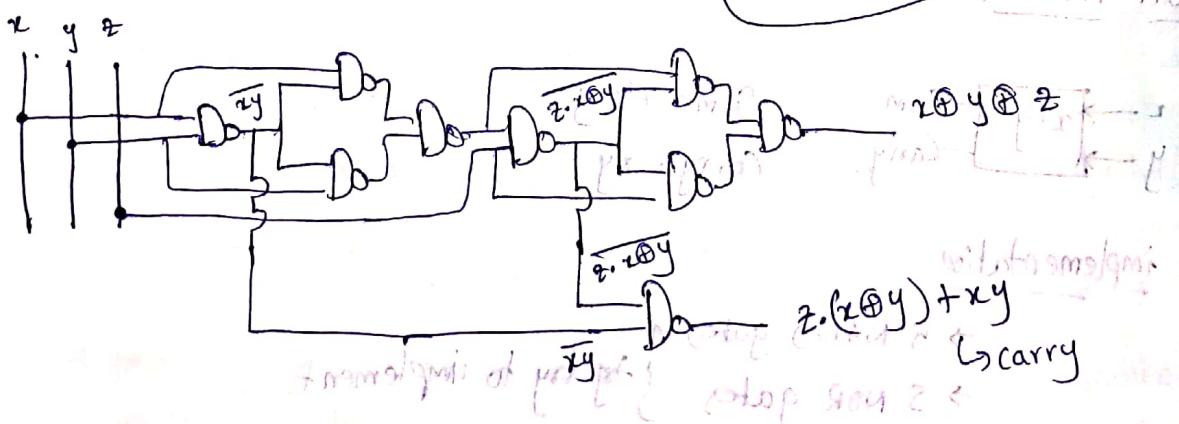
$$\text{Sum} = x \oplus y \oplus z$$

$$\text{Carry} = z \cdot (x \oplus y) + xy \quad [\text{By writing min terms}]$$

$$= yz + xz + xy \quad [\text{from K-map}]$$

Implementation with NAND & NOR:

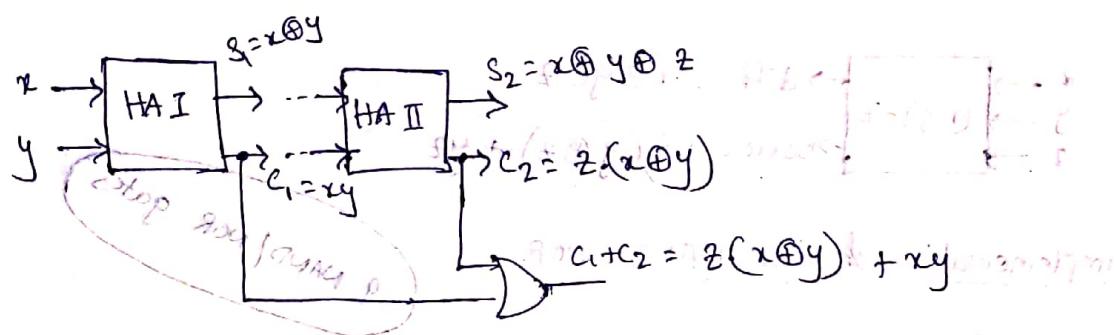
9 NAND/NOR gates



Note:

Above circuit produces same o/p even if we replace all NAND gate with NOR gates.

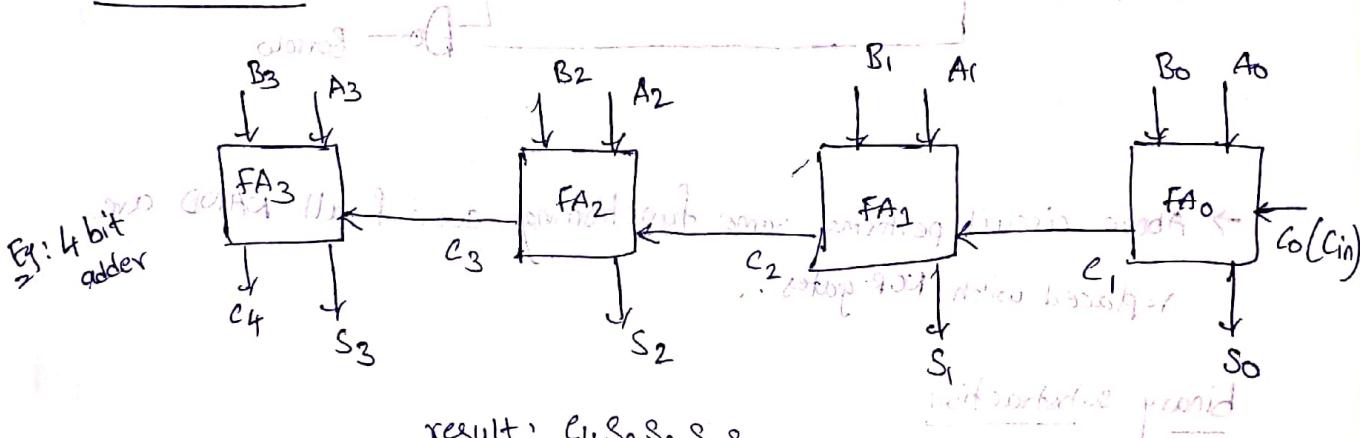
FA with HA:



* FA is known as one full bit adder

* HA is known as one half bit adder

n-bit adder:



$$\text{result: } C_4 S_3 S_2 S_1 S_0$$

(x and y are given to partial adder) \rightarrow FA = 2-bit AC

C_{in} : initial carry

partial sum are given to these below step of passing AC
 c_1, c_2, \dots, c_{n-1} : Ripple carries

question: n-bit adder to n-bit adder how many steps for n sum AC

\rightarrow $n-1$ ripple carries are req to start operation in MSB adder.

\rightarrow total Total ripple carry time: $(n-1) T_{cy}$

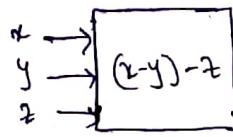
~~due to both side~~
 T_{cy} : time taken by FA to produce carry.

\rightarrow worst time to stabilize result

$$T_{worst} = [(n-1) T_{cy}] + \max(T_{sum}, T_{cy})$$

T_{sum} : time taken by FA to produce sum

Full Subtractor:

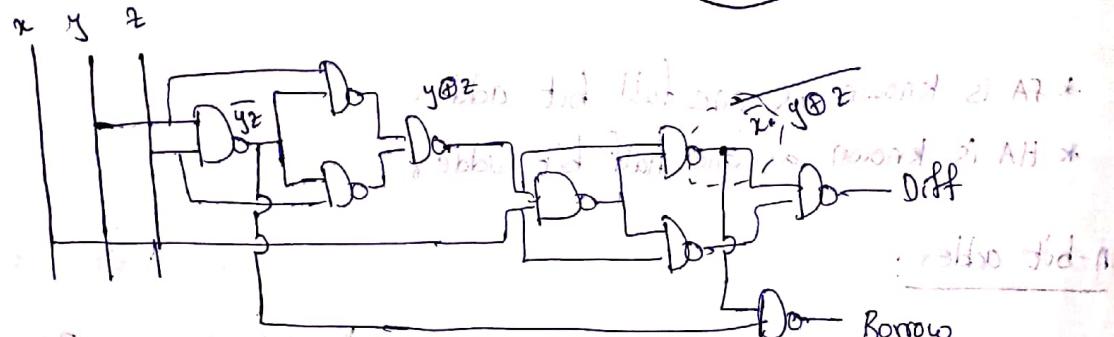


$$\text{diff} : x \oplus y \oplus z$$

$$\text{borrow} : \bar{x}(y \oplus z) + yz$$

implementation of with NAND & NOR:

a NAND/NOR gates



→ Above circuit performing same functioning even if all NAND are replaced with NOR gates.

binary subtraction:

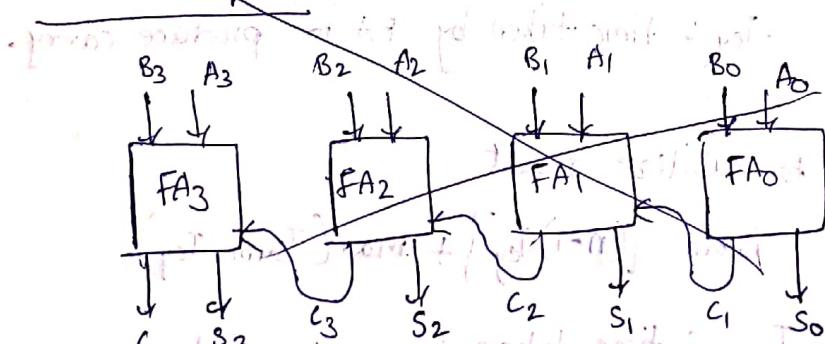
$$A - B = A + \bar{B} + 1 \quad (\text{i.e., adding 2's comp of } B \text{ to } A)$$

→ If carry is generated result is +ve & available in normal form.

→ If carry is not generated result is -ve & available in 2's comp.

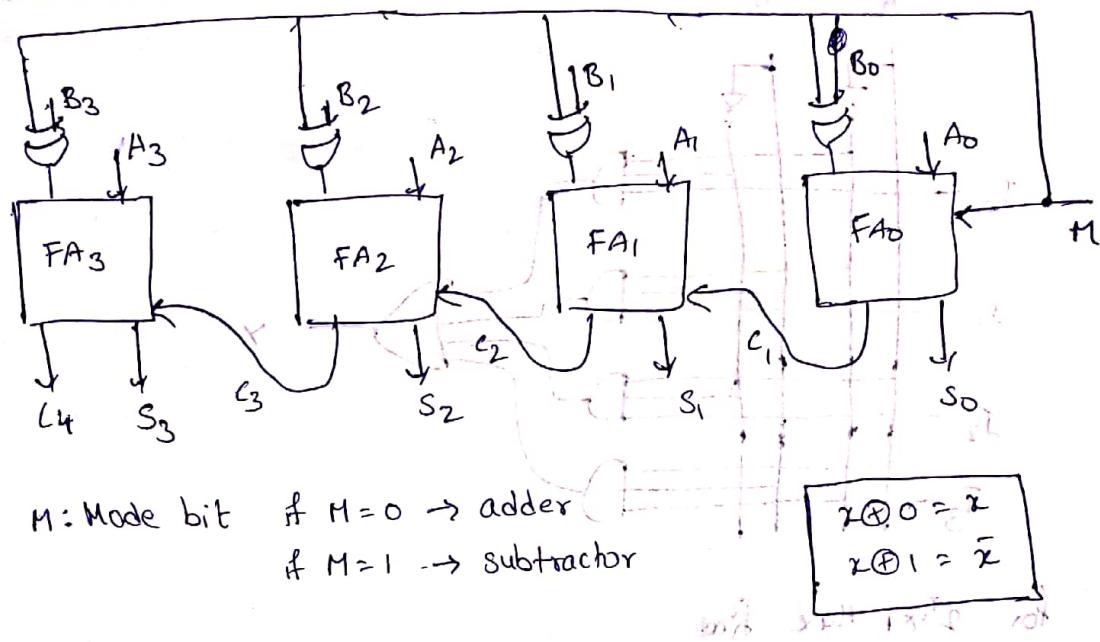
(Check Eg. if needed)

~~4 bit adder/sub:~~



Carry summing of FA7 for modulo result required

4 bit adder/subtractor:



for $M=0$

$$\left. \begin{array}{c} A_3 \ A_2 \ A_1 \ A_0 \\ B_3 \ B_2 \ B_1 \ B_0 \\ O(M) \end{array} \right\} \text{adder}$$

for $M=1$

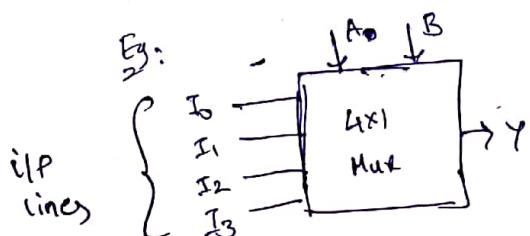
$$\left. \begin{array}{c} A_3 \ A_2 \ A_1 \ A_0 \\ B_3 \ B_2 \ B_1 \ B_0 \\ O(M) \end{array} \right\} \text{sub}$$

Multiplexer:

*also known as universal combinational circuit, data selector

many to one selector, parallel to serial converter

→ Its size is of form $2^n \times 1$ if addition is from left to right



$A, B \rightarrow$ selection logic I_0, I_1, I_2, I_3

$\therefore 1 \times 4$

A	B	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

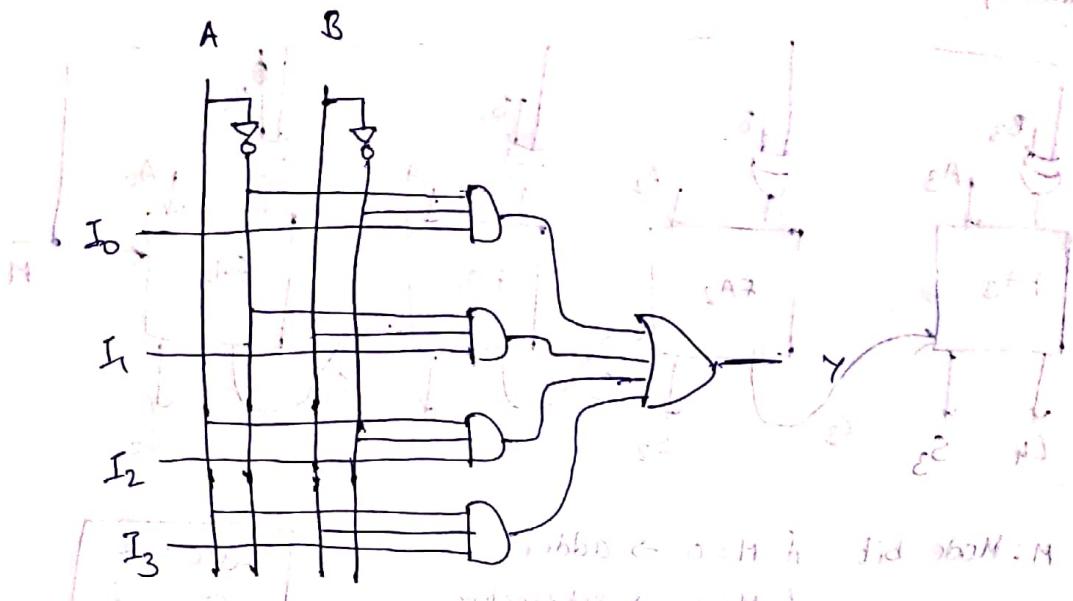
$$(A, B, S, I)_{AB} = (A, B)I$$

$$Y = \bar{A}\bar{B}I_0 + \bar{A}BI_1 + A\bar{B}I_2 + ABI_3$$



$$Y = \bar{A}\bar{B}I_0$$

Implementation:



for $2^n \times 1$ Mux find

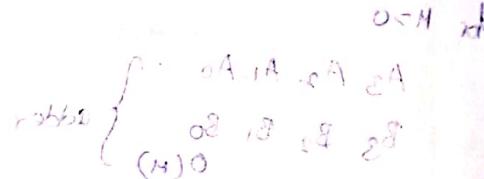
No of AND gates req $\geq 2^n$, size $= 2^n$

size of each AND gate = $n+1$

No of NOT gates req = n

No of OR gates req = 1

Size of the OR gate = 2^n



Example: realization of 3 variable function using 2³ x 1 Mux

Mux as Function Selector: Following are the steps of process

To implement n variable function make the following steps

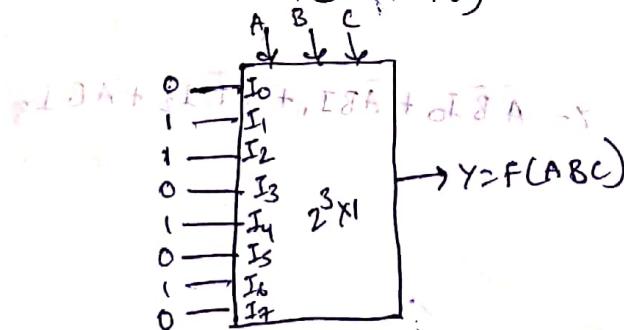
i) $2^n \times 1$ Mux

ii) $2^{n-1} \times 1$ Mux

$2^n \times 1$:



$$\text{Eq: } F(ABC) = \sum m(1, 2, 4, 6)$$



	0	1
0	0	0
1	0	0
2	0	1
3	1	0
4	1	1
5	0	1
6	0	0
7	1	0

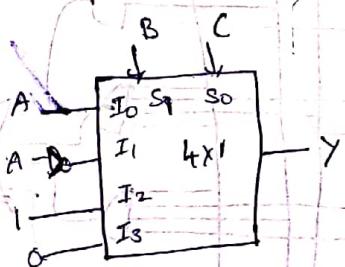
$2^{n-1} \times 1$ (Req. less gates compared to $2^n \times 1$)

* data i/p may be 0's, 1's, free i/p, complement of free i/p

$$\text{Ex: } F(ABC) = \sum m(1, 2, 4, 6)$$

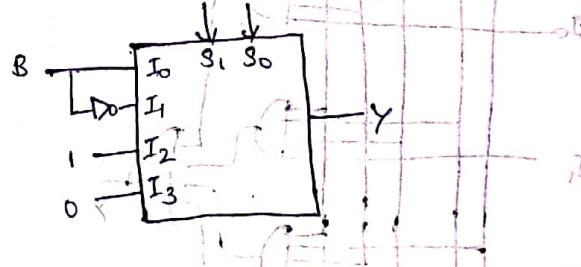
if A = free i/p

$$\begin{array}{c} \bar{A} \quad 0 \quad 1 \quad 2 \quad 3 \\ A \quad 4 \quad 5 \quad 6 \quad 7 \\ \hline A \quad \bar{A} \quad 1 \quad 0 \end{array}$$



if B = free i/p

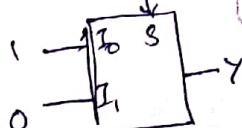
$$\begin{array}{c} \bar{B} \quad 0 \quad 1 \quad 4 \quad 5 \\ B \quad 2 \quad 3 \quad 6 \quad 7 \\ \hline B \quad \bar{B} \quad 1 \quad 0 \end{array}$$



* For clear check, notes; if you don't understand

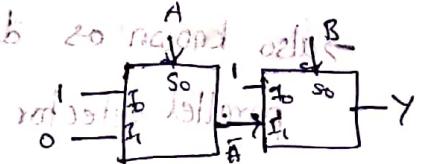
Logic Gates with Muxes ($2^{n-1} \times 1$):

NOT gate:



NAND gate

$$\begin{array}{c} \bar{A} \quad 0 \quad 1 \\ A \quad 2 \quad 3 \\ \hline 1 \quad \bar{A} \end{array}$$



↳ Not gate req. 2 Muxes.

* All basic gates req only 1×1 Mux

* All universal & derived gates req 2×1 Muxes.

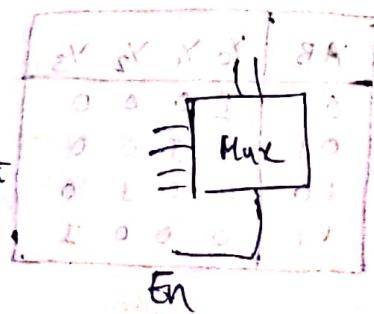
Enable i/p in Muxes:

→ Special i/p called enable i/p is used.

→ If mux is disabled, no matter what

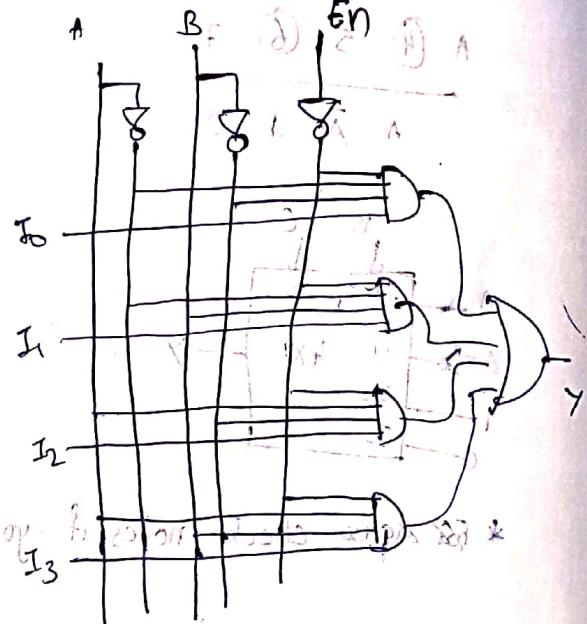
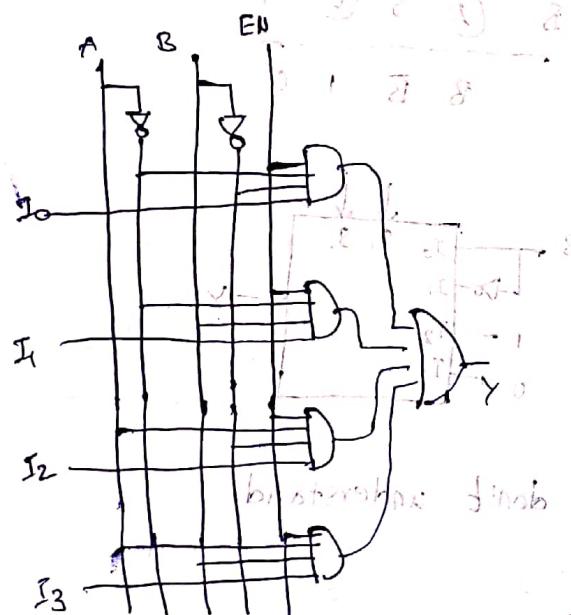
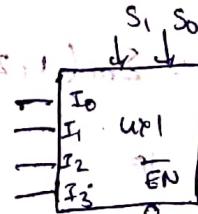
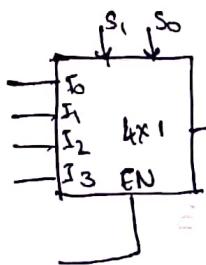
i/p & selection lines you give, o/p is

forced to zero.



two types of enable i/p: (i) active high enable and (ii) active low enable

(i) Active high enable (ii) Active low enable



(iii) circuit after enable input

Demux

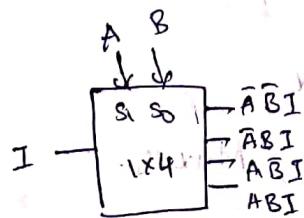
stop gun

stop 704

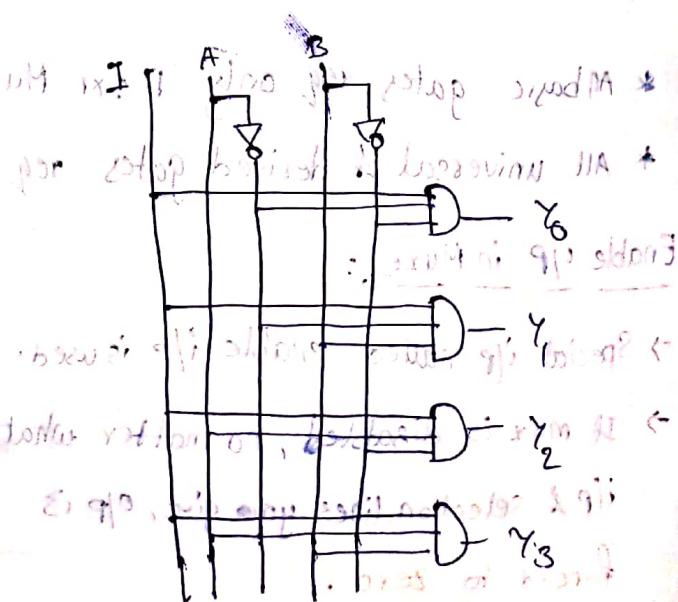
→ also known as data distributor, one to many selector, serial to parallel selector

→ size: 1×2^n

E:



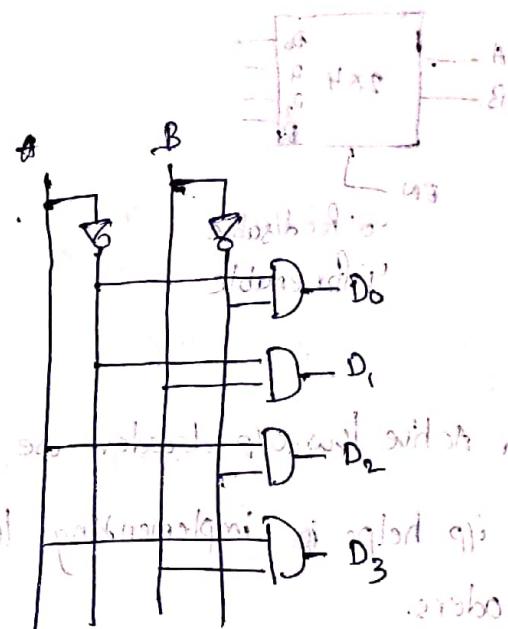
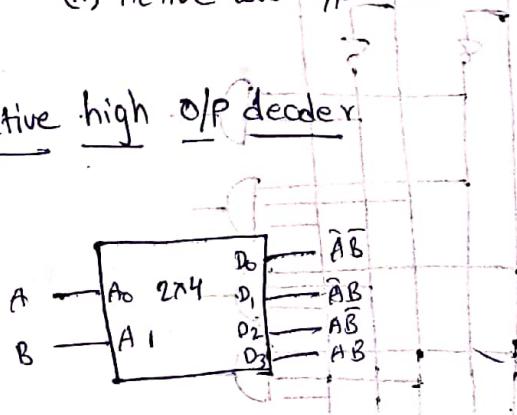
AB	Y ₀	Y ₁	Y ₂	Y ₃
00	1	0	0	0
01	0	1	0	0
10	0	0	1	0
11	0	0	0	1



Decoder:

- used for memory addressing & chip selection.
- No Selection lines, size: $n \times 2^n$, width of input no
- two types: i) $2^3 \times 8$ (using inputs A and B) no
 ii) Active high o/p decoder → only 1 o/p bit is high
 iii) Active low o/p decoder → only 1 o/p bit is low.

Active high o/p decoder:



A	B	D ₀	D ₁	D ₂	D ₃
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

→ Try to do the same above implementation for Active low o/p decoder (use NAND gates)

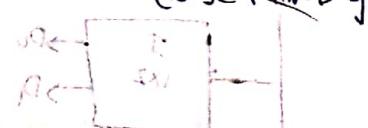
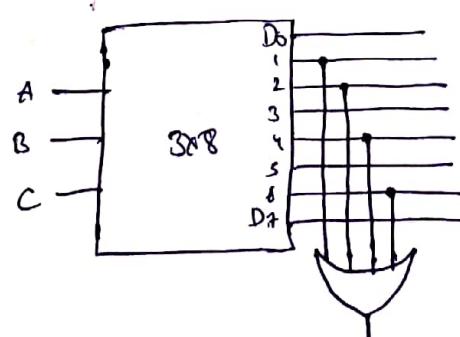
Decoder as function selector:

→ $n \times 2^n$ is used for n variable function

$$\text{Ex: } F(ABC) = \sum m(1, 2, 4, 6)$$

active, high o/p decoder

ABC	F
000	0
001	1
010	1
011	0
100	1
101	0
110	1
111	0



→ For Active low o/p decoder we use NAND gate (think why)

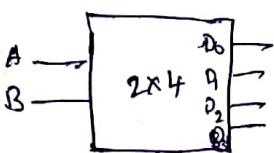
Enable i/p in decoder,

→ If a decoder is disabled

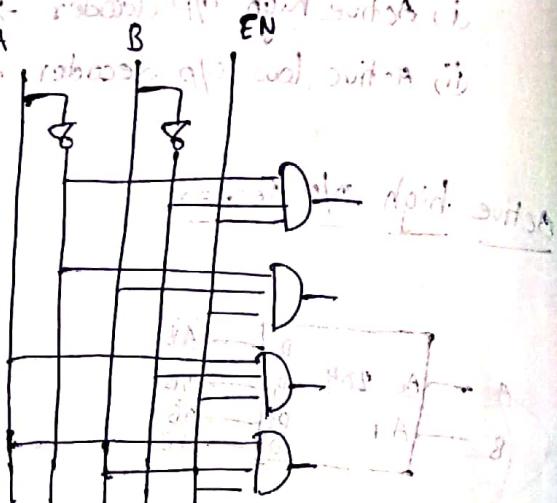
an active high decoder produces '0' o/p in all lines.

an active low o/p decoder produces '1' o/p. in all lines.

Active high o/p decoder



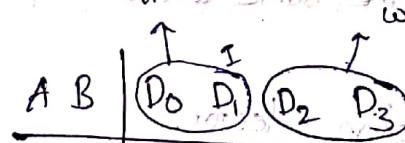
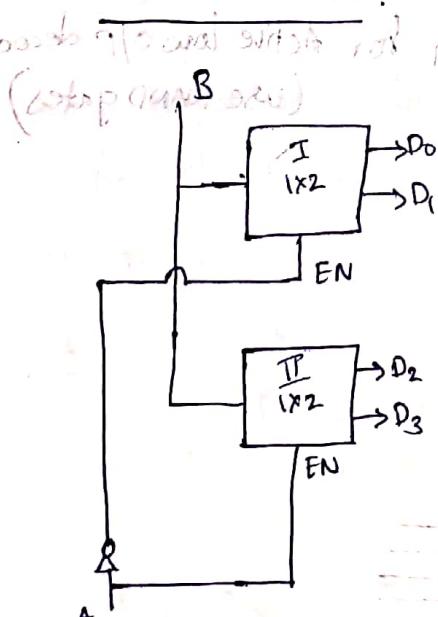
EN
0 for disable
1 for enable



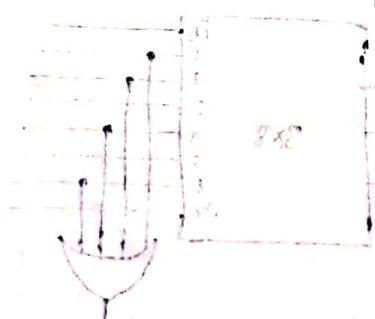
* For Active low o/p decoder we negate EN bcz we use NAND gates.

→ EN i/p helps in implementing larger size decoders with smaller size decoders.

2x4 with 1x2 :



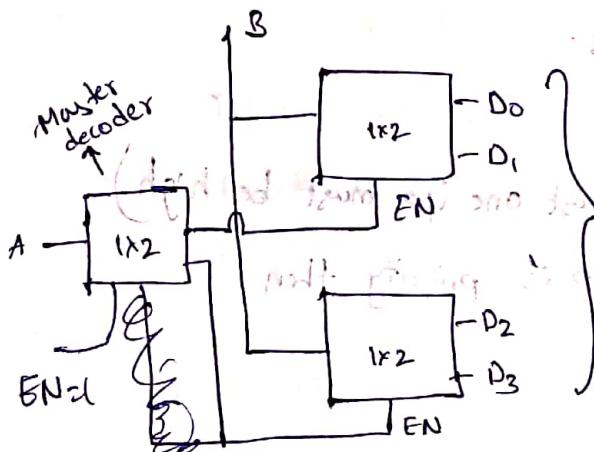
A	B	D ₀	D ₁	D ₂	D ₃
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1



	38A
0	000
1	100
2	010
3	110
4	001
5	101
6	011
7	111

(false start) stop carry 2nd no rebook off val switch not

without using NOT gate :



→ design of 4-to-16 decoder without using NOT gate

→ (v) find q10 binary & decimal 5 =

Slave decoders: pins below 2-to-4

are parallel and串行的

5 = 0101 0101

Req:	4x16	avail: 1x2	Req: 3x8	avail: 1x2	Req: 16x256	avail: 2x4
	$\frac{4 \times 16}{1 \times 2} = 4 + 2 + 1 = 7$		$\frac{3 \times 8}{1 \times 2} = 3 + 2 + 1 = 6$		$\frac{16 \times 256}{2 \times 4} = 64 + 16 + 4 + 1 = 85$	

Req:	16x256	avail: 2x4	$\frac{256}{4} = 64$ (from this divide it by 4 until you get 1)	No of 2x4 req	$= 64 + 16 + 4 + 1 = 85$

Encoder:

→ Reverse of decoder → size: $2^n \times n$

→ If two or more i/p line have high signal then it is invalid i/p.



I ₀	I ₁	I ₂	I ₃	E ₀
1	0	0	0	0
0	1	0	0	1
0	0	1	0	1
0	0	0	1	1

↓ sum of product

$$E_1 = I_2 + I_3$$

$$E_0 = I_0 + I_1 + I_3$$

X	1	X	X	X
0	X	X	X	X
X	X	X	X	X
0	X	X	X	X



Priority Encoder

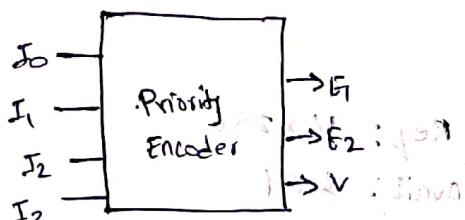
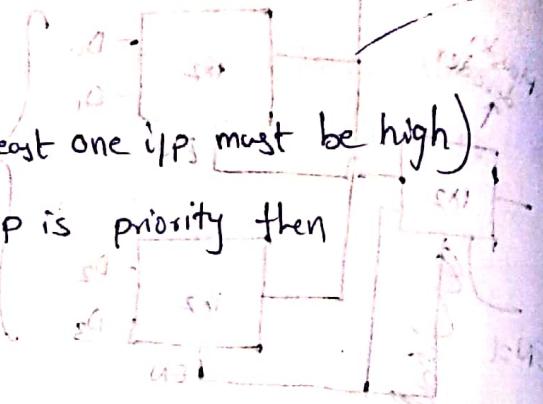
→ Permits one or more high i/p signals.

→ Consists of special o/p bit (v).

→ O/p is valid only if $N \geq 1$ (i.e., atleast one i/p must be high).

→ If i/p lines are I_0, I_1, I_2, I_3 and P is priority then

$$P_0 > P_1 > P_2 > P_3$$



fish with each priority \rightarrow $I_0 > I_1 > I_2 > I_3$
Priority is lost
 $(I_0 > I_1)$

I_0	I_1	I_2	I_3	E_1	E_0	V
0	0	0	0	x	x	0 (invalid)
1	0	0	0	0	0	1 (1st : I_0)
0	1	x	x	0	1	1
0	0	1	x	1	0	1 (2nd : I_2)
0	0	0	1	1	1	1 (3rd : I_3)

For E_1 : $I_0 + I_1 + I_2 + I_3$

I_0	I_1	I_2	I_3
00	01	11	10
00	x	1	1
01	0	0	0
11	0	0	0
10	0	0	0

Now find E_2

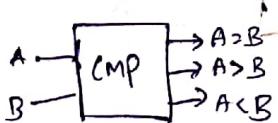
$$V = I_0 + I_1 + I_2 + I_3$$

if $E_1 = 1$ then compare I_0 with I_1 if $I_0 > I_1$ then $E_2 = 1$ else $E_2 = 0$

Magnitude Comparator:

→ Used to compare two magnitudes

one bit cmp:



A	B	$A = B$	$A < B$	$A > B$
0	0	1	0	0
0	1	0	1	0
1	0	0	0	1
1	1	1	0	0

$$F_{A=B} = A \oplus B$$

$$F_{A < B} = \overline{A} \cdot B$$

$$F_{A > B} = \overline{A} \cdot \overline{B}$$

silly find out how design a 2-bit comparator.

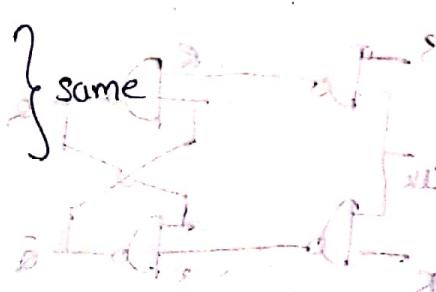
For a n-bit comparator \Rightarrow it adds a provision for various \leq

$$\text{Total no. of Combinations} = 2^{2n}$$

$$\text{No. of } A=B \text{ combinations} = 2^n$$

$$\text{No. of } A > B \text{ combinations} = \frac{2^{2n} - 2^n}{2}$$

$$\text{No. of } A < B \text{ combinations} = \frac{2^{2n} - 2^n}{2}$$



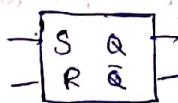
5. Sequential Circuits

→ Seq. circuit is at clocked ckt (and o/p arrival time) depends on frequency of the clock pulse.

→ If clock ip is '0', no changes are present at the o/p.

→ For seq. ckt reliable operation, max. clock frequency must be less than $1/t_{pd}$. $t_{pd} \rightarrow$ ckt propagation delay.

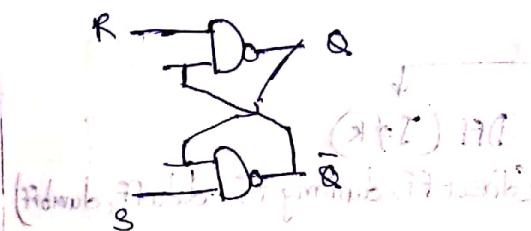
SR latch:



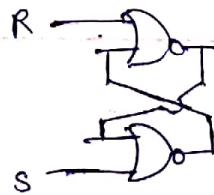
'S' sets Q
'R' reset Q

(O/P is invalid (X) if $S=Q=R$)

SR NAND latch



SR NOR latch



S	R	Q
0	0	X → invalid
0	1	0
1	0	1
1	1	N.C

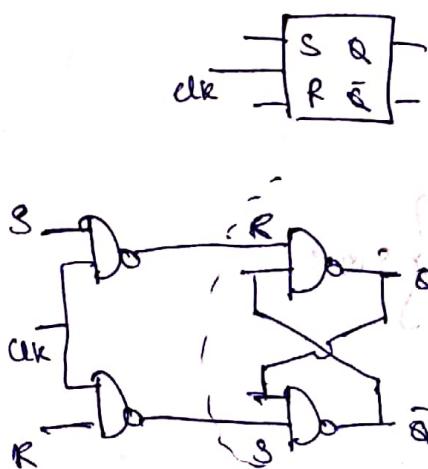
N.C → No change

S	R	Q
0	0	N.C
0	1	0
1	0	1
1	1	X

0	0	T
0	1	0
1	0	1
1	1	1

SR Flipflop

→ Obtained by connecting a clock to SR latch



S	R	Q
0	0	N.C.
0	1	0
1	0	1
1	1	X

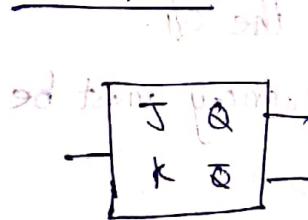
S	R	Q _{t+1}
0	0	Q _t
0	1	0
1	0	1
1	1	X

Q_{t+1} → Next state

Q_t → Present state

→ To get all the above o/p in table - the clk must be high
otherwise o/p will be unchanged irrespective of i/p

JK Flipflop



J	K	Q _{t+1}
0	0	Q _t
0	1	0
1	0	1
1	1	Q _t

Toggle

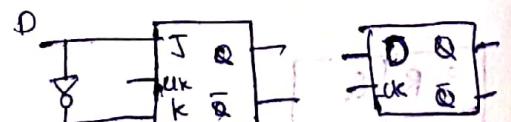
Single i/p flipflops

JKFF



DFF (J=K)

(direct FF, dummy FF, data FF, dummy FF)



T	JK	Q _{t+1}
0	00	Q _t
1	11	Q _t

T	Q _{t+1}
0	Q _t
1	Q _t

D	J	K	Q _{t+1}
0	0	1	0
1	1	0	1

D	Q _{t+1}
0	0
1	1

FlipFlop Tables

i) General tables: Tables that were written so far.

ii) Characteristic tables: Tells relation b/w Q_t & Q_{t+1}

iii) Excitation tables: Tells what has to be given to get desired Q_{t+1}

from given Q_t and what to do from given Q_t to get Q_{t+1}

Characteristic Tables:

S	R	Q_t	Q_{t+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	x
1	1	1	x

J	K	Q_t	Q_{t+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

D	Q_t	Q_{t+1}
0	0	0
0	1	0
1	0	1
1	1	1

$$Q_{t+1} = D$$

T	Q_t	Q_{t+1}
0	0	0
0	1	1
1	0	1
1	1	0

$$Q_{t+1} = T \oplus Q_t$$

Excitation Tables:

Q_t	Q_{t+1}	S	R
0	0	0	0
	1		
	x		
0	1	1	0
1	0	0	1
1	1	0	0
	0		
	1		
	x		

Q_t	Q_{t+1}	J	K
0	0	0	x
0	1	1	x
1	0	x	1
1	1	x	0

Q_t	Q_{t+1}	D
0	0	0
0	1	1
1	0	1
1	1	0

Q_t	Q_{t+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Counter

two types:

Synchronous Counter

Asynchronous Counter (Ripple Counter)

→ used to count states and each state is available for one clock cycle.

→ If n bit counter is used to count all 2^n states, it is called binary counter, else it is mod counter.

→ Mod k counter is used to count k no. of states.

→ Design of Mod k counter is expensive, cuz it req. more gates.

Design of Synchronous Counter:

3 bit upcounter (binary counter)

$Q_2 Q_1 Q_0$	T ₂	T ₁	T ₀
0 0 0	0	0	1
0 0 1	0	1	1
0 1 0	0	0	1
0 1 1	1	1	1
1 0 0	0	0	1
1 0 1	0	1	1
1 1 0	0	0	1
1 1 1	1	1	1

0 0 0 0	0 0 1 0
0 0 1 0	0 1 1 0
0 1 0 0	1 0 0 1
0 1 1 0	1 1 0 1
1 0 0 0	X 0 1 1
1 0 1 0	X 1 1 1

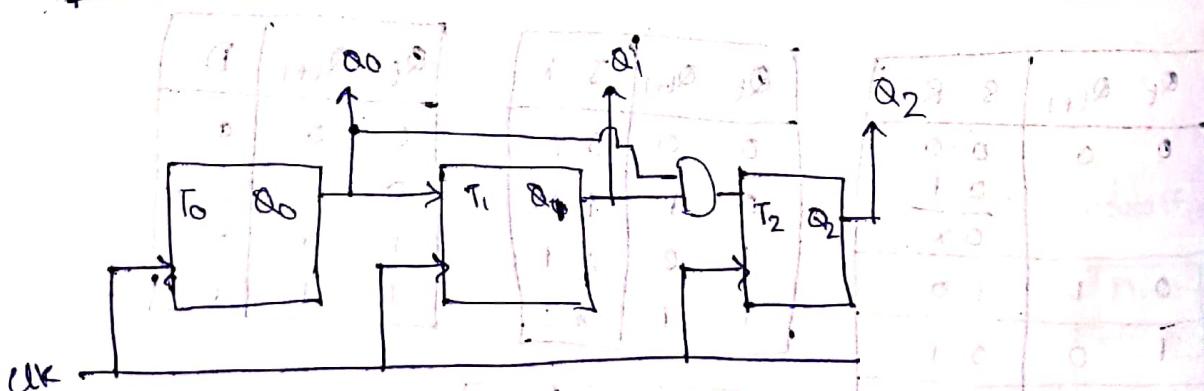
T₀ = 1

T₁ = Q₀

T₂ = Q₁Q₀

if you are designing n-bit binary-up counter then we can use general formula for $T_i = Q_{i-1}Q_{i-2}\dots Q_1Q_0$

$$\text{Ex: } T_5 = Q_4Q_3Q_2Q_1Q_0$$



Mod 6 upcounter

Mod 6 = 0, 1, 2, 3, 4, 5 (\therefore we req. 3 bits)

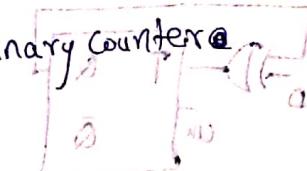
Q_2	Q_1	Q_0	T_2	T_1	T_0
0	0	0	0	0	0
0	0	1	0	1	x
0	1	0	0	0	1
0	1	1	1	1	1
1	0	0	0	0	1
1	0	1	1	1	x
1	1	0	x	x	x
1	1	1	x	x	x

$$T_2 = \overline{Q_0}(Q_1 + Q_2)$$

slly T_1 & T_2

Now design a seq ckt as it was

designed for 3-bit binary counter @



When it is asked to design a counter for given states check if any state is repeated. If no state is repeated then design process as you have seen above. But if any state is repeated then follow below procedure.

Eg: Design counter for 0, 1, 2, 1, 3

To distinguish two 1's we use extra flip-flop (dummy ff)

Circuit will be same as 7792 difference will be at

Q_0	Q_1	Q_2	T_D	T_1	T_0
0	0	0	0	0	1
0	0	1	0	1	1
0	1	0	1	1	1
1	0	1	1	1	0

$$T_D = \overline{Q_0} + Q_1 + Q_2$$

$$T_1 = Q_0 + Q_1$$

$$T_0 = \overline{Q_0}$$

Flip flop Conversions:

Any FF can be implemented with a given FF.

For this, first write characteristic table for target FF.

Then write excitation table for available FF.

Report level out in

Report level in

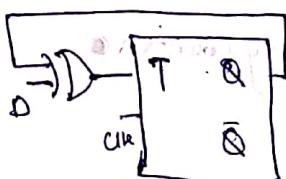
Report sps out in

Report sps in

DFF using TFF

D	Q	Q _{t+1}	T
0 0	0	0	
0 1	0	1	
1 0	1	1	
1 1	1	0	

$$T = D \oplus Q$$



JKFF using SRFF

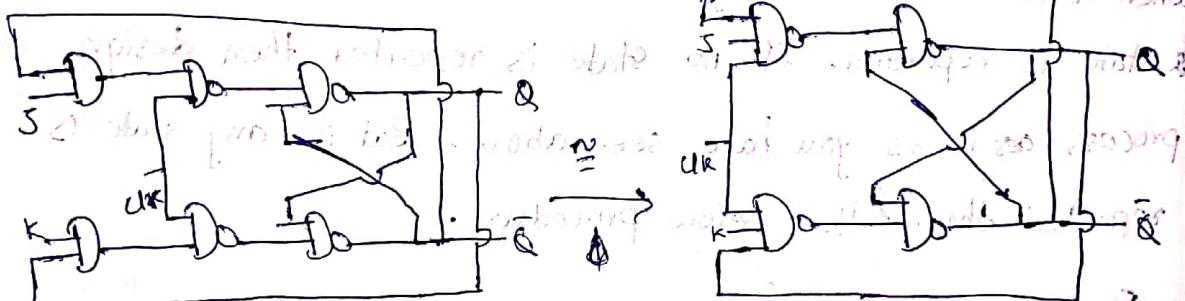
Initial state $Q=0, Q_t=0, S=R=0$ \rightarrow it toggles.

J	K	Q _t	Q _{t+1}	S	R
0	0	0	0	0	X
0	0	1	1	X	0
0	1	0	0	0	X
0	1	1	1	0	1
1	0	0	1	1	0
1	0	1	0	X	0
1	1	0	1	1	0
1	1	1	0	0	1

Initial state $Q=0, Q_t=0, S=R=0$ \rightarrow it toggles.

$$S = J\bar{Q}$$

$$R = KQ$$



→ Observe above fig. A and understand.

→ In 2nd fig which represents JKFF, if we remove 3ip NAND gates and place zip NAND-gate then the figure represents SRFF.

→ JKFF : two 3ip NAND, two 2ip NAND

SRFF : four 2ip NAND.

Flipflop Triggering

→ It process of applying high clock signal at desired time.

→ 4 techniques

- i) the level trigger
- ii) -ve level trigger
- iii) the edge trigger
- iv) -ve edge trigger

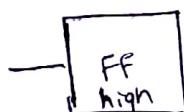
→ RAC occurs in level triggered FF, hence edge triggered are used most.

→ To use edge triggered FF, we ~~need~~ have to use pulse train.

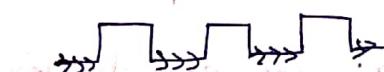
the level trigger:



→ Enabled during high signal.



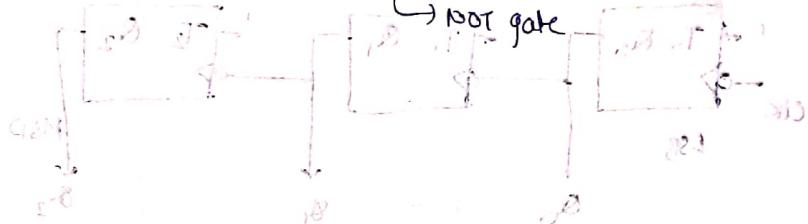
the level trigger



→ Enabled during low signal.



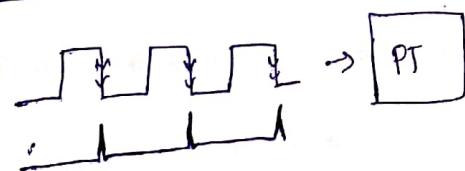
the edge trigger



→ Enabled once for each cycle for a short period of time.

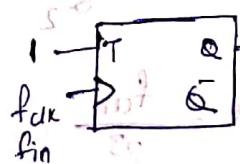


the edge trigger



TFF:

→ It known as frequency divider or mod 2 counter.



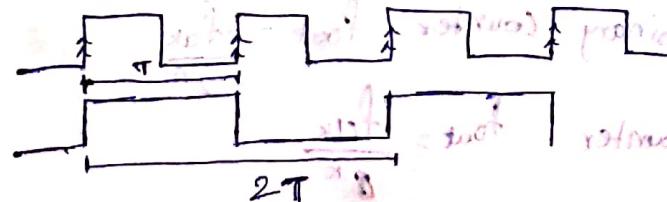
$$f_{out} = \frac{f_{in}}{2}$$

since it is the edge trigger FF, O/P will

be toggled at every rising edge of a cycle.

Let.
 Q_{20}
initially

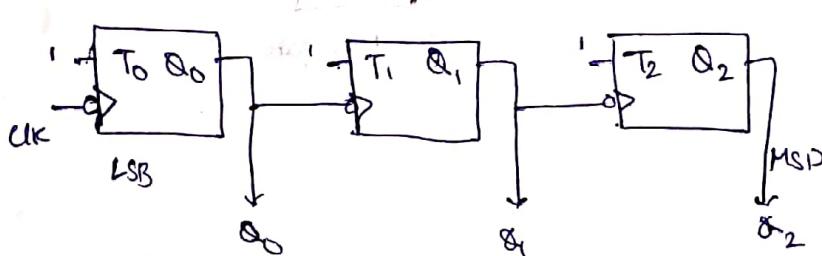
f_{in}
 f_{out}



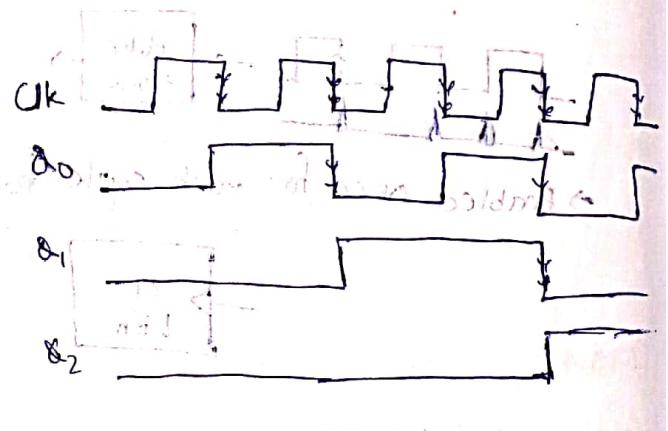
Design of Asynchronous Counter (Ripple Counter):

- Main clock generator is connected to LSB FF only.
- Design cost is cheaper and is used for freq division application.
- up counter; r^e edge trigger
down counter; f^u edge trigger

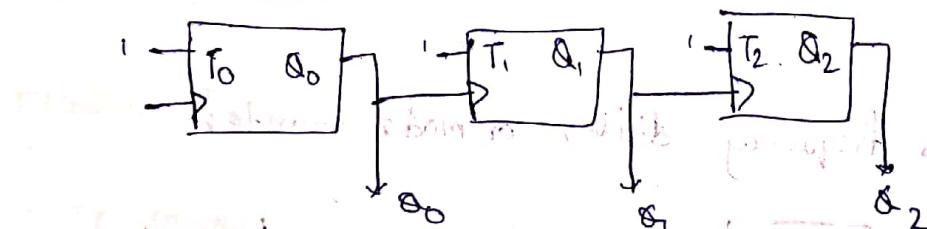
3bit up Counter



Code	Q_2	Q_1	Q_0
000	0	0	0
001	0	0	1
010	0	1	0
011	0	1	1
100	1	0	0
101	1	0	1
110	1	1	0
111	1	1	1



3 bit down counter



→ for 3 bit binary counter, $f_{out} = \frac{f_{clock}}{2^3}$

→ for n bit binary counter $f_{out} = \frac{f_{clock}}{2^n}$

→ for mod k counter $f_{out} = \frac{f_{clock}}{2^k}$

Design of asynchronous mod counters:

→ we use two special i/p

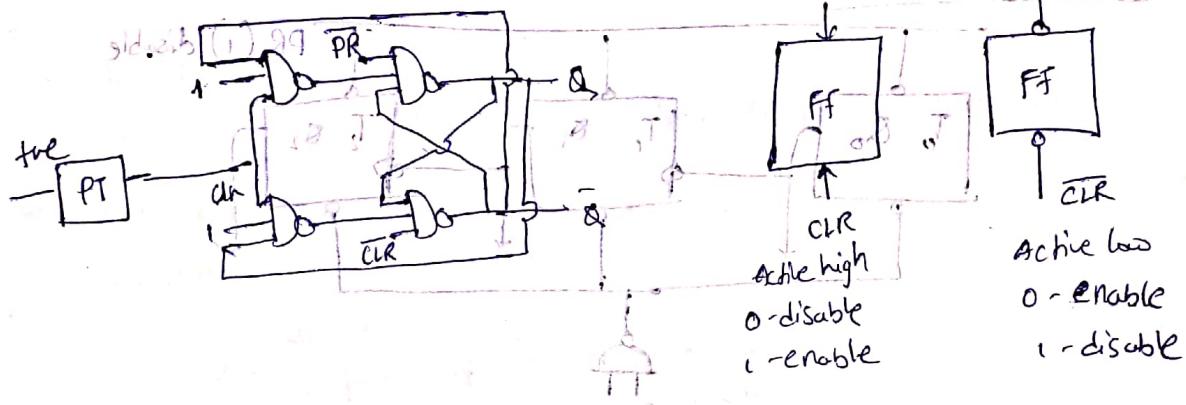
i) preset (pr) ii) clear (clr)

→ ~~both~~ Both are generally disabled.

→ When preset is enabled it forces o/p to high regardless of i/p

and clock and hence used in down counter.

→ similarly clear forces o/p to low and hence is used in upcounters.



Mod 6 upcounter

→ 0 to 5 accept

$$6 = (110)_2$$

if we have to stop counting exactly at 6 (binary 110)

Binary value of 6 is having 2 bits that are $Q_2 Q_1$. So if we want to stop at 6, then $Q_2 Q_1$ should be 11.

For active low enable \overline{CLR} should be given if $Q_2 Q_1$ when 11 occurs.

Hence we connect $Q_2 Q_1$ to NAND gate and connect it to \overline{CLR} .

For 6 $Q_2 Q_1 = 11 = 0 = \overline{CLR}$ so \overline{CLR} is at low level.

777 branching level $\Rightarrow 777$ by \overline{CLR} of timer is enable and preset to 000.

Also we can use OR gate in place of NAND gate and give

if $Q_2 Q_1 = 11 \Rightarrow \overline{D}$

Mod 83 upcounter:

→ 0 to 82 accept

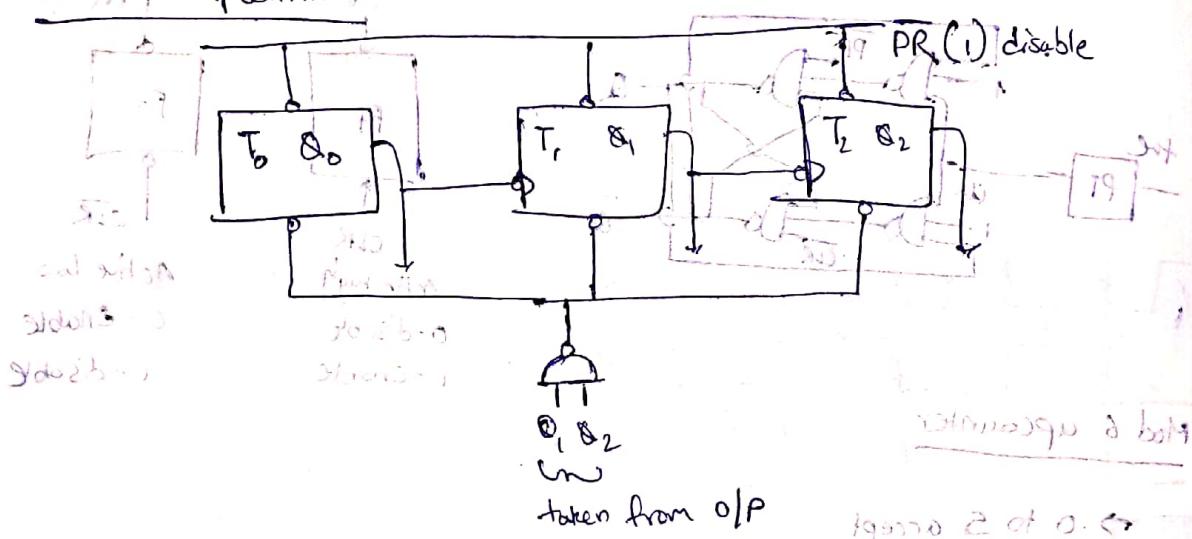
$$83 = (1010011)_2$$

$\oplus \oplus \oplus \oplus \oplus \oplus \oplus$

4 1's. Hence 4 ip NAND or OR TS reqd to ~~enable~~ \rightarrow
 4 flip-flops need to be set for bottom 2 bits now \leftarrow



For Mod 6 upcounter:



Race Around Condition (RAC):

→ when FF o/p is toggled more than once in one clk cycle then we

say RAC has occurred. If 3d state 315 olders and others not

→ RAC is diff from toggling. RAC is uncontrolled toggling.

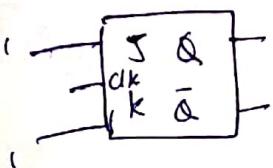
→ SRFF & DFF are free from RAC.

→ RAC never occurs in edge-triggered FFs.

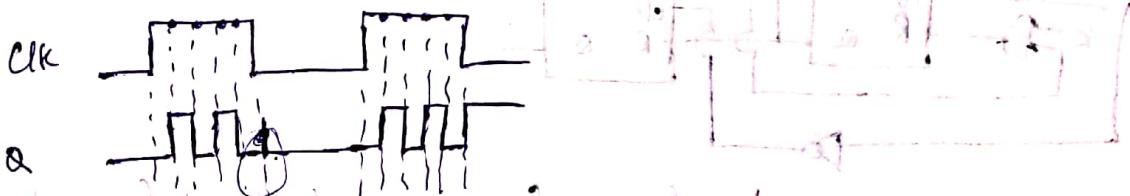
→ So RAC occurs only in level triggered JKFF & level triggered TFF.

→ Master slave FF is used to prevent RAC.

Eg for RAC:



$$t_{clk} = 10 \text{ ns} \Rightarrow f_{clk} = 0.1 \text{ GHz} \text{ or } 100 \text{ MHz}$$

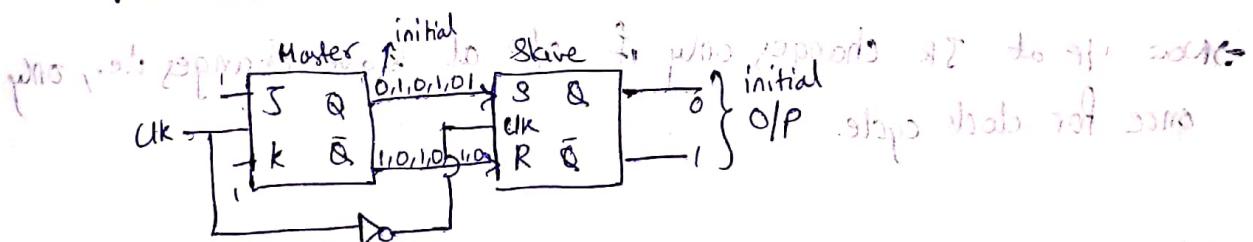


Master-Slave JKFF

→ Master FF: JKFF

Slave FF : JKFF or SRFF (gets i/p from Master FF & gives final o/p)

→ when clock is high master is enabled & slave is disabled and vice versa



→ But the above circuit works good only if Master toggles off for odd

no of times. If it toggles even, no of times i/p for slave remains

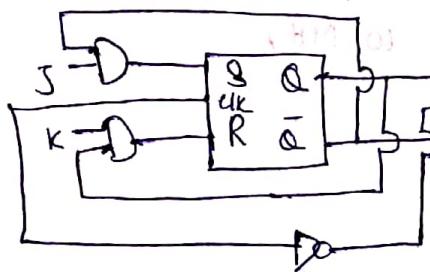
Some all time it is enabled and hence o/p is same i.e., straight line.

→ so we modify it as shown below · so that flip at Master toggles odd no of times (i.e., one time)

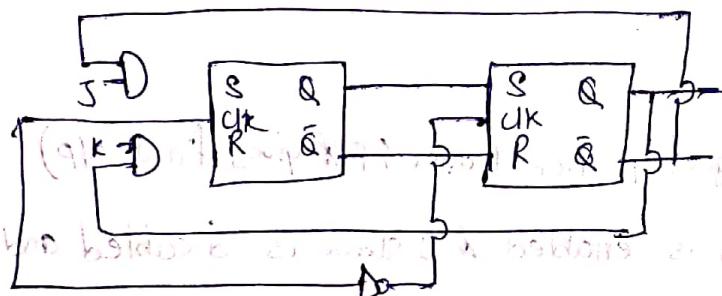
old ← find tip; e = old ←
(3) (4)

Modification,

The previous diagram can be seen as



↓ Modification (i.e., getting feedback from slave instead of getting it from itself)



This is stable because we have an additional feedback signal at slave's clock vertex.

Now i/p at JK changes only if o/p at slave changes i.e., only once for clock cycle.

Registers:

→ Used to save applied data and hence we use DFF.

→ Data shifting can perform binary mul & binary division.

→ types of shifting:

(i) Serial in Serial out.

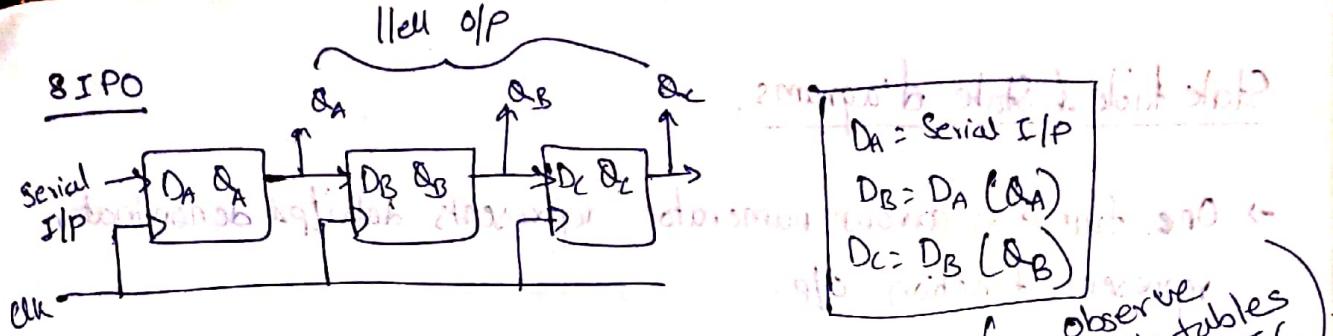
(ii) Serial in Parallel out (most used in real time)

(iii) Parallel in Serial Out

(iv) Parallel in Parallel Out

→ 010 → left shift → 100

→ 100 → right shift → 010

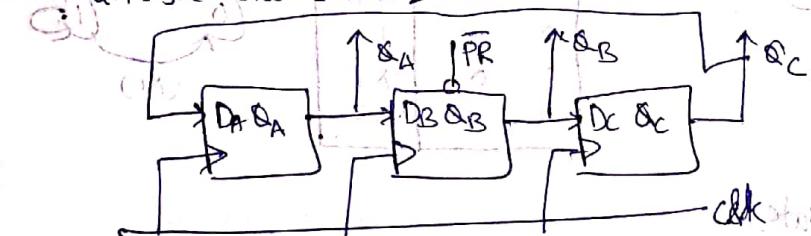


Ring Counter (RC): It is a ring of n flip flops connected in a loop.

→ Same as SIPO except Q_C is connected to serial i/p and one of the flip flops is connected with preset.

→ Only one o/p bit is high and hence it counts n states

→ Useful states $\rightarrow n$
 (n)
 (n) unused states $\rightarrow 2^n - n$



Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0
3	0	0	1

Final State: $Q_A = 0, Q_B = 0, Q_C = 1$

State	Q _A	Q _B	Q _C
1	1	0	0
2	0	1	0

Initial State: $Q_A = 1, Q_B = 0, Q_C = 0$

State	Q_{A</sub}

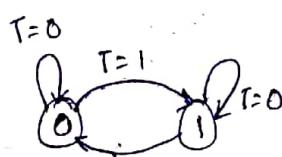
State table & State diagrams:

- On a transition arrow numerator represents data/o/p & denominator represents function o/p.
- To draw state diagram of n-bit binary counter we req. 2^n states.
(try to draw)

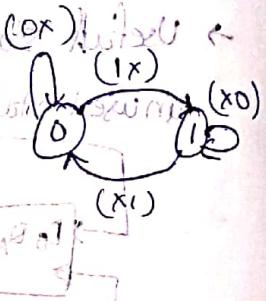
state diagrams for ffs

TFF

Q	Q_{t+1}	T
0 0	0	
0 1	1	
1 0	1	
1 1	0	



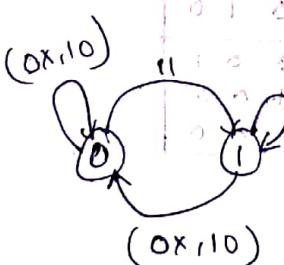
Q_t	Q_{t+1}	Jk
0 0	0	0x
0 1	1	1x
1 0	0	x1
1 1	1	x0



state diagrams for logic gates

AND gate

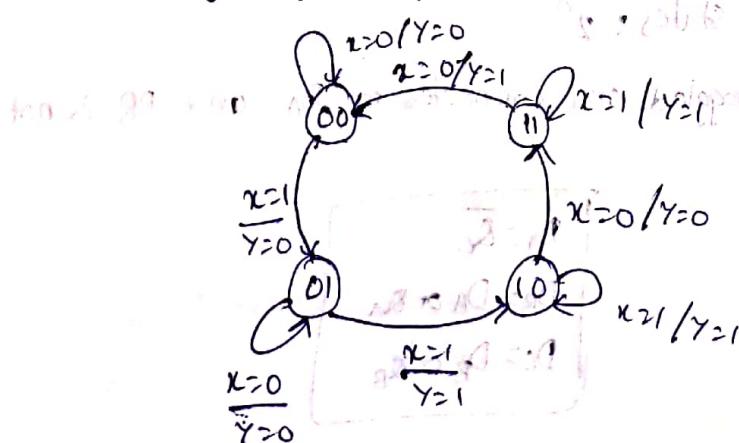
Q_t	Q_{t+1}	AB
0 0	0	(0x, 10)
0 1	1	11
1 0	0	(0x, 10)
1 1	1	11



Q_t	Q_{t+1}	AB
0 0	0	00
0 1	1	01
1 0	0	10
1 1	1	11

: (Q) shows next state

Eg: Design logic diagram for below state diagram with Tff



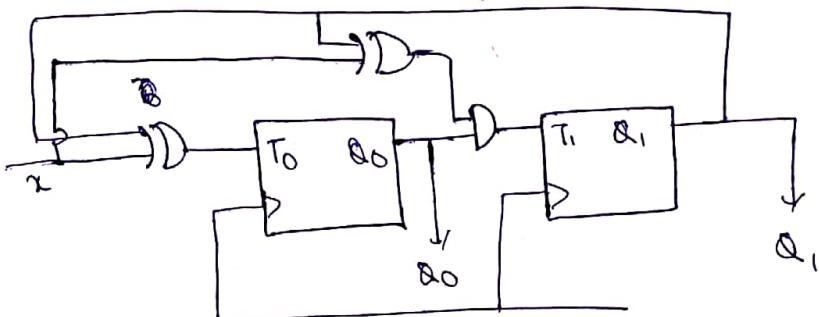
Q_1	Q_2	Q_3	Q_4
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

P.S & T		N.S & T'			
Q_1, Q_0	X	Q_1, Q_0	T_1	T_0	Y
0 0	0	0 0	0	0	0
0 0	1	0 1	0	1	0
0 1	0	0 1	0	0	0
0 1	1	1 0	1	1	1
1 0	0	1 1	0	1	0
1 0	1	1 0	0	0	1
1 1	0	0 0	1	1	1
1 1	1	1 1	0	0	1

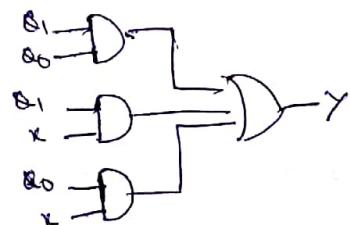
from fig excitation table from fig

$$T_1 = Q_0(Q_1 \oplus X)$$

$$T_0 = Q_1 \oplus X$$



for O/P Y connect below ckt to the above ckt



→ The sequential synchronous circuit is also known as finite State Machine (FSM).