EE521 : DSP Lab

Assignment 2

Submitted by:
- Sqn Ldr Monark Singh
- 214102322

Ques 6.

Random Process $X(t) = 5\sin(2t+\phi_1) + 6\cos(3t+\phi_2)$

$$\phi_1 \sim U[-\pi,\pi] \quad , \quad \phi_2 \sim U[0,2\pi]$$

(a)

→ Finding mean:

$$\mu_x(t) = \int_{-\infty}^{\infty} x(t)\, f_x(x)\, dx$$

$$= \int_{-\infty}^{\infty} 5\sin(2t+\phi_1)\, f_{\phi_1}(\phi_1)\, d\phi_1 + \int_{-\infty}^{\infty} 6\cos(3t+\phi_2)\, f_{\phi_2}(\phi_2)\, d\phi_2$$

$$= \int_{-\pi}^{\pi} 5\sin(2t+\phi_1)\cdot\frac{1}{2\pi}\, d\phi_1 + \int_{0}^{2\pi} 6\cos(3t+\phi_2)\cdot\frac{1}{2\pi}\, d\phi_2$$

$$= \frac{5}{2\pi}\int_{-\pi}^{\pi} \sin(2t+\phi_1)\, d\phi_1 + \frac{6}{2\pi}\int_{0}^{2\pi} \cos(3t+\phi_2)\, d\phi_2$$

$$= \frac{5}{2\pi}\times 0 + \frac{6}{2\pi}\times 0 = 0$$

$$\Rightarrow \mu_x(t) = 0 \quad \text{--- (1)}$$

⇒ Mean is constant (i.e. independent of time).

→ Finding auto correlation:

$$R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, x(t-\tau)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [5\sin(2t+\phi_1) + 6\cos(3t+\phi_2)]\cdot[5\sin(2(t-\tau)+\phi_1) + 6\cos(3(t-\tau)+\phi_2)]\, dt$$

$$\Rightarrow R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\left[\int_{-T}^{T} 25\sin(2t+\phi_1)\cdot\sin(2t-2\tau+\phi_1)\,dt\right.$$

$$+\int_{-T}^{T} 30\sin(2t+\phi_1)\cdot\cos(3t-3\tau+\phi_2)\,dt$$

$$+\int_{-T}^{T} 30\cos(3t+\phi_2)\cdot\sin(2t-2\tau+\phi_1)\,dt$$

$$\left.+\int_{-T}^{T} 36\cos(3t+\phi_2)\cos(3t-3\tau+\phi_2)\,dt\right]$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[\frac{25}{2}\int_{-T}^{T}\{\cos 2\tau - \cancel{\cos(4t-2\tau+2\phi_1)}^{\,0}\}\,dt\right.$$

$$+\frac{30}{2}\int_{-T}^{T}\{\cancel{\sin(5t-3\tau+\phi_1+\phi_2)}^{\,0}+\cancel{\sin(-t+3\tau+\phi_1-\phi_2)}^{\,0}\}\,dt$$

$$+\frac{30}{2}\int_{-T}^{T}\{\cancel{\sin(5t-2\tau+\phi_1+\phi_2)}^{\,0}-\cancel{\sin(t+2\tau+\phi_2-\phi_1)}^{\,0}\}\,dt$$

$$+\frac{36}{2}\int_{-T}^{T}\{\cancel{\cos(6t-3\tau+\phi_1+\phi_2)}^{\,0}+\cos 3\tau\}\,dt$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[\frac{25}{2}\int_{-T}^{T}\cos 2\tau\,dt + \frac{36}{2}\int_{-T}^{T}\cos 3\tau\,dt\right]$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left(\frac{25}{2}\cos 2\tau\times 2T + \frac{36}{2}\cos 3\tau\times 2T\right)$$

$$R_{xx}(\tau) = \frac{25}{2}\cos 2\tau + \frac{36}{2}\cos 3\tau \qquad \text{— (2)}$$

$\Rightarrow$ Autocorrelation depends only on the time lag 'τ'.

So from eqn (1) and eqn (2), it implies that $X(t)$ is a Wide Sense Stationary (WSS) Process.

(b) $Y(t) = M(t) \cdot X(t)$

$M(t)$ is W.S.S and independent of ϕ_1 and ϕ_2.

- Mean of $Y(t)$

$$E[Y(t)] = E[M(t) \cdot X(t)]$$
$$= E[M(t)] \cdot E[X(t)]$$
$$= E[M(t)] \cdot 0 = 0$$

- Autocorrelation of $Y(t)$

$$R_{YY}(t, t+\tau) = E[Y(t)Y(t+\tau)]$$
$$= E[\{M(t) \cdot X(t)\}\{M(t+\tau)X(t+\tau)\}]$$
$$= E[X(t)X(t+\tau) \cdot M(t)M(t+\tau)]$$
$$= E[X(t) \cdot X(t+\tau)] \cdot E[M(t)M(t+\tau)]$$
$$R_{YY}(t, t+\tau) = R_{XX}(\tau) \cdot R_{MM}(\tau) \quad (\because X(t) \;\&\; M(t) \text{ are WSS})$$

$\Rightarrow R_{YY}$ is a function of time lag τ only.

$\therefore Y(t)$ is also a WSS process.