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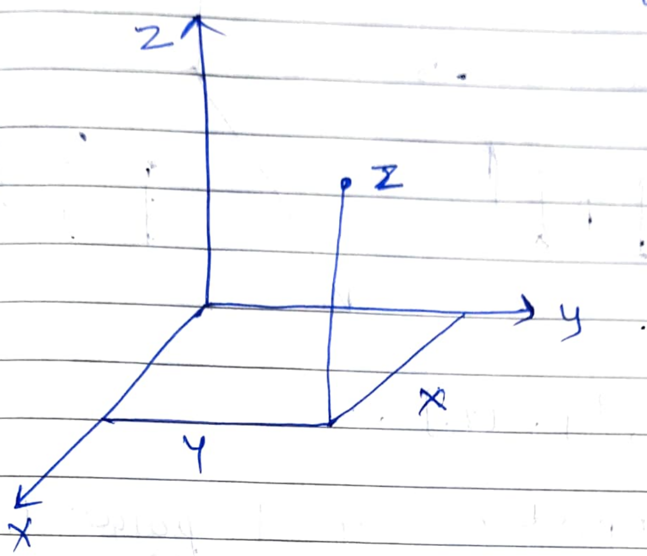
Chap 5.

3D Transformation.

- In 2D, the point is discribed by (x, y) co-ordinates in xy plane, whereas in 3D the point (x, y, z) is defined.



2D geometry divides the co-ordinate system into four quadrants.
3D divides the co-ordinate sys into eight octants.

- Distance between two points $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ is computed by -

$$d(A, B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

- When the point is projected on any plane using a perpendicular projection, the 3rd co-ordinate becomes zero.

- Any point on the xy plane will be $(x, y, 0)$ & any point on x-axis will be $(x, 0, 0)$ similar observation can be made for other planes.

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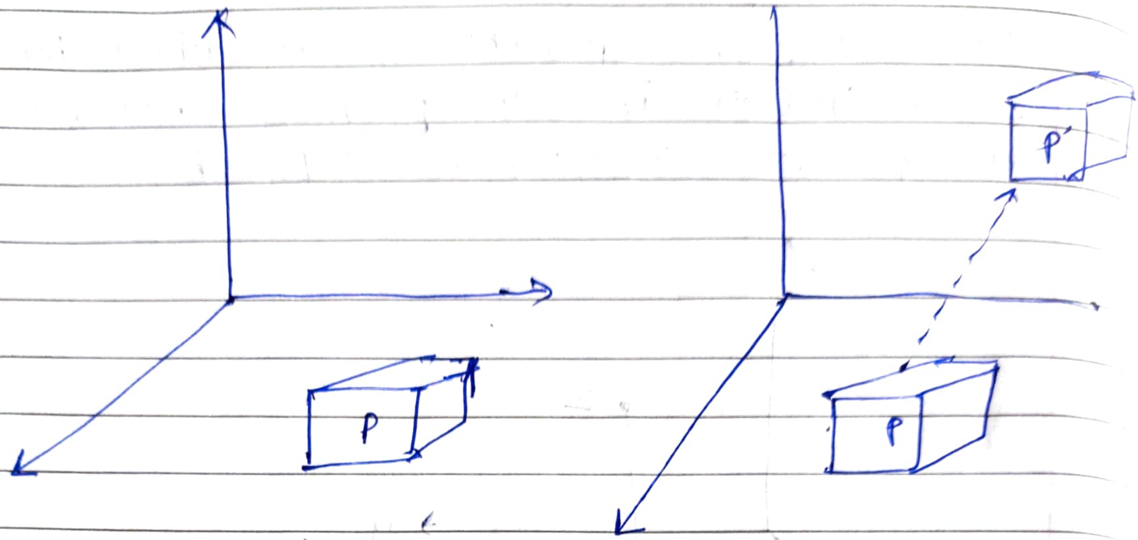
① Translation



$T = [t_x, t_y, t_z]$.

- Let us consider original point $P(x, y, z)$ which becomes $P'(x', y', z')$

$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z$$

Homogeneous Matrix –

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}.$$

$$P' = T \cdot P$$

Inverse Matrix

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & -t_y \\ 0 & 0 & 1 & -t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Q. Translate a triangle with vertices A(2,2,2) B(3,4,7) & C(8,9,12) by translation vector T [2,4,5].

$$P' = T \cdot P = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 8 \\ 2 & 4 & 9 \\ 2 & 7 & 12 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 & 10 \\ 6 & 8 & 13 \\ 7 & 12 & 17 \\ 1 & 1 & 1 \end{bmatrix}$$

A (2, 2, 2) — A' (4, 6, 7)
B (3, 4, 7) — B' (5, 8, 12
C (8, 9, 12) — C' (10, 13, 17)

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* Scaling

- Multiplying all co-ordinates of the Object by some constants called Scaling parameters S

$$S = [S_x, S_y, S_z]$$

- Scaling alters the shape & size of the obj.

- scaling can be performed in either Two ways

① with Respect to the origin.
② with Respect to reference point.

① With Respect to origin.

$$P' = S \cdot P$$

$$x' = S_x \cdot x$$
$$y' = S_y \cdot y$$
$$z' = S_z \cdot z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

2. With Respect to Reference point

(x_r, y_r, z_r)

- Translate ref point to the origin
- Apply scaling with respect to origin.
- Perform inverse translation to move the ref point back to the original position.

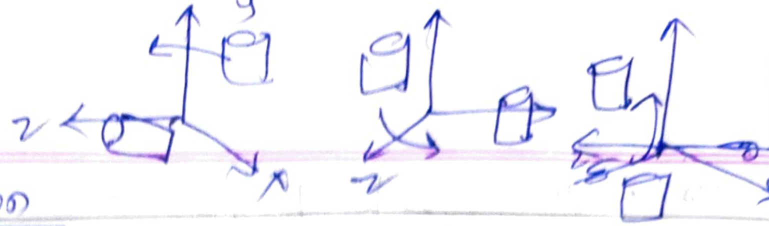
$$P' = T^{-1} \cdot S \cdot T \cdot P$$

$$P' = T(x_r, y_r, z_r) \cdot S(s_x, s_y, s_z) \cdot T(-x_r, -y_r, -z_r) \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x_r \\ 0 & 1 & 0 & y_r \\ 0 & 0 & 1 & z_r \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -x_r \\ 0 & 1 & 0 & -y_r \\ 0 & 0 & 1 & -z_r \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & (1-S_x) \cdot x_r \\ 0 & S_y & 0 & (1-S_y) \cdot y_r \\ 0 & 0 & S_z & (1-S_z) \cdot z_r \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Rotation

- For anticlockwise rotation, angle is considered +ve

- For clockwise rotation, angle is -ve

① Rotation about x-axis
- It does not alter x-coordinate.
- It only affects y & z co-ordinates.

$$x' = x$$
$$y' = y\cos\theta - z\sin\theta$$
$$z' = y\sin\theta + z\cos\theta$$

$$P' = R_x(\theta) \cdot P$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

② Rotation about y-axis

- It does not alter the y-coordinate.
It only affects z & x co-ordinates.

$$x' = z\sin\theta + x\cos\theta$$
$$y' = y$$
$$z' = z\cos\theta - x\sin\theta.$$

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$$P' = R_y(\theta) \cdot P$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) Rotation about z-axis

- It does not alter z-coordinate. It only affects x & y co-ordinates

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$P' = R_z(\theta) \cdot P$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Rotation Matrix

$$R_z(-\theta) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\because$ ~~cos θ~~ $\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

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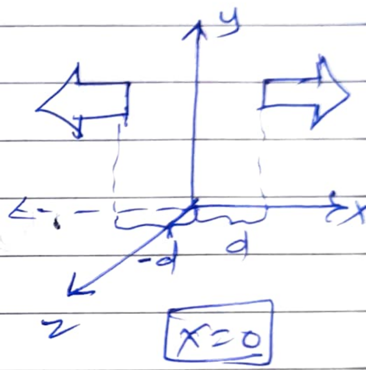
$$\therefore R_z(-\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* Reflection

- It can be performed either with respect to the selected axis or with respect to the selected plane.

$$Ref_{(x=0)} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Ref_{(y=0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$Ref_{(z=0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P' = M \cdot P = Ref_{(x=0)} \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = -x$$
$$y' = y$$
$$z' = z$$

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* Shearing

- Alter the shape of obj.
- It is used in 3D viewing to derive projection.

$$SH_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ b & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Sh_y = \begin{bmatrix} 1 & c & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & d & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Sh_z = \begin{bmatrix} 1 & 0 & e & 0 \\ 0 & 1 & f & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$SH = \begin{bmatrix} 1 & c & e & 0 \\ a & 1 & f & 0 \\ b & d & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

off diagonal values specify the shear amount in a particular direction.

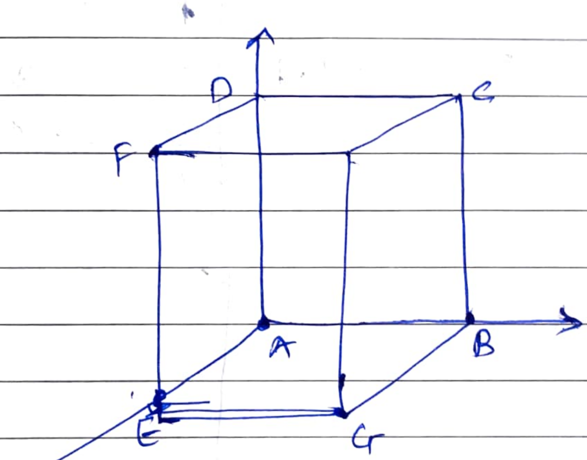
$$\underline{P' = M \cdot P = SH \cdot P}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & c & e & 1 \\ a & 1 & f & 1 \\ b & d & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$p' = \left[(x + cy + ez) \; (ax + y + fz) \; (bx + dy + z) \right]$$

Q. A cube is defined by 8 vertices
A (0,0,0) B (2,0,0) C (2,2,0) E (0,0,2)
F (0,2,2) G (2,0,2) H (2,2,2) D (0,2,0)
perform 3D Transformatn on the cube.

(i) Translation [5, 3, 4]
(ii) scaling [1, 2, 0.5]
(iii) Rotation abt x-axis by 90° in clkwise direction.



$$p' = T \cdot P = \begin{bmatrix} 1 & 0 & 0 & \overset{5}{\cancel{t_x}} \\ 0 & 1 & 0 & \overset{3}{\cancel{t_y}} \\ 0 & 0 & 1 & \overset{4}{\cancel{t_z}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$p' = \begin{bmatrix} 5 & 7 & 7 & 5 & 5 & 5 & 7 & 7 \\ 3 & 3 & 5 & 5 & 3 & 5 & 3 & 5 \\ 4 & 4 & 4 & 4 & 6 & 6 & 6 & c \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

② Scaling

$$P' = S \cdot P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 4 & 4 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

③ Rotation

$$P' = R_x(\theta = -90^\circ) \cdot P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & -2 & -2 & 0 & -2 & 0 & -2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Q. Rotate a triangle ABC with vertices A (2, 2, 2) B (3, 4, 7) & C (8, 9, 12) about y axis in anticlockwise direction by angle 90°

→

$$P' = R_y(\theta) \cdot P = \begin{bmatrix} \cos 90 & 0 & \sin 90 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90 & 0 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 8 \\ 2 & 4 & 9 \\ 2 & 7 & 12 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 8 \\ 2 & 4 & 9 \\ 2 & 7 & 12 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 7 & 12 \\ 2 & 4 & 9 \\ -2 & -3 & -[illegible] \end{bmatrix}$$

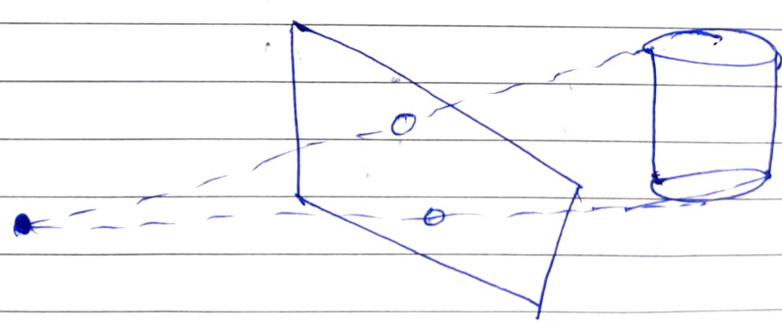
* Projection

- It is the process of converting a 3D object into 2D object.

Types

Projection
- Parallel
  - orthographic
    - Multiview
      - Front
      - Rear
      - Top
      - Bottom
      - Right
      - Left
    - Axometric
      - Isometric
      - Dimetric
      - Trimetric
  - oblique
    - Cavalier
    - Cabinet
- Perspective
  - one pt
  - 2 pt
  - 3 pt

* Par.



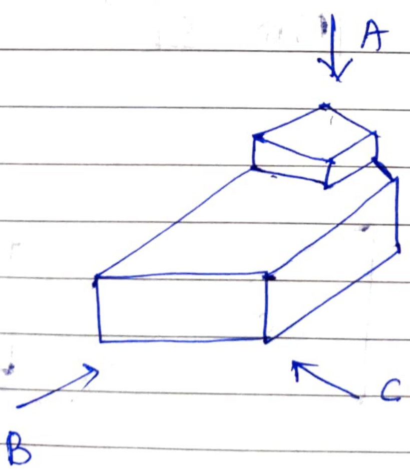
In 3D we map points from 3-space to the projectⁿ plane along projectors from center of projection (C.O.P)

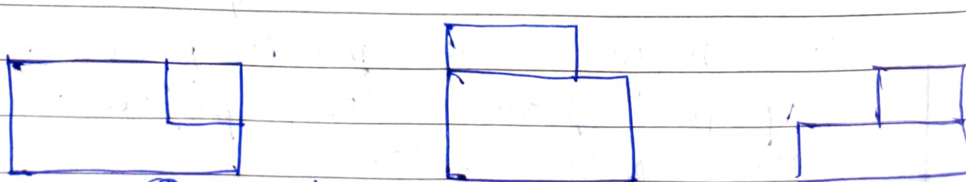
* Parallel Projection

- Parallel Projection
  - Orthographic parallel projection
    - Multiview parallel projection
    - Axonometric parallel projection
  - Oblique parallel projection
    - Cavalier projection
    - Cabinet projec[tion]

- Parallel projection use to display picture its true shape & size.

- When projectors are perpendicular to view plane then it called othorgraphic projection





Top (A) direction   Top (B) direction   Top (C) direc[tion]

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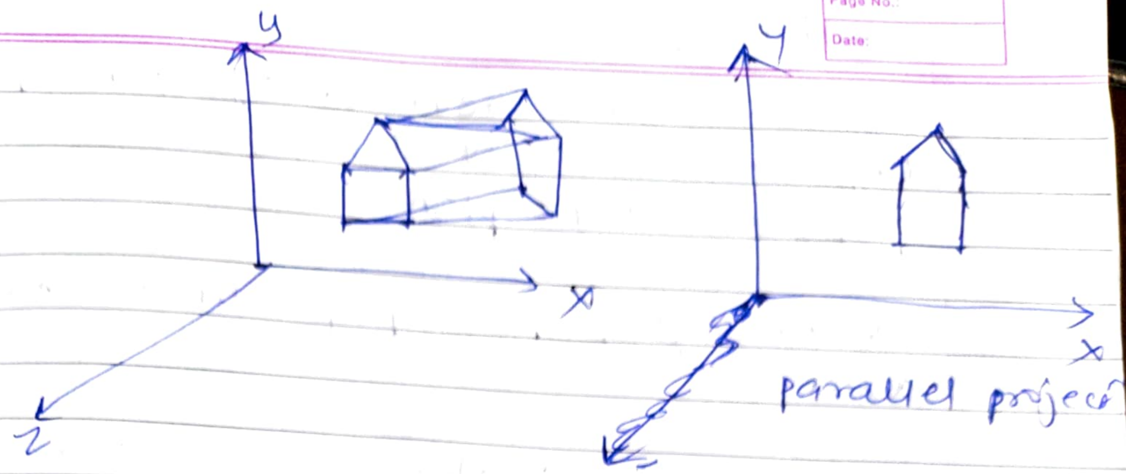
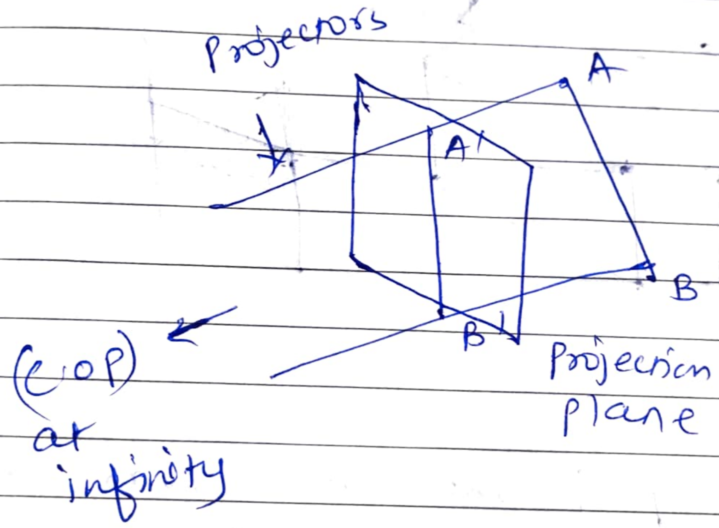
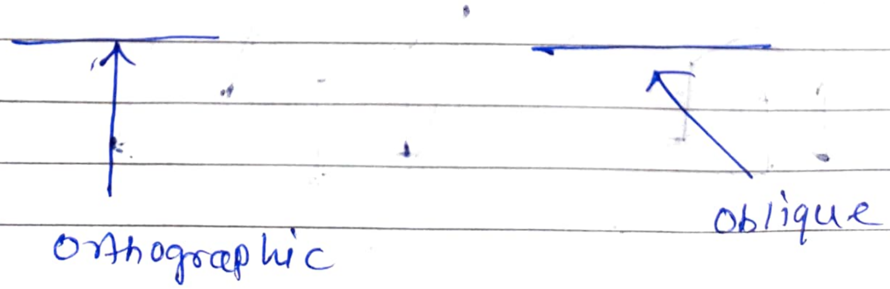


In parallel proj, coordinate positions are transformed to the view plane along parallel lines.



① Orthographic – when the projection is perpendicular to the view plane.

② Oblique – when proj is not perpendicular to the view plane.



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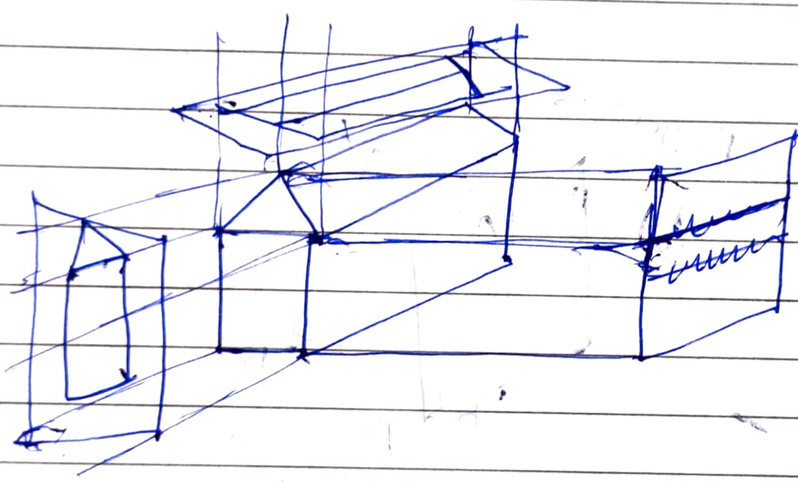
Orthographic

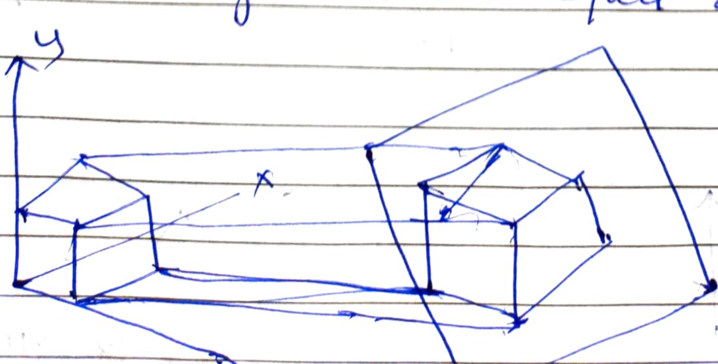
- Front, side and rear orthographic proj of obj are called elevations & the top proj is called plan view.

- All have proj plan perpendicular to a prin axies.

- Here length & angles are accurately depicted & measured, so engg & architect drawings its commonly use.



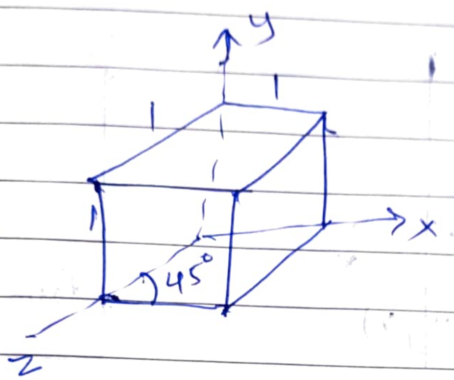
- Orthographic proj show more than one face of an obj are called axonometric orthographic projects.
most common axonometric proj is an isometric proj where the proj plane intersect each coordinate axis in the model coordinate sys at an equal distance.

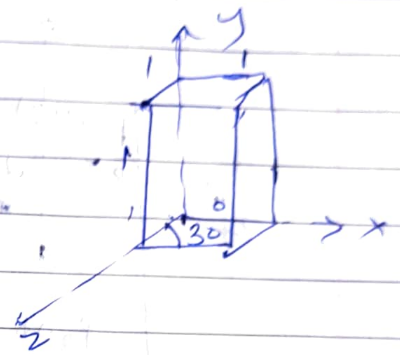


Oblique

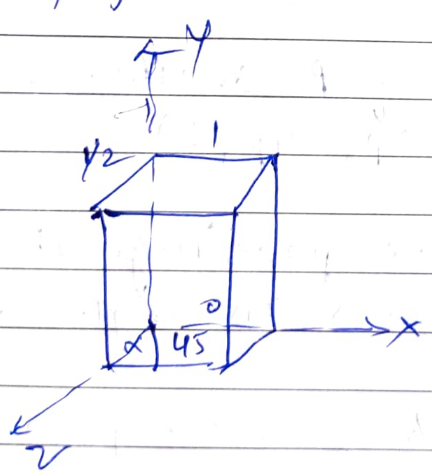
- common oblique parallel projs - canvalier & cabinet.

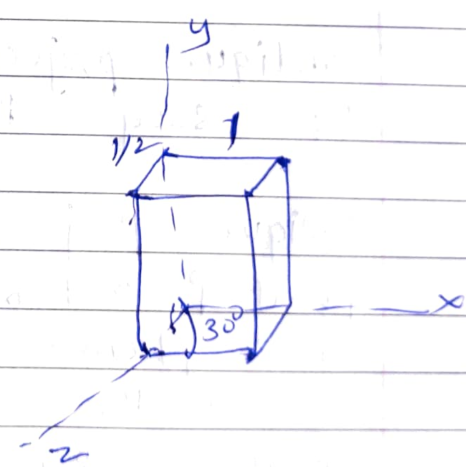
- **Cavalier** -
All lines not perpendicular to the projection plane are projected with no change in length.





- **Cabinet** -
Lines are not perpendicular to proj plane are projected 1/2 the length.

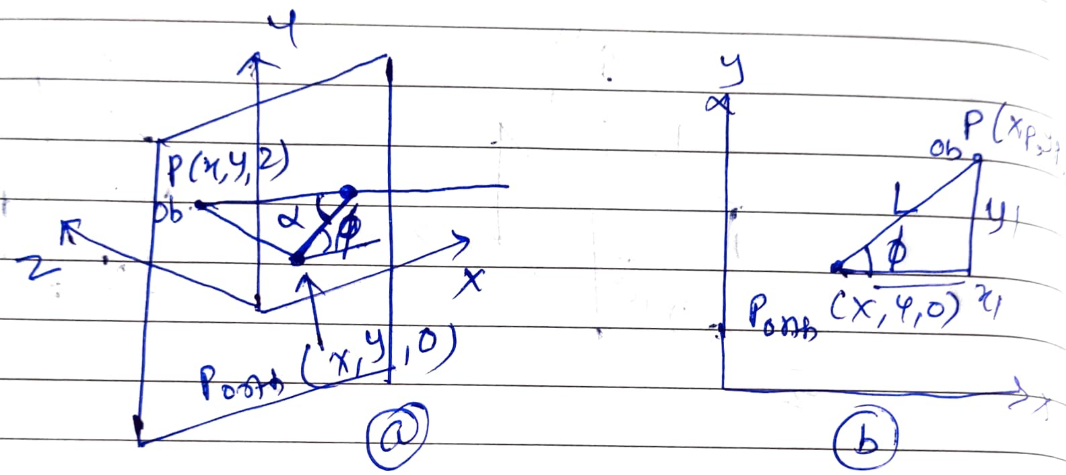




* Oblique parallel projection.

- Projectors are parallel to each other & they are not perpendicular to the view plane.

- view in oblique projection is controlled by two parameters ϕ and α.



- Let $P(x, y, z)$ be the point in space. orthographic projection of point P on view plane xy is $P(x, y, 0)$

- oblique projection of P on the same plane is say $P_{ob}(x_p, y_p, 0)$.

- oblique proj vector passing through point P & P_{ob} makes an angle α with view plane.

  Let the length of line joining points P_{orth} & P_{ob} is L

- $$x_p = x + L\cos\phi$$
  $$y_p = y + L\sin\phi$$

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$\tan \alpha = \frac{z}{L}$ ... from fig (a)

$\therefore L = \frac{z}{\tan \alpha} = zL_1$

$= zL_1$

by solving the eq^n of x_p & y_p for L

$x_p = x + zL_1 \cos \phi$

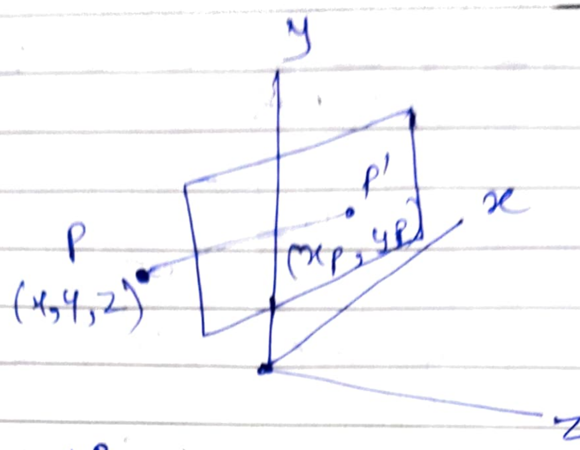
$y_p = y + zL_1 \sin \phi$

so the transformation matrix for any parallel projection on view plane x_v, y_v is -

$$M = \begin{bmatrix} 1 & 0 & L_1 \cos\alpha & 0 \\ 0 & 1 & L_1 \sin\alpha & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

L_1 is foreshortening factor.

* Parallel Projection Derivation

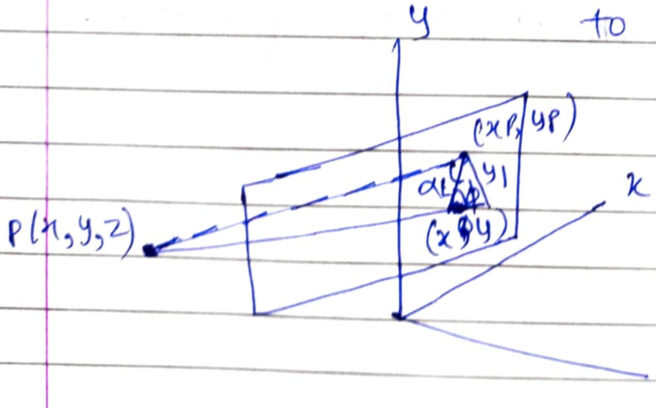


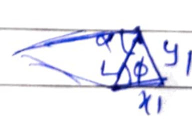
① Orthographic

$z_p = 0$ In xy plain z axis is
$x_p = x$ always zero. $\therefore z = 0$
$y_p = y$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

② oblique projection - Line is not perpendicular to view plane.





$x_p = x + L\cos\phi$ → $\cos\phi = \frac{b}{H} = \frac{x_1}{L}$ $\left(\frac{\text{base}}{\text{Hyp}}\right)$
$y_p = y + L\sin\phi$

$x_1 = L\cos\phi$

$\frac{\text{Perpendicular}}{\text{Hypotenious}}$ → opp to angle

$$\sin\phi = \frac{P}{H} = \frac{y_1}{L} \qquad \therefore \; y_1 = L\sin\phi$$

$$x_p = x + x_1$$
$$y_p = y + y_1$$

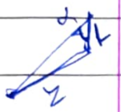
distance of x_1 & y_1 are equal. so x_p will be plot using $x + x_1$ & $y_p = y + y_1$

$$\therefore \; x_p = x + L\cos\phi$$
$$y_p = y + L\sin\phi$$

(90° angle)

$$\tan\alpha = \frac{P}{B} = \frac{z}{L} \qquad \therefore \; L = \frac{z}{\tan\alpha}$$

opp to angle



$$L = z L_1 \quad \text{where} \quad L_1 = \frac{1}{\tan\alpha}$$

$$x_p = x + L\cos\phi \qquad y_p = y + L\sin\phi$$
$$= x + zL_1\cos\phi \qquad = y + zL_1\sin\phi$$

$$\begin{bmatrix} x_p \\ y_p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & L_1\cos\phi & 0 \\ 0 & 1 & L_1\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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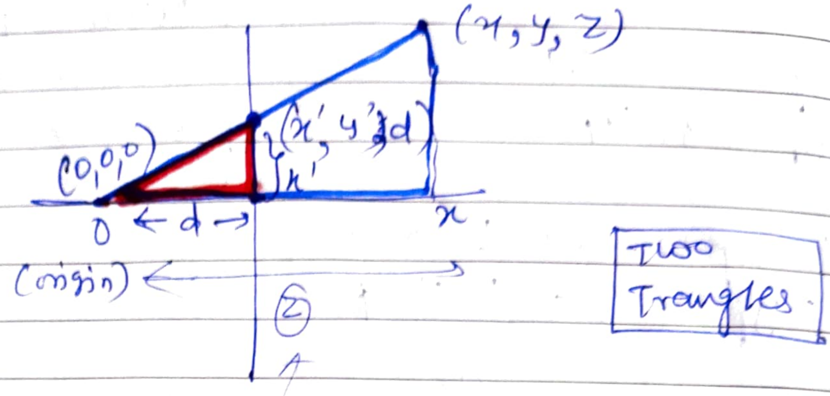
* Prospective Projection.



$$\Rightarrow \quad \frac{x'}{d} = \frac{x}{z}$$

(when two triangles are similar then there ratio is ~~similar~~ equal)

$$\therefore x' = \frac{xd}{z} = \frac{x}{z/d}$$

$$\frac{y'}{d} = \frac{y}{z}$$

$$\therefore y' = \frac{yd}{z} = \frac{y}{z/d}$$

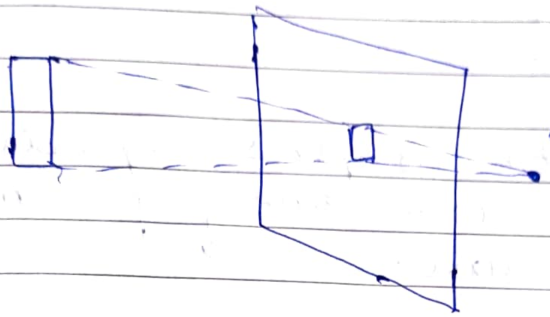
$$z' = z$$

$$x' = \frac{x}{z/d}, \quad y' = \frac{y}{z/d}, \quad z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Homoge $1/d$ coordinate.

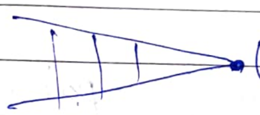
* Prespective projection.



finite

Projection or center
Ref point of
Project^n
(COP)

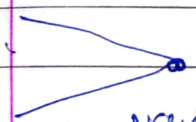
- No Relative proportion presence.



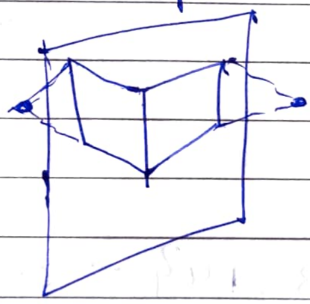
(vanishing pt)
infinite.

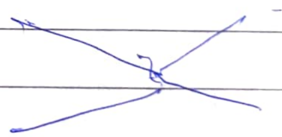
Ex:- Railway track.

① 1 pt ② 2 pt ③ 3 pt.



vanishing
point)





* Curves & Fractals

1. implict
2. explict
3. parametric

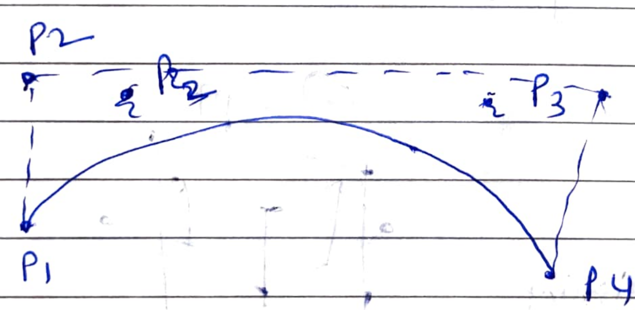
} Types of curves

Most computer graphics depends on parametric curve. one of parametric curve is Bezier curve.

* Bezier curve

- Is define by control points. By using this point we draw the curve.

Ex:-



{P1, P2, P3, P4} - control points.

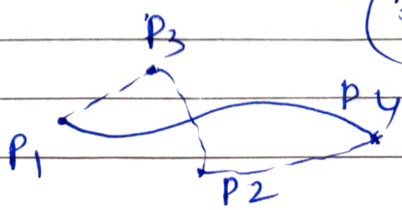
2 pt curve - 

2 pt (liner curve)

3 pt curve - 

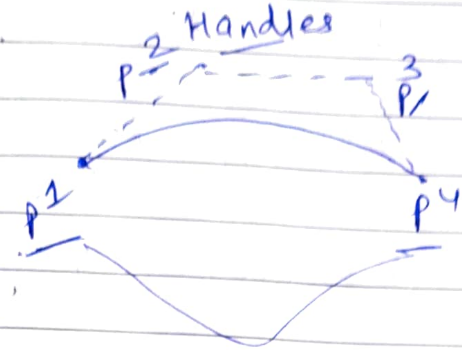
(quadratic curve
Parabolic curve)

(3 control points
degree is 2 (n-1))

4 pt curve 

cubic

- points are not always on curve.
- (n-1) degree on curve.



n-1

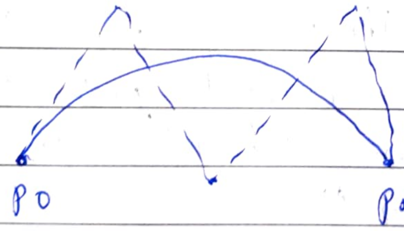
4-1 = 3. cubic curve

↑

degree.

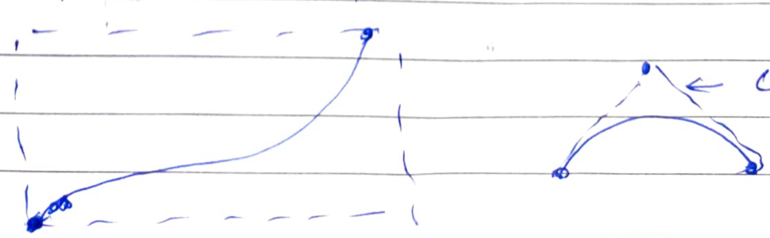
* Properties of Bezier curve

- They always pass through the first & last control points.
- They are contained in the convex hull of their defining control points.
- The degree of polynomial defining the curve segments is one less than the no. of defining polygon points.

① 

It pass through first & last control points

② convex hull - boundary outside to control point



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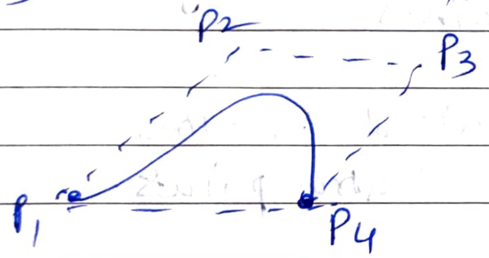
(3) If we having 4 control points
∴ degree is 3 (n-1) so it is
cubic polygon.

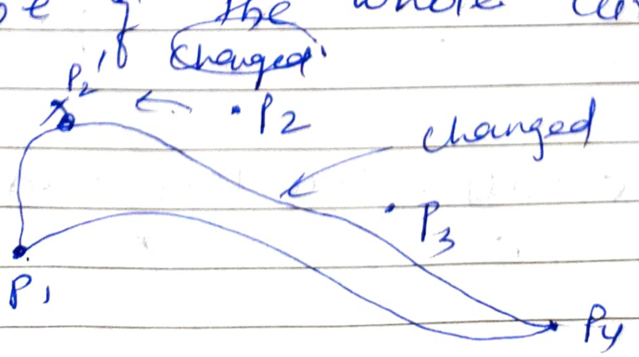
If degree is 1 – linear poly
—"— 2 - quadratic poly
—"— 3 - cubic poly

(4) The shape of defining polygon is
usually followed by Bezier curve.



(5) Bezier curves are tangent to their
first & last edges of control polyline

(6) Bezier curves exibit global control points
means moving a control point alters
the shape of the whole curve.



* Derivation of Bezier curve.

- Blending or basis function.

$$\boxed{P(u) = \sum_{i=0}^{n} P_i \, B_{i,n}(u)}$$

$\boxed{0 \leq u \leq 1}$

control points. $n+1 \Rightarrow$ control point.

where

$$B_{i,n}(u) = c(n,i)\, u^i (1-u)^{n-i}$$

$$\Downarrow$$

$$c(n,i) = \frac{n!}{i!\,(n-i)!}$$

$$\boxed{B_{i,n}(u) = \frac{n!}{i!\,(n-i)!}\, u^i (1-u)^{n-i}}$$

cubic Bezier - Degree is 3. $n=3$
control pt $= 4$

$$P(u) = P_0 B_{0,3}(u) + P_1 B_{1,3}(u) + P_2 B_{2,3}(u) + P_3 B_{3,3}(u)$$

$\uparrow$ control point

- Put values.

$\Rightarrow B_{0,3}(u) = c(3,0) \cdot u^0 (1-u)^{3-0}$

$$= \frac{3!}{0!\,3!} (1-u)^3 \Rightarrow (1-u)^3 \quad -(1)$$

$\Rightarrow B_{1,3}(u) = c(3,1) \cdot u^1 \cdot (1-u)^{3-1}$

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$$3u \cdot (1-u)^2 \quad — ②$$

$$\Rightarrow B_{2,3}(u) = C(3,2) \cdot u^2 \cdot (1-u)^{3-2}$$

$$= 3u^2 \cdot (1-u) \quad — ③$$

$$\Rightarrow B_{3,3}(u) = C(3,3) \cdot u^3 \cdot (1-u)^{3-3}$$

$$= u^3 \quad — ④$$

so, blending f^n is -
substitute values

$$\boxed{P(u) = P_0 \cdot (1-u)^3 + P_1 \cdot 3u (1-u)^2 + P_2 \cdot 3u^2 (1-u) + P_3 \cdot u^3}$$

x - cordinates

$$P(ux) = P_{x_0} (1-u)^3 + P_{x_1} \cdot 3u (1+u)^2 + P_{x_2} \cdot 3u^2 (1-u) + P_{x_3} \cdot u^3$$

y - cordinates

$$P(uy) = P_{y_0} (1-u)^3 + P_{y_1} \cdot 3u (1-u)^2 + P_{y_2} \cdot 3u^2 (1-u) + P_{y_3} \cdot u^3$$

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z co-ordinates.

$$P(u z) = P_{z_0}(1-u)^3 + P_{z_1} \cdot 3u(1-u)^2 + P_{z_3} \cdot 3u^2(1-u) + P_{z_3} \cdot u^3.$$

For Three ~~control~~ points dimensional take 3 co-ordinates.

ex:- Design a Bezier Curve controlled by 4 points A(1, 1) B(2, 3) C(4, 3) D(6, 4)

→ control point = 4
$n = 4-1 = 3$

$A(1,1) = P_0$ $\quad C(4,3) = P_2$
$B(2,3)\ P_1$ $\quad D(6,4) = P_3$

eqn of bezier curve.

$$P(u) = \sum_{i=0}^{n} P_i B_{i,n}(u)$$

$$B_{i,n}(u) = \frac{n!}{i!(n-i)!} \cdot u^i (1-u)^{n-i}$$

$\boxed{n=3}$

$$P(u) = P_0 B_{0,3}(u) + P_1 B_{1,3}(u) + P_2 \cdot B_{2,3}(u) + P_3 B_{3,3}(u)$$

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$$= P_0(1-u)^3 + P_1 \cdot 3u(1-u)^2 + P_2\, 3u^2(1-u) + P_3 u^3$$

To find $x(u)$ & $y(u)$

$P_0(1,1)$
$P_1(2,3)$
$P_2(4,3)$
$P_3(6,4)$

$$x(u) = 1\cdot(1-u)^3 + 2\times 3u(1-u)^2 + 4\times 3u^2(1-u) + 6\cdot u^3$$

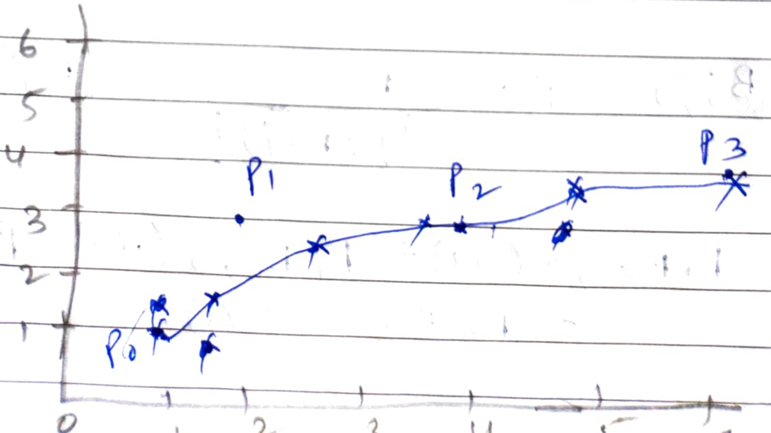
$$y(u) = (1-u)^3 + 9u(1-u)^2 + 9u^2(1-u) + 4u^3$$

$0 \le u \le 1$

0 to 1

take u 0.2 interval

u	$x(u)$	$y(u)$
0 substitute in $x(u)$ & $y(u)$	1	1
0.2	1.71	1.98
0.4	2.616	2.63
0.6	3.66	3.088
0.8	4.80	3.49
1	6	4

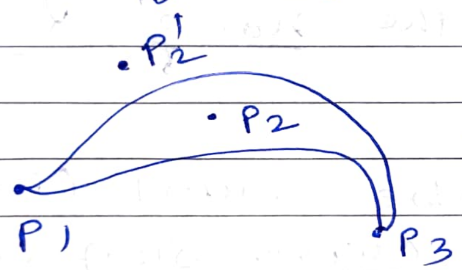


* B-Spline curve.

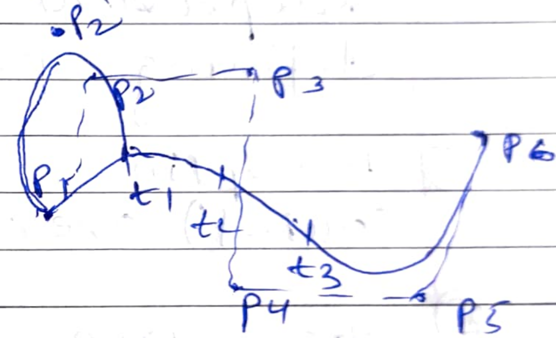
- Bezier curve (global control) — Depend on control point
- B-spline curve (local control)

Degree is depends on control points in Bezier curve.
Where in B-spline curve it does not depends on control points. It depends on order of polynomial. i.e. based on the no. of control points if order varies then blending f^n degree will be diff.

eg:- Bezier (global)



B-spline (Local)



If P2 change shape of curve will change

- only some part of portion will change.

Properties of B-spline curve

① The sum of B-spline basis function at any parameter 'u' equal to 1 i.e.

$$\sum_{i=1}^{n+1} N_{i,k}(u) = 1$$

equal segments.

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$n+1$ = no. of Control point
K = order of B-Spline curve.

② The basis f^n is +ve or zero for all parameter values i.e. $N_{i,K}(u) \geq 0$
Except for $K=1$ each basis f^n has one max value.

③ The maximum order of the curve is equal to the no. of vertices of defining polygon.

④ The degree of B-Spline polynomial is independent on the no. of vertices of defining polygon.

⑤ B-Spline allow the control (i.e. local control) over the curve surface.

⑥ The curve lies within the convex hull of its defining polygon.

⑦ The curve generally follow the shape of defining polygon.

* It is Represented as - Blending f^n.

$$\boxed{P(u) = \sum_{i=1}^{n+1} P_i N_{i,K}(u)}$$

where $u_{min} \leq u \leq u_{max}$

~~$1 \leq K \leq n+1$~~

$2 \leq K \leq n+1$

↳ Two partition.

$N_{i,K}(u) = (u - x_i) \cdot N_{i,K-1}(u) + (x_{i+K} - u) \cdot N_{i+1,K-1}(u)$

$x_{i+K-1} - x_i$ | $x_{i+K} - x_{i+1}$

$N_{i,K}(u) = \begin{cases} 1 & x_i \leq u \leq x_{i+1} \\ 0 & \text{otherwise} \end{cases}$

$x_i = 0$ if $i < K$

$x_i = i - K + 1$ if $K \leq i \leq n$

$x_i = n - K + 2$ if $i > n$.

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* Fractals

- Shapes like line, circle, ellips, curves defined using Eucliden geometry or mathematical eq^n.

- Natural objs like plants, tree, mountains, waves etc cannot described by such mathematical notion.

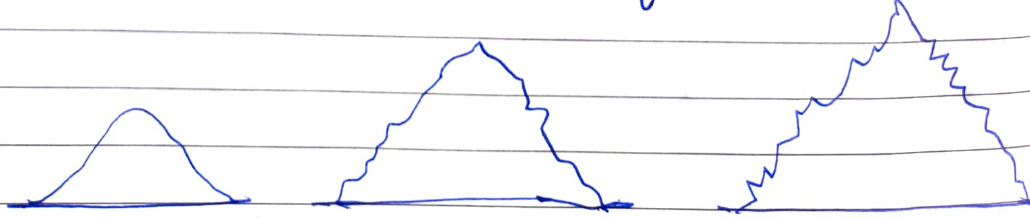
- Natural shapes have many irregular & self-similarities.

- Shapes generated using euclidean geom have a smooth surface. but they are not suitable for generating a realistic display of the natural scene.

- Natural objs are well described using fractal geometry methods.

* Properties of Fractal

1. Infinite detail at every point.
2. Self-similarity betn obj parts.
3. Natural objs have infinite details.



Distant mountain — closer view — more closer

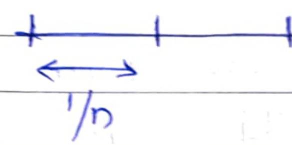
Fractal Dimension

- Amount of variation in obj detail is described by a number called fractal dimension.

- D can be measure of the roughness of the obj.

- obj with jaggy boundary have larger D value.

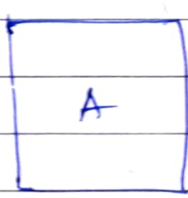
- Self - similar fractals are obtained by recursively applying the same scaling factor to Euclidean shapes.

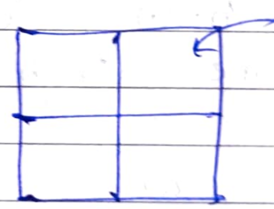


$D_E = 1$

$s = 1/n \,,\quad n = 2$

$$ns^1 = 1$$





$A' = \frac{A}{n}$

$D_E = 2$

$s = \frac{1}{\sqrt{n}} \,,\quad n = 4$

$$ns^2 = 1$$

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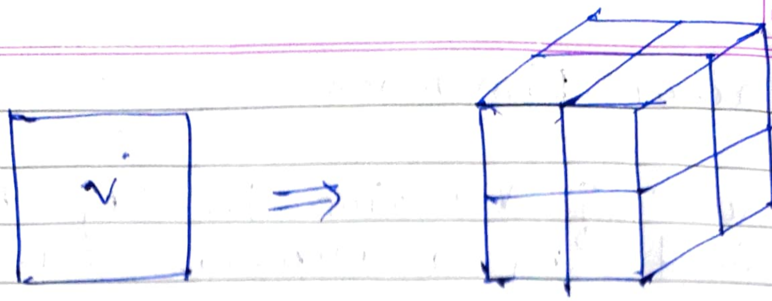


$D_E = 3$, $s = \frac{1}{3\sqrt{n}}$, $n = 8$

$$ns^3 = 1$$

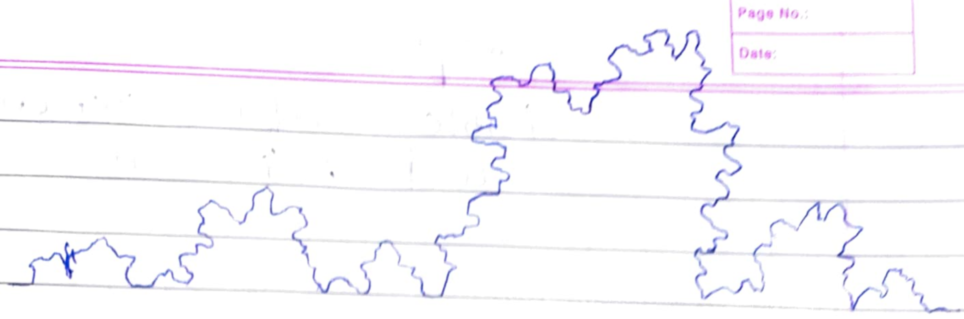
* Konch curve.

- Iterative construction is used to create this fractal shape.

- Start with a section of straight line.
- This line should be divided into 3 equal parts.

- without the base, an equilateral triangle is formed by removing the middle section & substituting it with two segments of the same length.

- For each segment, keep going through these stages indefinitely.

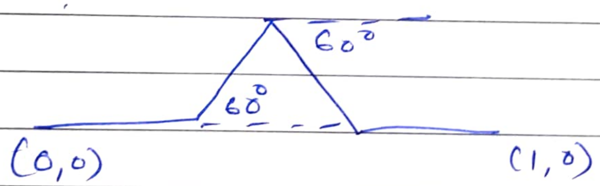


$$\sum_{K=1}^{4} r^d = 1 \Rightarrow d = \frac{\log(1/4)}{\log(1/3)} = \frac{\log(4)}{\log(3)}$$

$$= 1.2618$$

The koch curve is self similar with 4 non-overlapping copies of itself, each scaled by the factor $r < 1$. Therefore the similarity dimension, d, of the attractor of the IFs is the solution.

$r = 1/3$



It scaled by $r = 1/3$. Two segments must be rotated by 60°, one counterclockwise & one clockwise.