Graph Theory Important Questions

Spanning trees in complete graph is equal to $n^{(n-2)}$(where n is no of sides or regularity in complete graph). So, spanning trees in complete graph K_4 will be $4^{(4-2)}$. i.e. $4^2 = 16$. Spanning trees in a bipartite graph $K_{m,n}$ is equal to $m^{(n-1)} * n^{(m-1)}$. So, spanning trees in $K_{2,2}$ will be $2^{(2-1)} * 2^{(2-1)}$. i.e. $2^1 * 2^1 .= 4$. So, option (C) is correct.

The 2^n vertices of a graph G corresponds to all subsets of a set of size n, for n >= 6. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.

The maximum degree of a vertex in G is:

(A) $(n/2)C2 *2^{n/2}$

(B) 2^{n-2}

(C) $2^{n-3} *3$

(D) 2^{n-1}

Answer: (C)
Explanation:
$\max_k(kC2*2^{(n-k)})=3C2*2^{(n-3)}=3*2^{(n-3)}$.

Ans: C

Let S = {1, 2, 3, . . . n} and G be a simple graph where every subset of S is a vertex in G. There will be an edge between two sub-set of S when they intersect in exactly 3 elements. What will be the degree of a vertex containing 4 elements?

C(4,3) × 2^(n−4)

Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to

A 15

B 30

C 45

D 360

Ans: C

$C(6,4) \times ((4-1)!)/2$.

As vertices are labeled nd they are circular, we need to consider arrangements too.

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G is a graph on n vertices and 2n - 2 edges. The edges of G can be partitioned into two edge-disjoint spanning trees. Which of the following is NOT true for G?

- A. For every subset of k vertices, the induced subgraph has at most 2k-2 edges
- B. The minimum cut in G has at least two edges
- C. There are two edge-disjoint paths between every pair to vertices
- ✓ There are two vertex-disjoint paths between every pair of vertices

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Which of the following graphs has an Eulerian circuit?

Any k-regular graph where kis an even number.

B A complete graph on 90 vertices

The complement of a cycle on 25 vertices

D None of the above

Not every k–regular graph is connected even if degree is even.

A graph G = (V, E) satisfies $|E| \leq 3|V| - 6$. The min-degree of G is defined as $\min_{v \in V} \{\text{degree}(v)\}$. Therefore, min-degree of G cannot be

- A 3
- B 4
- C 5
- ✓ 6

$e \leq 3n-6$ —— ①

~~[illegible]~~ wkT

$$\delta(G) \leq \frac{2e}{n}$$

$$e \geq \frac{n \cdot \delta(G)}{2} \quad \text{— ②}$$

from ① & ②

$$\frac{n\,\delta(G)}{2} \leq e \leq 3n-6$$

a) $\delta(G) = 3$

$$\frac{3n}{2} \leq e \leq 3n-6$$

b) $\delta(G) = 4$

$$2n \leq e \leq 3n-6$$

c) $\delta(G) = 5$

$$\frac{5n}{2} \leq e \leq 3n-6$$

d) $\delta(G) = 6$

$$3n \leq e \leq 3n-6$$

$$3n \leq 3n-6$$

$$0 \leq -6$$

This is not possible

$\therefore$ opt (d)

In an undirected graph G with n vertices, vertex 1 has degree 1, while each vertex $2, \ldots, n-1$ has degree 10 and the degree of vertex n is unknown, Which of the following statement must be TRUE on the graph G?

a. There is a path from vertex 1 to vertex n.
b. There is a path from vertex 1 to each vertex $2, \ldots, n-1$.
c. Vertex n has degree 1.
d. The diameter of the graph is at most $\frac{n}{10}$
e. All of the above choices must be TRUE

Ans: a
There can't exist a single vextex of odd degree.
So vertex 'n' must have odd degree so it must be connect to some vertices from 2 to 'n−1'.
When a graph has exactly 2 vertices of odd degree, there exists a path b/w those two vertices.

Let $G=(V, E)$ be a simple undirected graph on n vertices. A colouring of G is an assignment of colours to each vertex such that endpoints of every edge are given different colours. Let $\chi(G)$ denote the chromatic number of G, i.e. the minimum numbers of colours needed for a valid colouring of G. A set $B \subseteq V$ is an independent set if no pair of vertices in B is connected by an edge. Let $a(G)$ be the number of vertices in a largest possible independent set in G. In the absence of any further information about G we can conclude.

A. $\chi(G) \geq a(G)$
B. $\chi(G) \leq a(G)$
C. $a(G) \geq \frac{n}{\chi(G)}$
D. $a(G) \leq \frac{n}{\chi(G)}$
E. None of the above.

Independence number : Size of largest maximum independent set.$a(G)$ (it covers all adjacent vertices)
Chromatic Number : Minimum No. of color required to properly color the graph .$\chi(G)$

The vertices of G can be partitioned into $\chi(G)$ monochromatic classes. Each class is an independent set, and hence cannot have size larger than $\alpha(G)$.

$\alpha(G)\chi(G) \geq n$. (its a theorem),
option C.

2.11 Line Graph(2) top

2.11.1 Line Graph: GATE2013-26 top

https://gateoverflow.in/1537

The line graph $L(G)$ of a simple graph G is defined as follows:

There is exactly one vertex $v(e)$ in $L(G)$ for each edge e in G.

For any two edges e and e' in G, $L(G)$ has an edge between $v(e)$ and $v(e')$, if and only if e and e' are incident with the same vertex in G.

Which of the following statements is/are TRUE?

- (P) The line graph of a cycle is a cycle.
- (Q) The line graph of a clique is a clique.
- (R) The line graph of a planar graph is planar.
- (S) The line graph of a tree is a tree.

A. P only
B. P and R only
C. R only
D. P, Q and S only

gate2013 graph-theory normal line-graph

Answer

2.11.2 Line Graph: TIFR2017-B-13 top

https://gateoverflow.in/95821

For an undirected graph $G=(V,E)$, the line graph $G'=(V',E')$ is obtained by replacing each edge in E by a vertex, and adding an edge between two vertices in V' if the corresponding edges in G are incident on the same vertex. Which of the following is TRUE of line graphs?

A. the line graph for a complete graph is complete
B. the line graph for a connected graph is connected

C. the line graph for a bipartite graph is bipartite
D. the maximum degree of any vertex in the line graph is at most the maximum degree in the original graph
E. each vertex in the line graph has degree one or two

tifr2017 graph-theory line-graph

Answer

Answers: Line Graph

2.11.1 Line Graph: GATE2013-26 top https://gateoverflow.in/1537

P) **True**. Because every edge in cycle graph will become a vertex in new graph $L(G)$ and every vertex of cycle graph will become an edge in new graph.

R) False. We can give counter example. Let G has 5 vertices and 9 edges which is a planar graph. Assume degree of one vertex is 2 and of all others are 4. Now, $L(G)$ has 9 vertices (because G has 9 edges) and 25 edges. (See below). But for a graph to be planar $|E| <= 3|V| - 6$.

For 9 vertices $|E| \leq 3 * 9 - 6$

$\Rightarrow |E| \leq 27 - 6$

$\Rightarrow |E| \leq 21$. But $L(G)$ has 25 edges and so is not planar.

As R) is False option (B), (C) are eliminated.
http://www.personal.kent.edu/~rmuhamma/GraphTheory/MyGraphTheory/planarity.htm

S) False. By counter example. Try drawing a simple tree which has a Root node ,Root node has one child A, node A has two child B and C. Draw its Line graph acc. to given rules in question you will get a cycle graph of 3 vertices.

So (D) also not correct.

$\therefore$ option (**A**) is correct.

For a graph *G* with *n* vertices and *m* edges, the number of vertices of the line graph *L*(*G*) is *m*, and the number of edges of *L*(*G*) is half the sum of the squares of the degrees of the vertices in *G*, minus *m*.

34 votes -- prashant singh

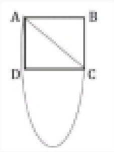
2.11.2 Line Graph: TIFR2017-B-13 top https://gateoverflow.in/95821

option **B** is the right answer

We can solve this question by eliminating options

Option (A) False

Let us take a complete graph of 4 vertices:



In a line graph, no. of edges is

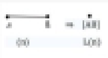
$\sum_{i=0}^{n} {}^{d_i}C_2,\quad d_i$ is degree of each vertex

$= \frac{3\times 2}{2} + \frac{3\times 2}{2} + \frac{3\times 2}{2} + \frac{3\times 2}{2} = 3\times 4 = 12$

No. of vertices in line graph = No. of edges in original graph
No. of vertices in line graph $= 6$
So, no. of edges to make complete graph with 6 vertices $= \frac{6\times 5}{2} = 3\times 5 = 15$
But for given line graph from complete graph of 4 vertices we have only 12 edges.
Contradiction.

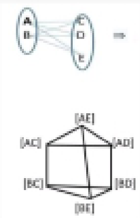
Option (B) True

1. Smallest line graph for original graph one edge

 which is also connected graph

If a graph is connected with more then one edge, it will never be disconnected

Option (C) False

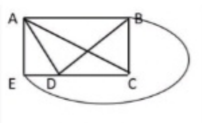


This cannot be 2-colorable and hence is not bipartite.

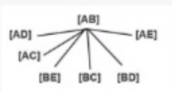
Option (D) False

Because line graph degree of vertex depends on the attribute

e.g., $[A, B]$ as point in line graph, then degree of this vertex depends on degree of A and degree of B in the original graph.



I'm drawing degree for a point $[AB]$ in line graph.



So, this is wrong.

Option (E) False (wrong as proved in above option (D))

Last modified: May 21, 2020