BEM114_HW2_Andrew_Daniel_Kyle

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1 BEM114 Homework 2 - Statistical Arbitrage

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1.1 Setup

Imports, Helper Functions, and DataFrames

[1]:
```
'''
Imports
'''

import pandas as pd
import numpy as np
from statsmodels.regression.rolling import RollingOLS
import statsmodels.api as sm

import matplotlib.pyplot as plt
```

[2]:
```
'''
Helper functions
'''

# Given a group of stocks, calculate equal-weighted and value-weighted weights
def calc_weights(group):
    # Calc equal weights
    group['weights_eq'] = 1 / float(group['decile'].count())
    # Calc total market equity of group
    group['TMV'] = group['MV'].sum()
    # Calc value weights
    group['weights_val'] = group['MV'] / group['TMV']
    return group

# Part b of problems 2,3,4: Calculates monthly returns from monthly weights
def part_b(df):
    # Calculate weights times returns
    df['weighted_val_ret'] = df['weights_val_lag'] * df['RET']
```

```python
    df['weighted_eq_ret'] = df['weights_eq_lag'] * df['RET']

    # Sum up portfolio returns
    eqports = df.groupby(['date', 'decile_lag'])['weighted_eq_ret'].sum()
    eqports = eqports.unstack()

    # Drop if missing accounting data in early years
    eqports = eqports.dropna(axis=0)

    # Match data format of FF factors
    eqports = eqports * 100
    eqports = eqports.reset_index()

    valports = df.groupby(['date', 'decile_lag'])['weighted_val_ret'].sum()
    valports = valports.unstack()
    valports = valports.dropna(axis=0)
    valports = valports * 100
    valports = valports.reset_index()

    return eqports, valports

# Creates a graph of the decile mean returns
def graph_deciles(df, title):
    df_mean = df.mean(numeric_only=True)
    df_mean.plot(kind='bar', x='Decile Portfolio', y='values', legend=False)
    plt.xlabel('Decile Portfolio')
    plt.ylabel('Mean Monthly Returns')
    plt.title(title)
    plt.show()

# Calculates returns and prints the returns mean, vol, and Sharpe ratio for a␣
↪strategy
def analyze(df, strat_name, ret_col_name, reverse=False):
    df[ret_col_name] = df[10.0] - df[1.0] if reverse else df[1.0] - df[10.0]

    strat_mean = df[ret_col_name].mean()
    strat_vol = df[ret_col_name].std()
    strat_sharpe = strat_mean / strat_vol

    print(f"{strat_name} monthly returns have mean {strat_mean}%, vol␣
↪{strat_vol}%, and Sharpe {strat_sharpe}")

# Estimates the CAPM and FF3 models on df_old using the returns found in␣
↪ret_col_name
def estimate_capm_and_ff3(df_old, ret_col_name, ff3):
    # Merge in ff3 data. Keep separate from ff5 because there is a larger data␣
↪range available in ff3.
```

```python
    # May lose a few rows since ff3 goes back to July 1926 and our data starts␣
↪Jan 1926
    df = pd.merge(df_old, ff3, how='inner', on=['date'])

    # Estimate CAPM
    print('CAPM')
    print(sm.OLS(df[ret_col_name] - df['RF'], sm.add_constant(df[['Mkt-RF']])).
↪fit().summary())

    # Estimate FF3
    print('FF3')
    print(sm.OLS(df[ret_col_name] - df['RF'], sm.add_constant(df[['Mkt-RF',␣
↪'SMB', 'HML']])).fit().summary())

# Estimates the FF5 model on df_old using the returns found in ret_col_name,␣
↪optionally adding momentum
def estimate_ff5(df_old, ret_col_name, ff5, add_momentum=False, mom_rets=None):
    # Merge in ff5 data. Truncates dates so create a df separate from ff3.
    df = pd.merge(df_old, ff5, how='inner', on=['date'])

    # Estimate FF5
    print('FF5')
    print(sm.OLS(df[ret_col_name] - df['RF'], sm.add_constant(df[['Mkt-RF',␣
↪'SMB', 'HML', 'RMW', 'CMA']])).fit().summary())

    if add_momentum:
        # Estimate FF5 + Momentum
        print('FF5 + Momentum')
        df = pd.merge(df, mom_rets, how='inner', on=['date'])
        print(sm.OLS(df[ret_col_name] - df['RF'], sm.add_constant(df[['Mkt-RF',␣
↪'SMB', 'HML', 'RMW', 'CMA', 'MOM']])).fit().summary())
```

[3]:
```python
'''
Load CRSP data
'''

df = pd.read_csv('crsp_1926_2020.zip')

# Convert prices and returns to numeric and drop NaNs
df['PRC'] = pd.to_numeric(df['PRC'], errors='coerce')
df['RET'] = pd.to_numeric(df['RET'], errors='coerce')
df = df.dropna(subset=['PRC', 'RET'])

# Set types for relevant columns
df = df.astype({'date': 'string', 'SHRCD': 'int', 'EXCHCD': 'int'})

# Drop day information in the dates
```

```
df['date'] = df['date'].str[:-3]
```

[4]:
```
'''
Load FF3 and FF5 data
'''

ff3 = pd.read_csv('ff3_factors.csv')
ff3 = ff3.astype({'date': 'string'})
ff3['date'] = ff3['date'].apply(lambda x: x[:4] + '-' + x[4:])

ff5 = pd.read_csv('ff5_factors.csv')
ff5 = ff5.astype({'date': 'string'})
ff5['date'] = ff5['date'].apply(lambda x: x[:4] + '-' + x[4:])
```

1.2 Problem 1

1.2.1 Part A

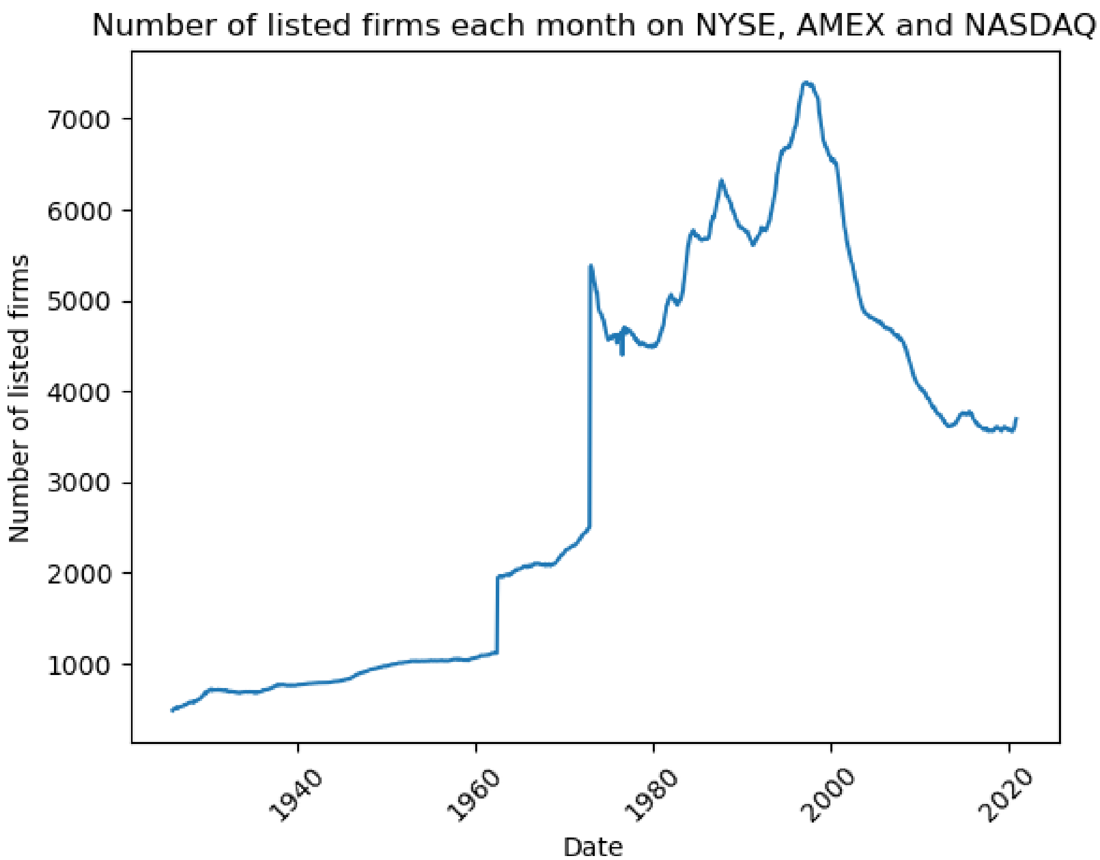
[5]:
```
# Filter SHRCD and EXCHCD, set negative prices to NA
df = df[df['SHRCD'].isin([10, 11])]
df = df[df['EXCHCD'].isin([1, 2, 3])]
df.loc[df['PRC'] < 0, 'PRC'] = 'NA'
df
```

[5]:
```
         PERMNO     date  SHRCD  EXCHCD        PRC       RET    SHROUT
2         10000  1986-02     10       3         NA -0.257143    3680.0
3         10000  1986-03     10       3         NA  0.365385    3680.0
4         10000  1986-04     10       3         NA -0.098592    3793.0
5         10000  1986-05     10       3         NA -0.222656    3793.0
6         10000  1986-06     10       3         NA -0.005025    3793.0
…           …       …    …      …          …         …         …
4705164   93436  2020-08     11       3  498.32001  0.741452  931809.0
4705165   93436  2020-09     11       3  429.01001 -0.139087  948000.0
4705166   93436  2020-10     11       3  388.04001 -0.095499  947901.0
4705167   93436  2020-11     11       3  567.59998  0.462736  947901.0
4705168   93436  2020-12     11       3  705.66998  0.243252  959854.0

[3563041 rows x 7 columns]
```

1.2.2 Part B

[6]:
```
# Group by year and month
by_month = df.groupby(df['date'])['PERMNO'].nunique().reset_index()
by_month['date'] = pd.to_datetime(by_month['date'])

# Create plot of number of firms listed each month
plt.plot(by_month['date'], by_month['PERMNO'])
```

```
plt.title('Number of listed firms each month on NYSE, AMEX and NASDAQ')
plt.xlabel('Date')
plt.ylabel('Number of listed firms')
plt.xticks(rotation=45)
plt.show()
```



1.3 Problem 2

1.3.1 Part A

[7]:
```
# Drop NA pricees and create MV column
df['PRC'] = pd.to_numeric(df['PRC'], errors='coerce')
df = df.dropna(subset=['PRC'])
df['MV'] = df['PRC'] * df['SHROUT']

sortdf = df.copy()
sortdf['rank'] = sortdf.groupby('date')['MV'].rank(pct=True)

# Label decile portfolios
sortdf['decile'] = np.ceil(sortdf['rank']*10)
```

```
# Calculate weights
# Introduces Nans because some rows do not have BM_decile
sortdf = sortdf.groupby(['date', 'decile'], group_keys=False).
 ↪apply(calc_weights)

# Move weights and deciles forward one month
# We form portfolios at the end of this month, and then earn returns over
# the next month.
sortdf['decile_lag'] = sortdf.groupby('PERMNO')['decile'].shift(1)
sortdf['weights_val_lag'] = sortdf.groupby('PERMNO')['weights_val'].shift(1)
sortdf['weights_eq_lag'] = sortdf.groupby('PERMNO')['weights_eq'].shift(1)

sortdf = sortdf.drop(['rank', 'decile', 'weights_eq', 'TMV', 'weights_val'],␣
 ↪axis=1)
sortdf
```

```
/var/folders/24/b7637qzn5pz9_l33fc30f4240000gn/T/ipykernel_92939/2236265313.py:4
: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  df['MV'] = df['PRC'] * df['SHROUT']
```

[7]:

	PERMNO	date	SHRCD	EXCHCD	PRC	RET	SHROUT	\
28	10001	1986-09	11	3	6.37500	-0.003077	991.0	
29	10001	1986-10	11	3	6.62500	0.039216	991.0	
30	10001	1986-11	11	3	7.00000	0.056604	991.0	
31	10001	1986-12	11	3	7.00000	0.015000	991.0	
32	10001	1987-01	11	3	6.75000	-0.035714	991.0	
…	…	… …	…	…	…	…		
4705164	93436	2020-08	11	3	498.32001	0.741452	931809.0	
4705165	93436	2020-09	11	3	429.01001	-0.139087	948000.0	
4705166	93436	2020-10	11	3	388.04001	-0.095499	947901.0	
4705167	93436	2020-11	11	3	567.59998	0.462736	947901.0	
4705168	93436	2020-12	11	3	705.66998	0.243252	959854.0	

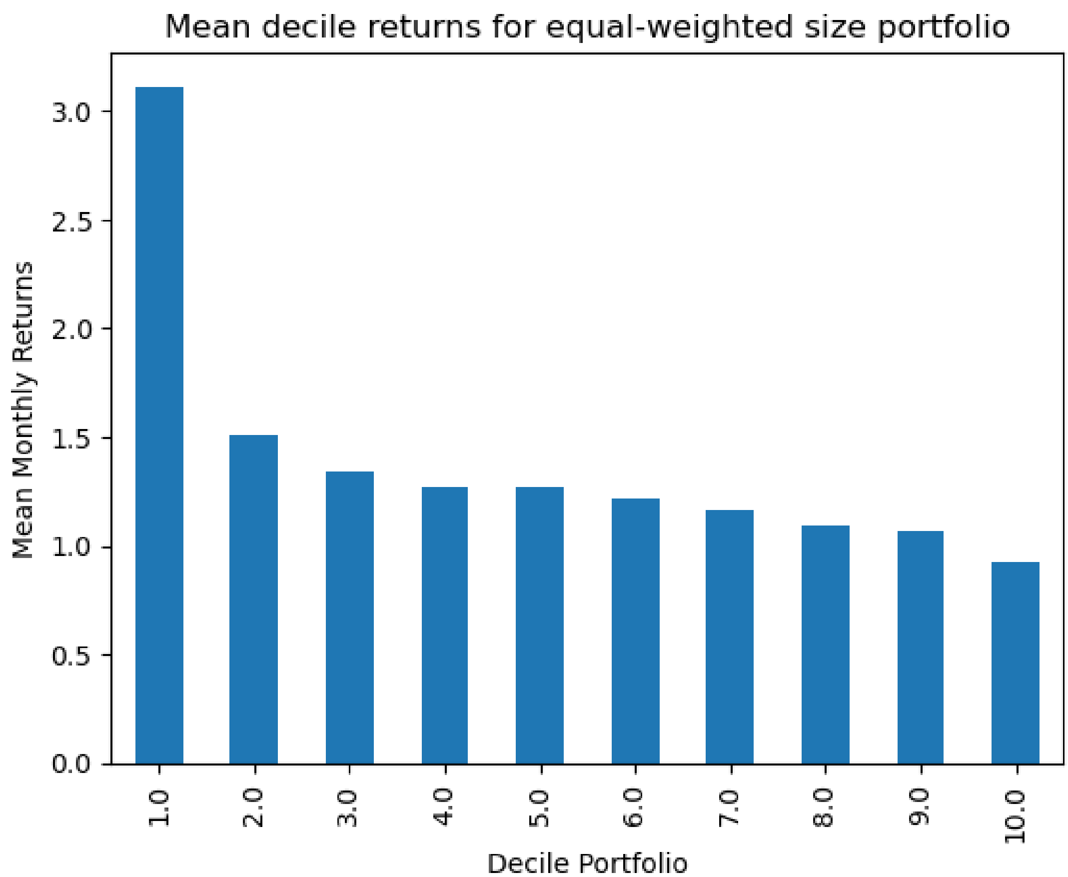
	MV	decile_lag	weights_val_lag	weights_eq_lag
28	6.317625e+03	NaN	NaN	NaN
29	6.565375e+03	1.0	0.001939	0.002494
30	6.937000e+03	1.0	0.001996	0.002469
31	6.937000e+03	1.0	0.002139	0.002551
32	6.689250e+03	1.0	0.002337	0.002309
…	…	…	…	…
4705164	4.643391e+08	10.0	0.009859	0.002825

```
4705165  4.067015e+08       10.0         0.015874        0.002809
4705166  3.678235e+08       10.0         0.014459        0.002801
4705167  5.380286e+08       10.0         0.013439        0.002793
4705168  6.773402e+08       10.0         0.017566        0.002740

[2855895 rows x 11 columns]
```

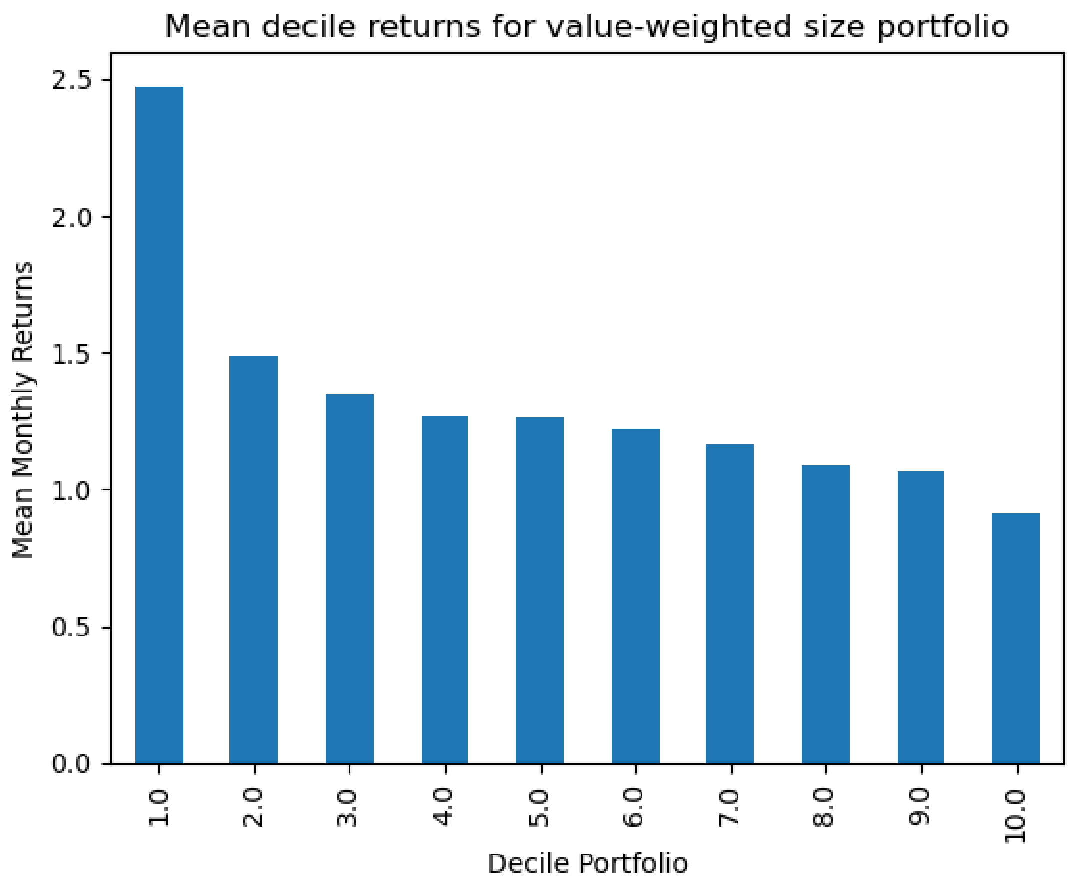
1.3.2 Part B

[8]:
```python
eqports, valports = part_b(sortdf)

display(eqports.mean(numeric_only=True))
graph_deciles(eqports, 'Mean decile returns for equal-weighted size portfolio')

display(valports.mean(numeric_only=True))
graph_deciles(valports, 'Mean decile returns for value-weighted size portfolio')
```

```
decile_lag
1.0     3.110296
2.0     1.509020
3.0     1.337370
4.0     1.272587
5.0     1.267089
6.0     1.215686
7.0     1.162114
8.0     1.092401
9.0     1.070809
10.0    0.924893
dtype: float64
```




```
decile_lag
1.0     2.471250
2.0     1.486205
3.0     1.343516
4.0     1.268714
5.0     1.264068
6.0     1.217934
7.0     1.160938
8.0     1.084152
9.0     1.064744
10.0    0.912878
dtype: float64
```



The mean monthly returns appear to be monotonic in both the equal-weighted and value-weighted portfolios. The mean returns decrease as size increases.

1.3.3 Part C

[9]:
```
analyze(eqports, 'Equal-weighted Size', 'RET')
analyze(valports, 'Value-weighted Size', 'RET')
```

```
Equal-weighted Size monthly returns have mean 2.1854033471142524%, vol
15.054678304411805%, and Sharpe 0.14516440025648475
Value-weighted Size monthly returns have mean 1.5583727812713861%, vol
12.915460300728482%, and Sharpe 0.12065948444620962
```

1.3.4 Part D

[10]:
```
print('Equal-Weighted Size:')
print('--------------------------------------------------------------------------------------')
estimate_capm_and_ff3(eqports, 'RET', ff3)

print('\n\n\n\nValue-Weighted Size:')
```

```python
print('-------------------------------------------------------------------------------------------')
estimate_capm_and_ff3(valports, 'RET', ff3)
```

```
Equal-Weighted Size:
--------------------------------------------------------------------------------
--
CAPM
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.170
Model:                            OLS   Adj. R-squared:                  0.169
Method:                 Least Squares   F-statistic:                     231.7
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           9.47e-48
Time:                        00:13:34   Log-Likelihood:                -4581.7
No. Observations:                1134   AIC:                             9167.
Df Residuals:                    1132   BIC:                             9177.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.1508      0.412      2.793      0.005       0.342       1.959
Mkt-RF         1.1627      0.076     15.223      0.000       1.013       1.313
==============================================================================
Omnibus:                     1632.470   Durbin-Watson:                   1.706
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           597390.160
Skew:                           8.042   Prob(JB):                         0.00
Kurtosis:                     114.286   Cond. No.                         5.44
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF3
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.584
Model:                            OLS   Adj. R-squared:                  0.583
Method:                 Least Squares   F-statistic:                     529.6
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          7.41e-215
Time:                        00:13:34   Log-Likelihood:                -4189.4
No. Observations:                1134   AIC:                             8387.
Df Residuals:                    1130   BIC:                             8407.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
```

```
==============================================================================
const          0.6092      0.292      2.083      0.037       0.035       1.183
Mkt-RF         0.4166      0.058      7.122      0.000       0.302       0.531
SMB            2.5175      0.096     26.121      0.000       2.328       2.707
HML            1.6717      0.085     19.674      0.000       1.505       1.838
==============================================================================
Omnibus:                     1208.710   Durbin-Watson:                   1.825
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           152550.865
Skew:                           4.857   Prob(JB):                         0.00
Kurtosis:                      58.984   Cond. No.                         5.71
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.




Value-Weighted Size:
--------------------------------------------------------------------------------
--
CAPM
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.195
Model:                            OLS   Adj. R-squared:                  0.194
Method:                 Least Squares   F-statistic:                     274.4
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           2.34e-55
Time:                        00:13:34   Log-Likelihood:                -4390.6
No. Observations:                1134   AIC:                             8785.
Df Residuals:                    1132   BIC:                             8795.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.5797      0.348      1.665      0.096      -0.103       1.263
Mkt-RF         1.0692      0.065     16.566      0.000       0.943       1.196
==============================================================================
Omnibus:                     1453.246   Durbin-Watson:                   1.709
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           327541.754
Skew:                           6.574   Prob(JB):                         0.00
Kurtosis:                      85.215   Cond. No.                         5.44
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
```

```
specified.
FF3
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.646
Model:                            OLS   Adj. R-squared:                  0.645
Method:                 Least Squares   F-statistic:                     686.2
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          6.08e-254
Time:                        00:13:34   Log-Likelihood:                -3925.5
No. Observations:                1134   AIC:                             7859.
Df Residuals:                    1130   BIC:                             7879.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1060      0.232      0.457      0.648      -0.349       0.561
Mkt-RF         0.4015      0.046      8.664      0.000       0.311       0.492
SMB            2.3121      0.076     30.275      0.000       2.162       2.462
HML            1.4252      0.067     21.169      0.000       1.293       1.557
==============================================================================
Omnibus:                     1005.072   Durbin-Watson:                   1.876
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            65012.508
Skew:                           3.748   Prob(JB):                         0.00
Kurtosis:                      39.328   Cond. No.                         5.71
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

For both the equal-weighted and value-weighted portfolios, the alphas for the CAPM model are greater than that of the FF3 model. In the equal-weighted portfolio, the alpha for the CAPM model is 1.15 while the alpha for the FF3 model is 0.61. In the value-weighted portfolio, the alpha for the CAPM model is 0.58 while the alpha for the FF3 model is 0.11.

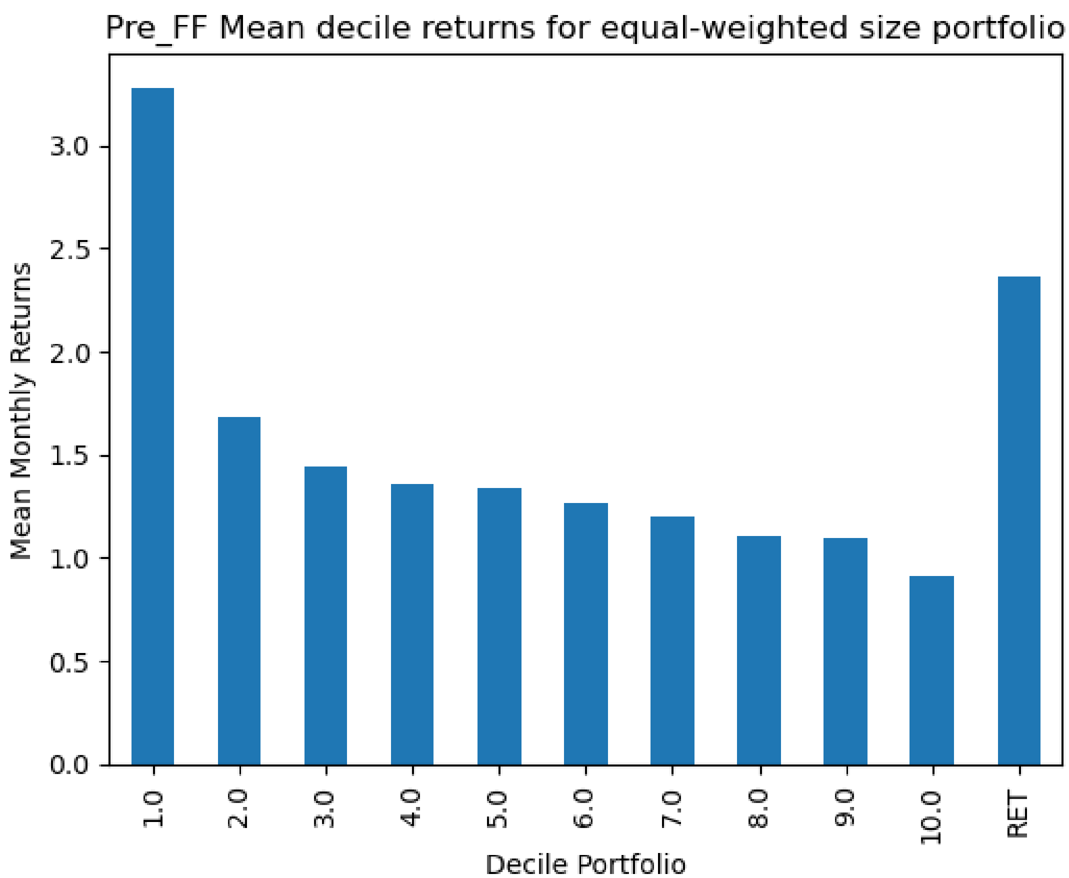
This makes sense with our strategy formulation because the equal-weight portfolio, if we are able to execute it, would put higher weight on the stocks we believe are undervalued (low size), causing higher alpha. From these numbers, we can also see that the alphas calculated from CAPM are higher than those from FF3.

1.3.5 Part E

[11]:
```python
pre_FF = eqports[eqports['date'] < '1992-00'].copy()

analyze(pre_FF, 'Equal-weighted Size', 'RET')
display(pre_FF.mean(numeric_only=True))
```

```
graph_deciles(pre_FF, 'Pre_FF Mean decile returns for equal-weighted size␣
 ↪portfolio')
estimate_capm_and_ff3(pre_FF, 'RET', ff3)
```

```
Equal-weighted Size monthly returns have mean 2.366380613074474%, vol
17.34598770401589%, and Sharpe 0.13642236195789625

decile_lag
1.0     3.278559
2.0     1.684028
3.0     1.444906
4.0     1.361792
5.0     1.335321
6.0     1.266390
7.0     1.202053
8.0     1.107576
9.0     1.092938
10.0    0.912178
RET     2.366381
dtype: float64
```



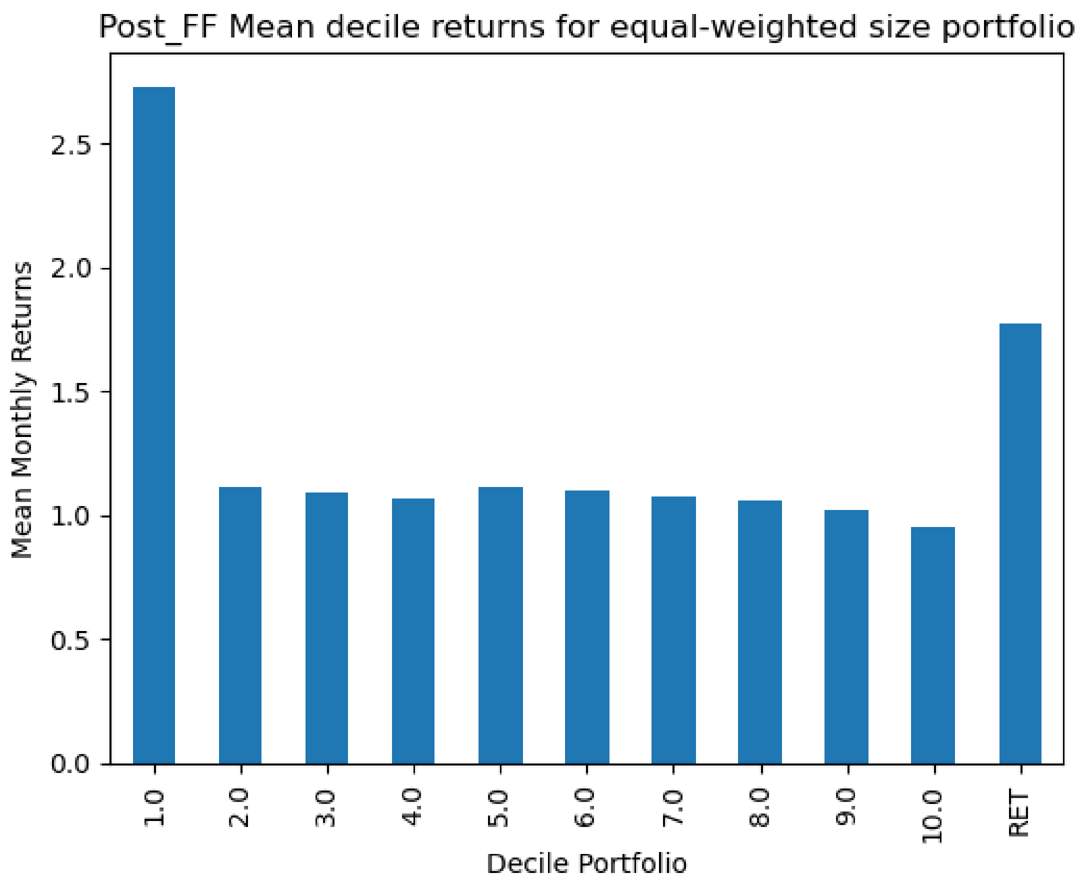
```
CAPM
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.216
Model:                            OLS   Adj. R-squared:                  0.215
Method:                 Least Squares   F-statistic:                     216.2
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           2.09e-43
Time:                        00:13:34   Log-Likelihood:                -3265.2
No. Observations:                 786   AIC:                             6534.
Df Residuals:                     784   BIC:                             6544.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.1710      0.554      2.113      0.035       0.083       2.259
Mkt-RF         1.4090      0.096     14.704      0.000       1.221       1.597
==============================================================================
Omnibus:                     1085.296   Durbin-Watson:                   1.705
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           263830.027
Skew:                           7.333   Prob(JB):                         0.00
Kurtosis:                      91.548   Cond. No.                         5.82
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF3
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.698
Model:                            OLS   Adj. R-squared:                  0.697
Method:                 Least Squares   F-statistic:                     602.6
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          8.38e-203
Time:                        00:13:34   Log-Likelihood:                -2890.3
No. Observations:                 786   AIC:                             5789.
Df Residuals:                     782   BIC:                             5807.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.3366      0.345      0.974      0.330      -0.342       1.015
Mkt-RF         0.4024      0.066      6.091      0.000       0.273       0.532
SMB            3.0060      0.117     25.685      0.000       2.776       3.236
HML            1.9846      0.100     19.784      0.000       1.788       2.182
```

```
==============================================================================
Omnibus:                      782.943   Durbin-Watson:                   1.772
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            68582.445
Skew:                           4.312   Prob(JB):                         0.00
Kurtosis:                      47.942   Cond. No.                         6.21
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[12]:
```
post_FF = eqports[eqports['date'] > '1992-00'].copy()

analyze(post_FF, 'Equal-weighted Size', 'RET')
display(post_FF.mean(numeric_only=True))
graph_deciles(post_FF, 'Post_FF Mean decile returns for equal-weighted size␣
 ↪portfolio')
estimate_capm_and_ff3(post_FF, 'RET', ff3)
```

```
Equal-weighted Size monthly returns have mean 1.7740441017851276%, vol
7.61795357856323%, and Sharpe 0.23287672778385687
```

```
decile_lag
1.0     2.727837
2.0     1.111228
3.0     1.092943
4.0     1.069827
5.0     1.112000
6.0     1.100437
7.0     1.071333
8.0     1.057907
9.0     1.020510
10.0    0.953793
RET     1.774044
dtype: float64
```



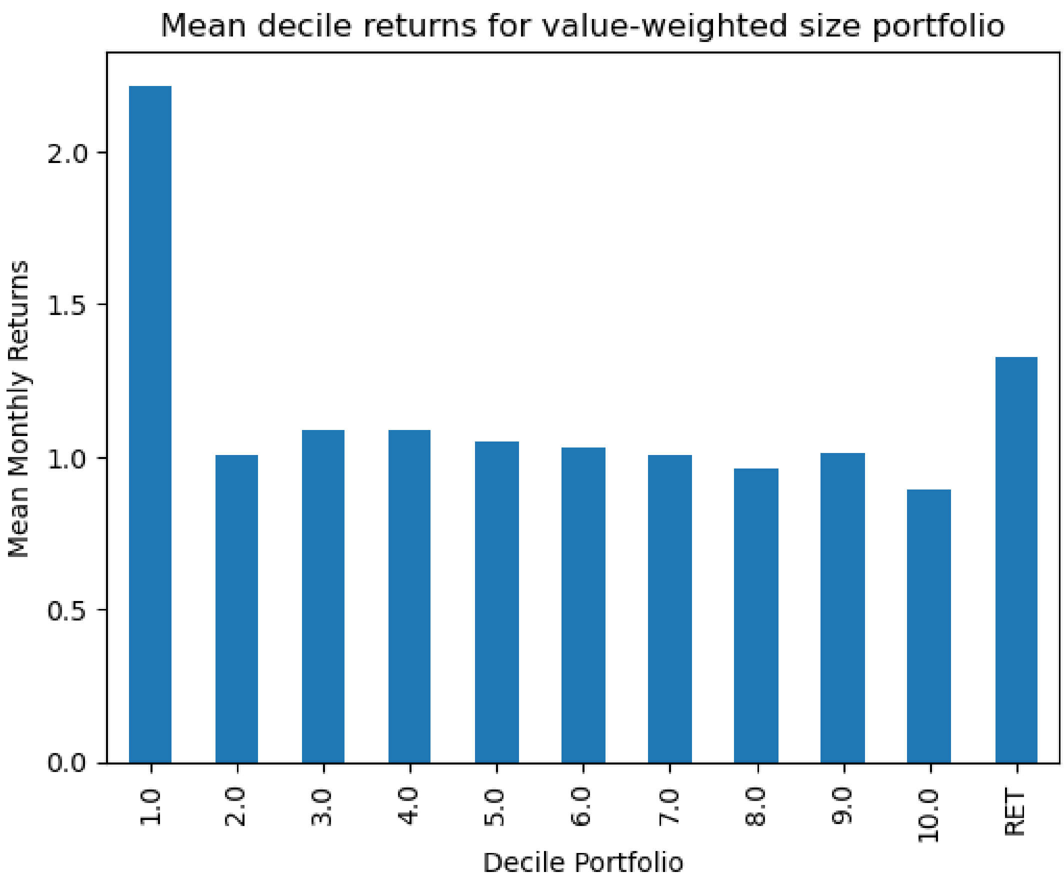

```
CAPM
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.011
Model:                            OLS   Adj. R-squared:                  0.009
Method:                 Least Squares   F-statistic:                     4.021
Date:                Mon, 22 Apr 2024   Prob (F-statistic):             0.0457
Time:                        00:13:34   Log-Likelihood:                -1198.0
No. Observations:                 348   AIC:                             2400.
Df Residuals:                     346   BIC:                             2408.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.4425      0.412      3.498      0.001       0.631       2.254
Mkt-RF         0.1881      0.094      2.005      0.046       0.004       0.373
==============================================================================
Omnibus:                      188.311   Durbin-Watson:                   1.764
```

```
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1789.617
Skew:                           2.067   Prob(JB):                         0.00
Kurtosis:                      13.312   Cond. No.                         4.46
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF3
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.190
Model:                            OLS   Adj. R-squared:                  0.183
Method:                 Least Squares   F-statistic:                     26.91
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           1.17e-15
Time:                        00:13:34   Log-Likelihood:                -1163.4
No. Observations:                 348   AIC:                             2335.
Df Residuals:                     344   BIC:                             2350.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.4251      0.375      3.801      0.000       0.688       2.163
Mkt-RF        -0.0007      0.088     -0.008      0.993      -0.174       0.172
SMB            1.0341      0.121      8.514      0.000       0.795       1.273
HML           -0.0022      0.121     -0.018      0.986      -0.240       0.236
==============================================================================
Omnibus:                      161.889   Durbin-Watson:                   1.979
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1257.695
Skew:                           1.769   Prob(JB):                    7.85e-274
Kurtosis:                      11.615   Cond. No.                         4.64
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[13]:
```
post_DC = eqports[eqports['date'] > '2002-00'].copy()

analyze(post_DC, 'Equal-weighted Size', 'RET')
display(post_DC.mean(numeric_only=True))
graph_deciles(post_DC, 'Mean decile returns for value-weighted size portfolio')
estimate_capm_and_ff3(post_DC, 'RET', ff3)
```

```
Equal-weighted Size monthly returns have mean 1.3233240130717987%, vol
6.181493202819268%, and Sharpe 0.21407837389002618
```

```
decile_lag
1.0     2.215561
2.0     1.002400
3.0     1.084754
4.0     1.090113
5.0     1.046541
6.0     1.027662
7.0     1.007764
8.0     0.961050
9.0     1.009594
10.0    0.892237
RET     1.323324
dtype: float64
```




```
CAPM
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.040
Model:                            OLS   Adj. R-squared:                  0.036
Method:                 Least Squares   F-statistic:                     9.454
```

```
Date:                Mon, 22 Apr 2024   Prob (F-statistic):            0.00237
Time:                        00:13:34   Log-Likelihood:                -734.24
No. Observations:                 228   AIC:                             1472.
Df Residuals:                     226   BIC:                             1479.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.0134      0.409      2.480      0.014       0.208       1.819
Mkt-RF         0.2814      0.092      3.075      0.002       0.101       0.462
==============================================================================
Omnibus:                       46.372   Durbin-Watson:                   1.640
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               73.628
Skew:                           1.133   Prob(JB):                     1.03e-16
Kurtosis:                       4.617   Cond. No.                         4.53
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF3
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.142
Model:                            OLS   Adj. R-squared:                  0.130
Method:                 Least Squares   F-statistic:                     12.32
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           1.73e-07
Time:                        00:13:34   Log-Likelihood:                -721.50
No. Observations:                 228   AIC:                             1451.
Df Residuals:                     224   BIC:                             1465.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.9456      0.390      2.422      0.016       0.176       1.715
Mkt-RF         0.1224      0.094      1.301      0.195      -0.063       0.308
SMB            0.8697      0.169      5.140      0.000       0.536       1.203
HML           -0.0554      0.148     -0.375      0.708      -0.347       0.236
==============================================================================
Omnibus:                       43.676   Durbin-Watson:                   1.768
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               68.040
Skew:                           1.080   Prob(JB):                     1.68e-15
Kurtosis:                       4.579   Cond. No.                         4.74
==============================================================================

Notes:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

Comparing the alpha of the size strategy before the 1992 FF paper, after the paper and before the Dot-Com Bubble, and after the Dot-Com Bubble, we conclude that the size strategy still works.

Before the Fama French 1992 paper was published, the CAPM alpha was 1.17 and the FF3 alpha was 0.34, and after the paper the CAPM alpha was 1.44 and the FF3 alpha was 1.42, showing an improvement. Additionally, the graphs of the mean monthly returns of each decile since the publication of the 1992 Fama French paper show that the 1st decile's returns are still larger than that of the 10th decile. So, the size portfolio still works.

We can see that the same still holds for the size factor since the Dot-Com Bubble burst (post-2002), with CAPM alpha of 1.01 and FF3 alpha of 0.95. However, as we can see from the graph, although the 1st decile's mean monthly returns are still higher than the 10th decile's returns, the difference as well as the alpha values are smaller than before. So, while size still works, it does not work as well as before the Bubble.

1.4 Problem 3

1.4.1 Part A

[14]:
```python
# Calculate cumulative returns from month t-11 to month t-1
cum_rets = df.groupby('PERMNO')['RET'].rolling(window=11, min_periods=11).
 ↪apply(lambda x: np.prod(1+x)-1).shift(1)
idx = cum_rets.index.get_level_values(1)
cum_rets = pd.Series(cum_rets.values, index=idx).rename('11M_RET')
cum_rets
```

[14]:
```
28              NaN
29              NaN
30              NaN
31              NaN
32              NaN
             …
4705164    5.341741
4705165    9.344169
4705166    5.811416
4705167    4.880464
4705168    5.784121
Name: 11M_RET, Length: 2855895, dtype: float64
```

[15]:
```python
# Merge 11 month rolling returns into df
df_11m_rets = pd.merge(df, cum_rets, left_index=True, right_index=True)
assert(len(df_11m_rets) == len(df))
df_11m_rets = df_11m_rets.dropna(subset=['11M_RET'])
df_11m_rets
```

```
[15]:          PERMNO     date  SHRCD  EXCHCD        PRC       RET    SHROUT  \
41          10001  1987-10     11       3    6.37500  0.020000     992.0
42          10001  1987-11     11       3    6.18750 -0.029412     992.0
43          10001  1987-12     11       3    5.87500 -0.033535     992.0
44          10001  1988-01     11       3    6.25000  0.063830     992.0
45          10001  1988-02     11       3    6.75000  0.080000     992.0
…             …       …    …     …       …          …         …
4705164     93436  2020-08     11       3  498.32001  0.741452  931809.0
4705165     93436  2020-09     11       3  429.01001 -0.139087  948000.0
4705166     93436  2020-10     11       3  388.04001 -0.095499  947901.0
4705167     93436  2020-11     11       3  567.59998  0.462736  947901.0
4705168     93436  2020-12     11       3  705.66998  0.243252  959854.0

                   MV   11M_RET
41       6.324000e+03  0.169797
42       6.138000e+03  0.196875
43       5.828000e+03  0.117836
44       6.200000e+03  0.022473
45       6.696000e+03  0.071663
…                 …       …
4705164  4.643391e+08  5.341741
4705165  4.067015e+08  9.344169
4705166  3.678235e+08  5.811416
4705167  5.380286e+08  4.880464
4705168  6.773402e+08  5.784121

[2648337 rows x 9 columns]
```

```
[16]: # Add deciles
df_11m_rets['rank'] = df_11m_rets.groupby('date')['11M_RET'].rank(pct=True)
df_11m_rets['decile'] = np.ceil(df_11m_rets['rank']*10)

# Use calc_weights helper to get the equal- and value-weighted portfolios
df_weights = df_11m_rets.groupby(['date', 'decile'], group_keys=False).
 ↪apply(calc_weights)
df_weights['decile_lag'] = df_weights.groupby('PERMNO')['decile'].shift(1)
df_weights['weights_val_lag'] = df_weights.groupby('PERMNO')['weights_val'].
 ↪shift(1)
df_weights['weights_eq_lag'] = df_weights.groupby('PERMNO')['weights_eq'].
 ↪shift(1)

# Process final portfolio weights
df_weights = df_weights.drop(['rank', 'decile', 'weights_eq', 'TMV',␣
 ↪'weights_val'], axis=1)
df_weights
```

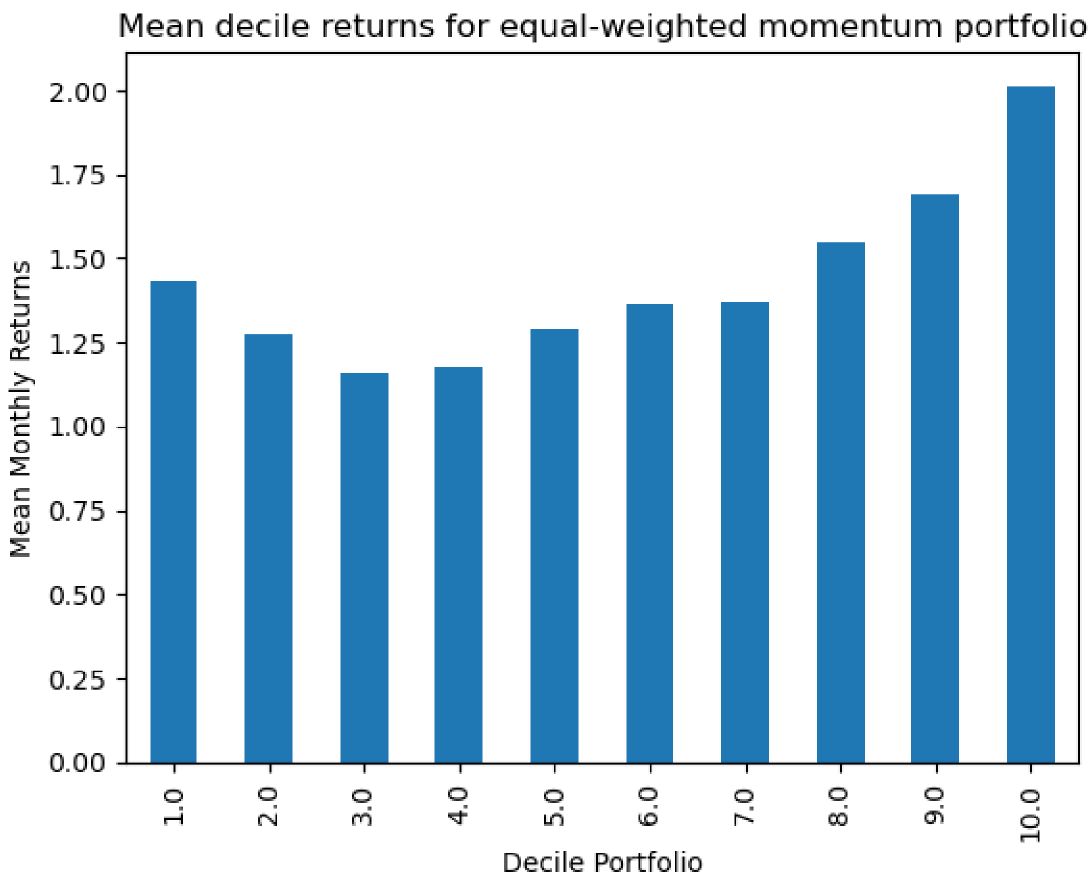
[16]:
```
              PERMNO     date  SHRCD  EXCHCD        PRC       RET    SHROUT  \
41             10001  1987-10     11       3    6.37500  0.020000     992.0
42             10001  1987-11     11       3    6.18750 -0.029412     992.0
43             10001  1987-12     11       3    5.87500 -0.033535     992.0
44             10001  1988-01     11       3    6.25000  0.063830     992.0
45             10001  1988-02     11       3    6.75000  0.080000     992.0
…                …       …   …      …       …         …         …
4705164        93436  2020-08     11       3  498.32001  0.741452  931809.0
4705165        93436  2020-09     11       3  429.01001 -0.139087  948000.0
4705166        93436  2020-10     11       3  388.04001 -0.095499  947901.0
4705167        93436  2020-11     11       3  567.59998  0.462736  947901.0
4705168        93436  2020-12     11       3  705.66998  0.243252  959854.0

                   MV   11M_RET  decile_lag  weights_val_lag  weights_eq_lag
41       6.324000e+03  0.169797         NaN              NaN             NaN
42       6.138000e+03  0.196875         6.0         0.000024        0.002525
43       5.828000e+03  0.117836         9.0         0.000012        0.002538
44       6.200000e+03  0.022473         9.0         0.000013        0.002469
45       6.696000e+03  0.071663         8.0         0.000013        0.002653
…                 …         …           …                …               …
4705164  4.643391e+08  5.341741        10.0         0.064252        0.002967
4705165  4.067015e+08  9.344169        10.0         0.071977        0.002959
4705166  3.678235e+08  5.811416        10.0         0.058150        0.002967
4705167  5.380286e+08  4.880464        10.0         0.060722        0.002959
4705168  6.773402e+08  5.784121        10.0         0.185959        0.002941

[2648337 rows x 12 columns]
```

1.4.2 Part B

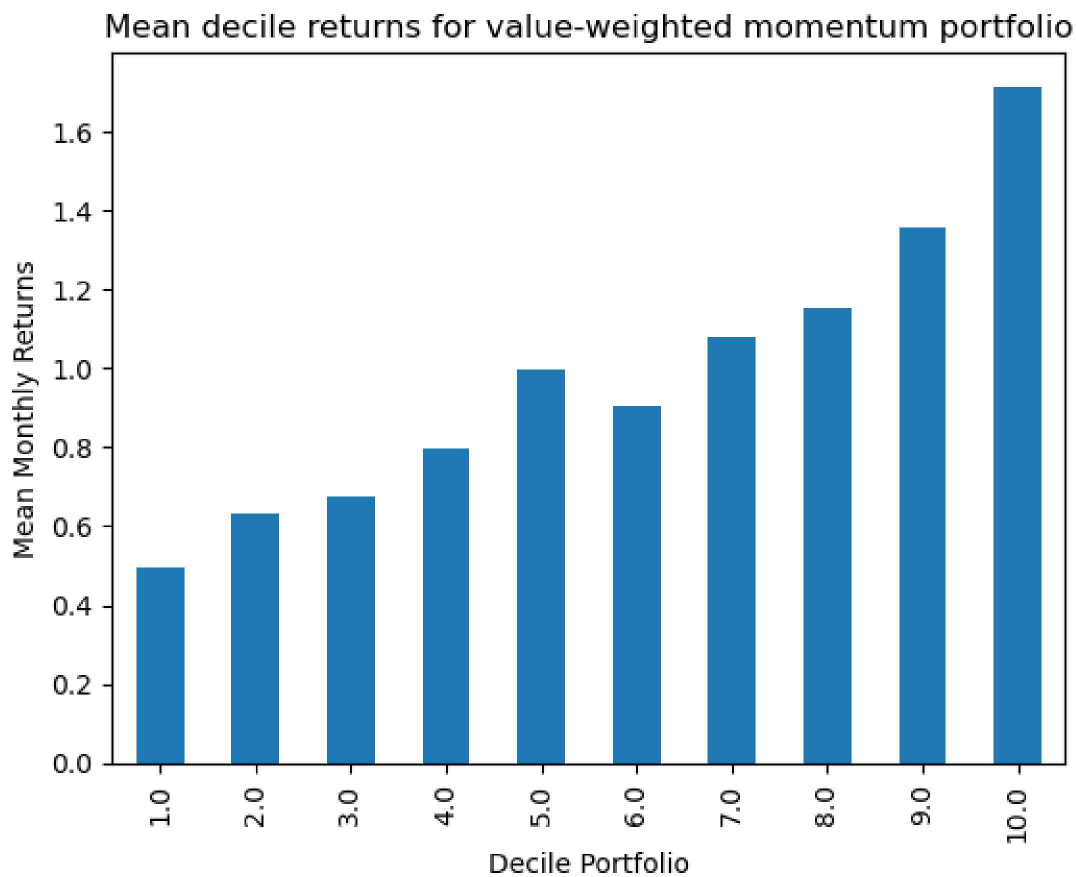
[17]:
```python
eq_decile_returns, val_decile_returns = part_b(df_weights)

display(eq_decile_returns.mean(numeric_only=True))
graph_deciles(eq_decile_returns, 'Mean decile returns for equal-weighted␣
 ↪momentum portfolio')

display(val_decile_returns.mean(numeric_only=True))
graph_deciles(val_decile_returns, 'Mean decile returns for value-weighted␣
 ↪momentum portfolio')
```

```
decile_lag
1.0      1.433417
2.0      1.275422
3.0      1.158702
4.0      1.177848
5.0      1.289182
6.0      1.366047
```

```
7.0     1.367513
8.0     1.547550
9.0     1.688353
10.0    2.012180
dtype: float64
```




```
decile_lag
1.0     0.496700
2.0     0.629950
3.0     0.674610
4.0     0.795372
5.0     0.994576
6.0     0.905602
7.0     1.080470
8.0     1.149786
9.0     1.357799
10.0    1.712453
dtype: float64
```



The equal-weighted decile returns are not monotonic, and instead follow a curved pattern where the 10th decile is higher than the 1st decile. The value-weighted decile returns are roughly monotonic, increasing as the decile increases with one exception from the 5th to 6th decile.

1.4.3 Part C

[18]:
```
analyze(eq_decile_returns, 'Equal-weighted Momentum', 'MOM', reverse=True)
analyze(val_decile_returns, 'Value-weighted Momentum', 'MOM', reverse=True)
```

```
Equal-weighted Momentum monthly returns have mean 0.5787633266410294%, vol
10.412249945766787%, and Sharpe 0.055584847622326995
Value-weighted Momentum monthly returns have mean 1.215753149735365%, vol
9.423347313797782%, and Sharpe 0.12901499958037674
```

1.4.4 Part D

[19]:
```
print('Equal-Weighted Momentum:')
print('--------------------------------------------------------------------------------')
estimate_capm_and_ff3(eq_decile_returns, 'MOM', ff3)
estimate_ff5(eq_decile_returns, 'MOM', ff5)
```

```python
print('\n\n\n\nValue-Weighted Momentum:')
print('--------------------------------------------------------------------------------------')
estimate_capm_and_ff3(val_decile_returns, 'MOM', ff3)
estimate_ff5(val_decile_returns, 'MOM', ff5)

# Save momentum returns for part 4
eq_mom_returns = eq_decile_returns[['date', 'MOM']]
val_mom_returns = val_decile_returns[['date', 'MOM']]
```

```
Equal-Weighted Momentum:
--------------------------------------------------------------------------------
--
CAPM
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.129
Model:                            OLS   Adj. R-squared:                  0.129
Method:                 Least Squares   F-statistic:                     167.4
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           8.30e-36
Time:                        00:17:22   Log-Likelihood:                -4166.2
No. Observations:                1129   AIC:                             8336.
Df Residuals:                    1127   BIC:                             8346.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.7817      0.291      2.686      0.007       0.211       1.353
Mkt-RF        -0.6967      0.054    -12.940      0.000      -0.802      -0.591
==============================================================================
Omnibus:                     1213.422   Durbin-Watson:                   2.003
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           126441.503
Skew:                          -5.013   Prob(JB):                         0.00
Kurtosis:                      53.866   Cond. No.                         5.45
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF3
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.343
Model:                            OLS   Adj. R-squared:                  0.342
Method:                 Least Squares   F-statistic:                     196.1
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          2.65e-102
```

```
Time:                        00:17:22   Log-Likelihood:                -4006.9
No. Observations:                1129   AIC:                             8022.
Df Residuals:                    1125   BIC:                             8042.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.0947      0.253      4.319      0.000       0.597       1.592
Mkt-RF        -0.3496      0.051     -6.908      0.000      -0.449      -0.250
SMB           -0.8548      0.083    -10.248      0.000      -1.018      -0.691
HML           -1.1486      0.074    -15.619      0.000      -1.293      -1.004
==============================================================================
Omnibus:                      797.451   Durbin-Watson:                   2.005
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            27077.932
Skew:                          -2.804   Prob(JB):                         0.00
Kurtosis:                      26.327   Cond. No.                         5.72
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF5
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.089
Model:                            OLS   Adj. R-squared:                  0.083
Method:                 Least Squares   F-statistic:                     13.44
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           1.62e-12
Time:                        00:17:22   Log-Likelihood:                -2341.8
No. Observations:                 690   AIC:                             4696.
Df Residuals:                     684   BIC:                             4723.
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.5562      0.288      1.930      0.054      -0.010       1.122
Mkt-RF        -0.1504      0.070     -2.140      0.033      -0.288      -0.012
SMB           -0.2230      0.100     -2.226      0.026      -0.420      -0.026
HML           -0.5734      0.133     -4.310      0.000      -0.835      -0.312
RMW            0.6210      0.139      4.453      0.000       0.347       0.895
CMA            0.4629      0.205      2.254      0.024       0.060       0.866
==============================================================================
Omnibus:                      419.119   Durbin-Watson:                   2.130
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            12101.288
Skew:                          -2.192   Prob(JB):                         0.00
Kurtosis:                      23.042   Cond. No.                         5.19
```

```
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

Value-Weighted Momentum:

```
--------------------------------------------------------------------------------
--
CAPM
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.153
Model:                            OLS   Adj. R-squared:                  0.152
Method:                 Least Squares   F-statistic:                     203.0
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           1.77e-42
Time:                        00:17:22   Log-Likelihood:                -4038.8
No. Observations:                1129   AIC:                             8082.
Df Residuals:                    1127   BIC:                             8092.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.4110      0.260      5.428      0.000       0.901       1.921
Mkt-RF        -0.6853      0.048    -14.248      0.000      -0.780      -0.591
==============================================================================
Omnibus:                      529.207   Durbin-Watson:                   1.947
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7027.991
Skew:                          -1.808   Prob(JB):                         0.00
Kurtosis:                      14.676   Cond. No.                         5.45
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF3
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.280
Model:                            OLS   Adj. R-squared:                  0.278
Method:                 Least Squares   F-statistic:                     145.6
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           9.96e-80
Time:                        00:17:22   Log-Likelihood:                -3947.1
No. Observations:                1129   AIC:                             7902.
```

```
Df Residuals:                    1125   BIC:                             7922.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.6345      0.240      6.799      0.000       1.163       2.106
Mkt-RF        -0.4645      0.048     -9.676      0.000      -0.559      -0.370
SMB           -0.4053      0.079     -5.123      0.000      -0.560      -0.250
HML           -0.8953      0.070    -12.837      0.000      -1.032      -0.758
==============================================================================
Omnibus:                      281.001   Durbin-Watson:                   1.955
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1616.654
Skew:                          -1.018   Prob(JB):                         0.00
Kurtosis:                       8.497   Cond. No.                         5.72
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF5
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.124
Model:                            OLS   Adj. R-squared:                  0.117
Method:                 Least Squares   F-statistic:                     19.34
Date:                Mon, 22 Apr 2024   Prob (F-statistic):           4.94e-18
Time:                        00:17:22   Log-Likelihood:                -2410.9
No. Observations:                 690   AIC:                             4834.
Df Residuals:                     684   BIC:                             4861.
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.2522      0.319      3.931      0.000       0.627       1.878
Mkt-RF        -0.4269      0.078     -5.494      0.000      -0.580      -0.274
SMB           -0.0109      0.111     -0.099      0.922      -0.228       0.207
HML           -0.8105      0.147     -5.512      0.000      -1.099      -0.522
RMW            0.6429      0.154      4.171      0.000       0.340       0.946
CMA            0.5730      0.227      2.524      0.012       0.127       1.019
==============================================================================
Omnibus:                      165.141   Durbin-Watson:                   2.013
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              782.467
Skew:                          -1.000   Prob(JB):                    1.23e-170
Kurtosis:                       7.818   Cond. No.                         5.19
==============================================================================
```

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

Comparing the (CAPM, FF3, and FF5) alphas of the equal-weighted momentum portfolio (0.78, 1.09, 0.56) with those of the value-weighted portfolio (1.41, 1.63, 1.25), it is clear that the value-weighted momentum portfolios have far better alphas. Perhaps the momentum signal is stronger in high value firms.

Under the CAPM model, the FF3 model, and the FF5 model, the alpha that the equal-weighted and value-weighted momentum strategies are generating is statistically significant at the 5% level, with the exception of the equal-weighted FF5 alpha.

As can be seen from the results, the alpha is consistently smaller under the FF5 model, indicating that FF5 does a decent job of pricing momentum. We note a large drop in alpha, especially from the FF3 to the FF5 model, but the alpha does not completely disappear.

1.4.5 Part E

We find it likely that the positive momentum alphas are indicative of some unpriced risk. For one, since the momentum papers have been out for a while and this strategy is relatively easy to implement, it stands to reason that the market has priced momentum into the equation already, and that there are many people who have run or are running this strategy.

Second, it is not hard to imagine that a momentum strategy could face extreme risk in many ways: if the portfolio is rebalanced monthly and happens to buy into hot stocks at the start of the month, there is no guarantee that this momentum will carry through the whole month, as just one example.

1.5 Problem 4

1.5.1 Part A

Notes: Losing ~3000 (<0.1% of) rows since FF website starts July 1926, Professor Sinclair's data starts January 1926. We also assume that once data starts for a given stock, then it continues to be present every month until it stops.

[20]:
```python
# Merge ff3 data and edit columns
df_merged = pd.merge(df, ff3, how='inner', on=['date'])
df_merged['Ret-RF'] = df_merged['RET']*100 - df_merged['RF']
df_merged = df_merged.drop(['SHRCD', 'EXCHCD', 'PRC', 'SHROUT', 'SMB', 'HML',␣
↪'RF'], axis=1)
df_merged
```

[20]:

	PERMNO	date	RET	MV	Mkt-RF	Ret-RF
0	10001	1986-09	-0.003077	6317.625	-8.60	-0.7577
1	10003	1986-09	-0.057692	40149.375	-8.60	-6.2192
2	10008	1986-09	-0.155963	33867.500	-8.60	-16.0463
3	10009	1986-09	-0.092157	10315.500	-8.60	-9.6657
4	10016	1986-09	-0.020000	224959.000	-8.60	-2.4500
…	…	…	…	…	…	…
2853548	84241	1984-12	0.000000	21850.000	1.84	-0.6400

```
2853549   85033  1984-12  0.000000   80750.000    1.84  -0.6400
2853550   85762  1984-12  0.093750  110687.500    1.84   8.7350
2853551   89552  1984-12 -0.027778   45543.750    1.84  -3.4178
2853552   90617  1984-12 -0.086420   49052.750    1.84  -9.2820

[2853553 rows x 6 columns]
```

[21]:
```python
# Applies the rolling ols and returns the betas where there are enough months
# or np.NaN where there are not enough months
def rolling_ols(group):
    dates = np.array(group['date'])

    if len(group) >= 36:
        y = group['Ret-RF']
        x = sm.add_constant(group['Mkt-RF'])
        model = RollingOLS(y, x, window=36)

        beta_vals = np.array(model.fit().params[['Mkt-RF']].values)
        return np.column_stack((dates, beta_vals))
    else:
        return np.column_stack((dates, np.full(len(group), np.nan)))

# Get the market beta for stock i from time t-36 to time t
beta_i_t = df_merged.groupby('PERMNO').apply(rolling_ols)
beta_i_t = pd.DataFrame(beta_i_t, columns=['beta'])
beta_i_t = beta_i_t.explode('beta')
beta_i_t[['date', 'beta']] = beta_i_t['beta'].apply(lambda el: pd.Series(el))
beta_i_t
```

[21]:
```
             beta     date
PERMNO
10001         NaN  1986-09
10001         NaN  1986-10
10001         NaN  1986-11
10001         NaN  1986-12
10001         NaN  1987-01
…           …      …
93436    1.768065  2020-08
93436    1.847295  2020-09
93436    1.895699  2020-10
93436    2.097154  2020-11
93436    2.120909  2020-12

[2853553 rows x 2 columns]
```

[22]:
```python
# Merge betas into df and drop NaN betas
df_betas = pd.merge(df_merged, beta_i_t, how='inner', on=['PERMNO', 'date'])
```

```python
assert(len(df_betas) == len(df_merged))
df_betas = df_betas.dropna()
df_betas
```

[22]:

```
          PERMNO     date       RET           MV  Mkt-RF   Ret-RF      beta
147437     10001  1990-06  0.014103   10052.2500   -1.09   0.7803  0.025533
147438     10003  1990-06 -0.178571   12615.5000   -1.09 -18.4871  0.883344
147442     10020  1990-06 -0.105357  254355.7500   -1.09 -11.1657  0.186161
147444     10026  1990-06 -0.043478   68763.7500   -1.09  -4.9778  1.741824
147447     10034  1990-06 -0.011111   64013.2500   -1.09  -1.7411  1.057335
…            …       …         …            …       …        …
2853546    84180  1984-12  0.045455   22302.8125    1.84   3.9055  1.922804
2853548    84241  1984-12  0.000000   21850.0000    1.84  -0.6400  1.860227
2853549    85033  1984-12  0.000000   80750.0000    1.84  -0.6400  3.746187
2853550    85762  1984-12  0.093750  110687.5000    1.84   8.7350  1.295810
2853551    89552  1984-12 -0.027778   45543.7500    1.84  -3.4178  0.789196

[2191529 rows x 7 columns]
```

[23]:

```python
# Add deciles
df_betas['rank'] = df_betas.groupby('date')['beta'].rank(pct=True)
df_betas['decile'] = np.ceil(df_betas['rank']*10)

# Use calc_weights helper to get the equal- and value-weighted portfolios
df_weights = df_betas.groupby(['date', 'decile'], group_keys=False).
 ↪apply(calc_weights)
df_weights['decile_lag'] = df_weights.groupby('PERMNO')['decile'].shift(1)
df_weights['weights_val_lag'] = df_weights.groupby('PERMNO')['weights_val'].
 ↪shift(1)
df_weights['weights_eq_lag'] = df_weights.groupby('PERMNO')['weights_eq'].
 ↪shift(1)

# Process final portfolio weights
df_weights = df_weights.drop(['rank', 'decile', 'weights_eq', 'TMV',␣
 ↪'weights_val'], axis=1)
df_weights
```

[23]:

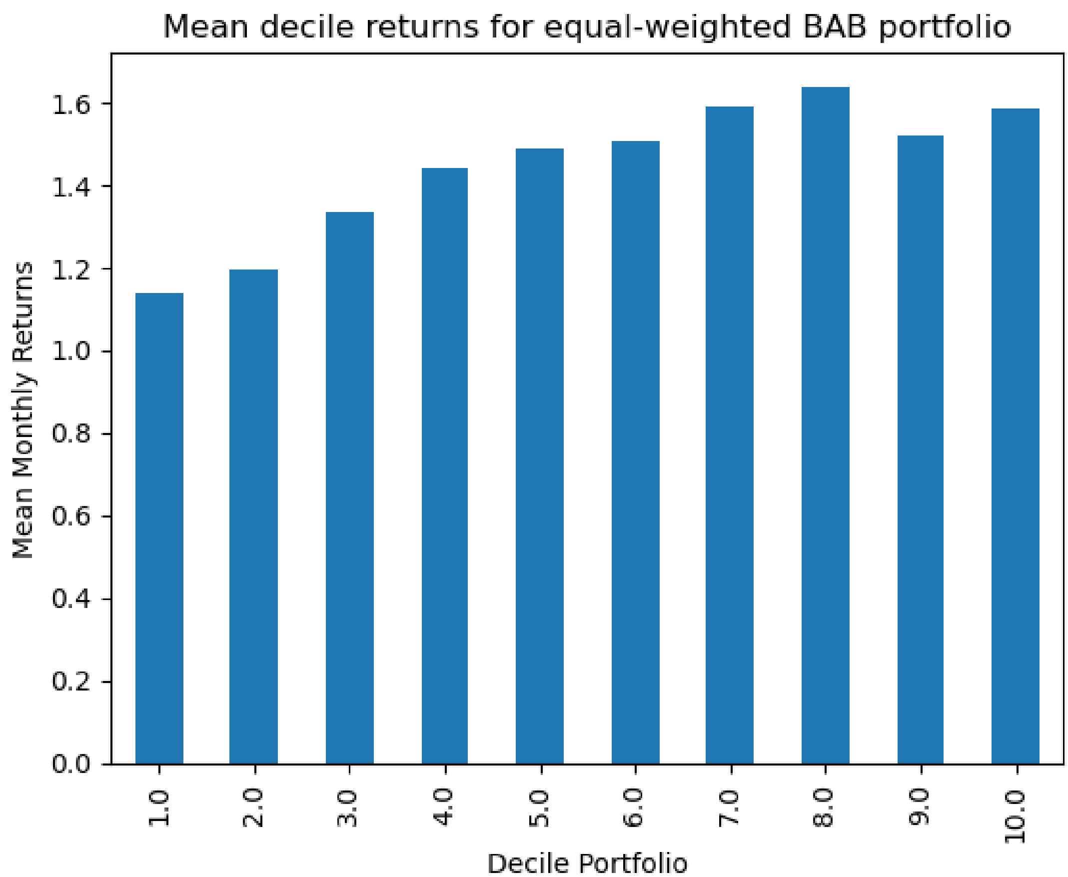
```
          PERMNO     date       RET           MV  Mkt-RF   Ret-RF      beta  \
147437     10001  1990-06  0.014103   10052.2500   -1.09   0.7803  0.025533
147438     10003  1990-06 -0.178571   12615.5000   -1.09 -18.4871  0.883344
147442     10020  1990-06 -0.105357  254355.7500   -1.09 -11.1657  0.186161
147444     10026  1990-06 -0.043478   68763.7500   -1.09  -4.9778  1.741824
147447     10034  1990-06 -0.011111   64013.2500   -1.09  -1.7411  1.057335
…            …       …         …            …       …        …
2853546    84180  1984-12  0.045455   22302.8125    1.84   3.9055  1.922804
2853548    84241  1984-12  0.000000   21850.0000    1.84  -0.6400  1.860227
2853549    85033  1984-12  0.000000   80750.0000    1.84  -0.6400  3.746187
```

```
2853550   85762  1984-12  0.093750  110687.5000    1.84   8.7350  1.295810
2853551   89552  1984-12 -0.027778   45543.7500    1.84  -3.4178  0.789196

         decile_lag  weights_val_lag  weights_eq_lag
147437          NaN              NaN             NaN
147438          NaN              NaN             NaN
147442          NaN              NaN             NaN
147444          NaN              NaN             NaN
147447          NaN              NaN             NaN
…             …               …              …
2853546         9.0         0.000277        0.003484
2853548         9.0         0.000283        0.003484
2853549        10.0         0.001482        0.003472
2853550         7.0         0.000976        0.002770
2853551         3.0         0.000193        0.003484

[2191529 rows x 10 columns]
```

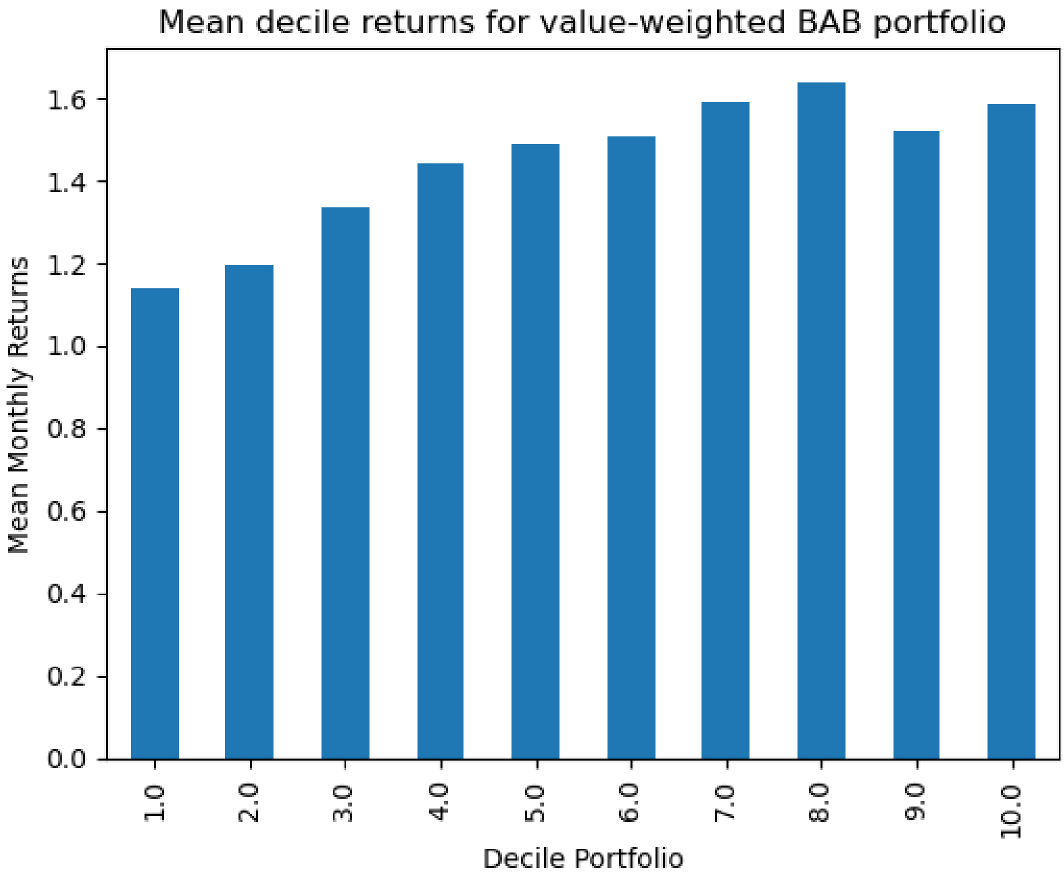
1.5.2 Part B

[24]:
```
eq_decile_returns, val_decile_returns = part_b(df_weights)

display(eq_decile_returns.mean(numeric_only=True))
graph_deciles(eq_decile_returns, 'Mean decile returns for equal-weighted BAB␣
↪portfolio')

display(val_decile_returns.mean(numeric_only=True))
graph_deciles(eq_decile_returns, 'Mean decile returns for value-weighted BAB␣
↪portfolio')
```

```
decile_lag
1.0     1.139108
2.0     1.193500
3.0     1.332549
4.0     1.440455
5.0     1.489458
6.0     1.507843
7.0     1.591752
8.0     1.637702
9.0     1.520740
10.0    1.585494
dtype: float64
```




```
decile_lag
1.0     0.811529
2.0     0.878040
3.0     0.966308
4.0     1.104858
5.0     1.056267
6.0     1.050534
7.0     1.182881
8.0     1.108010
9.0     1.075136
10.0    1.266046
dtype: float64
```



Neither the mean equal-weighted BAB portfolio returns nor the mean value-weighted BAB portfolio returns are perfectly monotonic, though they follow a general upward trend with a plataeu around the 6th decile. This makes sense; as the market has gone up on average from 1926, stocks with higher beta would have higher returns on average.

1.5.3 Part C

[25]:
```
analyze(eq_decile_returns, 'Equal-weighted BAB', 'BAB')
analyze(val_decile_returns, 'Value-weighted BAB', 'BAB')
```

```
Equal-weighted BAB monthly returns have mean -0.4463859627601404%, vol
8.364446592723462%, and Sharpe -0.05336706473186975
Value-weighted BAB monthly returns have mean -0.4545175762216238%, vol
8.4328931069834%, and Sharpe -0.05389817829485248
```

1.5.4 Part D

[26]:
```python
print('Equal-Weighted Betting-Against-Beta:')
print('--------------------------------------------------------------------------------')
estimate_capm_and_ff3(eq_decile_returns, 'BAB', ff3)
estimate_ff5(eq_decile_returns, 'BAB', ff5, add_momentum=True,␣
↪mom_rets=eq_mom_returns)

print('\n\n\n\nValue-Weighted Betting-Against-Beta:')
print('--------------------------------------------------------------------------------')
estimate_capm_and_ff3(val_decile_returns, 'BAB', ff3)
estimate_ff5(val_decile_returns, 'BAB', ff5, add_momentum=True,␣
↪mom_rets=val_mom_returns)
```

```
Equal-Weighted Betting-Against-Beta:
--------------------------------------------------------------------------------
--
CAPM
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.538
Model:                            OLS   Adj. R-squared:                  0.537
Method:                 Least Squares   F-statistic:                     1275.
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          8.20e-186
Time:                        00:23:34   Log-Likelihood:                -3464.1
No. Observations:                1098   AIC:                             6932.
Df Residuals:                    1096   BIC:                             6942.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0673      0.173      0.390      0.697      -0.272       0.406
Mkt-RF        -1.1476      0.032    -35.701      0.000      -1.211      -1.085
==============================================================================
Omnibus:                      543.422   Durbin-Watson:                   1.814
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6826.466
Skew:                          -1.961   Prob(JB):                         0.00
Kurtosis:                      14.569   Cond. No.                         5.42
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF3
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.670
```

```
Model:                            OLS   Adj. R-squared:                  0.669
Method:                 Least Squares   F-statistic:                     739.3
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          1.56e-262
Time:                        00:23:34   Log-Likelihood:                -3279.5
No. Observations:                1098   AIC:                             6567.
Df Residuals:                    1094   BIC:                             6587.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.2418      0.147      1.650      0.099      -0.046       0.529
Mkt-RF        -0.9033      0.030    -30.499      0.000      -0.961      -0.845
SMB           -0.8305      0.048    -17.257      0.000      -0.925      -0.736
HML           -0.4648      0.042    -10.959      0.000      -0.548      -0.382
==============================================================================
Omnibus:                      390.398   Durbin-Watson:                   1.885
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3758.299
Skew:                          -1.354   Prob(JB):                         0.00
Kurtosis:                      11.650   Cond. No.                         5.72
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF5
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.610
Model:                            OLS   Adj. R-squared:                  0.607
Method:                 Least Squares   F-statistic:                     203.0
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          5.17e-130
Time:                        00:23:34   Log-Likelihood:                -1815.9
No. Observations:                 654   AIC:                             3644.
Df Residuals:                     648   BIC:                             3671.
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.3256      0.159     -2.043      0.041      -0.638      -0.013
Mkt-RF        -0.7074      0.040    -17.651      0.000      -0.786      -0.629
SMB           -0.5994      0.055    -10.848      0.000      -0.708      -0.491
HML            0.2353      0.073      3.215      0.001       0.092       0.379
RMW            0.5455      0.076      7.167      0.000       0.396       0.695
CMA            0.1648      0.113      1.456      0.146      -0.058       0.387
==============================================================================
Omnibus:                      254.102   Durbin-Watson:                   1.941
```

```
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2380.543
Skew:                          -1.463   Prob(JB):                         0.00
Kurtosis:                      11.877   Cond. No.                         5.08
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF5 + Momentum
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.703
Model:                            OLS   Adj. R-squared:                  0.700
Method:                 Least Squares   F-statistic:                     255.3
Date:                Mon, 22 Apr 2024   Prob (F-statistic):         7.09e-167
Time:                        00:23:34   Log-Likelihood:                -1727.1
No. Observations:                 654   AIC:                             3468.
Df Residuals:                     647   BIC:                             3500.
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.5475      0.140     -3.908      0.000      -0.823      -0.272
Mkt-RF        -0.6623      0.035    -18.835      0.000      -0.731      -0.593
SMB           -0.5430      0.048    -11.209      0.000      -0.638      -0.448
HML            0.3715      0.065      5.745      0.000       0.245       0.498
RMW            0.3913      0.067      5.808      0.000       0.259       0.524
CMA            0.0534      0.099      0.538      0.591      -0.141       0.248
MOM            0.2574      0.018     14.210      0.000       0.222       0.293
==============================================================================
Omnibus:                       48.883   Durbin-Watson:                   1.898
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              143.963
Skew:                          -0.328   Prob(JB):                     5.48e-32
Kurtosis:                       5.203   Cond. No.                         8.43
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.




Value-Weighted Betting-Against-Beta:
--------------------------------------------------------------------------------
--
CAPM
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.543
Model:                            OLS   Adj. R-squared:                  0.543
Method:                 Least Squares   F-statistic:                     1305.
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          8.32e-189
Time:                        00:23:34   Log-Likelihood:                -3466.1
No. Observations:                1098   AIC:                             6936.
Df Residuals:                    1096   BIC:                             6946.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0697      0.173      0.403      0.687      -0.270       0.409
Mkt-RF        -1.1632      0.032    -36.119      0.000      -1.226      -1.100
==============================================================================
Omnibus:                      208.198   Durbin-Watson:                   1.862
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1185.641
Skew:                          -0.743   Prob(JB):                    3.48e-258
Kurtosis:                       7.869   Cond. No.                         5.42
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF3
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.612
Model:                            OLS   Adj. R-squared:                  0.611
Method:                 Least Squares   F-statistic:                     575.7
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          1.93e-224
Time:                        00:23:34   Log-Likelihood:                -3376.5
No. Observations:                1098   AIC:                             6761.
Df Residuals:                    1094   BIC:                             6781.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1532      0.160      0.957      0.339      -0.161       0.467
Mkt-RF        -1.0065      0.032    -31.108      0.000      -1.070      -0.943
SMB           -0.7186      0.053    -13.670      0.000      -0.822      -0.615
HML           -0.0926      0.046     -1.999      0.046      -0.184      -0.002
==============================================================================
Omnibus:                      192.058   Durbin-Watson:                   1.904
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1764.380
```

```
Skew:                          -0.510   Prob(JB):                         0.00
Kurtosis:                       9.126   Cond. No.                         5.72
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF5
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.597
Model:                            OLS   Adj. R-squared:                  0.594
Method:                 Least Squares   F-statistic:                     192.2
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          2.10e-125
Time:                        00:23:34   Log-Likelihood:                -1934.0
No. Observations:                 654   AIC:                             3880.
Df Residuals:                     648   BIC:                             3907.
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.5084      0.191     -2.664      0.008      -0.883      -0.134
Mkt-RF        -0.7893      0.048    -16.441      0.000      -0.884      -0.695
SMB           -0.6070      0.066     -9.172      0.000      -0.737      -0.477
HML            0.3649      0.088      4.161      0.000       0.193       0.537
RMW            0.6802      0.091      7.461      0.000       0.501       0.859
CMA            0.3662      0.136      2.700      0.007       0.100       0.633
==============================================================================
Omnibus:                      129.386   Durbin-Watson:                   1.900
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              740.879
Skew:                          -0.743   Prob(JB):                    1.32e-161
Kurtosis:                       7.998   Cond. No.                         5.08
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
FF5 + Momentum
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.624
Model:                            OLS   Adj. R-squared:                  0.621
Method:                 Least Squares   F-statistic:                     179.1
Date:                Mon, 22 Apr 2024   Prob (F-statistic):          6.65e-134
Time:                        00:23:35   Log-Likelihood:                -1911.4
No. Observations:                 654   AIC:                             3837.
Df Residuals:                     647   BIC:                             3868.
```

```
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.7502      0.188     -3.992      0.000      -1.119      -0.381
Mkt-RF        -0.7201      0.048    -15.154      0.000      -0.813      -0.627
SMB           -0.6105      0.064     -9.540      0.000      -0.736      -0.485
HML            0.4800      0.086      5.553      0.000       0.310       0.650
RMW            0.5872      0.089      6.583      0.000       0.412       0.762
CMA            0.2845      0.132      2.160      0.031       0.026       0.543
MOM            0.1483      0.022      6.801      0.000       0.105       0.191
==============================================================================
Omnibus:                       62.838   Durbin-Watson:                   1.870
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              269.473
Skew:                          -0.310   Prob(JB):                     3.05e-59
Kurtosis:                       6.083   Cond. No.                         9.65
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

The CAPM and FF3 alphas are generally small and positive, but the alphas become negative when using FF5 and decrease even more when using FF5 + Momentum. Perhaps this can be explained in part due to more sophisticated models like FF5 and FF5 + Momentum taking into account sources of risk that are unaccounted for in simpler models such as CAPM (CAPM may mistakenly incorporate returnes due to this risk as alpha instead of beta). With the exception of CAPM, the equal-weighted portfolio has the higher alphas.

1.5.5 Part E

Fundamentally, decreasing the proportion of volatility to returns in this strategy requires the use of better hedging and diversification techniques. One thing we might do is minimize our holdings in assets that are highly correlated with one another, diversifying our portfolio by replacing them with assets of a desired beta that have lower correlation with the rest of our portfolio. If we diversify some of our assets but maintain the same or similar beta deciles, we can decrease our volatility and keep our returns roughly constant, resulting in a better Sharpe Ratio.

[ ]: