Project Euler Problem 17
Number Letter Counts

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June 8, 2013

1 Problem Statement

If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are $3 + 3 + 5 + 4 + 4 = 19$ letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

NOTE: Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of "and" when writing out numbers is in compliance with British usage.

2 Method

Since we are converting each number to a string in the range 1,1000, we might as well save ourselves a great deal of work and find common quantities.

2.1 Redifining the numbers

The following table shows all of the most common numbers (which also make up parts of larger numbers) as the integer value of the letters present.

Number	Letters
One	3
Two	3
Three	5
Four	4
Five	4
Six	3
Seven	5
Eight	5
Nine	4

Number	Letters
Ten	3
Eleven	6
Twelve	6
Thirteen	8
Fourteen	8
Fifteen	7
Sixteen	7
Seventeen	9
Eighteen	8
Nineteen	8

Number	Letters
Twenty	6
Thirty	6
Forty	5
Fifty	5
Sixty	5
Seventy	7
Eighty	6
Ninety	6

2.2 Summation

The next logical step was to find the values for repeating ranges such as 1-9, 1-99 and so on.

2.2.1 { 1-9 }

The first range to have its values summed was 1 through 9:

$$\begin{aligned} a &= \sum_{i=1}^{9} Letters[i] \\ a &= \boxed{36} \end{aligned}$$

2.2.2 { 10-19 }

Next, the teens

$$\begin{aligned} b &= \sum_{i=10}^{19} Letters[i] \\ b &= \boxed{70} \end{aligned}$$

2.2.3 { 20-90 }

Next, a corner case was considered; every ten integers were summed as c.

$$\begin{aligned} c &= \sum_{i=20}^{90} Letters[i \times 10] \\ c &= \boxed{46} \end{aligned}$$

So the number of letters in the range {1,99} is:

$$\begin{aligned} 9a + b + 10c &= d \\ d &= \boxed{854} \end{aligned}$$

With this in mind, and given that

$$\text{One Thousand} = \boxed{11}$$

the final equation can be set up:

$$\overbrace{10d}^{1\to 99} + \underbrace{(900 \times 7)}_{\text{hundred}} + \overbrace{(891 \times 3)}^{\text{and}} + 100a + 11$$

Final Answer : $\boxed{21124}$