ENGINEERING MATHEMATICS

CALCULUS

→ Signum function $\begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$

↳ discont at 0.

→ Step function: $[1.5] = 1$; $[-1.5] = 2$;

↳ discontinous at every integer

→ $f(x,y) = f(y,x) \Rightarrow$ curve symmetric at $y = x$

Limit:

~~limit exists~~ $\cancel{\lim_{x \to a} f(x)}$

left limit: $\lim_{x \to a^-} f(x)$; right limit: $\lim_{x \to a^+} f(x)$

limit exist $\Leftrightarrow \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

→ $\boxed{\lim_{x \to \infty} (1 + a/x)^x = e^a \; ; \; \lim_{x \to 0} (1 + ax)^{1/x} = e^a}$

Note:

$\frac{0}{0}, \frac{\infty}{\infty}$ } → L'Hospital rule.

$0 \times \infty$ → reduce to $0/0$ (or) $\frac{\infty}{\infty}$ (keep log in numerator).

$\infty - \infty$ → Take LCM and reduce to $\frac{0}{0}$ (or) $\frac{\infty}{\infty}$.

$0^0, \infty^0, 1^\infty$ → take log on b.s (Do not forget e^{result})

Continuity:

$f(x)$ is continuous at $x = a$ iff

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

i.e., $f(a)$ is defined and limit exists.

left continuous: $\lim_{x \to a^-} f(x) = f(a)$

right continous: $\lim_{x \to a^+} f(x) = f(a)$

→ $f(x)$ is continuous in (a,b) iff $f(x)$ is cont $\forall x \in (a,b)$

→ $f(x)$ is cont in $[a,b]$ iff

(i) $f(x)$ is cont in (a,b)

(ii) $\lim_{x \to a^+} f(x) = f(a)$

(iii) $\lim_{x \to b^-} f(x) = f(b)$

Differentiability:

→ $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

→ $f(x)$ is differentiable at $x = a$ iff

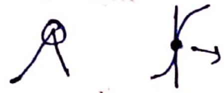
$f'(a^-) = f'(a^+)$

(or)

$\lim_{x \to a^-} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$

(left hand derivative) (RHD)

→ $f(x)$ is not differentiable under below cases



$\frac{dy}{dx} = \infty$ (undefined)

So no differentiable

$\boxed{\begin{array}{l} \text{diff} \Rightarrow \text{cont} \\ \text{not cont} \Rightarrow \text{not diff} \end{array}}$

Note:

$|ax - b|$ is cont everywhere

is diff everywhere except at $x = b/a$

Note:

→ $f(x)$ is diff in (a,b) iff $f'(c)$ exists $\forall c \in (a,b)$

→ $f(x)$ is diff in $[a,b]$ iff

(i) $f(x)$ is diff (a,b)

(ii) $f'(a^+)$ exists

(iii) $f'(b^-)$ exists.

Mean Value Theorems:

Rolle's Theorem / Fundamental MVT of calculus:

→ $f(x)$ is cont on $[a,b]$ and diff on (a,b) such that $f(a) = f(b)$ then there exists atleast one c such that $f'(c) = 0$, $c \in (a,b)$

→ Converse of rolle's need not to be true.

Lagrange's MVT or MVT (or) First MVT of calculus

→ $f(x)$ is cont on $[a,b]$ and diff on (a,b) then there exists atleast one $c \in (a,b)$ such that

$f'(c) = \frac{f(b) - f(a)}{b - a}$

Cauchy's MVT (or) 2nd MVT of calculus

let $f(x)$ and $g(x)$ be 2 funct such that

(i) f and g are cont on $[a,b]$

(ii) f and g are diff on (a,b)

(iii) $g(x) \neq 0$, $\forall x \in [a,b]$ then there exist atleast on c such that $\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$

Maxima and Minima

stationary point $\rightarrow f'(x)=0$

Critical point $\rightarrow f'(x)=0$ (or) $f'(x)=\infty$
↳ i.e., not diff

Inflection point : $f''(x)$ is undefined
(or)
$f''(x)=0$ and $f'''(x)\neq 0$

Global Maxima/Global Minima (Absolute)

→ find all critical points. (not just $f'(x)=0$, check $f'(x)=\infty$ also)
→ let a,b be end points.

Global Max : Max ($f(a), f(b), f$(critical pts))

Global Min : Min ($f(a), f(b), f$(critical pts))

Local Maxima & Local Minima

→ Find critical points. ($f'(x)=0 \vee f'(x)=\infty$)

→ $f''(x)>0 \Rightarrow$ local minimum
$f''(x)<0 \Rightarrow$ local maximum

→ $f''(x)=0 \Rightarrow$ neither maxima nor minima

→ $f''(x)=0 \wedge f'''(x)\neq 0 \Rightarrow$ inflection point

→ $f''(x)=0 \wedge f'''(x)=0$
Now ~~to~~ take $f'''(x)$ as $f'(x)$ and repeat.

Maxima & Minima in 2 variables:

→ $p=\frac{\partial f}{\partial x}$; $q=\frac{\partial f}{\partial y}$.
Solve $p=0$ & $q=0$ to obtain stationary points.

→ $r=\frac{\partial^2 f}{\partial x^2}$; $s=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$; $t=\frac{\partial^2 f}{\partial y^2}$

For each stationary point find r,s,t

$rt-s^2>0 \wedge r<0 \Rightarrow$ maximum

$rt-s^2>0 \wedge r>0 \Rightarrow$ Minimum

$rt-s^2<0 \Rightarrow$ saddle point

$rt-s^2=0 \Rightarrow$ cannot decide.

Note:

→ $\frac{d}{dx}\left(\frac{u}{v}\right)' = \frac{vu'-uv'}{v^2}$

→ $\frac{d}{dx}(a^x) = a^x \log_e a$

→ $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

→ $\frac{d}{dx}(\log_a x) = \frac{1}{x}\frac{1}{\log_e a}$

• $\sin x \rightarrow \cos x$; $\cos x \rightarrow -\sin x$

• $\tan x \rightarrow \sec^2 x$; $\sec x \rightarrow \sec x \cdot \tan x$

• $\cot x \rightarrow -\mathrm{cosec}^2 x$; $\mathrm{cosec}\, x \rightarrow -\mathrm{cosec}\, x \cdot \cot x$

• $\sin^{-1} x \rightarrow \frac{1}{\sqrt{1-x^2}}$; $\cos^{-1} x \rightarrow \frac{-1}{\sqrt{1-x^2}}$

• $\tan x \rightarrow \frac{1}{1+x^2}$; $\cot x \rightarrow \frac{-1}{1+x^2}$

• $\sec x \rightarrow \frac{1}{x\sqrt{x^2-1}}$; $\mathrm{cosec}\, x \rightarrow \frac{-1}{x\sqrt{x^2-1}}$

• $\sinh x \rightarrow \cosh x$; $\cosh x \rightarrow \sinh x$

• $\tanh x \rightarrow \mathrm{sech}^2 x$; $\mathrm{sech}\, x \rightarrow -\mathrm{sech}\, x \tanh x$

• $\coth x \rightarrow -\mathrm{cosech}^2 x$; $\mathrm{cosech}\, x \rightarrow -\coth x\, \mathrm{cosech}\, x$

Definite Integrals:

→ $\int_{-a}^{a} f(x) = \begin{cases} 2\int_0^a f(x)\,dx & , f(x) \text{ is even} \\ 0 & , f(x) \text{ is odd} \end{cases}$

→ $\int_0^{2a} f(x) = \begin{cases} 2\int_0^a f(x) & , f(2a-x)=f(x) \\ 0 & , f(2a-x)=-f(x) \end{cases}$

→ $\int_0^a f(x) = \begin{cases} 2\int_0^{a/2} f(x)\,dx & , f(a-x)=f(x) \\ 0 & , f(a-x)=-f(x) \end{cases}$

→ $\int_0^a f(x) = \int_0^a f(a-x)\,dx$

→ $\int_a^b f(x) = \int_a^b f(a+b-x)$

→ $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx = \frac{b-a}{2}$

→ $\int_0^{\pi/2} \sin^n x = \int_0^{\pi/2} \cos^n x = \begin{cases} \frac{(n-1)(n-3)\cdots 1}{n(n-2)\cdots 2}\cdot\frac{\pi}{2} & , n \text{ is even} \\ \frac{(n-1)}{n}\frac{(n-3)}{n-2}\cdots\frac{2}{3} & , n \text{ is odd} \end{cases}$

→ $\int_0^{\pi/2} \sin^m x \cos^n x = \frac{[(m-1)(m-3)\cdots 1 \text{ (or) } 2][(n-1)(n-3)\cdots 1 \text{ (or) } 2]}{(m+n)(m+n-2)(m+n-4)\cdots 1 \text{ (or) } 2}\, k$

$k=\pi/2$ if m,n are even
$=1$ otherwise.

Leibnitz's rules

$$\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)\,dt\right] = v'\cdot f(v(x)) - u' f(u(x))$$

(Learn all the formulae in Ace Material)

Note:

→ $\int uv = u\int v - \int\left(u'\int v\,dx\right)dx$

'u' is choose in order ILATE

→ $\int_0^{na} f(x)\,dx = n\int_0^{a} f(x)\,dx$

where a is periodicity of $f(x)$

Note:

→ $\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\mathrm{cosec}^2\theta - \cot^2\theta = 1$

→ $\sin(x\pm y) = \sin x\cos y \pm \cos x\sin y$

$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$

$\cos(x-y) = \cos x\cos y + \sin x\sin y$

$\tan(x+y) = \dfrac{\tan x + \tan y}{1-\tan x\tan y}$

$\tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x\tan y}$

→ $\sin 2\theta = 2\sin\theta\cos\theta$

~~cos 2θ =~~

→ $1+\cos\theta = 2\cos^2\theta/2$

→ $1-\cos\theta = 2\sin^2\theta/2$

→ $\sinh x = \dfrac{e^x - e^{-x}}{2}$

→ $\cosh x = \dfrac{e^x + e^{-x}}{2}$

LINEAR ALGEBRA

Types of Matrices:

→ Real matrix ; Complex matrix.

→ Row matrix ; column matrix

→ Null matrix / Zero matrix (all entries = 0)

→ rectangular matrix, square matrix.

→ Diagonal matrix

Scalar matrix   Unit matrix

→ upper triangular, Lower triangular.

Trace: sum of diagonal elements

$tr(AB) \neq tr(A)\,tr(B)$

$tr(AB) = tr(BA)$

Transpose:

• $(AB)^T = B^TA^T$ • $(A\pm B)^T = A^T \pm B^T$

• $(A^2)^T = (A^T)^2$

Symmetric & Skew-Symmetric matrices

Symmetric: $A^T = A$

Skew-Symmetric: $A^T = -A$
• all diagonal elements = 0

→ Given any matrix and if asked to prove whether it is sym or skew sym just try to find transpose and check.

→ det(skew sym) = 0, if order is odd

Idempotent Matrix: $A^2 = A \Rightarrow |A| = 0/1$

• $AB = A$ and $BA = B \Rightarrow A$ & B are idempotent

Involuntary matrix: $A^2 = I$

Nilpotent matrix: $A^m = 0$ (m is +ve int)

Such least m is called index of nilpotent matrix

Orthogonal matrix: $AA^T = A^TA = I$ (or) $A^{-1} = A^T$

det(orthogonal mat) = ±1

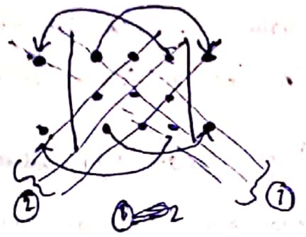
→ A, B are orthogonal ⇒ AB & BA are orthogonal.

Periodic Matrix: $A^{k+1} = A$; k is period.

Determinants:

3x3 matrix shortcut:

① - ②



→ $|AB| = |A||B|$; $|A^m| = |A|^m$; $|A^T| = |A|$

→ $|A^{-1}| = \frac{1}{|A|}$

Singular: $|A| = 0$
Non-singular: $|A| \neq 0$

→ $|kA| = k^n |A|$

→ $\begin{vmatrix} a+x & b+y & c+z \\ m & n & o \\ p & q & r \end{vmatrix} = \begin{vmatrix} a & b & c \\ m & n & o \\ p & q & r \end{vmatrix} + \begin{vmatrix} x & y & z \\ m & n & o \\ p & q & r \end{vmatrix}$

→ $R_i \leftrightarrow R_j$: ~~sing~~ sign of det changes

$R_i \leftarrow R_i + kR_j$: det doesn't change

$R_i \leftarrow kR_i$: det raises by k times.

→ In lower ~~triangle~~, upper triangular, diagonal
det(A) = product of diagonal elements.

→ If 2 rows or 2 colums are proportionate then det(A) = 0.

Inverse:

Minor: Minor(a_{ij}) = det of matrix remaining after row & col deletion.

Cofactor (a_{ij}) = $(-1)^{i+j} M_{ij}$

Adjoint matrix = [Cofactor Matrix]^T

→ Inverse exists ⟺ $|A| \neq 0$

$$A^{-1} = \frac{1}{|A|}(\text{adj} A)$$

→ If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Note:

- $|A| = 0 \Rightarrow |\text{adj} A| = 0$
- $|\text{adj} A| = |A|^{n-1}$
- $\text{adj}(\text{adj} A) = |A|^{n-2} \cdot A$
- $|\text{adj}(\text{adj} A)| = |A|^{(n-1)^2}$
- $|\underbrace{\text{adj}(\text{adj}(\cdots(\text{adj} A)\cdots))}_{k \text{ times}}| = |A|^{(n-1)^k}$
- $(kA)^{-1} = \frac{1}{k} \cdot A^{-1}$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(A^T)^{-1} = (A^{-1})^T$
- If product of 2 non-zero square matrices is a zero matrix then both A and B must be singular.
- If AB = 0 and B is non-singular then A = 0

Shortcut to find adj(A):

$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ $\begin{bmatrix} e & h & b & e \\ f & i & c & f \\ d & g & a & d \\ e & h & b & e \end{bmatrix}$

If same is written as rows: cofactor matrix

Vectors:

If $x_1, x_2 \ldots x_n$ are vectors then
$k_1x_1 + kx_2 + \cdots + k_nx_n$ is called linear combination

linearly dependent:

$k_1x_1 + k_2x_2 + \cdots + k_nx_n = 0$
$k_1, k_2, \ldots k_n$ are all not zero

linearly independent

$k_1x_1 + k_2x_2 + \cdots + k_nx_n = 0 \Leftrightarrow k_i = 0 \; \forall i$

→ If vectors are linearly dependent then atleast one of the vectors can be expressed as linear combination of others.

→ Subset of LI is LI
Subset of LD is LI or LD.

→ If matrix A is treated as vectors, then
$|A| = 0 \Rightarrow$ linearly dependent vectors
$|A| \neq 0 \Rightarrow$ linearly independent vectors.

→ No of LI vectors (rows/cols) = Rank(A)

Rank:

- Rank(A) = order of largest non-vanishing minor.
- $|A| \neq 0 \Leftrightarrow$ Rank(A) = n
- If all rows/cols are proportionate then Rank(A) = 1
- Rank(A) = 0 ⟺ A is null matrix.
- Rank(A) = no of non-zero rows in row echelon form.
- $\text{Rank}(A^T) = \text{Rank}(A)$
- $\rho(A_{m \times n}) \leq \min\{m, n\}$
- $\rho(AB) \leq \min\{\rho(A), \rho(B)\}$
- $\rho(A+B) \leq \rho(A) + \rho(B)$
- $\rho(A-B) \geq \rho(A) - \rho(B)$

- If

$\rho(A)=n$, then $\rho(\text{adj}A)=\rho(A^{-1})=n$

- $\rho(A)=n-1$, then $\rho(\text{adj}A)=1$

$\rho(A)\leq n-2$, then $\rho(\text{adj}A)=0$

System of Non-Homogeneous equations:

→ $AX=B$ $(B\neq 0)$

$\rho(AB)=\rho(A)=n$ → unique soln } Consistent
$\rho(AB)=\rho(A)\neq n$ → infinite soln }
$\rho(AB)\neq\rho(A)$ → no solution } inconsistent

$$X=[x_1h_1+x_2h_2+\dots+x_{n-r}h_{n-r}]+P$$
↳ particular soln

→ If $\rho(AB)=\rho(A)=r$, then
no of linearly independent solns $=n-r$

Homogeneous System of Linear equations

→ $AX=0$

→ $\rho(A)=n$ → unique & trivial soln

$\rho(A)\neq n$ → non-trivial & infinite solns

$$X=[x_1h_1+x_2h_2+\dots+x_{n-r}h_{n-r}]$$

→ Here inconsistency is not possible.

→ If $\rho(A)=r$, then no of linearly independent soln $=n-r$

Note: In both the cases $n-r$ is no of free variables (corresponds to non-basic cols)

Note: Presence of zero row need not imply infinite solns. Only free variables does.

Eigen Values & Eigen Vectors

If A is a square matrix,

- $A-\lambda I$ is called characteristic matrix of A.
- $|A-\lambda I|=0$ is called characteristic eqn of A.

roots of this eqn are called characteristic roots or eigen values.

- Corresponding to each eigen value we have eigen vector X such that

$$(A-\lambda I)(X)=0 \text{ i.e. } AX=\lambda X \text{ \& } X\neq 0$$

no of LI eigen-vectors corresponding to eigen value λ is given by $n-r$. r is rank of $A-\lambda I$.

- If n^{th} order matrix has n different eigen values then the matrix will have 'n' linearly independent eigen vectors.

Properties of eigen values:

- For lower triangular, upper triangular, diagonal matrices, eigen values are same as diagonal elements.
- $\lambda_1+\lambda_2+\dots+\lambda_n=tr(A)$
  $\lambda_1\cdot\lambda_2\cdots\lambda_n=|A|$
- 0 is eigen value ⇒ $|A|=0$
- If all eigen values are non-zero then $|A|\neq 0$ & $\text{Rank}(A)=n$.
- Eigen values of A^{-1} are $\frac{1}{\lambda_1},\frac{1}{\lambda_2},\dots\frac{1}{\lambda_n}$
- Eigen values of (adj A) are $\frac{|A|}{\lambda_1},\frac{|A|}{\lambda_2},\dots\frac{|A|}{\lambda_n}$
- Eigen values of $a_0I+a_1A+a_2A^2$ are $a_0+a_1\lambda_1+a_2\lambda_1^2, a_0+a_1\lambda_2+a_2\lambda_2^2, \dots$
- If $a+ib$ or $a+\sqrt{b}$ is an eigen value then $a-ib$ or $a-\sqrt{b}$ is another eigen value.
- Eigen values of real symmetric matrix are real.
- Eigen values of real skew-sym matrix are either 0 or purely imaginary.
- Eigen values of orthogonal matrix are unit modulus. i.e., $|\lambda|=1$.

Properties of eigen vectors:

- One eigen vector cannot correspond to more than one eigen value.
- For each eigen value there are infinitely many eigen vectors.
- Eigen vectors of all the below matrices are same:
  $A, KA, A^m, A^{-1}, a_0I+a_1A+a_2A^2$
- Eigen vectors of A & A^T are different
- Eigen vectors of real symmetric matrix are pairwise orthogonal.

Algebraic multiplicity of eigen value:

It multiplicity of eigen value λ.

Geometric multiplicity of eigen value:

It is no of LI eigen vectors associated with that eigen value.

i.e., $n-r$.

Algebraic Multiplicity $\geq$ Geometric multiplicity.

Caley-Hamilton's theorem:

Every square matrix satisfies its own characteristic eqn.

- Used to find high powers & inverse.

LU Decomposition:

A matrix is said to be LU decomposible if it can be written as $A = LU$

where L is lower triangular,

U is upper triangular.

- A matrix is LU decomposible iff all leading principle minors of matrix are non-zero.
- LU decomposition need not be unique.

~~Dolt~~ Dolittle's method:

If $A = LU$ then

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$$

Crout's method:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

Leading principle minors:

If A is $n \times n$ square matrix then

$\Delta_1 = |a_{11}|$

$\Delta_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$\Delta_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ ....

are called leading principle minors.

Probability

Mutually Exclusive (Disjoint) Events: occurance of one event prevents other.

Mutually ~~Exclusive~~ Exhaustive events: set of events from which atleast one must occur (events from sample space)

Mutually independent: result of one event doesn't affect result of other $[P(A\cap B) = P(A)P(B)]$

- $\rightarrow P(A\cup B) = P(A)+P(B) - P(A\cap B)$ (Silly for 3 variables)
- $\rightarrow P(A-B) = P(A) - P(A\cap B)$
- $\rightarrow$ If $A\subset B$, then $P(A)\le P(B)$

Conditional Prob:

$P(A|B) = \frac{P(A\cap B)}{P(B)}$

- what fraction of B is A

$P(A/B) + P(\bar{A}/B) = 1$

Baye's Theorem:

$$P(B_i/A) = \frac{P(B_i)\,P(A/B_i)}{\sum_{k=1}^{n} P(B_k)\,P(A/B_k)} = \frac{P(B_i\cap A)}{P(A)}$$

Random Variables:

Discrete Random Variable:

- PMF; $P(x)\ge 0$ & $\sum P(x) = 1$
- $E(x) = \sum x P(x)$

Cont. Random variable

- PDF $f(x)\ge 0$ & $\int_{-\infty}^{\infty} f(x) = 1$
- $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

Note:

- $Var(x) = E(x^2) - [E(x)]^2$; $SD = \sqrt{var} = \sigma$;
  $= E[(x-\mu)^2]$
- $E(k) = k$
- $E(ax) = aE(x)$
- $E(ax\pm b) = aE(x)\pm b$
- $E(ax\pm by) = aE(x)\pm bE(y)$
- $E(x^n) = \sum x^n P(x)$ (nth moment of x)
- $E(x^2) \ge [E(x)]^2$
- $Var(k) = 0$
- $Var(ax) = a^2 Var(x)$
- $Var(ax\pm b) = a^2 Var(x)$
- $E(x^n) = \sum x_i^n f(x_i)$ is called nth moment of x
- $Var(ax\pm by) = a^2 Var(x) + b^2 Var(y) \pm 2ab\,Cov(x,y)$
- $Var(ax\pm by) = a^2 Var(x) + b^2 Var(y)$ if x, y are independent
- $Var(ax\pm by\pm cz)$
  $= a^2 Var(x) + b^2 Var(y) + c^2 Var(z)$
  if x, y, z are independent

- Covariance: +ve : positive relation; -ve : negative relation
- $Cov(x,y) = E[(x-\mu_x)(y-\mu_y)]$
- $Cov(x,y) = E(xy) - E(x)E(y)$
- If x, y are independent
  $E(xy) = E(x)E(y)$
  $\Rightarrow Cov(x,y) = 0$

Probability Distributions:

(i) Binomial Distribution

- $P(x) = {}^nC_x\, p^x q^{n-x}$
- Mean $= np$
- var $= npq$
- $\mu >$ var

$n\to\infty$, $p\to 0$ $\rightarrow$ Poisson

$n\to\infty$, p is not too small $\rightarrow$ Normal

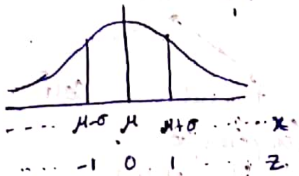
(ii) Poisson Distribution

- $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$
- $\mu = \sigma^2 = \lambda$
- $SD = \sqrt{\lambda}$

(iii) Normal/Gaussian Distribution:

- $f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $(-\infty<0<\infty)$ $(\sigma>0)$
- $f(z) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}z^2}$

$z = \frac{x-\mu}{\sigma}$; $\sigma^2 = 1$; $\mu = 0$



Areas $\rightarrow 0.6827$
$\rightarrow 0.9545$
$\rightarrow 0.9973$

- Points of inflection $\rightarrow \mu-\sigma, \mu+\sigma$
- If $x_1, x_2, \dots x_n$ are independent normal random variables then $c_1x_1 + c_2x_2 + \dots + c_nx_n$ is also a normal random variable

(iv) Uniform Distribution:

- $f(x) = \frac{1}{b-a}$, $a\le x\le b$
- $\mu = \frac{a+b}{2}$; $\sigma^2 = \frac{(b-a)^2}{12}$

CDF:

- $F_X(x) = P(X\le x) = \int_{-\infty}^{x} f(x)\,dx$
- Increasing function

(v) Exponential Distribution

- $f(x) = \lambda e^{-\lambda x}$, $x\ge 0$
- $\mu = \frac{1}{\lambda}$; $\sigma^2 = \frac{1}{\lambda}$
- $P(X\ge x) = e^{-\lambda x}$
- $\min(x_1, x_2, \dots x_n)$'s mean $= \frac{1}{\lambda_1+\lambda_2+\dots\lambda_n}$

Mean: $\frac{\sum f_i x_i}{\sum f_i}$ & $\frac{\sum f_i m_i}{\sum f_i}$

Median: (middle/avg); $\left(\frac{N+1}{2}\right)$; $l + \left(\frac{\frac{N}{2} - m}{f}\right)c$ (Refer notes for median in discrete freq)

$P(X\le a) = P(X\ge a) = 1/2$ where $a\rightarrow$ median

Mode: $P(x)$ is max at mode; $l + \left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right)*c$

$\Delta_1 \rightarrow f - f_{-1}$ $\Delta_2 \rightarrow f - f_1$

- $\rightarrow$ we may have more than one modal class
- $\rightarrow$ Mode may be undefined if nthg repeats
- $\rightarrow$ Mode = 3Median - 2Mean