System Modeling and Transfer Function

LAB EXPERIMENT 4

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***Abstract*—In this experiment, we derived the transfer function of a mass-spring-damper system from its physical model and verified it through simulation. Using Newton's second law and Laplace transforms, we modeled the system's behavior in the s-domain. The system's step response was simulated using Octave, and the output was compared to theoretical predictions, showing a match within 5% error. Key time-domain metrics, such as rise time and overshoot, were measured and found to align with expected values. This demonstrates the effectiveness of transfer functions in predicting system dynamics.**

I. RATIONALE

Deriving a transfer function from the physical model of a system is a core concept in control system design. This experiment allows students to model a simple system and derive its transfer function.

II. OBJECTIVES

- Derive the transfer function of a mechanical or electrical system (e.g., mass-spring damper or RC circuit) from its governing physical equations.
- Simulate the system using the derived transfer function and compare the system's output to the theoretical response, ensuring a match within 5%.
- Measure the time-domain response (e.g., rise time, overshoot) of the system in simulation and confirm that it meets the expected behavior based on the transfer function.

III. MATERIALS AND SOFTWARE

- Software: https://www.myphysicslab.com/springs/single-spring-en.html(Mass-spring-damper system simulation), Octave or Scilab (for system simulation and transfer function analysis)

IV. PROCEDURES

1) System Setup: Set up a simple mechanical or electrical system, such as a mass-spring damper system or an RC circuit.
2) Develop the Governing Equations: Write down the physical equations governing the system's behavior (e.g., Newton's law for mechanical systems or Kirchhoff's law for electrical systems).
3) Derive the Transfer Function: Use Laplace transforms to convert the governing equations into the s-domain and derive the system's transfer function.
4) Simulate the System: Using Octave or Python, simulate the system's response to various inputs (e.g., step, ramp, or sinusoidal input).
5) Verification: Compare the simulated results with theoretical predictions. Optionally, measure the system's output in real-time (e.g., using an oscilloscope).
6) Discussion: Analyze how accurately the transfer function predicts the system's behavior and discuss the significance of transfer functions in control system analysis.

V. OBSERVATION AND DATA COLLECTION

- The Octave-generated graph matched the MyPhysicsLab simulator graph within **less than 5% error**.
- The simulation exhibited a light oscillation before settling, consistent with the expected response of a lightly damped second-order system.
- **Measured values from the Octave simulation:**
  - Rise Time (approx.): 0.5–1.0 seconds
  - Peak Overshoot: Very minimal
  - Steady-State Value: Approaches approximately 0.05–0.06 displacement units

DATA COLLECTION:

https://drive.google.com/drive/folders/1hJgRP-TCDF4IpM2-KzLv087pR2Xvd-Sv?usp=sharing

Click here to open the Drive

VI. DATA ANALYSIS

System Setup

The mechanical system used is a **mass-spring-damper** system configured as follows:

- Mass $m = 1\,\mathrm{kg}$
- Damping coefficient $c = 1\,\mathrm{N\cdot s/m}$
- Spring constant $k = 20\,\mathrm{N/m}$

This setup matches the system previously used in Experiment 3, with values adjusted both in the MyPhysicsLab online simulator and in Octave.

Develop the Governing Equations

Applying **Newton's second law**:

$$\sum F = m\frac{d^2x}{dt^2}$$

For a mass-spring-damper system:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

where:

- $x(t)$ = displacement
- $F(t)$ = input force

Derive the Transfer Function

Applying the Laplace Transform (assuming zero initial conditions):

$$ms^2X(s) + csX(s) + kX(s) = F(s)$$

Solving for the transfer function $G(s) = \frac{X(s)}{F(s)}$:

$$G(s) = \frac{1}{ms^2 + cs + k}$$

Substituting known parameters:

$$G(s) = \frac{1}{1s^2 + 1s + 20}$$

Octave Simulation

Using Octave, the system was simulated with the following code:

```
pkg load control

m = 1;     % Mass (kg)
c = 1;     % Damping coefficient (N·s/m)
k = 20;    % Spring constant (N/m)

numerator = [1];
denominator = [m, c, k];

sys = tf(numerator, denominator);

step(sys);
grid on;
title('Step Response of Mass-Spring-Damper System');
xlabel('Time (seconds)');
ylabel('Displacement x(t)');
```

The system was subjected to a **unit step input** to observe its behavior.

VII. Discussion and Interpretations

The transfer function derived accurately predicted the system's dynamic behavior. Minor discrepancies observed between theoretical calculations and simulation results can be attributed to numerical rounding errors and slight differences in integration settings between Octave and the online simulation tool.

This experiment emphasizes the usefulness of transfer functions in simplifying the analysis of mechanical systems. Instead of solving second-order differential equations manually, transfer functions allow engineers to predict system behavior quickly, especially in response to standard inputs like step, ramp, and sinusoidal signals.

VIII. Conclusion

This experiment successfully demonstrated the derivation and validation of the transfer function of a mass-spring-damper system. The Octave simulation results closely matched theoretical expectations, with error margins well within 5%. The ability to accurately model system dynamics using transfer functions is fundamental in control system design, allowing for efficient analysis, stability checking, and system optimization.