GAUS Junior AG: Maps between spherical group rings

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In this GAUS Junior AG, we want to learn about the recent development on spherical group rings by Carmeli-Nikolaus-Yuan [CNY24].

The schedule is planned as follows:

Contents

The content of the talks should be roughly the following:

Talk 1: δ-rings I (Timo Weiß) 09:30 - 10:30

This section covers [CNY24, Section 2.1].

- Define δ-rings [CNY24, Definition 2.1].

- Define rank-1-units in a δ-ring [CNY24, Definition 2.4].

- Explain the short exact sequences of [CNY24, Proposition 2.10 and 2.15]. Give sketches of the proofs if time permits.

- Define animated δ-rings [CNY24, Definition 2.2].

- Explain as much as possible about the Artin-Schreier sequence for Frobenius lifts [CNY24, Proposition 2.23].

Talk 2: δ-rings II (Luca Passolunghi) 10:45 - 11:45

This section covers [CNY24, Section 2.2 - 2.4].

- Define reduced units [CNY24, Definition 2.25].

- State [CNY24, Theorem 2.26], give a sketch of the torsion case, i.e. (quickly!) cover (parts of) [CNY24, Section 2.2.1].

- Define global δ rings [CNY24, Definition 2.31].

- Prove the baby case of the main Theorem, i.e. prove [CNY24, Theorem 2.34].

- If time permits, sketch [CNY24, Proposition 2.41].

Lunch Break 11:45 - 13:00

Talk 3: Staticity for reduced units (Xiaowen Dong) 13:00 - 14:00

This section covers [CNY24, Section 3].

- Define strict and reduced units of a(n augmented) ring spectrum [CNY24, Definitions 3.2 and 3.3]. Explain the associated adjunction of the free augmented R-algebra with the reduced units [CNY24, Remark 3.4].

- Explain the construction of the tautological units [CNY24, Construction 3.11 and before].

- Define M-rigid and p-rigid ring spectra [CNY24, Definition 3.12], state [CNY24, Proposition 3.15].

- Define F-static ring spectra [CNY24, Definition 3.16].

- Sketch the proof of staticity away from the characteristic [CNY24, Proposition 3.20].

- Sketch the proof of staticity for reduced units [CNY24, Proposition 3.21 and 3.22].

The focus of this talk should be the last bullet point, so if time is short, shorten the sketch of the proof of staticity away from the characteristic.

Talk 4: Rigidity for finite p-groups (Ruth Wild) 14:15 - 15:15

This talk covers [CNY24, Section 4]. The first part of this talk should recall concepts from chromatic homotopy theory.

- Recall the notion of $T(n)$-local spectra, as e.g. in the quick rundown of [LMMT24, Section 2.1].

- Recall the chromatic Fourier transformation [CNY24, Section 4.1.1]. In particular, state [CNY24, Theorems 4.3 and 4.4].

- Sketch the proof of [CNY24, Theorem 4.11].

- If time permits, cover the material of [CNY24, Section 4.2].

Coffee Break 15:15 - 15:45

Talk 5: Rigidity for Free Abelian Groups (Anton Engelmann) 15:45 - 16:45

This talk covers [CNY24, Sections 5].

- Sketch the proof of [CNY24, Proposition 5.5].

- Explain how to deduce [CNY24, Proposition 5.13] out of this.

- Introduce the spherical Witt Vectors of [Lur18, Example 5.2.7].

- Prove in more detail [CNY24, Propositions 5.16, 5.17, 5.18 and 5.19].

Talk 6: Assembling the proof (Klaus Mattis) 17:00 - 18:00

This talk covers [CNY24, Section 6] and the proof of the main Theorem.

- Try to prove as much as possible about [CNY24, Section 6]. In particular, cover the main result, Corollary 6.6.

Subsequent dinner 18:00 - open end

References

[CNY24]   Shachar Carmeli, Thomas Nikolaus, and Allen Yuan. Maps between spherical group rings, 2024.

[LMMT24] Markus Land, Akhil Mathew, Lennart Meier, and Georg Tamme. Purity in chromatically localized algebraic K-theory. *Journal of the American Mathematical Society*, February 2024.

[Lur18]   Jacob Lurie. Elliptic cohomology ii: Orientations. 2018.