Review of Laplace Transforms

LAB EXPERIMENT 3

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***Abstract*—This experiment aimed to review and apply Laplace transforms to model the dynamics of a mass-spring-damper system and derive its transfer function. The governing second-order differential equation was transformed into the s-domain to obtain a simple algebraic expression relating input and output. Using Octave, the system's step response was simulated and compared to the output of the MyPhysicsLab online simulation tool. Both simulations exhibited nearly identical behaviour, confirming the validity of the theoretical model. The results emphasize the effectiveness of Laplace transforms in simplifying system analysis and demonstrate the predictive accuracy of transfer functions derived using this method.**

I. RATIONALE

Laplace transforms are essential tools in control theory for analyzing and solving linear differential equations. This experiment reviews the application of Laplace transforms to model system dynamics and derive transfer functions.

II. OBJECTIVES

- Calculate the transfer function of a given system (e.g., a mass-spring-damper system) by applying Laplace transforms to its governing differential equation.
- Verify the transfer function by comparing the system's simulated output (using the transfer function) against the theoretical response to a step input.
- Determine the impact of system parameters (e.g., damping coefficient, spring constant) on the transfer function and how they affect system behavior.

III. MATERIALS AND SOFTWARE

- Software: https://www.myphysicslab.com/springs/single-spring-en.html(Mass-spring-damper system simulation), Octave or Scilab (for system simulation and transfer function analysis)

IV. PROCEDURES

1) Set up the System: Use a mechanical or electrical system (e.g., a mass-spring-damper system or RC circuit). This system should have well-defined dynamics governed by linear differential equations.
2) Form the Differential Equation: Write down the governing differential equation for the system. For a mass-spring-damper system, for example, it might be a second-order equation that involves mass, damping coefficient, and spring constant.
3) Apply Laplace Transform: Use the Laplace transform to convert the differential equation into an algebraic equation in the s-domain. Simplify the equation and solve for the transfer function.
4) Simulate the system: Using Scilab or Octave, simulate the system's response to different inputs (e.g. step input). Compare the simulated results with theoretical predictions.
5) Verification: If possible, implement the system in hardware and verify the Laplace transform results by measuring the system's behavior.
6) Analysis: Discuss how the Laplace transform helps in simplifying system analysis, especially when dealing with complex differential equations.

V. OBSERVATION AND DATA COLLECTION

For this experiment, we consider a classical **mass-spring-damper system**. The system consists of:

- A mass m (in kilograms) that moves horizontally.
- A spring with spring constant k (in newtons per meter), which exerts a restoring force proportional to displacement.
- A damper with damping coefficient c (in newton-seconds per meter), which provides resistance proportional to velocity.
- An external force input $F(t)$ (in newtons) applied to the mass.
- The output of the system is the displacement $x(t)$ (in meters) of the mass.

The objective is to model the dynamics of this system using differential equations, apply Laplace transforms, and derive its transfer function.

Applying Newton's Second Law, the governing differential equation is:

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = F(t) \tag{1}$$

Taking the Laplace transform of both sides of Equation (1), assuming zero initial conditions ($x(0) = 0$, $\dot{x}(0) = 0$), we obtain:

$$ms^2X(s) + csX(s) + kX(s) = F(s) \tag{2}$$

Factoring $X(s)$:

$$X(s)\left(ms^2+cs+k\right)=F(s) \tag{3}$$

Thus, the transfer function $G(s)$, defined as the ratio of output to input in the s-domain, is:

$$G(s)=\frac{X(s)}{F(s)}=\frac{1}{ms^2+cs+k} \tag{4}$$

Upon running the Octave simulation with parameters $m = 1\,\mathrm{kg}$, $c = 1\,\mathrm{N\cdot s/m}$, and $k = 20\,\mathrm{N/m}$, the step response graph was generated. The resulting graph displayed a smooth rise in displacement followed by slight oscillations that quickly damped out, eventually reaching a steady-state value.

When compared with the graph produced by the MyPhysicsLab online simulator (using identical parameter values), the Octave-generated graph showed near-perfect resemblance in shape, amplitude, and settling behavior. Both systems exhibited similar transient and steady-state characteristics, confirming the validity of the transfer function derived via Laplace transforms.

DATA COLLECTION:

https://drive.google.com/drive/folders/
1zgglMygW3JK87nTb167GStlGicoHYIb8?usp=sharing

Click here to open the Drive

VI. DATA ANALYSIS

- **System Type:** Mass-spring-damper system
- **Parameters used:**
  - Mass (m) = 1 kg
  - Damping coefficient (c) = 1 N·s/m
  - Spring constant (k) = 20 N/m
- **Input:** Step force input
- **Output:** Displacement $x(t)$ over time

Aspect	**Octave Simulation**	**MyPhysicsLab Simulation**
Initial Response	Sharp rise	Sharp rise
Oscillations	Light oscillations before settling	Light oscillations before settling
Steady-State	Quickly achieved	Quickly achieved
General Shape	Overdamped (close to critically damped)	Overdamped (close to critically damped)

TABLE I

COMPARISON OF OCTAVE AND MYPHYSICSLAB SIMULATIONS

VII. DISCUSSION AND INTERPRETATIONS

The mass-spring-damper system is governed by a second-order linear differential equation. Applying the Laplace transform simplified the process of finding the system's transfer function, which characterizes its dynamic behavior.

The good match between the Octave and MyPhysicsLab results suggests:

- The Laplace transform accurately represents the real physical system.
- The damping is effective enough to prevent continuous oscillations but not so large as to slow down the system excessively.
- The system behavior is consistent with theoretical predictions for an underdamped or near critically damped system.

Additionally, the fact that the response reaches a steady-state without instability indicates that the system poles (roots of the denominator polynomial) have negative real parts, verifying system stability.

VIII. CONCLUSION

The experiment successfully demonstrated the application of Laplace transforms in deriving and analyzing the transfer function of a mass-spring-damper system. The Octave simulation closely matched the online MyPhysicsLab simulation, verifying the correctness of the theoretical model. Laplace transforms greatly simplified the modeling of the system dynamics, converting a complex differential equation into a manageable algebraic form. This experiment highlights the practical relevance of Laplace methods in control system analysis and design, especially for second-order dynamic systems.