Contents

"Smack my pitch up."

The Prodigy (1997)

1 Optimal control of pitch/travel without feedback

1.1 State space form

We want to write the model in equation 1.1.1 in continuous time state space form with $x = [\lambda \ r \ p \ \dot{p}]^T$ and $u = p_c$

$$\dot{x} = A_c x + B_c u \tag{1.1.1}$$

1.2 Model discussion

The states are travel, travel rate, pitch, and pitch rate. The controller output is the pitch setpoint used by the given controller which in turn calculates voltage inputs for the plant. Thus, we are modelling not the helicopter alone, but a system that consists of the helicopter along with the given controller. This corresponds with Figure 7 (proper ref here?) in the exercise text (add ref).

"I've got 99 problems, but a pitch ain't one."

Ice-T (1993)

2 Optimal control of pitch/travel with LQ control

3 Optimal control of pitch/travel and elevation with and without feedback

4 Pastebin (remove before handing in lol)

4.1 Copied source for system description equations

$$\ddot{e} + K_3 K_{ed}\dot{e} + K_3 K_{ep} e = K_3 K_{ep} e_c \tag{4.1.1a}$$
$$\ddot{p} + K_1 K_{pd}\dot{p} + K_1 K_{pp} p = K_1 K_{pp} p_c \tag{4.1.1b}$$
$$\dot{\lambda} = r \tag{4.1.1c}$$
$$\dot{r} = -K_2 p \tag{4.1.1d}$$

4.2 10.2.1

$$\mathbf{x} = \begin{bmatrix} \lambda & r & p & \dot{p} \end{bmatrix}^{\mathrm{T}} \tag{4.2.1a}$$
$$u = p_c \tag{4.2.1b}$$

$$\ddot{p} = K_1 K_{pp}(p_c - p) - K_1 K_{pd}\dot{p} \tag{4.2.2a}$$
$$\dot{r} = -K_a p \tag{4.2.2b}$$
$$\dot{\lambda} = r \tag{4.2.2c}$$
$$\dot{p} = \dot{p} \tag{4.2.2d}$$

$$K_1 = \frac{K_f l_n}{J_p} \tag{4.2.3a}$$
$$K_2 = \frac{K_p l_a}{J_t} \tag{4.2.3b}$$

$$\dot{\mathbf{x}} = \boldsymbol{A}_c + \boldsymbol{B}_u u \tag{4.2.4}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -K_2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -K_1 K_{pp} & -K_1 K_{pd} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_1 K_{pp} \end{bmatrix} p_c \tag{4.2.5}$$

4.3 10.2.2

$$
\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{x}_k + \left(\boldsymbol{A}_c \mathbf{x}_k + \boldsymbol{B}_c u_k\right) h && \text{(4.3.1a)} \\
&= \mathbf{x}_k + h\boldsymbol{A}_c \mathbf{x}_k + h\boldsymbol{B}_c u_k && \text{(4.3.1b)} \\
&= \left(\mathbf{I} + h\boldsymbol{A}_c\right) \mathbf{x}_k + h\boldsymbol{B}_c u_k && \text{(4.3.1c)} \\
&= \boldsymbol{A}\mathbf{x}_k + \boldsymbol{B} u_k && \text{(4.3.1d)}
\end{aligned}
$$

$$
\boldsymbol{A} = \mathbf{I} + h\boldsymbol{A}_c = \begin{bmatrix} 1 & h & 0 & 0 \\ 0 & 1 & -K_2 h & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & -K_1 K_{pp} h & 1 - K_1 K_{pd} h \end{bmatrix} \tag{4.3.2a}
$$

$$
\boldsymbol{B} = h\boldsymbol{B}_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_1 K_{pp} h \end{bmatrix} \tag{4.3.2b}
$$