chap 3.

2D Geometric Transformations.

* Transformation

- The process of changing the scale, shape or position of the object is called Transform.

  - Translation
  - Scaling
  - Rotation.

* Matrix Representation & Homogeneous Coordina

① Translation Homogeneous Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$x' = x + tx$
$y' = y + ty$
$1 = 1$

② Rotation –

x-axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

y-axis

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

z-axis

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

③ Scaling

$$\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$x' = x \cdot Sx$ $y' = y \cdot Sy$

* Translation

- Shifting of an object along a straight path.
- It does not alter the shape or size of the object. It just moves the entire object from one location to another location along a straight path.

$$T = [t_x, t_y]$$

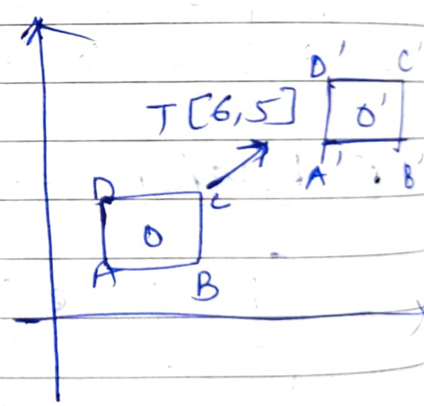
Translation operation -

$$x' = x + t_x$$
$$y' = y + t_y$$

where (x, y) is the original point, $[t_x, t_y]$ is shift vector & (x', y') is translated point.

$$P' = T + P$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$



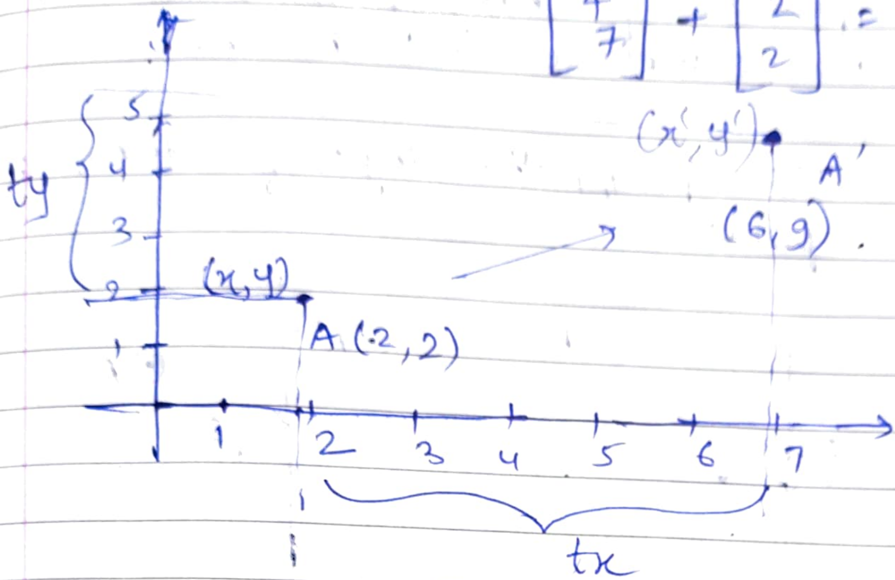
Ex:- Translate a ~~line~~ pixel with co-ordinates A (2,2) with translation vector [4 7].

$$T = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$$\boxed{P' = T + P}$$

$\therefore A' = T + A$

$$= \begin{bmatrix} 4 \\ 7 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$



ex:- consider the line with endpoints $A(x_1, y_1)$ & $B(x_2, y_2)$

$\therefore x_1' = t_x + x_1 \qquad x_2' = t_x + x_2$

$y_1' = t_y + y_1 \qquad y_2' = t_y + y_2$

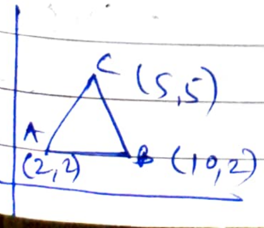
$$\boxed{P' = T \cdot P}$$

Homogeneous Representation of transformation would be

$$P' = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \\ 1 & 1 \end{bmatrix}$$

1) $A(2,2)$, $B(10,2)$ & $C(5,5)$ Translate the triangle with $t_x = 5$ & $t_y = 6$

→



$\boxed{P' = P + T}$

$A' = A + T$

$A' = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

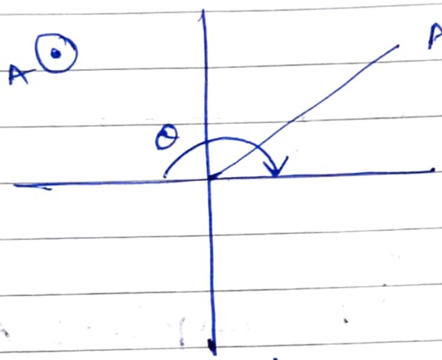
$B' = \begin{bmatrix} 10 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 15 \\ 8 \end{bmatrix}$

$C' = \begin{bmatrix} 5 \\ 5 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \end{bmatrix}$

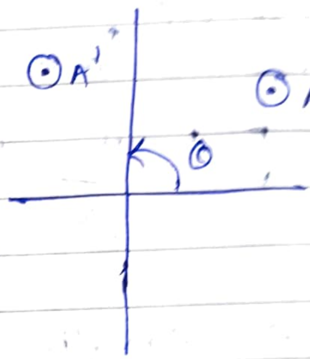
* Rotation

- It is a process of changing ~~to~~ of the object. It can be clockwise or anticlockwise.

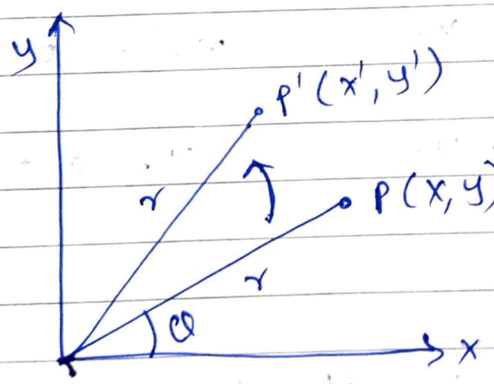
- We have to specify the angle of rotation & rotation point.



Anticlockwise



clockwise



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x = r\cos\theta$

$y = r\sin\theta$

$x' = x\cos\theta - y\sin\theta$

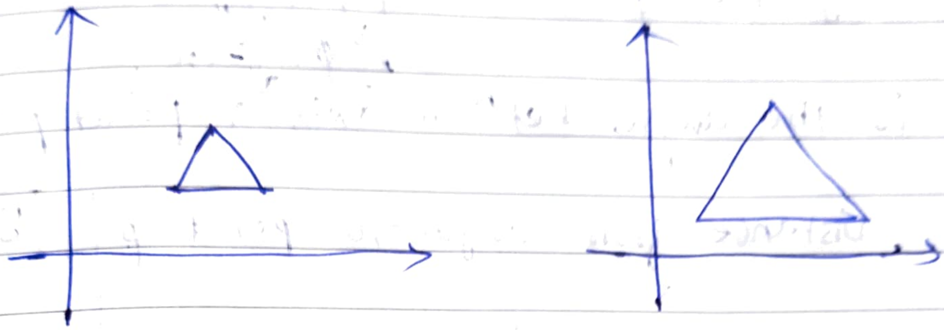
$y' = x\sin\theta + y\cos\theta$

$\boxed{P' = R \cdot P}$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

* Scaling

- It is used to alter or change the size of objects. Change is done using scaling factors.



$$x' = S_x \cdot x$$
$$y' = S_y \cdot y$$

multiplying the vertex coordinates by scaling parameters S_x & S_y.

$$\boxed{P' = S \cdot P}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

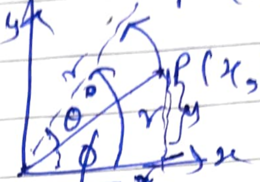
Anticlockwise direction (+ve)
clockwise direction (−ve)

(2) Rotation

Let consider of P in the polar. (Anticlockwise

$x = r\cos\phi$

$y = r\sin\phi$



$\cos\phi = \frac{x}{r}$

$\boxed{x = r\cos}$

ϕ is the angle betⁿ x axis & point P

$\sin\phi = \frac{y}{r}$

$\boxed{y = r\sin\theta}$

r = Distance from origin to point P

$\therefore x' = r\cos(\phi + \theta)$

$y' = r\sin(\phi + \theta)$

$x' = r[\cos\phi\cos\theta - \sin\phi\sin\theta]$

$= r\cos\phi\cos\theta - r\sin\phi\sin\theta$

$x = r\cos\phi$

$y = r\sin\phi$

$\therefore \boxed{x' = x\cos\theta - y\sin\theta}$

$y' = r[\sin\phi\cos\theta + \cos\phi\sin\theta]$

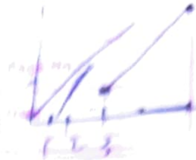
$= r\sin\phi\cos\theta + r\cos\phi\sin\theta$

$\therefore \boxed{y' = y\cos\theta + x\sin\theta}$

$\hookrightarrow y' = x\sin\theta + y\cos\theta$

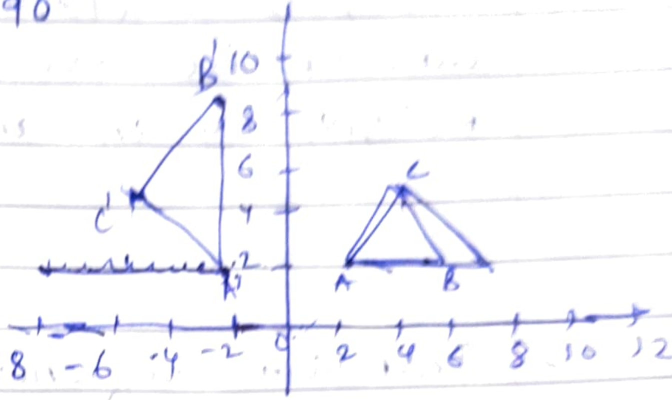
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{P' = R \cdot P}$$



Ex:- One triangle is given (2,2) (8,2) (5,5)
Rotate the triangle 90°

→ $\theta = 90°$



$$R = \begin{bmatrix} \cos 90° & -\sin 90° \\ \sin 90° & \cos 90° \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$A' = R \cdot A$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 8 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$$

$$C' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

* **Clockwise** (-ve direction)

$$x' = x\cos(-\theta) - y\sin(-\theta) \Rightarrow \boxed{x\cos(\theta) + y\sin\theta}$$

$$y' = x\sin(-\theta) + y\cos(-\theta) \Rightarrow \boxed{-x\sin\theta + y\cos\theta}$$

$\cos\theta$ Always +ve

Q1 consider a square P(0,0), Q(0,10) R(10,10) S(10,0). Rotate the square anticlockwise about fixed point R(10,10) by an angle 45°.

→

1. Translate refrence point R to the origin
2. Perform a rotation by 45° in anticlockwise direction.
3. Inverse translation of point R.

$$\boxed{P' = R \cdot P}$$

Reference point (10,10)

$$P' = \begin{bmatrix} 1 & 0 & 10(tx) \\ 0 & 1 & 10(ty) \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 10 & 10 \\ 0 & 10 & 10 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Multiply first two & last two matrixes

$$= \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.70 & -0.70 & 0 \\ 0.70 & 0.70 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

10 10

$$= \begin{bmatrix} 0.70 & -0.70 & 10 \\ 0.70 & 0.70 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -10 & -10 & 0 & 0 \\ -10 & 0 & 0 & -10 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 10 & 2.91 & 10 & 17.09 \\ -4.18 & 2.91 & 10 & 2.91 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

original co-ordinates	Transformed co-ordinates.
P (0,0)	P' (10, -4.18)
Q (0,10)	Q' (2.91, 2.91)
R (10,10)	R' (10, 10)
S (10,0)	S' (17.09, 2.91)

Q.2 consider a triangle with vertices A(1,1) B(5,2) & C(3,4) find out the transformation matrix which rotates given triangle about point C(3,4) by an angle 30° clockwise. find the co-ordinates of the rotated triangle.

→ 30° clockwise (-ve direction)

Ref point (3,4)

$$P' = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 30 & \sin 30 & 0 \\ -\sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & 1/2 & \frac{2-3\sqrt{3}}{2} \\ -1/2 & \frac{\sqrt{3}}{2} & \frac{11-4\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P' = M \cdot P = \begin{bmatrix} \frac{3-2\sqrt{3}}{2} & \frac{2\sqrt{3}+4}{2} & 3 \\ \frac{10-3\sqrt{3}}{2} & \frac{6-2\sqrt{3}}{2} & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

original co-ordinates	Transformed co-ordinates
$A(1,1)$	$A'\left(\frac{3-2\sqrt{3}}{2}, \frac{10-3\sqrt{3}}{2}\right)$
$B(5,2)$	$B'\left(\frac{2\sqrt{3}+4}{2}, \frac{6-2\sqrt{3}}{2}\right)$
$C(3,4)$	$C'(3,4)$

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* Scaling

$x' = S_x \cdot x$

$y' = S_y \cdot y$

$$\boxed{P' = S \cdot P}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

S_x S_y are any +ve values.

ex:- $A(1,1)$ & $B(1,4)$

$S_x = 3$, $S_y = 2$

$$A' = S \cdot A = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$B' = S \cdot B = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$

* Scaling with Respect to Reference point.

1. Translate the ref point to origin

$$T = \begin{bmatrix} -x_r \\ -y_r \end{bmatrix}$$

2. Apply scaling on the translated obj.

$$S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

3) Translate ref point back to its actual location.

$$T^{-1} = \begin{bmatrix} x_r \\ y_r \end{bmatrix}$$

triangle

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(2) scale a triangle with vertices A(2, 2) B(6, 2) & C(4, 4) with respect to a ref point (3, 4) with $S_x = 2$ & $S_y = 3$

→ Scaling is with respect to a ref point.

First translate to origin
then apply scaling
then translate back

$$x' = S_x \cdot x + x_r(1 - S_x)$$

$$y' = S_y \cdot y + y_r(1 - S_y)$$

$x_r(1 - S_x)$ & $y_r(1 - S_y)$ are constant for all points.

$$x_r(1 - S_x) = 3(1 - 2) = -3$$
$$y_r(1 - S_y) = 4(1 - 3) = -8$$

$$A' = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ -8 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} + \begin{bmatrix} -3 \\ -8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$B' = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ -8 \end{bmatrix} = \begin{bmatrix} 9 \\ -2 \end{bmatrix}$$

$$C' = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix} + \begin{bmatrix} -3 \\ -8 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

original coordinates	Transformed co-ordinates
A (2, 2)	A' (1, -2)
B (6, 2)	B' (9, -2)
C (4, 4)	C' (5, 4)

Q. scaling on triangle (1, 1) (8, 1) & (1, 9) with scaling factor 2 in both x & y direction.

→ $P' = S \cdot P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 8 & 1 \\ 1 & 1 & 9 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 16 & 2 \\ 2 & 2 & 18 \\ 1 & 1 & 1 \end{bmatrix}$$

original	
A (1, 1)	A' (2, 2)
B (8, 1)	B' (16, 2)
C (1, 9)	C' (2, 18)

Q. Scale a triangle $A(4,4)$ $B(12,4)$ & $C(8,10)$ with scaling factor $S_x = 2$ & $S_y = 1$ [Ex - may23]

* **Reflection**

- Produces a mirror image of an object about a given axis.
- Reflection is achieved by rotation an obj by 180° around ref axis, perpendicular to the xy plane.

- It does not alter the shape & size of the object

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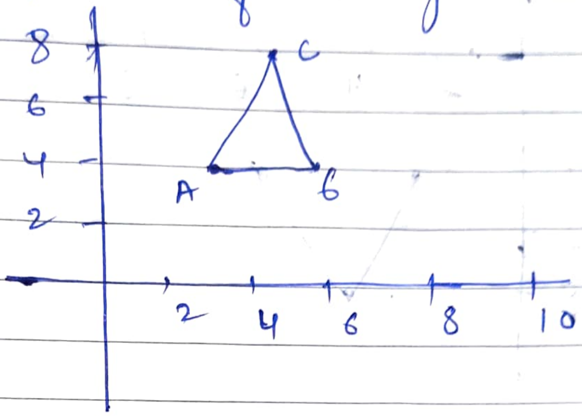
Ex:- A triangle ABC is given A (3,4) B (6,4) C (4,8) find Reflected position of triangle. to the x-axis



→ Ref to the x-axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A point after Reflection.

$$(x, y) = (3, 4) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [3, -4]$$

B pt

$$(x, y) = (6, 4) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [6, -4]$$

C pt

$$(x, y) = (4, 8) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [4, -8]$$

* Ref about origin

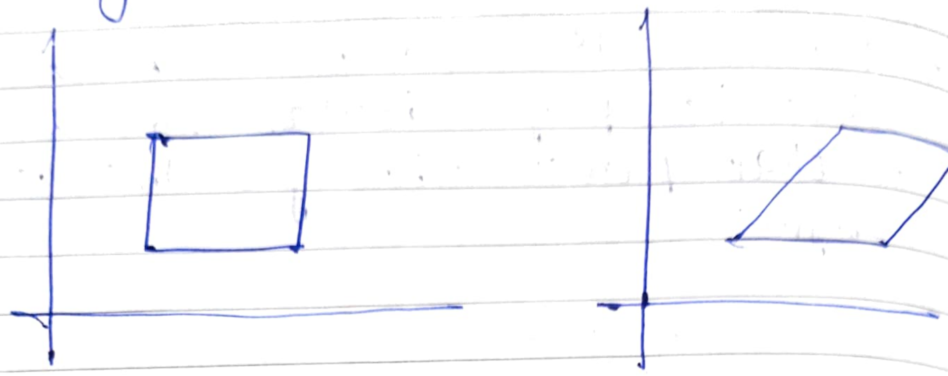
- This is 180° rotation about an axis that is perpendicular to the xy plane & that passes through the coordinate origin.

$$x' = -x$$
$$y' = -y$$

$$\text{Ref (origin)} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Shearing

x-direction



$$\boxed{x' = x + Shx \cdot y}$$

Shx is shear parameter in x-direction & y is hight of the internal layer from the base.

$$Shx = \begin{bmatrix} 1 & Shx & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{P' = SHx \cdot P}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Shx & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

ex: Apply shearing on cube with vertices A(0,0) B(2,0), C(2,2) & D(0,2) with $Shx = 3$

→ For x-direction shearing with $y = 0$ as ref line

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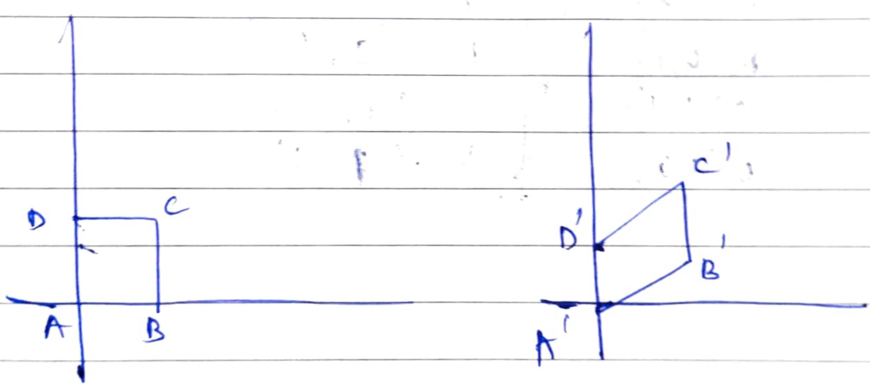
$x' = x + Sh_x \cdot y$
$y' = y$

$A' = 0 + 3(0) = 0$ $(0, 0)$
$B' = 2 + 3(0) = 2$ $(2, 0)$
$C' = 2 + 3(2) = 8$ $(2, 2)$
$D' = 0 + 3(2) = 6$ $(0, 2)$

A (0, 0)	A' (0, 0)
B (2, 0)	B' (2, 0)
C (2, 2)	C' (8, 0)
D (0, 2)	D' (6, 0)

y - direction



$x' = x$
$y' = y + x \cdot sh_y$

$$SH_y = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ex:- Apply y direction shearing on unit square with the base on $x=0$ line & one corner at the origin with $sh_y = 1/2$
$A(0,0)$ $B(1,0)$, $C(1,1)$ & $D(0,1)$. For y direction shear:

→ $x' = x$

$y' = y + x \cdot sh_y$

$A' = 0 + (0) 1/2 = 0$

$B' = 0 + (1) 1/2 = 1/2$

$C' = 1 + (1) 1/2 = 3/2$

$D' = 1 + (0) 1/2 = 1$

$A(0,0)$	$A'(0,0)$
$B(0,1)$	$B'(0, 1/2)$
$C(1,1)$	$C'(1, 3/2)$
$D(1,0)$	$D'(0, 1)$

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Q. Translate ABCD with co-ordinates A (0,0) B(5,0) C (5,5) D (0,5) by 2 units x-direc & 3 unit in y-direc

$$P' = M \cdot P = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} - t_x \\ - t_y \\ \ \end{matrix} \begin{bmatrix} 0 & 5 & 5 & 0 \\ 0 & 0 & 5 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 2 & 7 & 7 & 2 \\ 3 & 3 & 8 & 8 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

) Triangle (10,10) (40,10) (30,30) Apply scaling with scale factor 5 in x & y direc^n Draw triangle

⇒ $S_x = S_y = 5$

$$P' = S \cdot P = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 40 & 30 \\ 10 & 10 & 30 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 50 & 200 & 150 \\ 50 & 50 & 150 \\ 1 & 1 & 1 \end{bmatrix}$$

3) Rotate a triangle ABC by 30°. where the triangle is A(0,0) B(10,2) & C(7,4)

$$P' = M \cdot P = R(\theta = 30) \times P = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 10 & 7 \\ 0 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{10\sqrt{3}-2}{2} & \frac{7\sqrt{3}-4}{2} \\ 0 & \frac{10+2\sqrt{3}}{2} & \frac{7+4\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Q. ① Find transformatⁿ matrix to rotate the obj abt the origin by 45° in counter clockwise directⁿ.

② Find new co-ordinates of the point (8,4) after rotation.

$$\Rightarrow \quad M = \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 2\sqrt{2} \\ 6\sqrt{2} \end{bmatrix}$$

Q. Explain the steps for 2D reflection w.r.t. line $y = mx$ & also derive a composite transformation matrix. — Exam Dec 19

- Ref abt any arbitrary line $y = mx + c$ can be accomplished by the combinatⁿ of translatⁿ, ~~scaling~~ rotation & reflection.
- First translate the line which passes through origin.
- Then rotate the line so that it aligns with one of the principal axes.

- Perform reflection operation.
- Finally, restore the line to its actual position by performing inverse rotation & inverse translatn

- The composite transformatn matrix for this seq of operatn is obtained by multiplying following matrices.

$$M = T^{-1} \cdot R^{-1} \cdot Ref \cdot R \cdot T$$