Solution to Task2 of Part 2

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Explanation 1 (Via substitution):

According to given Lemma 1,

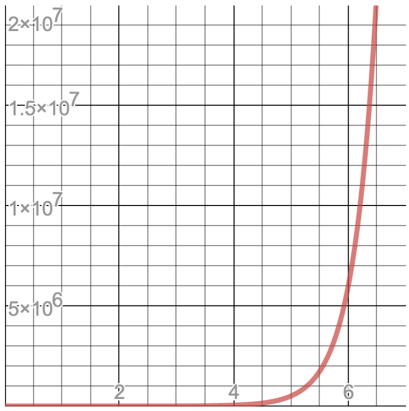
Lemma 1: The barest B-tree of height H contains $N = 2K^H - 1$ elements, where $K = \lceil M/2 \rceil$

For a B-tree of order $M = 23$

$K = \lceil M/2 \rceil = \lceil 23/2 \rceil = \lceil 11.5 \rceil = 12$

If we calculate the bare minimum number of elements in a B-tree(N) with increasing heights H

H	N
2	$((2(12)^2)-1) = 287$
3	$((2(12)^3)-1) = 3455$
4	$((2(12)^4)-1) = 41471$
5	$((2(12)^5)-1) = 497663$
6	$((2(12)^6)-1) = 5971967$
7	$((2(12)^7)-1) = 71663615$



We notice that a B-tree of height(H) = 7 has at least 71663615 elements. which is greater than 10^7 i.e. $N > 10^7$ at $H = 7$ but not at $H = 6$.

Hence, the upper bound for a B-tree of order 23 which has 10,000,000 = 10^7 elements is of a height H = 7.

Explanation 2 (Via derivation):

Lemma 1: The barest B-tree of height H contains $N = 2K^H - 1$ elements, where $K = \lceil M/2 \rceil$

Hence, if we derive H from Lemma1

i.e. $(N + 1)/2 = \lceil M/2 \rceil^H$

i.e. $\log((N+1)/2) = H \cdot \log(2\lceil M/2 \rceil)$

i.e. $H = \log((N+1)/2)/\log(\lceil M/2 \rceil)$

For a B-tree of order 23 and has 10,000,000 elements, the height calculated is

$H = \log(5000000.5)/\log(\lceil 23/2 \rceil) = \log_{12}(5000000) = 6.207455911$

Since minimum number of nodes increases with height.

Lemma1(H=6) < Lemma1(H=6.207455911) < Lemma1(H=7)

i.e. Lemma1(H=6) < 10,000,000 < Lemma1(H=7)

Since H is an integer, Lemma1: the barest B-tree of height H, order 23 and has 10,000,000 elements is of height H = 7.