FORMULA SHEET, ECEN 3400

Maxwell's equations:

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{S} \quad \left[emf = -\frac{d\Phi}{dt}\right], \quad \Phi_m = \int_S \vec{B}\cdot d\vec{S} \ [Wb]$$

$\nabla\times\vec{E} = -\partial\vec{B}/\partial t$

$\nabla\times\vec{H} = \vec{J} + \partial\vec{D}/\partial t$

$\nabla\cdot\vec{D} = \rho$

$\nabla\cdot\vec{B} = 0$

$$\oint_C \vec{H}\cdot d\vec{l} = \int_S\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)\cdot d\vec{S}$$

$$\oint_S \vec{D}\cdot d\vec{S} = \int_{vol}\rho\, dv = Q$$

$$\oint_S \vec{B}\cdot d\vec{S} = 0$$

$\vec{B} = \mu_0(\vec{H}+\vec{M})$, $\vec{D} = \varepsilon_0\vec{E}+\vec{P}$

linear: $\vec{B} = \mu_0\mu_r\vec{H}$, $\vec{D} = \varepsilon_0\varepsilon_r\vec{E}$

$E\left[\frac{V}{m}\right]$, $H\left[\frac{A}{m}\right]$, $D\left[\frac{C}{m^2}\right]$, $B[T]$

$\varepsilon_0\left[\frac{F}{m}\right]$, $\mu_0\left[\frac{H}{m}\right]$

Statics: $C = \frac{Q}{V}$ [F], $\underset{[J]}{W_e} = \frac{1}{2}CV^2$, $\underset{[J/m^3]}{w_e} = \frac{1}{2}\varepsilon E^2$, $F_x = \pm\frac{dW_e}{dx}\Big|_{V=const,\ Q=const}$

$\vec{F}_e = Q\vec{E}$, $V_{AB} = \int_A^B \vec{E}\cdot d\vec{l}$, $\vec{F}_{e_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}\vec{u}_{r_{12}}$

DC Currents: $\underset{[A/m^2]}{\vec{J}} = NQ\vec{v}$, $I = \frac{dq}{dt} = \int_S \vec{J}\cdot d\vec{S}$, $\oint_S \vec{J}\cdot d\vec{S} = 0$ (KCL), $\vec{J} = \sigma\vec{E} = \frac{1}{\rho}\vec{E}$, $\sigma\left[\frac{S}{m}\right]$, $\rho[\Omega m]$

$\left[\frac{W}{m^3}\right]$ $p_J = \sigma E^2 = J^2/\rho$

Static magnetic field:

$$d\vec{F}_{m12} = I_2 d\vec{l}_2 \times \underbrace{\frac{\mu_0}{4\pi}\frac{I_1 d\vec{l}_1\times\vec{u}_{r_{12}}}{r^2}}_{d\vec{B}}, \quad d\vec{F} = I d\vec{l}\times\vec{B}$$

$\vec{F}_m = Q\vec{v}\times\vec{B}$

Induced field: $\vec{E}_{ind} = \vec{v}\times\vec{B}$, $\left(\vec{F}_{TOT} = Q(\vec{E}+\vec{v}\times\vec{B}) = Q(\vec{E}_{stat}+\vec{E}_{ind})\right)$ → Lorentz force

$\underset{[H]}{L} = \frac{\Phi}{I}$ self-ind. $L_{12} = \frac{\Phi_{12}}{I_1} = L_{21}$ mutual inductance

$emf = -L\frac{di}{dt}$, $W_m = \frac{1}{2}\sum_j\sum_k L_{jk}I_jI_k = \frac{1}{2}L_{11}I_1^2 + \frac{1}{2}L_{22}I_2^2 + L_{12}I_1I_2$ for 2 contours

Single contour: $\underset{[J]}{W_m} = \frac{1}{2}LI^2$, $\underset{[J/m^3]}{w_m} = \frac{1}{2}\mu H^2$, $F_x = \pm\frac{dW_m}{dx}\Big|_{I=const,\ \Phi=const}$

Boundary conditions: $E_{1tang} = E_{2tang}$, $H_{1tang} = H_{2tang}$ $[H_{1t} - H_{2t} = J_s]$

$D_{1norm} = D_{2norm}$, $B_{1norm} = B_{2norm}$

$[D_{1n} - D_{2n} = \sigma]$ Also, $J_{1n} = J_{2n}$

Transmission lines: $V(z) = V_+ e^{-j\beta z} + V_- e^{+j\beta z}$ (for $\cos\omega t$ time-dep.)
$I(z) = \frac{V_+}{Z_0} e^{-j\beta z} - \frac{V_-}{Z_0} e^{+j\beta z}$

$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{L'C'}$, $c = \frac{1}{\sqrt{L'C'}}$, $Z_0 = \frac{V_+}{I_-} = -\frac{V_-}{I_-} = \sqrt{\frac{L'}{C'}}$, $\lambda = \frac{c}{f}$ Lossless lines

$\rho = \frac{V_-}{V_+}$, $\tau = \frac{V_{load}}{V_+}$, At load: $\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$, $\tau = 1 + \rho$, $VSWR = \frac{1+|\rho|}{1-|\rho|}$

$Z_{in}(z=\ell) = Z_0 \frac{Z_L \cos\beta\ell + jZ_0 \sin\beta\ell}{Z_0 \cos\beta\ell + jZ_L \sin\beta\ell}$, $Z_{in}(\ell = \frac{\lambda}{4}) = \frac{Z_0^2}{Z_L}$
at $-\ell = z$ away from load

Lossy lines: $Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$, $\gamma = \alpha + j\beta = \sqrt{(R'+j\omega L')(G'+j\omega C')}$, $P(z) = P(0)e^{-2\alpha z}$
α [Np, dB]

Transmission line (1-D) wave equation: $\frac{\partial^2 v(z,t)}{\partial z^2} - L'C' \frac{\partial^2 v(z,t)}{\partial t^2} = 0$ | $\frac{d^2V}{dz^2} + \omega^2 L'C'V = 0$

Plane waves in 3-D: $\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$ | $\nabla^2 \vec{E} + \omega^2 \mu\varepsilon \vec{E} = 0$ Helmholtz

$\xrightarrow{\cos\omega t}$

Plane waves: $\vec{E} \perp \vec{H}$, both functions of z, t.
$c = \frac{1}{\sqrt{\varepsilon\mu}}$, $\eta = Z = \sqrt{\frac{\mu}{\varepsilon}}$, $\frac{E_x}{H_y} = \eta$, $\vec{E} \times \vec{H} = \vec{P}$ $\left[\frac{W}{m^2}\right]$ Pointing vector → direction and power flow

$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, $\tau = 1 + \rho$, Snell: $\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

Waveguides (TE modes) $\beta_{TE} = \omega\sqrt{\varepsilon\mu}\sqrt{1 - \frac{f_c^2}{f^2}}$, $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$, $f_{c_{TE10}} = \frac{c}{2a}$

$Z_{TE10} = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\sqrt{1 - f_c^2/f^2}} > \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}}$

Antennas: $D = \frac{S(\theta,\phi)}{S_i}$, $S_i = \frac{P_{Tr}}{4\pi r^2}$, $A_{eff} = \frac{P_{rec}}{S_{rec}(\theta,\phi)}$, $P_{rec} = P_{Tr} \frac{G_T A_R}{4\pi r^2}$ (D_T)

$\frac{D}{A} = \frac{4\pi}{\lambda^2}$ $\boxed{\text{Skin depth: } \delta = 1/\sqrt{\pi f \sigma \mu}}$

Constants: $\varepsilon_0 = 8.854 \cdot 10^{-12} \frac{F}{m}$, $\mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m}$, $\eta_0 = 120\pi\ \Omega \simeq 377\Omega$
$c_0 = 3 \cdot 10^8 \frac{m}{s}$, $q_e = e = 1.6 \cdot 10^{-19}$ C