

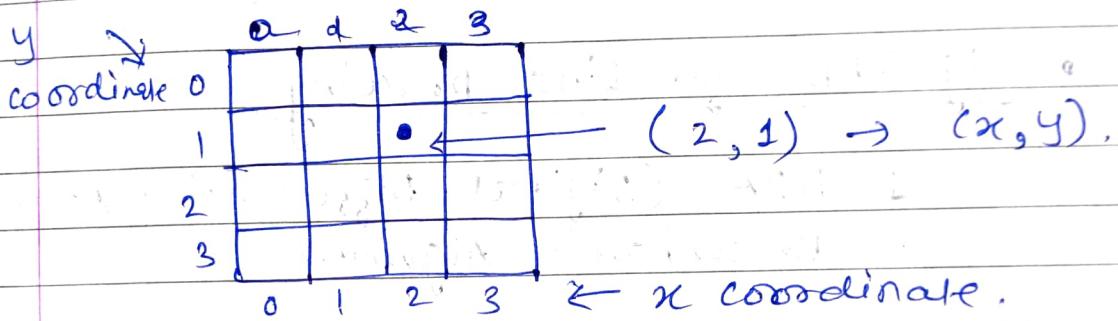
## Chap 2.

## Output Primitives

\* Scan conversion of point, line, circle & ellipse:

- Each pixel on the graphics display does not represent a mathematical point. Scan converting a point involves illuminating the pixel that contains the point.

Ex:- point  $p(x, y)$  represented by the integer part of  $x$  & the integer part of  $y$ .

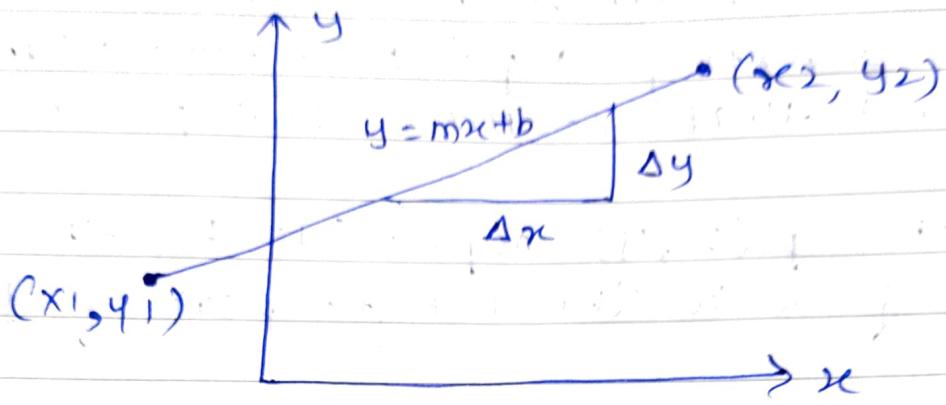


- Scan conversion is a process of representing graphics objects collection of pixels.
- The graphics objects are continuous. The pixels used are discrete. Each pixel can have either on or off state.
- The circuitry of the video display device of the computer is capable of converting binary values (0, 1) into a pixel on & off information.

Using this ability graphics Computer represent picture having discrete dots.

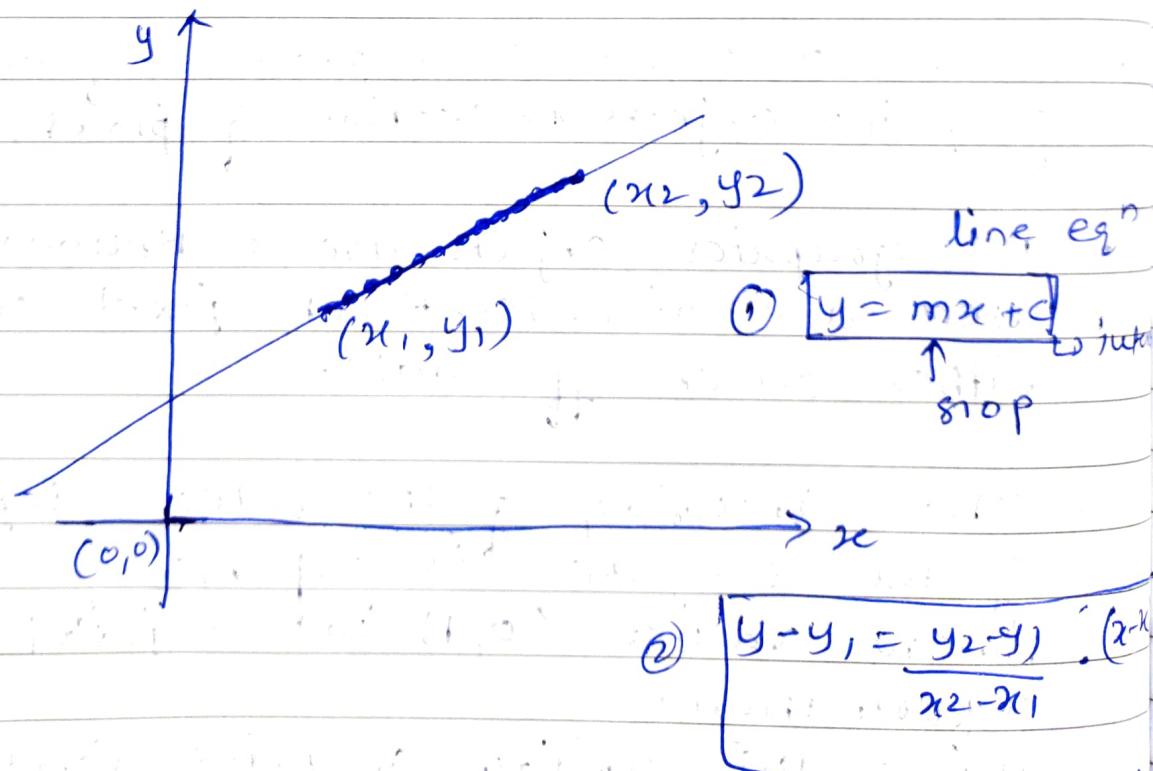
- Scan Conversion of Line

- A straight line may be defined by endpoints & an equation.  
The equation of the line is used to determine the x, y coordinates of all the points that lie between these two endpoints.



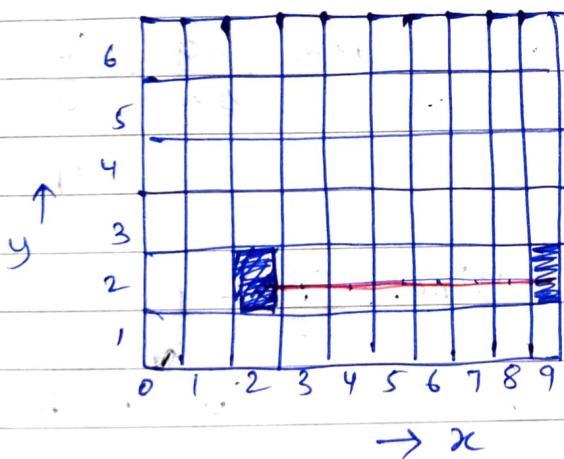
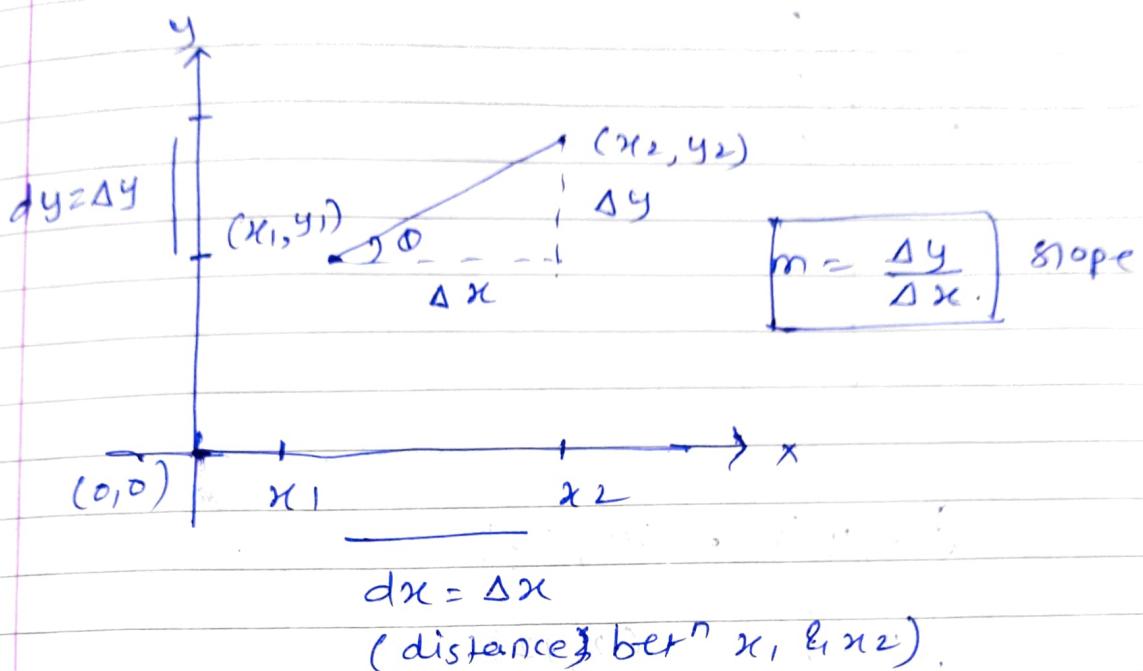
- Algorithm for Line Drawing

1. DDA (Digital Differential Analysis)
2. Bresenham's Algorithm.



## \* DDA Algorithm

- It is an incremental method of scan conversion of line.



$(x_1, y_1)$      $(x_2, y_2)$

$(2, 2)$      $(9, 2)$

$$\Delta x = 9 - 2 = 7$$

$$\Delta y = 2 - 2 = 0$$

$$m = \frac{\Delta y}{\Delta x} = \frac{0}{7}$$

Steps = 7

$$x_{\text{inc}} = \frac{7}{7} = 1$$

$$y_{\text{inc}} = \frac{0}{7} = 0$$

Inc by 1

↑ not inc.

x	y
2	2
3	2
4	2
5	2
6	2
7	2
8	2
9	2

$$\text{ex: } (x_1, y_1) = (x_2, y_2)$$

$$(2, 5) \rightarrow (2, 12)$$

$$\Delta x = 2 - 2 = 0$$

$$\Delta y = 12 - 5 = 7$$

$$m = \frac{\Delta y}{\Delta x} = \frac{7}{0} = \infty$$

steps = 7

(y val)

$$x_{\text{inc}} = \frac{0}{7} = 0$$

$$y_{\text{inc}} = \frac{7}{7} = 1$$

↑

x not inc.

↑

(y inc by 1)

x	y
2	5
2	6
2	7
2	8
2	9
2	10
2	11
2	12

$$\text{Ex: } (x_1, y_1) \quad (x_2, y_2)$$

$$(5, 4) \quad (12, 7)$$

$$\Delta x = 12 - 5 = 7$$

$$\Delta y = 7 - 4 = 3$$

$$m = \frac{\Delta y}{\Delta x} = \frac{3}{7} = 0.4$$

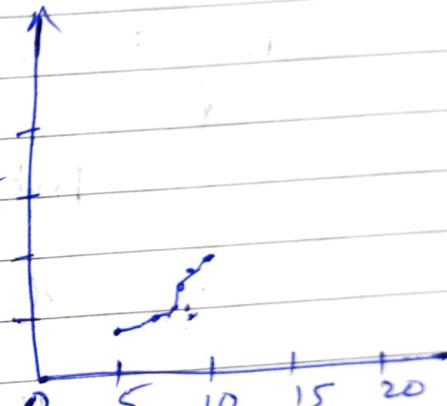
Steps 7

$$x_{\text{inc}} = \frac{7}{7} = 1$$

$$y_{\text{inc}} = \frac{3}{7} = 0.4$$

Here  $m < 1$  (slope)

x	y	round(y)
5	4	4
6	4.4	4
7	4.8	5
8	5.2	5
9	5.6	6
10	6	6
11	6.4	6
12	6.8	7



\* Algorithm DDA ( $x_1, y_1, x_2, y_2$ )

$$dx = x_2 - x_1$$

$$dy = y_2 - y_1$$

if ( $\text{abs}(dx) > \text{abs}(dy)$ )

$$\text{step} = \text{abs}(dx)$$

else

$$\text{step} = \text{abs}(dy)$$

$$x_{\text{inc}} = dx / \text{step}$$

$$y_{\text{inc}} = dy / \text{step}$$

for ( $i = 1; i < \text{step}; i++$ )

{

putpixel ( $x_1, y_1$ );

$$x_1 = x_1 + x_{\text{inc}}$$

$$y_1 = y_1 + y_{\text{inc}}$$

y

- Algorithm takes  $x_{\text{inc}}$  &  $y_{\text{inc}}$  in float calculation of floats take extra time than integer values. So Algo is slow.
- We are getting points which are not accurate one so we have to round off.
- Line will not smooth line.
- This is drawback of DDA Algo. So we go for Bresenham's Algorithm.

## \* (DDA) Algo (Practical)

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- It is a scan conversion method for drawing line.
- It has incremental approach.
- It analyses difference in pixel points.
- straight line equation is,  
 $y = mx + b$



if the line has two end points  
 $A(x_1, y_1)$  &  $B(x_2, y_2)$

slope of the line is  $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$\therefore m = \frac{\Delta y}{\Delta x}$$

- DDA based on calculation of values  $\Delta x, \Delta y$ .

$$\therefore \Delta y = m \Delta x \quad \text{--- (1)}$$

$$\Delta x = \frac{\Delta y}{m} \quad \text{--- (2)}$$

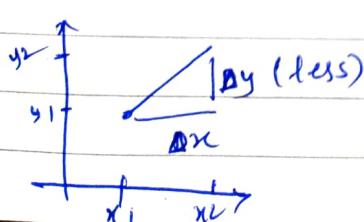
- Given line have the slope +ve or -ve

If +ve, the  $\Delta x$  &  $\Delta y$  values are increased else decreased.

case (1) if  $m < 1$

$$x_n = x_1 + 1$$

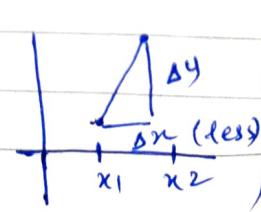
$$y_n = y_1 + m$$



(2) if  $m > 1$

$$x_n = x_1 + 1/m$$

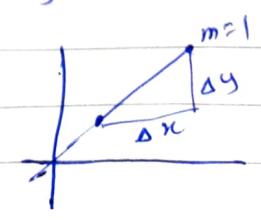
$$y_n = y_1 + 1$$



(3) if  $m = 1$

$$x_n = x_1 + 1$$

$$y_n = y_1 + 1$$



## DDA Algorithm

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- 1) Steps : Accept start & end point coordinates  $(x_1, y_1)$  &  $(x_2, y_2)$
- 2) calculate  $\Delta x = x_2 - x_1$   
 $\Delta y = y_2 - y_1$
- 3) if  $\text{abs}(\Delta x) > \text{abs}(\Delta y)$       ||  $m < 1$   
then  $K = \text{abs}(\Delta x)$   
else  
 $K = \text{abs}(\Delta y)$
- 4) calculate  $\Delta x = \frac{\Delta x}{K}$ ,  $\Delta y = \frac{\Delta y}{K}$
- 5) Initialize  $x = x_1$ ,  $y = y_1$
- 6) Display pixel  $(x, y)$
- 7)  $x = x + \Delta x$        $y = y + \Delta y$
- 8) Display pixel  $\text{round}(x)$ ,  $\text{round}(y)$
- 9) Repeat Step 7 & 8  $'K'$  times.

### Advantage

- faster method : when compared to direct use of line eq'n.

Q. Draw the line between  $(5, 5)$  &  $(10, 9)$  using DDA Algo.

$$\rightarrow (x_1, y_1) = (5, 5) \quad (x_2, y_2) = (10, 9)$$

$$dx = x_2 - x_1 = 10 - 5 = 5$$

$$dy = y_2 - y_1 = 9 - 5 = 4$$

$$(dx > dy) \rightarrow (5 > 4) \rightarrow K = dx$$

$$K = dx = 5$$

$$\Delta x = \frac{dx}{K} = \frac{5}{5} = 1$$

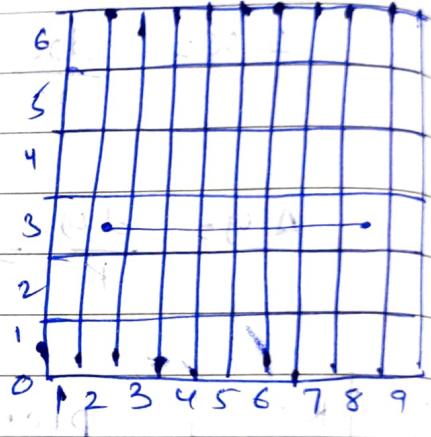
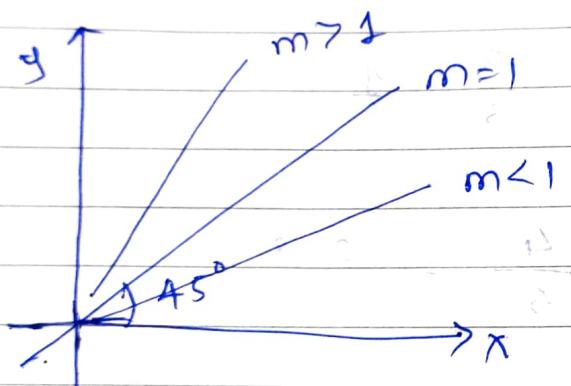
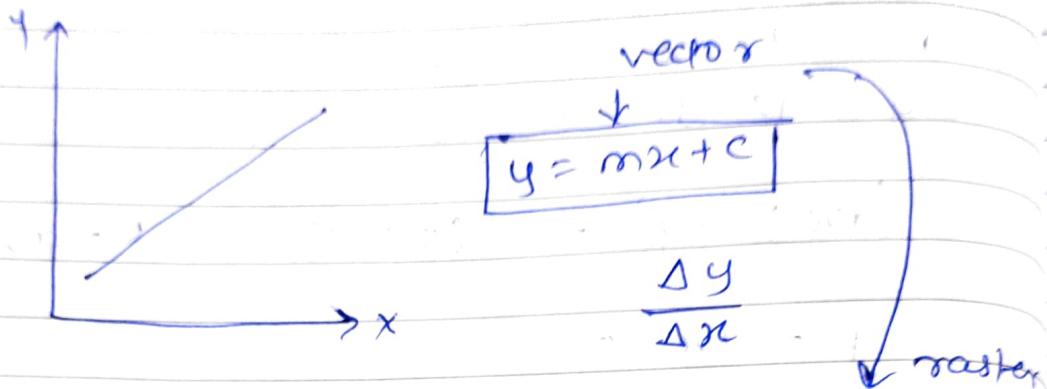
$$\Delta y = \frac{dy}{K} = \frac{4}{5} = 0.8$$

Step K=5 plot(round(x), round(y))  
 $x + \Delta x$        $y + \Delta y$

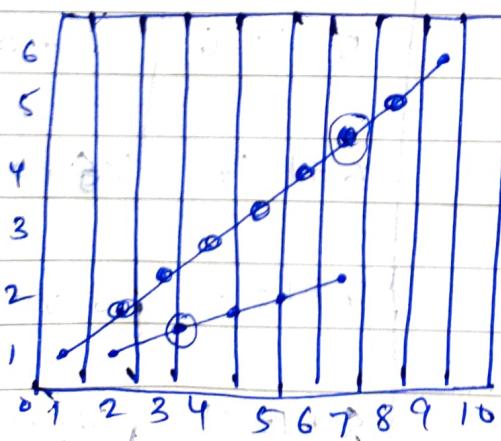
iteration	plot (x, y)	$x + \Delta x$	$y + \Delta y$
0	$(5, 5)$	$5 + 1 = 6$	$5 + 0.8 = 5.8$
1	$(6, 6)$	$6 + 1 = 7$	$5.8 + 0.8 = 6.8$
2	$(7, 7)$	$7 + 1 = 8$	$6.8 + 0.8 = 7.4$
3	$(8, 8)$	$8 + 1 = 9$	$7.4 + 0.8 = 8.2$
4	$(9, 8)$	$9 + 1 = 10$	$8.2 + 0.8 = 9$
5	$(10, 9)$	$10 + 1 = 11$	$9 + 0.8 = 9.8$

## \* Bresenham's Line Drawing Algorithm

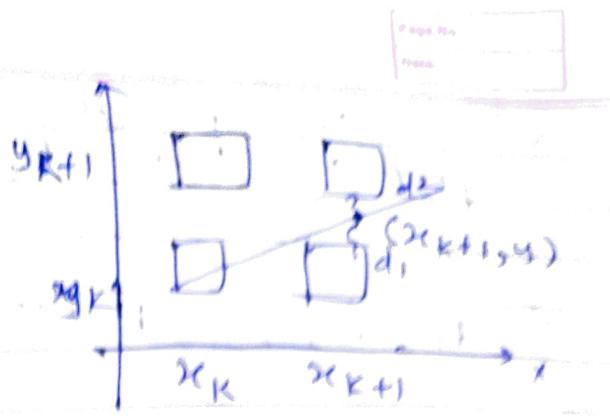
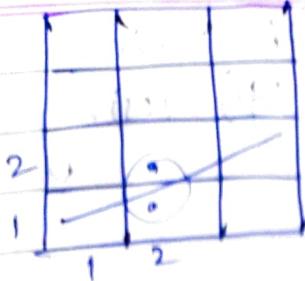
- Drawback of DDA algo i.e. floating point number (processing time is more)



• Converting  $y = mx + c$  into screen coordinates is called Rasterization.



Line is going partially to another pixel all



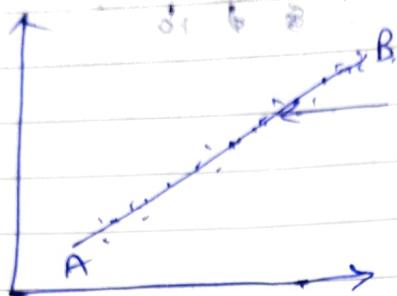
$$x_{inc} = x_{k+1}$$

$$y_{inc} = \begin{cases} y_k \\ y_{k+1} \end{cases}$$

parme | dan | (t, x<sub>k</sub>, y<sub>k</sub>) | fachanisti

1	1	(1, 1)	(1, 1)	0
2	1	t	(t, 1)	1
3	1	s	(t, s)	0
4	1	t	(t, t)	2
5	1	s	(t, s)	3
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289	1	s	(t, s)	287
290	1	t	(t, t)	288
291	1	s	(t, s)	289
292	1	t	(t, t)	290
293	1	s	(t, s)	291
294	1	t	(t, t)	292
295	1	s	(t, s)	293
296	1	t	(t, t)	294
297	1	s	(t, s)	295
298	1	t	(t, t)	296
299	1	s	(t, s)	297
300	1	t	(t, t)</td	

- \* Bresenham's Line Drawing Algo.
- It determines the point of an n-dimensional raster that should be selected in order to form a close approximation to a straight line between two points.



It will take close approximation of pixel.

$$\rightarrow A(x_1, y_1) \quad B(x_2, y_2)$$

- It is efficient because it only takes integer addn, sub, & multiplication.
- Operations performed rapidly

- Algorithm

1) cal start & end coordinates.

$$(x_1, y_1) \& (x_2, y_2)$$

2) cal  $\Delta x$  &  $\Delta y$

$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

3) cal decision parameter (It is used to find extra point to draw line)

$$P_k = 2\Delta y - \Delta x$$

- 4) current point  $(x_k, y_k)$   
 next point  $(x_{k+1}, y_{k+1})$   
 find next point depending on value of  
 decision parameter  $P_k$ .

case ①

$$\text{if } P_k < 0 \Rightarrow P_{k+1} = P_k + 2\Delta y$$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k$$

case ②

$$\text{if } P_k \geq 0 \Rightarrow P_{k+1} = P_k + 2\Delta y - 2\Delta x$$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k + 1$$

- 5) Repeat step ④ until end point is reached.

### Advantages

- It involves integer arithmetic, very simple.
- It avoids the generation of duplicate pts.
- It can be implemented using HW.
- It is faster than DDA.

Example

$$\text{I) } (x_1, y_1) = (9, 18) \\ (x_2, y_2) = (14, 22)$$

$$\rightarrow \Delta x = x_2 - x_1 \rightarrow 14 - 9 = 5 \\ \Delta y = y_2 - y_1 \rightarrow 22 - 18 = 4$$

cal decision parameter

$$P_K = 2\Delta y - \Delta x \\ = 2 \times 4 - 5 \\ \boxed{P_K = 3}$$

$\therefore P_K > 0 \rightarrow \text{case ②}$

$$P_{K+1} = P_K + 2\Delta y - 2\Delta x \\ = P_K + 2(\Delta y - \Delta x) \\ = 3 + 2(4 - 5) \\ = 3 + 2(-1) \\ = 3 - 2 \\ = 1$$

$$x_{K+1} = x_{K+1} = 9 + 1 = 10 \\ y_{K+1} = y_{K+1} = 18 + 1 = 19$$

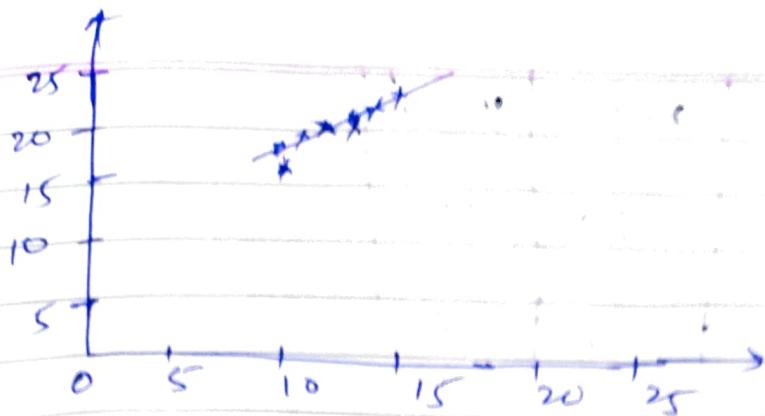
$P_K$	$P_{K+1}$	$x_{K+1}$	$y_{K+1}$	$P_{K+1} < 0$
3	1	9	18	
-1	-1	10	19	
-7	7	11	20	
5	5	12	20	
3	3	13	21	
0	0	14	22	
		15	23	

$P_K < 0$

$$P_{K+1} = P_K + 2(\Delta y - \Delta x)$$

$$x_{K+1} = x_K + \Delta x \\ y_{K+1} = y_K + \Delta y$$

$$= -1 + 2(4) \\ = -1 + 8 = 7$$



2) Explain Bresenham line drawing Algo. ~~with~~  
and identify the pixel positions of A(10, 10)  
& B(18, 16).

→ A (10, 10)  
B (18, 16)

$$\begin{aligned}\Delta x &= x_2 - x_1 & 18 - 10 &= 8 \\ \Delta y &= y_2 - y_1 & 16 - 10 &= 6\end{aligned}$$

Decision parameter

$$P_k = 2\Delta y - \Delta x = 2 \times 6 - 8 = \underline{\underline{4}}$$

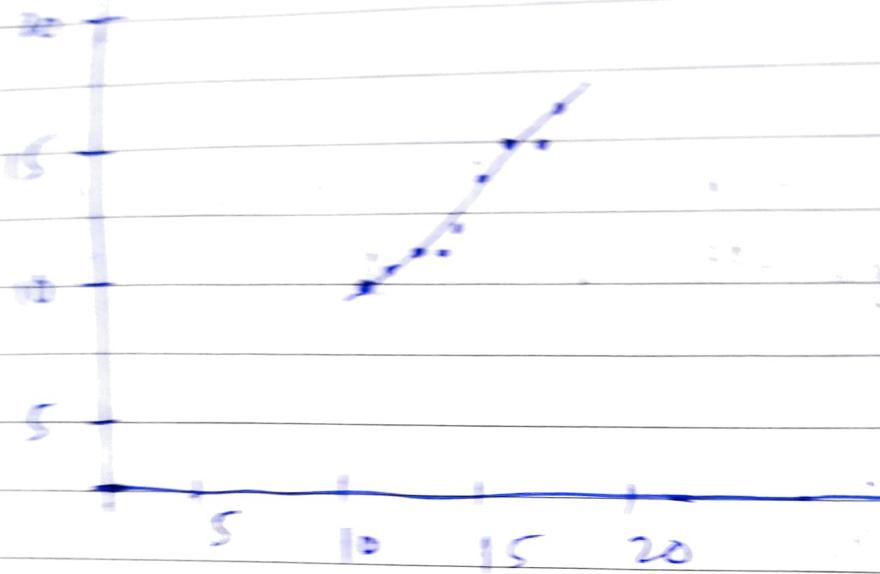
$P_k > 0 \rightarrow$  Case ②

$$\begin{aligned}P_{k+1} &= P_k + 2(\Delta y - \Delta x) \\ &= 4 + 2(6 - 8) \\ &= 4 - 4 = 0\end{aligned}$$

$P_k$	$P_{k+1}$	$x_{k+1}$	$y_{k+1}$
4	0	11	11
0	-4	12	12
-4	8	13	12
8	4	14	13
4	0	15	14

$$\begin{aligned}x_{k+1} &= x_k + 1 \\ y_{k+1} &= y_k + 1\end{aligned}$$

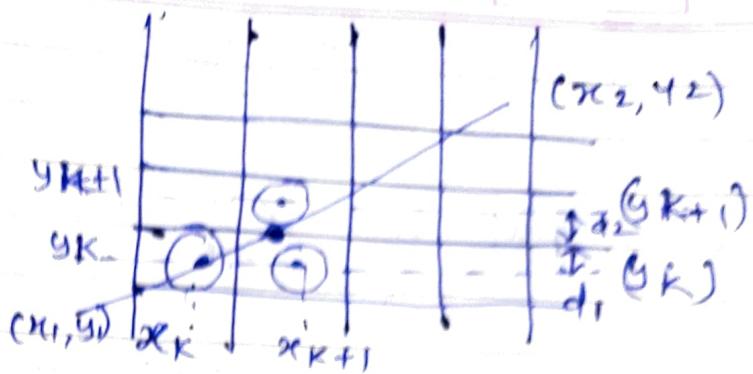
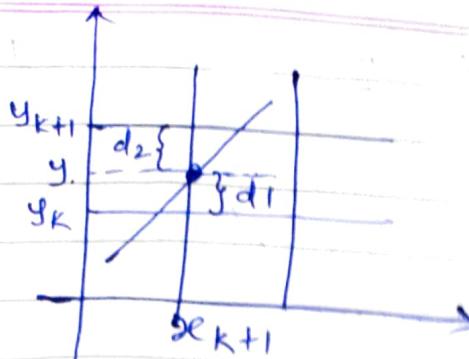
$P_{c1}$	$P_{c21}$	$x_{c21}$	$y_{c21}$
0	-4	16	15
-4	48	12	15
8	4	18	16



# Derivation of Bresenham's Line Drawing Algorithm

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$y = mx + c$ . At sampling position  $x_{k+1}$ ,  $y$  is computed as,

$$\boxed{y = m(x_{k+1}) + c \\ = m \cdot x_{k+1} + m + c}$$

$$dy = y_{k+1} - y_k \\ = (m \cdot x_{k+1} + m + c) - y_k$$

$$d_2 = y_{k+1} - y \\ = y_{k+1} - (m \cdot x_k + m + c)$$

$\therefore$  (for next scan line  
 $y_{k+1} = (y_k + dy)$ )

$$d_1 - d_2 = (m \cdot x_k + m + c - y_k) - (y_{k+1} - m \cdot x_k - m - c) \\ = m \cdot x_k + m + c - y_k - y_k - 1 + \\ m \cdot x_k + m + c \\ = \boxed{2m \cdot x_k + 2m - 2y_k + 2c - 1}$$

To simplify the computation, multiply it by  $\Delta x$ ,

decision parameter at step  $K$ ,  $P_K$

$$P_K = \Delta x(d_1 - d_2)$$

$$= \Delta x(2m \cdot x_k + 2m - 2y_k + 2c - 1) \text{ constant}$$

## Derivative

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$$d_1 = y - y_k \\ = [m(x_{k+1}) + c] - y_k$$

$$d_2 = y_{k+1} - y \\ = y_{k+1} - m(x_{k+1}) - c$$

if  $d_1 - d_2 < 0$        $y_{\text{next}} = y_k$   
 $d_1 - d_2 > 0$        $y_{\text{next}} = y_{k+1}$

$$d_1 - d_2 = [m(x_{k+1}) + c - y_k] - [y_{k+1} - m(x_{k+1}) - c] \\ = 2m(x_{k+1}) + c - y_k - y_{k+1} + c \\ = 2m(x_{k+1}) - 2y_k + 2c - 1$$

Here  $m = \frac{\Delta y}{\Delta x}$

Again a float value so remove float by multiplying  $\Delta x$  on both side.

$$\Delta x(d_1 - d_2) = \Delta x \left[ \frac{2\Delta y}{\Delta x} (x_{k+1}) - 2y_k + 2c - 1 \right]$$

$$\Delta x(d_1 - d_2) = 2\Delta y(x_{k+1}) - 2\Delta x y_k + \frac{2\Delta y + 2\Delta x c - 1}{\Delta x}$$

$$= \underbrace{2\Delta y x_k}_{\downarrow} - 2\Delta x y_k + \frac{2\Delta y + 2\Delta x c - 1}{\text{constant}} \quad \text{--- (A)}$$

$$\Delta x(d_1 - d_2) = 2\Delta y x_k - 2\Delta x y_k$$

$$\Delta x(d_1 - d_2) = P_k - \underline{\text{decision p}}$$

$$P_k = 2\Delta y x_k - 2\Delta x y_k$$

Calculate next decision parameters

$$P_{\text{next}} = 2 \Delta y x_{\text{next}} - 2 \Delta x y_{\text{next}}$$

$$P_{\text{next}} - P_K = [2 \Delta y x_{\text{next}} - 2 \Delta x y_{\text{next}}] \\ - [2 \Delta y x_K - 2 \Delta x y_K]$$

$$= 2 \Delta y x_{\text{next}} - 2 \Delta x y_{\text{next}} - 2 \Delta y x_K + 2 \Delta x y_K$$

$$= 2 \Delta y (x_{\text{next}} - x_K) - 2 \Delta x (y_{\text{next}} - y_K)$$

$$\text{Here } x_{\text{next}} = x_{K+1}$$

But  $y_{\text{next}}$  has  $y_K$

So take decision

$$P_{\text{next}} - P_K < 0$$

then  $y_{\text{next}} = y_K$ ,  $x_{\text{next}} = x_{K+1}$

$$P_{\text{next}} = P_K + 2 \Delta y (x_{K+1} - x_K) - 2 \Delta x (y_K - y_K)$$

$$\boxed{P_{\text{next}} = P_K + 2 \Delta y}$$

if  $P_{\text{next}} - P_K \geq 0$  then  
 $x_{\text{next}} = x_{K+1}$  &  $y_{\text{next}} = y_{K+1}$

$P_{next} - P_k \geq 0$  then

$$x_{next} = x_{k+1} \quad \& \quad y_{next} = y_{k+1}$$

$$P_{next} = P_k + 2\Delta y (x_{k+1} - x_k) - 2\Delta x (y_{k+1} - y_k)$$

$$P_{next} = P_k + 2\Delta y - 2\Delta x$$

$$\text{Now put } P_{next} = P_{k+1}$$

when  $P \leq 0 \rightarrow \therefore P_{k+1} = P_k + 2\Delta y \quad P_{k+1} - P_k < 0$

when  $P > 0 \rightarrow P_{k+1} = P_k + 2\Delta y - 2\Delta x \quad P_{k+1} - P_k > 0$

Now cal initial value of decision parameter  $P_0$  we eqn A

$$P_k = 2\Delta y x_k - 2\Delta x y_k + 2\Delta x c - \Delta x$$

$$\text{Put } k=1$$

$$P_1 = 2\Delta y x_1 - 2\Delta x y_1 + 2\Delta x c - \Delta x$$

$$c = y_1 - \frac{\Delta y}{\Delta x} x_1$$

$$P_1 = 2\Delta y x_1 - 2\Delta x y_1 + 2\Delta y + 2\Delta x \\ \left[ y_1 - \frac{\Delta y}{\Delta x} x_1 \right]$$

$$= 2\Delta y x_1 - 2\Delta x y_1 + 2\Delta y + 2\Delta x y_1 - 2\Delta y x_1 - \Delta x$$

$$Py = 2Ax - Ax$$

initial value  
of decision  
parameter.

- (Q1) A (1, 1)  
B (8, 5)

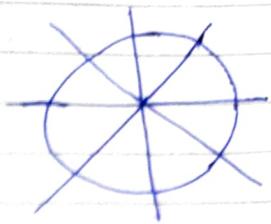
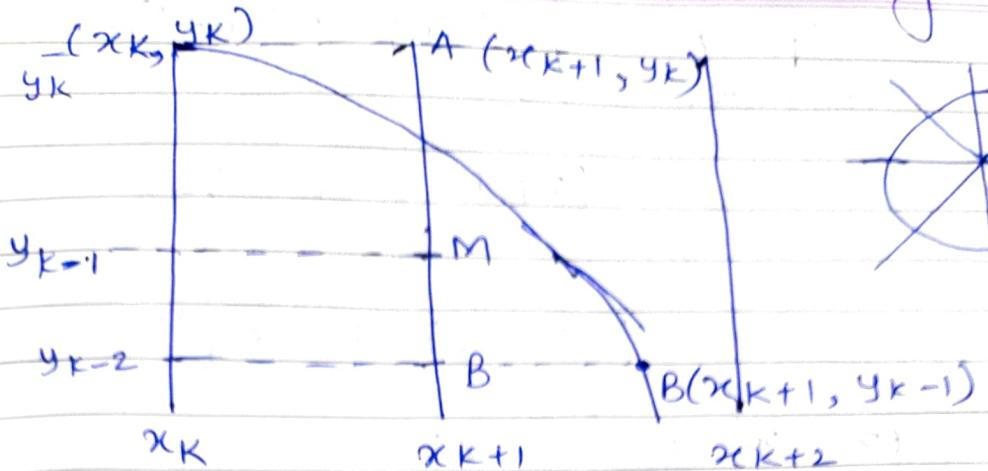
- (Q2) (4, 4), (12, 9)

# Mid- Point circle Drawing Algorithm

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End



$$\rightarrow x^2 + y^2 = r^2$$

$$\rightarrow x^2 + y^2 - r^2 = 0$$

$(x_m, y_m)$  → are midpoints co-ordinates

$$\downarrow \quad \quad \quad \downarrow$$

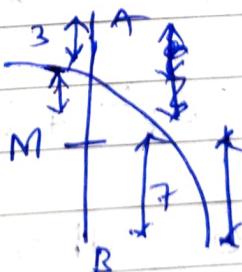
$$(x_m)^2 + (y_m)^2 - r^2$$

$0$ : point lies on circle

$A \rightarrow < 0$ : point lies inside

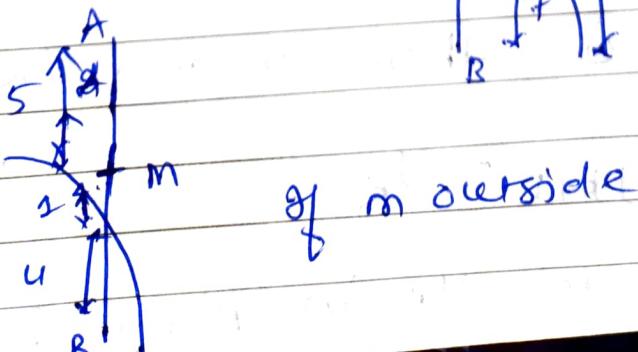
$B \rightarrow > 0$ : point lies outside

Find minimum distance from  $M$  of point  $A$  &  $B$  from midpoint to select for drawing circle.



select shortest  
distance.

$M$  inside.



of  $m$  outside

Date: \_\_\_\_\_

Midpoint =  $\left[ \frac{(x_{k+1} + x_k)}{2}, \frac{(y_{k+1} + y_k)}{2} \right]$

$$= \left( \frac{x_{k+1}}{2}, y_k - 1/2 \right)$$

Put above into circle eq<sup>n</sup>.

$\text{let } \frac{x^2 + y^2 - r^2}{d_k = (x_{k+1})^2 + (y_k - 1/2)^2 - r^2}$

$d_k$  is decision parameter.

$$\boxed{\frac{d_{k+1}}{d_{\text{inc}}} = \frac{(x_{k+1} + 1)^2}{\text{next}} + \frac{(y_{k+1} - 1/2)^2}{\text{next}}}$$

$$\boxed{d_{k+1} - d_k}$$

$x_{k+1}$  will same in next step.

Substitute  $x_{k+1}$  in eqn ② in place of  $x_{k+1}$

$$d_{k+1} = d_k + 2x_k + 3 + (y_{k+1})^2 - y_{k+1} - y_k^2 + y_k$$

If  $d_k < 0$  then y coordinate only update

$$y_{k+1} = y_k$$

Substitute  $y_k$  in place of  $y_{k+1}$

$$d_{k+1} = d_k + 2x_k + 3$$

If  $d_k > 0$  then  $y_{k+1} = y_k - 1$

$$d_{k+1} = d_k + 2x_k + 2y_k + 5$$

Now find initial decision parameter

initial coordinates  $(x_k, y_k)$  substitute  
this with  $(0, r)$  i.e.  $x_k = 0$  &  $y_k = r$

Now substitute  $(0, r)$  in eq<sup>n</sup> ①

$$d_0 = (0+1)^2 + (r - 1/2)^2 - r^2$$

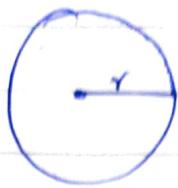
$$d_0 = \frac{5}{4} - r \Rightarrow 1 - r$$

$\frac{5}{4} \approx 1 \therefore r$  is an integer

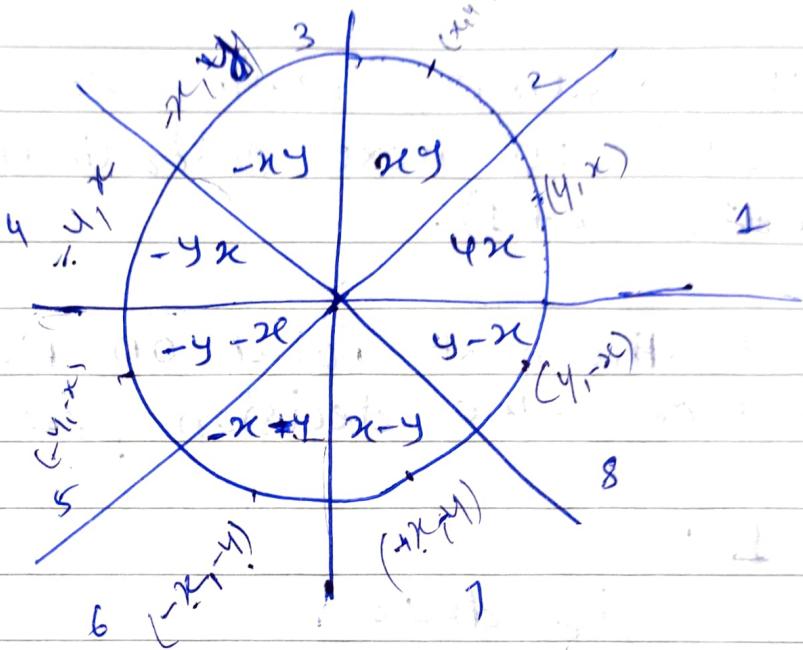
so

$$\boxed{\phi = 1 - r}$$

initial value  $(0, r)$



circle divide into 8 octants on 2D plane



## Algorithm

1. Consider a center coordinates  $(x_1, y_1)$  as  
 $x_1 = 0;$   
 $y_1 = r;$

2. cal the starting decision parameter  
 $d_1 = 1 - r^2$

3. Let us assume starting co-ordinates  $(x_k, y_k)$  so next coordinates are  $(x_{k+1}, y_{k+1})$ .

Find the next point on first octant based on the decision parameter  $(d_k)$

4. Case 1 :

if  $d_k < 0$   
then

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k$$

$$d_{k+1} = d_k + 2(x_{k+1}) + 1$$

$$= d_k + 2x_{k+1} \boxed{d_k + 2x_{k+1}}$$

- case 2 :

if  $d_k > 0$   
then

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k - 1$$

$$d_{k+1} = d_k - 2(x_{k+1} + 2(x_{k+1})) + 1$$

$$= d_k + 2x_{k+1} \boxed{d_k - 2y_{k+1} + 2x_{k+1}}$$

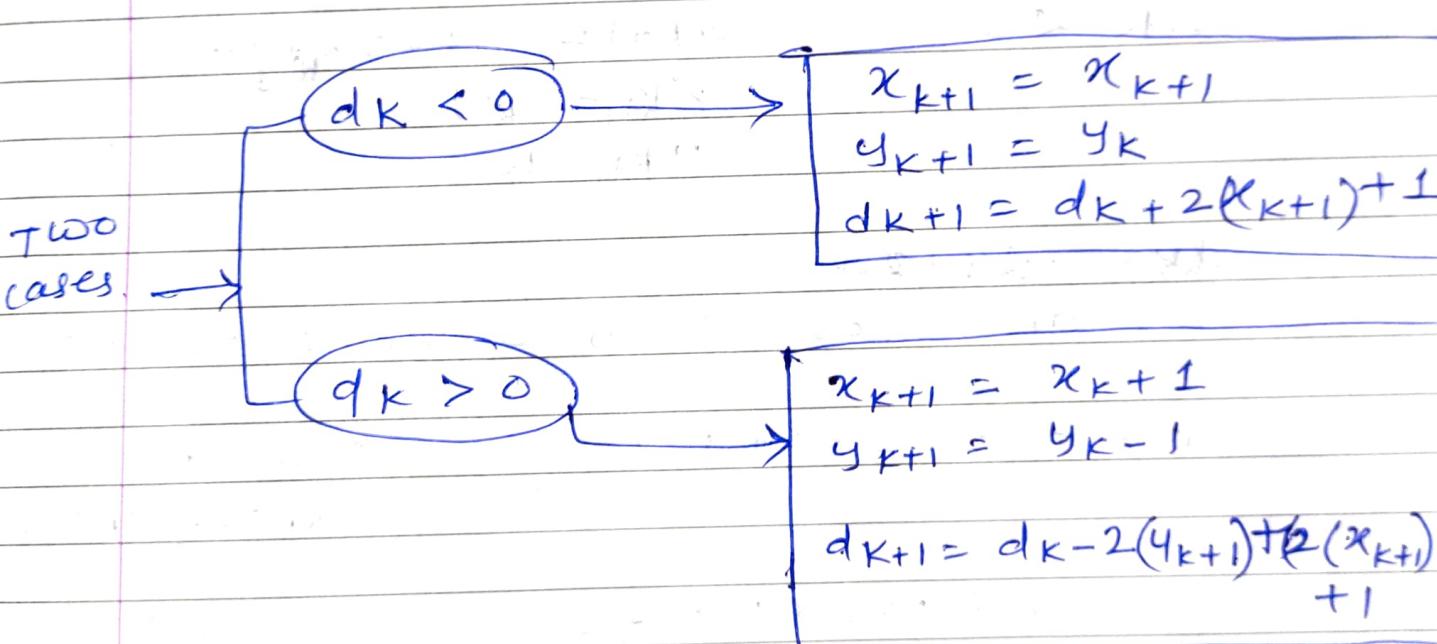
5. If the center co-ordinate pt  $(x_1, y_1)$  is not at the origin  $(0, 0)$  then finding the point as -

for x coordinate =  $x_c + x_1$

for y coordinate =  $y_c + y_1$

$\{ x_c \& y_c$  contains current value  
 $\{ x \& y \}$

6. Repeat step 4 & 5 till we get  $x \geq y$ .



## Example

- ① Draw a circle using mid-pt circle at origin with radius = 15.

$$\rightarrow (x, y) = (0, 0) \text{ & radius} = 15$$

$$x = 0$$

$$y = 0$$

$$r = 15$$

$$d_k = 1 - r \\ = 1 - 15 \\ = -14$$

$$d_k < 0$$

$$d_{k+1} = d_k + 2x_{k+1} + 1$$

$$x_{k+1} - x_k$$

$$4x_k + 4$$

$$d_k = -14 + 2 \times 1$$

As  $d_k$  is less than 0,  $x_n = 1$ ,  $y_n = 15$

$$d_k = -14 + 2 \times 1 + 1 = \underline{\underline{-11}}$$

again less than

$$\therefore x_n = 2, y_n = 15$$

$$d_k = -11 + 2 \times 2 + 1 = \underline{\underline{-6}}$$

again  $< 0$

$$x_n = 3, y_n = 15$$

$$d_k = -6 + 2 \times 3 + 1 = \underline{\underline{1}}$$

$d_k > 0$ , case ②

$$x_n = 4, y_n = 4$$

~~$$d_k = 1 + 2x_4 + 2y_4 \\ = 1 + 8 + 28 \\ = 37$$~~

$$dK > 0$$

$$x_{k+1} = 4 \quad y_{k+1} = 14$$

$$dK_{f1} = 1 - 2 \times 14 + \\ 2 \times 4 + 1$$

$$= 1 - 28 + 9$$

$$= -18$$

$$dK_{f2} = -7$$

$$x_{k+1} \in (x_{k+1}) + 1$$

$$y_{k+1} = y_{k+1} - 1$$

$$dK_{f1} = dK - 2x_{k+1} + 2x_{k+1}$$

$$dK < 0$$

$$x_{k+1} = 5, y_{k+1} = 14$$

$$dK_{f1} = dK + 2x_{k+1} + 1$$

$$= -18 + 2 \times 5 + 1 \\ = -18 + 10 = -7$$

$$dK_{f1} = -7 + 2 \times 6 + 1 \\ = -7 + 13 \\ = 6$$

$$dK_{f1} < 0$$

$$x_{k+1} = 6, y_{k+1} = 14$$

$$dK_{f1} = 6 - 2 \times 13 + 2 \times 7 + 1 \\ = 6 - 26 + 15 \\ = -5$$

$$dK_{f1} > 0$$

$$x_{k+1} = 7, y_{k+1} = 13$$

$$dK_{f1} = -5 + 2 \times 8 + 1 \\ = -5 + 17 \\ = 12$$

$$dK > 0$$

$$x_{k+1} = 9, y_{k+1} = 12$$

$$dK_{f1} = 12 - 2 \times 12 + 2 \times 9 + 1 \\ = 12 - 24 + 19 \\ = 12 - 5 \\ = 7$$

$$dK > 0$$

$$x_{k+1} = 10, y_{k+1} = 11$$

$$= -2 \times 11 + 2 \times 10 + 1$$

$$= 7 + 22 + 2 \cancel{1} \text{ PK}$$

$$= 7 - 1$$

$$= 6$$

$$\Rightarrow$$

$$14,$$

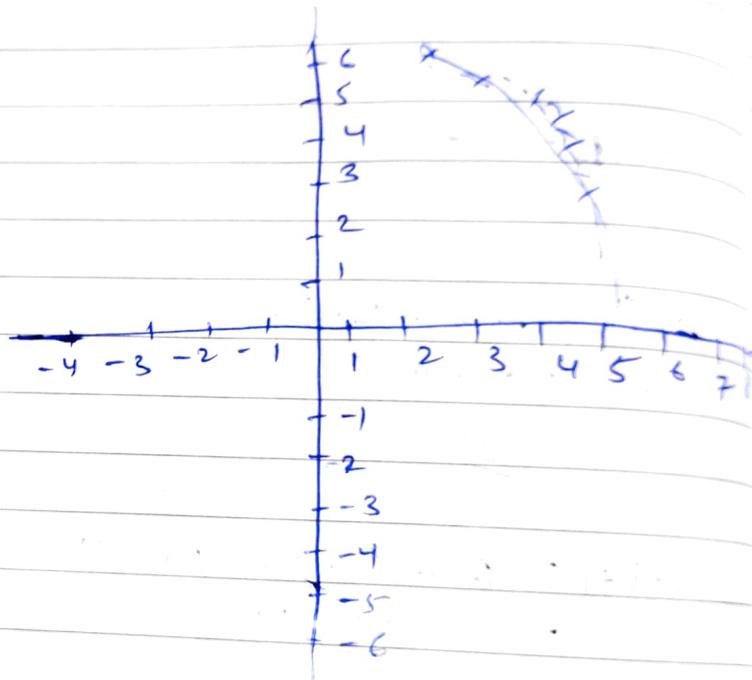
$$dk > 0$$

$$x(k+1) = 11 \quad y(k+1) = 10$$

Here we stop because  
 $\underline{x > y}$

points are -

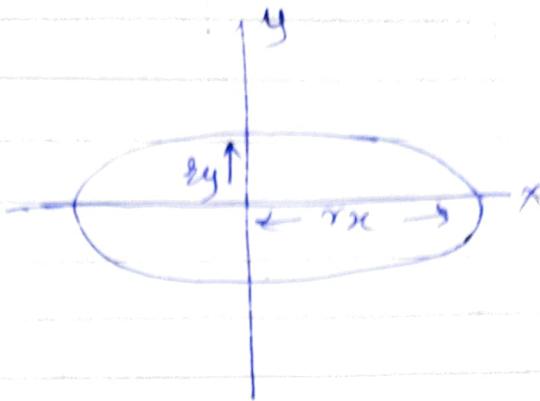
- (-1, 15)
- (2, 15)
- (3, 15)
- (4, 14)
- (5, 14)
- (6, 14)
- (7, 13)
- (8, 13)
- (9, 12)
- (10, 11)
- (11, 10)



# Midpoint Ellipse Algorithm

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ellipse is an elongated circle

eq<sup>n</sup> of ellipse is centered at (0,0)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where  $a = rx$  &  $b = ry$

Major Axis =  $2a = 2rx$  (longer axis)  
 Minor Axis =  $2b = 2ry$  (shorter axis)

Semi major Axis =  $a = rx$   
 Semi minor Axis =  $b = ry$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$x^2 b^2 + a^2 y^2 = a^2 b^2$$

$$x^2 b^2 + y^2 a^2 - a^2 b^2 = 0$$

put  $a = rx$  &  $b = ry$

$$\therefore r^2 x^2 + y^2 r^2 - r^2 r^2 = 0 \quad \text{---(1)}$$

If we put any point in eq ①

case 1  
 $\leq 0$   
on boundary

case 2  
 $< 0$

case 3  
 $> 0$

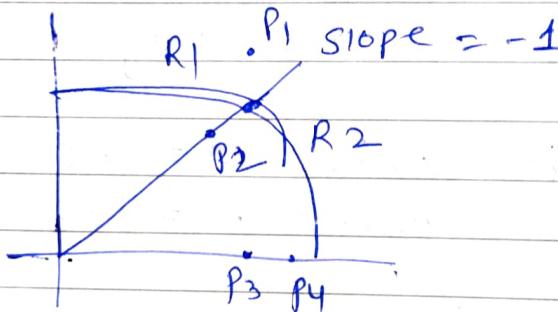
inside  
boundary

outside  
boundary

In Circle, 8 way symmetry

In Ellips 4 way symmetry

In circle, we need to plot one octant of any quadrant but in ellips we need to plot 2 octants i.e. one complete quadrant.



In Region R<sub>1</sub> we can have next point at  $x$  increment,  $y$  to take decision.

$(x_{k+1}, y_k)$  or  $(x_{k+1}, y_{k-1})$

But in Region R<sub>2</sub> we are considering 2 points on major axis P<sub>3</sub> & P<sub>4</sub>.

so here it depends on whether to increment  $x$  or decrement  $x$

In this Algo, we plot 1<sup>st</sup> quadrant which has 2 regions & have different slope in region R<sub>1</sub> & R<sub>2</sub> so we have to plot both regions by using different formulas.

Quadrant 1 → Region 1

- i) start point : (0, ry)
- ii) slope of ellips  $\leq -1$
- iii) Take until steps in positive x direction till boundary between 2 regions is reached.  $\rightarrow x = x + 1$

Quadrant 2 → Region 2

slope of ellips  $\geq -1$

Take until steps in negative y direction till the end of quadrant.

on the boundary between 2 regions, slope of the ellips is -1.

Here y is decremented by 1 so y-1 & will decide whether to increment x or not

## Example - midpoint ellipse algo

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- Read radius  $r_x$  &  $r_y$
- Initialize starting point of region 1 as  
 $x=0, y=r_y \rightarrow (0, \underline{r_y})$
- calculate  $P_{10} = r_y^2 - r_x^2 \cdot r_y + 1/4 r_x^2$
- calculate  $dx = 2r_y^2 \cdot x, dy = 2r_x^2 \cdot y$
- Repeat while ( $dx \leq dy$ ) - Region 1

\* plot  $(x, y)$

\* if  $(P_1 < 0)$

{

$x = x+1$

update  $dx = 2r_y^2 \cdot x \approx \text{old } (dx+2r_y^2)$

$$P_1 = P_1 + 2r_y^2 \cdot x + r_y^2$$

}

\* else

{

$x = x+1$

$y = y-1$

update  $dx \rightarrow 2r_y^2 \cdot x \quad \{ \text{old } dx + 2r_y^2 \cdot x \}$

update  $dy \rightarrow 2r_x^2 \cdot y \quad \{ \text{old } dy + 2r_x^2 \cdot y \}$

$$P_1 = P_1 + dx - dy + r_x^2 \cdot r_y^2$$

}

when ( $dx \geq dy$ ) plot Region 2 as -

- And  $P_{20} = r_y^2 \cdot (x+1/2)^2 + r_x^2 \cdot (y-1)^2 - r_x^2 \cdot r_y^2$

- Repeat till ( $y > 0$ )

\* Plot  $(x, y)$

\* if  $(P_2 > 0)$

{

$$x = x$$

$$y = y - 1$$

$$\text{update } dy = 2 \cdot 8x^2 y$$

$$P_2 = P_2 - dy + 8x^2$$

3

else if

{

$$x = x + 1$$

$$y = y - 1$$

$$dy = \cancel{dx} 2 \cdot 8x^2$$

$$dx = \cancel{dy} 2 \cdot 8y^2$$

$$P_2 = P_2 + dx - dy + 8x^2$$

4

Q. 1.

$$x_0 = 8, \quad y_0 = 6 \quad \text{given}$$

Iterat<sup>n</sup>  $(x_k, y_k)$

Q.  $(0, y_0)$

$$P_{10} = y_0^2 - 8x_0^2 \cdot y_0 +$$

$(0, 6)$

$$1/4 \cdot 8x_0^2$$

$$= 36 - 64 \times 6 +$$

$$1/4 \cdot 64$$

$$= 36 - 384 + 16$$

$$= -332$$

$\therefore P_1 < 0$

Q.  $(1, 6)$

$$P_1 = P_1 + \frac{dy}{dx} x_0 + y_0^2$$

$$= -332 + 2 \cdot 36 + 36$$

$$(x_{k+1}, y_{k+1}) \quad \begin{cases} dx \\ 2x_{k+1}^2 y \\ 2y_{k+1}^2 x \end{cases}$$

$(1, 6)$

$$x_{\text{inc}}, y_{\text{out}} = \frac{72}{64} = \underline{\underline{1}}$$

Now  $dx < dy$

dxdy

$$= \underline{92} \times 2$$

$$= \underline{144}$$

dx < dy768

2. (2, 6)  $P_1 = -224 + 144 + 36$   
 $= -\underline{44}$

$$(2, 6) = \underline{92} \times 2$$

$$= \underline{144}$$

dx < dy768

3. (3, 6)  $P_1 = -44 + 216 + 36$   
 $= \underline{208}$

$$(3, 6) = 6(36)$$
  
 $= \underline{216}$

dx < dy768

4. (4, 5)  $P_1 = 208 + 288 + 36 - 64^0$   
 $= \underline{823} - \underline{108}$

$$(4, 5) = 10(36)$$
  
 $= \underline{360}$

dx < dy648

5. (5, 5)  $P_1 = -108 + 360 + 36$   
 $= \underline{288}$

$$(5, 5) = 12(36)$$
  
 $= \underline{432}$

dx < dy512

6. (6, 4)  $P_1 = 288 + 432 - 512 + 36$   
 $= \underline{244}$

$$(6, 4) = 14(36)$$
  
 $= \underline{504}$

384Now  $dx > dy$ 

plot Region 2

R2

7. (7, 3)  $P_2 = xy^2(x+1/2)^2 + rx^2(y-1)^2 - rx^2 \cdot ry^2$   
 $= 36(7+1/2)^2 +$   
 $64(3-1)^2 - 36 \times 64$   
 $= 10824 + 256 - 2304$   
 $= \underline{8660} - \underline{23}$

$$(7, 3) = 16(36)$$
  
 $= \underline{576}$

$$4 \times 64$$
  
 $= \underline{256}$

8. (8, 2)  $P_2 = -23 + 576 - 256$   
 $+ 64$

$$P_2 = \underline{\underline{361}}$$

$$(8, 1)$$

$$16(36) = \underline{\underline{576}}$$

$$2 \times \underline{\underline{64}} = \underline{\underline{128}}$$

2.  $(8, 1)$

$$\begin{aligned} P_2 &= 361 - 128 + \\ &\quad \underline{\underline{64}} \\ &= \underline{\underline{297}} \end{aligned}$$

$(8, 0)$

Now  
8 top  
when  $y=0$

3.  $(8, 0)$

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