# Bayesian neural networks for option pricing

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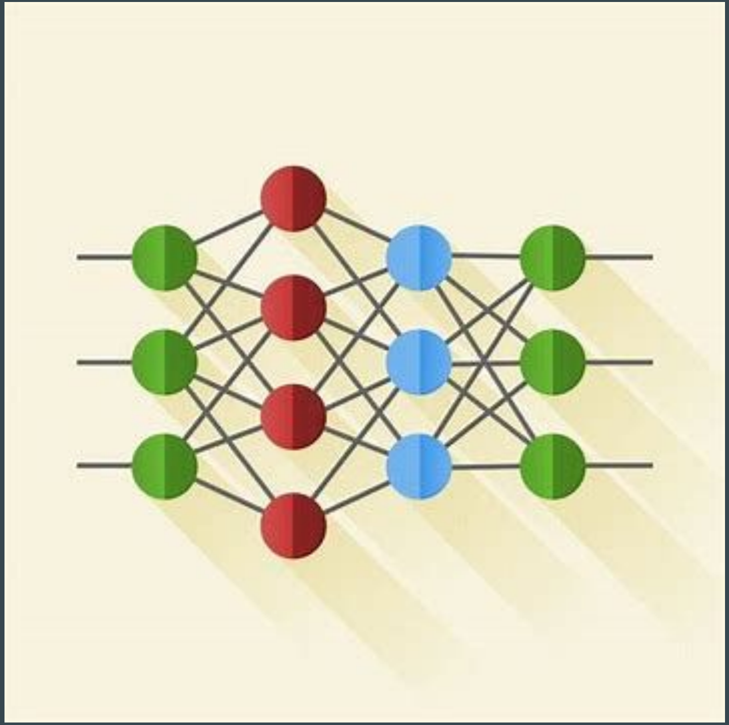
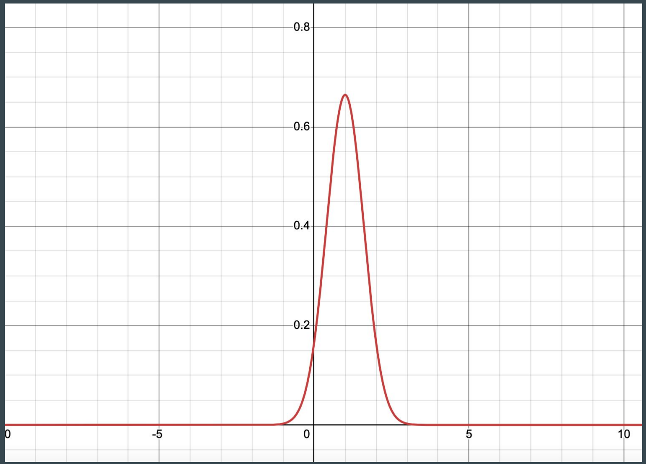
# Motivation

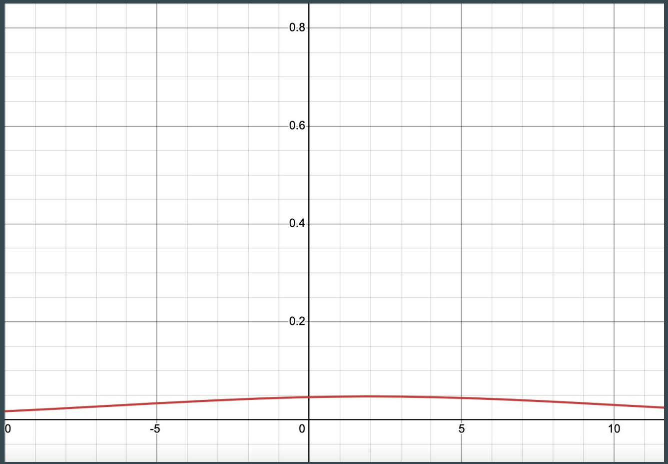
- Deterministic neural networks inadequate
  - Point estimate vs. probability distribution



# Motivation

- Strategy in HW5 is cool but naive
    - Could be improved given probability distributions

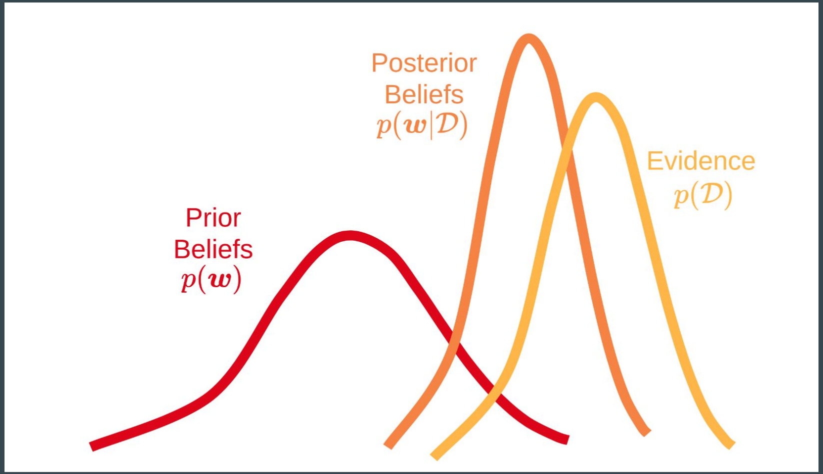



Prob. dist of returns for A vs. Prob. dist of returns for B

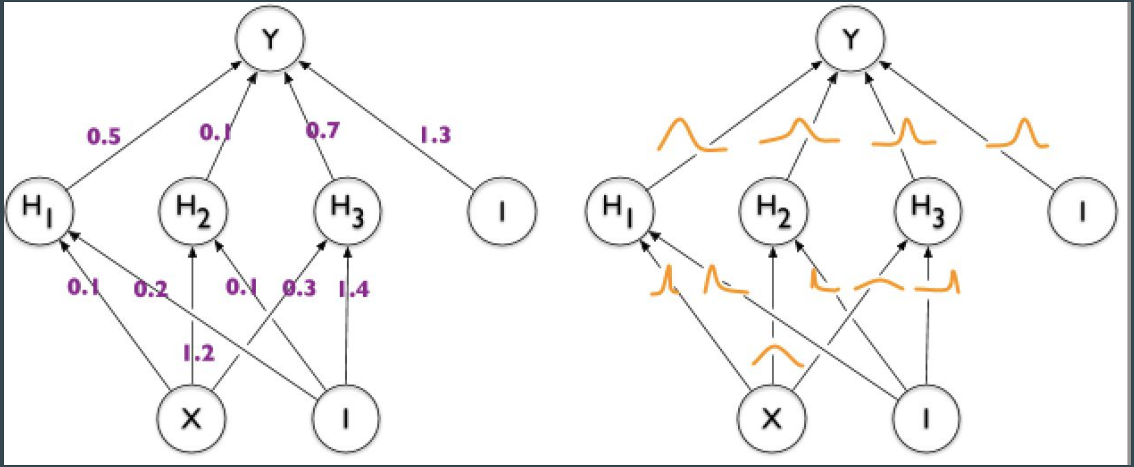
# Approach

- Bayesian neural networks
  - Output probability distribution rather than point estimate
- Use information to trade options
- Key idea:
  - Probability distributions provide more information rather than point estimates to trade options



# Bayesian neural networks

Bayesian neural networks, each parameter is a distribution rather than a single weight. It is trained using Bayesian Statistics, iteratively updating each parameter distribution using Bayes' Rule based on the training data. Thus, it outputs a posterior distribution for the asset price rather than a single value, allowing for more nuanced trading strategies.

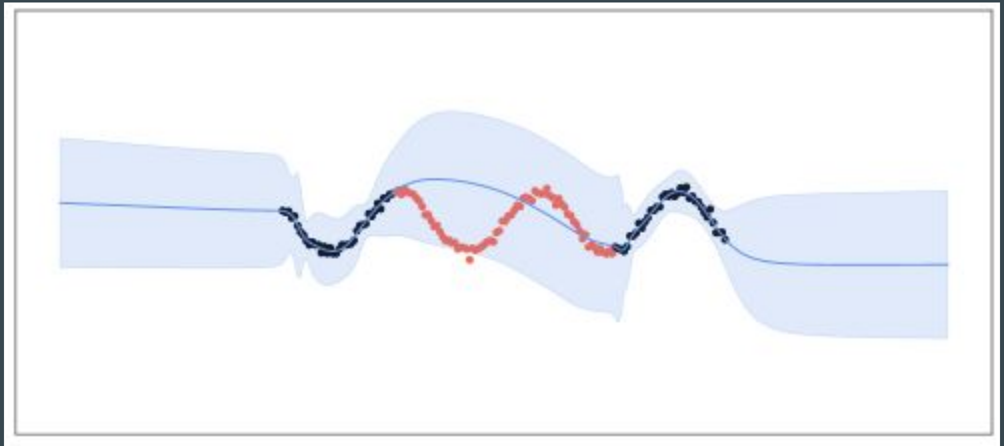


# Bayesian neural networks

Computing posteriors for every weight is intractable except for the simplest networks.

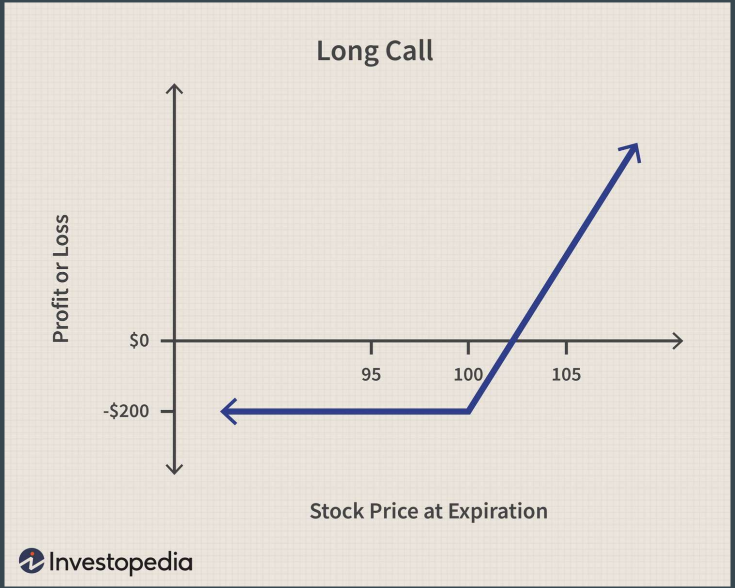
In practice approximate inference procedures are used:

- Hamiltonian Monte Carlo
- Linearized Laplace Approximation
- Deep ensembles
- Make only the last layer of the deep neural network is Bayesian



# Options

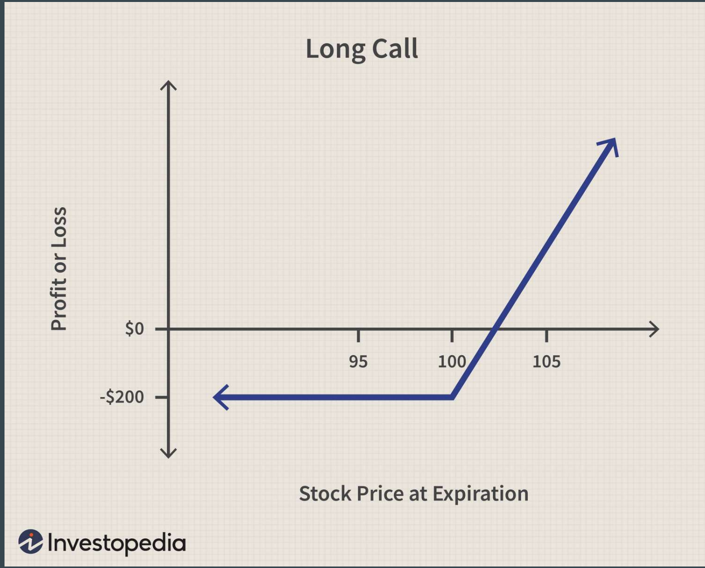
An option is a financial derivative that gives the holder the right, but not the obligation, to buy or sell an underlying asset at a specified price within a set period.



# Applying posteriors to option valuation

Having a posterior distribution rather than a singular predicted value can be particularly powerful for trading options due to the nonlinearity of their payoff

We can calculate the expectation and variance of the fair value of options using factor data, incorporating broad market information into our predictions unlike Black Scholes.



# Applying posteriors to option valuation

We will apply our strategy towards SPX options, but it should work on any option contracts given sufficient data

2 ideas:

- Long-Short strategy using Black-Scholes
- Analytically construct the optimal Markowitz Portfolio

Data used:

- SPX price data
- SPX option data
- FF5 data

# Idea #1:
# Long-short with Black Scholes

# Black-Scholes option pricing model

- Well-tested, widely used model for pricing options
- Assumptions:
  - Underlying asset returns ~ Brownian motion (Stochastic calculus)
  - No transaction costs

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-rT}$$

$$d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

| | |
|---|---|
| $C(S,t)$ | (call option price) |
| $N(\,)$ | (cumulative distribution function) |
| $T = (T_1 - t)$ | (time left til maturity (in years)) |
| $S$ | (stock price) |
| $K$ | (strike price) |
| $r$ | (risk free rate) |
| $\sigma$ | (volatility) |

# Black-Scholes option pricing model

- Well-tested, widely used model for pricing options
- Assumptions:
  - Underlying asset returns ~ Brownian motion (Stochastic calculus)
  - No transaction costs
  - **Known and fixed volatility σ (and $r_f$)**

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-rT}$$

$$d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

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| $\sigma$ | (volatility) |

# Forming the long-short

- Probability distribution will yield volatility parameter ($\sigma$)
- For a bunch of strike prices K, plug $\sigma$ into Black-Scholes to get estimated call option price
- Pick the highest magnitude price differential
  - Exactly replicate our estimated option using stocks and bonds
  - Long-short the estimated option and the actual option at strike K (and vice versa)

# Strengths and Opportunities

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- Black-Scholes is widely used
- Quick implementation
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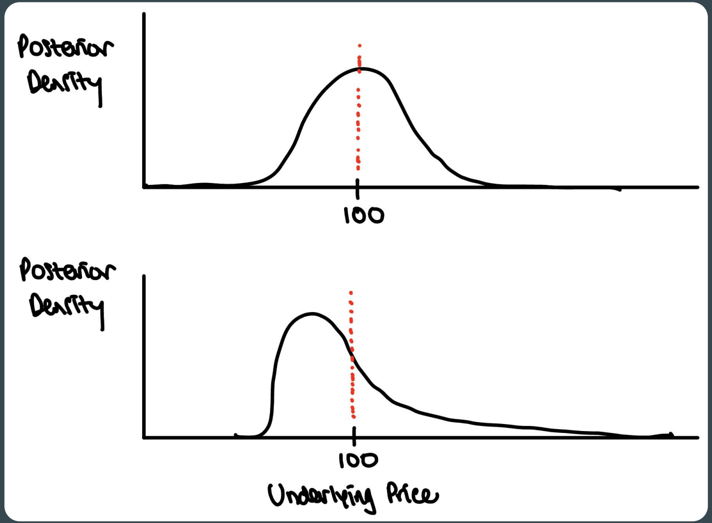
Opportunities

- Try put options and multiple expiration dates
- Still doesn't use the full posterior distribution
  - Ex: asymmetric distribution with a long right tail should bias us towards buying call options (given same mean and standard deviation)

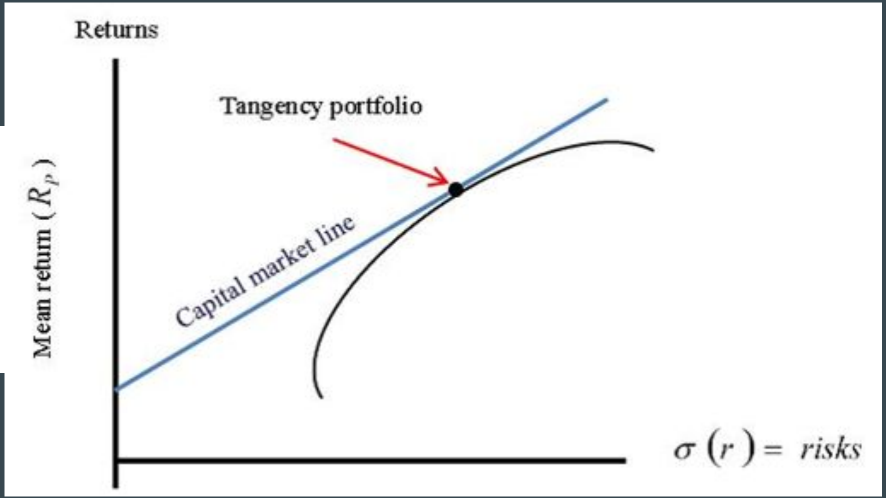


# Idea #2:
# Build Markowitz portfolio analytically

# Build Markowitz portfolio analytically

1. We can treat buying and selling each call and put at every strike price as its own asset.
2. Calculate the expectation vector and covariance matrix based on our model's distribution
3. Construct the "Markowitz Portfolio" for this option to maximize Sharpe Ratio

$$w^* = (\iota\Sigma^{-1}\mu)^{-1}\Sigma^{-1}\mu$$



# Build Markowitz portfolio analytically

Advantages:

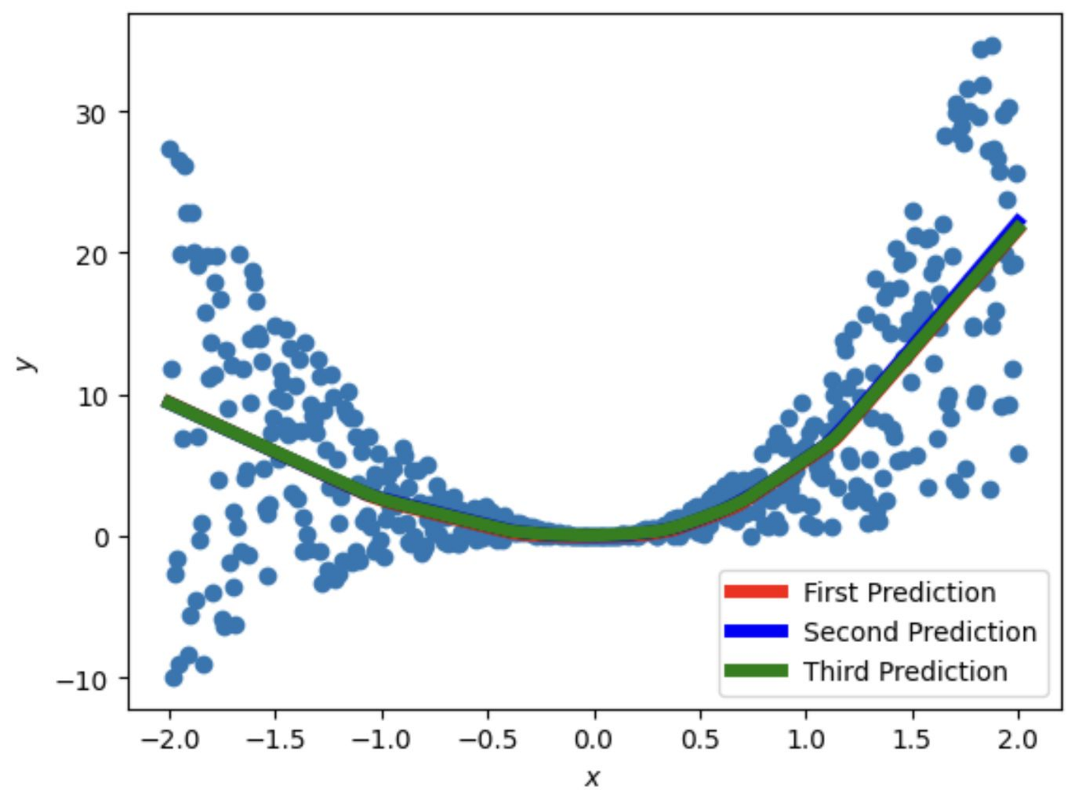
- Easily tunable to risk preferences
- Likely smaller tail risk compared to only trading the options with the largest mispricing

Disadvantages

- Potential problem is that options at different strike prices are not just correlated, but rather causally connected. This means that the markowitz portfolio may not be optimal.
    - Overcoming this is a difficult mathematical problem. Instead, we can attempt to compute a high sharpe ratio portfolio in a brute force manner or by training a neural network instead.

# BNN: Proof of Concept

# Posterior samples for BNN trained on toy dataset



# Intuitive analysis (execution-independent)

# Additional Improvements

Bayesian neural network

- **Add features:** idiosyncratic volatility, trading volume, sentiment score
- **Improve current features:** increase lag time

# Risks

- Implied leverage on options → take on minimal additional leverage
- Sudden changes in volatility that the BNN didn’t predict → keep expiration date <1 year in the future
- Market crash → Backtest against financial crises, consider volatility timing or outright pausing the strategy in periods of high volatility

# Summary

- Produce information using a novel neural network framework
- Implement an options long-short that will have low beta
- Research opportunities to improve our model and to make execution even more precise