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BE-IT 8084

ICS Assignment 1: Chinese remainder theorem

Title: Develop and program in C++ based on number theory such as Chinese remainder.

Objective:

Understand and implement the Chinese remainder theorem.

Theory:

Chinese Remainder Theorem states that there always exists an x that satisfies given congruences. Let $\text{num}[0], \text{num}[1], \dots, \text{num}[k-1]$ be positive integers that are pairwise coprime. Then, for any given sequence of integers $\text{rem}[0], \text{rem}[1], \dots, \text{rem}[k-1]$, there exists an integer x solving the following system of simultaneous congruences.

The first part is clear that there exists an x . The second part basically states that all solutions (including the minimum one) produce the same remainder when divided by-product of $\text{num}[0], \text{num}[1], \dots, \text{num}[k-1]$. In the above example, the product is $3*4*5 = 60$. And 11 is one solution, other solutions are 71, 131, .. etc. All these solutions produce the same remainder when divided by 60, i.e., they are of form $11 + m*60$ where $m \geq 0$.

A Naive Approach to find x is to start with 1 and one by one increment it and check if dividing it with given elements in $\text{num}[]$ produces corresponding remainders in $\text{rem}[]$. Once we find such an x , we return it.

Below is the implementation of the Naive Approach.

Time Complexity: $O(M)$, M is the product of all elements of $\text{num}[]$ array.

Auxiliary Space: $O(1)$

Algorithm:

1. Take divisor and subsequent remainders as input from the user.
2. Check if the remainders are relatively prime or not.
3. Calculate N_i for all divisors.
4. Calculate inverse.
5. Calculate the final output and print it.

Program:

```
#include<iostream>
#include<vector>
using namespace std;

class crt
{
    vector<int>bi, ni, Ni, yi;
    int n, N=1, x=0;

public:

    int pipeline()
    {
        input();
        if(relativePrime()==0)
            return 0;
        calcNi();
        inverse();
        return output();
    }

    void input()
    {
        int r, d;

        cout<<"Enter the number of divisors or remainders : ";
        cin>>n;

        cout<<endl<<"Enter the divisors and the corresponding remainders : "<<endl;
        for(int i=0; i<n; i++)
        {
            cin>>d>>r;
```

```

        ni.push_back(d);
    bi.push_back(r);
    }
}

```

```

int gcd(int num1, int num2)
{
    if (num2==0)
        return num1;
    return gcd(num2, num1%num2);
}

```

```

int relativePrime()
{
    for(int i=0; i<n-1; i++)
        for(int j=i+1; j<n; j++)
            if(gcd(ni.at(i), ni.at(j))!=1)
                return 0;
    return 1;
}

```

```

void calcNi()
{
    for(int i=0;i<n;i++)
        N*=ni.at(i);

    for(int i=0;i<n;i++)
        Ni.push_back(N/ni.at(i));
}

```

```

void inverse()
{
    for(int i=0;i<n;i++)
        for(int j=1;j<ni.at(i);j++)

```

```

        if((Ni[i]*j)%ni[i]==1)
        {
            yi.push_back(j);
            break;
        }
    }

int output()
{
    for(int i=0;i<n;i++)
        x+=bi.at(i)*Ni.at(i)*yi.at(i);
    return x%N;
}

};

int main()
{
    int solution;
    crt obj;

    solution=obj.pipeline();

    if(solution==0)
        cout<<"Solution not possible as divisors aren't relatively prime.";
    else
        cout<<endl<<"X="<<solution;

    return 0;
}

```

/*

OUTPUT 1:

Enter the number of divisors or remainders : 3

Enter the divisors and the corresponding remainders :

3 2

4 3

5 1

X=11

OUTPUT 2:

Enter the number of divisors or remainders : 3

Enter the divisors and the corresponding remainders :

3 2

3 4

5 1

The solution is not possible as divisors aren't relatively prime.