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KOALA: Estimating coalition probabilities in multi-party electoral systems

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Abstract 150 to 250 words
Common election poll reporting is often misleading as sample uncertainty is either not covered at all or only insufficiently. For a more comprehensive coverage, we propose shifting the focus towards reporting survey-based probabilities for specific election outcomes. We present such an approach for multi-party electoral systems, focusing on probabilities of coalition majorities. A Monte Carlo based Bayesian Multinomial-Dirichlet model is used for estimation. The method utilizes published opinion polls and is accompanied by a pooling approach to summarize multiple current surveys, accounting for dependencies between polling agencies. Sample uncertainty-based probabilities are estimated, assuming the election was held today. An implementation in `R` is freely available.

Keywords 4 to 6 keywords Election analysis · Opinion polls · Election reporting · Multinomial-Dirichlet · Pooling

1 Introduction and data

Election polls try to represent the public opinion based on a finite sample. Usually, polling agencies publish the shares of the electorate who would vote for the

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reported political parties *if the election was held today*, the number of overall respondents and – more or less prominent – information about the uncertainty of the results. Current reporting of general media on such surveys in the end is most often limited to the observed shares, while sample uncertainty is usually ignored or only covered insufficiently.

Prominent examples for inaccurate reporting can be found in multi-party electoral systems. In such, one party usually doesn't obtain enough votes for a majority. In these situations, multiple parties form a so-called coalition to jointly obtain the necessary majority share of seats in parliament. Often then, a coalition is stated to "lose" its majority just because the joint poll share drops under 50% from one opinion poll to the next (cf. FAZ.net, 2017). Such interpretations are clearly misleading as opinion polls are based on a finite sample of voters and only allow conclusions about the whole electorate with a specific certainty. Reporting results in this manner thus reinforces general misunderstandings of the public about which message to draw from published opinion polls. Additionally, the perception of the observed voter shares to be definite values that hold for the electorate can also lead to an – to some extent – unjustified criticism of general opinion polls if the final election result differs from the latest polls published before election day. One prominent example is XXX hier wenn moeglich a Beispiel fuer so a unberechtigte Kritik an Umfragen nach da Wahl XXX (QUELLE).

Beyond ensuring proper reporting of sample uncertainties, in our opinion, the whole focus in survey reporting should be shifted from describing the raw observed party shares. Instead, reporting should focus on the most relevant question, i.e. *how probable* specific events or election outcomes are. Events of interest can range from probabilities for parties passing a specific voter share via one party obtaining more seats in parliament than another through to probabilities for majorities of potential multi-party coalitions. As such probabilities combine both – the observed raw voter share and sample uncertainty – in one number, they are in theory easier to communicate to the general public. Evtl (hier oder in da Discussion) auf de geplante Graefe-Studie verweisen, der de Vermittelbarkeit vo soichane W'keiten untersuacha mecht.

We present our KOALA (Coalition Analysis) approach to estimate such probabilities to bring more value to opinion poll-based reporting, specifically focusing on multi-party electoral systems and the estimation of probabilities for coalition majorities. To estimate the probabilities, a Bayesian Multinomial-Dirichlet model with Monte Carlo simulations is used. Also, a pooling approach is presented to summarize multiple current opinion polls to reduce sample uncertainty. Prior to the German federal elections 2013 and 2017, results based on (an earlier iteration of) our approach already entered general media reporting (cf. ZEIT ONLINE, 2013; Gelitz, 2017).

As database, we use opinion polls conducted by established polling agencies, quantifying the electoral behavior *if an election was held today*. We focus on the question of quantifying current majority situations, not taking into consideration potential shifts until election day. Approaches for predicting future election outcomes based on past information can e.g. be found in Graefe (2017) or Norpoth and Gschwend (2010).

All methods were implemented in `R` (R Core Team, 2017) and are available in the open-source package `coalitions` on GitHub (Bender and Bauer, 2018). An interactive `shiny`-based (Chang et al., 2017) website `koala.stat.uni-muenchen.`

`de` visualizes estimated coalition probabilities and is used to communicate the results to the general public, covering German federal and state-wide elections. The process of fetching new polls, updating the website and sending out Twitter messages based on the newest results is automated and allows for an immediate transfer of the estimated event probabilities to media and pulic.

2 Calculation of probabilities

In the last opinion poll conducted before the German federal election 2013 (Forsa, 2013), special interest was on whether CDU/CSU-FDP (also "Union-FDP") would obtain enough votes to form the governing coalition:

Table 1 Observed voter shares in the Forsa opinion poll for the German federal election, published September 20th, 2013 with $n = 1995$ respondents

Union	SPD	Greens	FDP	The Left	Pirates	AfD	Others
40%	26%	10%	5%	9%	2%	4%	4%

The German election system mandates a 5% votes share for parties to enter the parliament. Votes for parties below this threshold are redistributed (proportionally) to parties above it, leading to the following redistributed party shares:

Table 2 Redistributed party shares based on the Forsa opinion poll for the German federal election, published September 20th, 2013 with $n = 1995$ respondents. Parties marked with "–" didn't pass the 5% hurdle.

Union	SPD	Greens	FDP	The Left	Pirates	AfD	Others
44.44%	28.89%	11.11%	5.56%	10.00%	–	–	–

As can be seen in Table 2, Union-FDP with its 45% raw voter share would get exactly 50% of parliament seats after redistribution. Thus, ingoring uncertainty one would conclude that a majority of the coalition is slightly missed. However, it is clear that this only holds with a certain probability and particularly depends on whether FDP, Pirates and AfD each pass the 5% hurdle.
No bissl rumlabern a la voter numbers are Multinomially distributed, party shares as shares that sum up to 1 are Dirichlet distributed.
Evtl a no: Using the prior one could steuern welche Parteien in de Surveys unter- oder uberschaetzt wern
To estimate coalition probabilities, we choose a Bayesian Multinomial-Dirichlet model with Jeffreys prior as uninformative prior for the true party shares θ_j (Gelman et al., 2013):

$$\begin{aligned} \theta = (\theta_1, \dots, \theta_k)^{\mathrm{T}} &\sim Dirichlet(\alpha_1, \dots, \alpha_k), \\ \text{with} \qquad \alpha_1 = \dots = \alpha_k &= \frac{1}{2} \end{aligned} \tag{1}$$

Given one (pooled) survey, the posterior also is a Dirichlet distribution with $\alpha_j = x_j + \frac{1}{2}$ for each party j and its observed vote counts x_j.
Using Monte Carlo simulations of election outcomes, one can obtain specific event probabilities by taking their relative frequency of occurence. E.g., if a specific event occurs in $2,600$ of $10,000$ simulations this equals an estimated probability of 26%.
Figure 1 shows the simulated parliament seat shares for the coalition Union-FDP, based on the observed voter shares in Table 1. The estimated density is clearly bimodal as the observed FDP share before redistribution is exactly 50% and so FDP only enters the parliament in half of the simulations. The corresponding probability for a seat majority of Union-FDP is 26.33%.

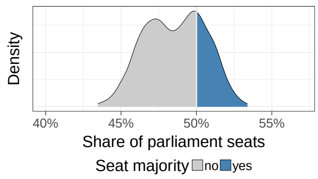

Fig. 1 Density of $10,000$ simulated parliament seat shares for the coalition Union-FDP before the German federal election in September 2013 based on the Forsa opinion poll in Table **??**. The part of the density encoding for seat majorities is colored blue.

As such density plots depict both the probability and the underlying uncertainty for specific coalitions, they are a nice possibility to communicate uncertainties underlying opinion polls to the general public. As the estimation of event probabilities with our approach, such plots can be created for all kinds of specific election outcomes.
To visualize the *development* of such probabilities for a specific coalition we recommend extending the visualization of Figure 1 by using ridgeline plots (Wilke, 2017) for the simulated seat distributions (Fig. 2).

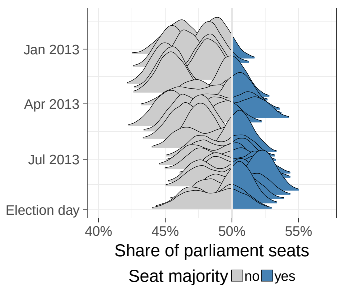

Fig. 2 Development of $10,000$ simulated parliament seat share densities for the coalition Union-FDP before the German federal election in September 2013 based on Forsa opinion polls. The parts of the densities encoding for seat majorities are colored blue.

Before applying the Bayesian approach and as vote shares are usually rounded before publication, we adjust the available data by adding uniformly distributed random noise to the observed voter shares x_j to avoid a potential bias caused by the use of rounded numbers:

$$\begin{aligned} x_{j,adj} &= x_j + r_{\gamma,j}, \\ \text{with} \quad r_{\gamma,j} &\sim U[-\gamma,\gamma]. \end{aligned} \tag{2}$$

E.g., for data rounded to 0.5% we use a correction term of $\gamma = 0.5\%$. Afterwards, the adjusted shares are rescaled to ensure a sum of 100% and the Bayesian approach is performed based on the adjusted shares.

3 Pooling approach

In the presence of multiple published opinion polls, pooling is used to summarize the observed results in order to reduce sample uncertainty. To assure a reliable pooling regarding the current public opinion, we only use polls published within the past 14 days and only use the most recent survey published by each polling agency.
Looking at a single poll i, the observed number of votes X_{ij} for each of k parties follow a multinomial distribution with sample size n_i and underlying, unknown party shares θ_j in the population. Pooling over multiple such polls as independent random samples leads to another multinomial distribution for the summed number of votes $\sum_i X_{ij}$:

$$\sum_i X_{i1}, \ldots, \sum_i X_{ik} \sim Multinomial\left(\sum_i n_i, \theta_1, \ldots, \theta_k\right). \tag{3}$$

Further analyses, however, show that polls from different polling agencies are correlated and the independency assumption does not hold. Therefore, we adjust the resulting multinomial distribution by using an *effective sample size* (Hanley et al., 2003), reflecting that the aggregation over multiple polls does not reflect the information from a sample with $\sum_i n_i$ observations.
Quantification of pairwise correlation is done based on the variance of the difference between two polls. The following equation holds for two independent random sample polls A and B:

$$\begin{aligned} & Var(X_A - X_B) = Var(X_A) + Var(X_B) - 2 \cdot Cov(X_A, X_B) \\ \Leftrightarrow \quad & Cov(X_{Aj}, X_{Bj}) = \frac{1}{2} \cdot \left(Var(X_{Aj}) + Var(X_{Bj}) - Var(X_{Aj} - X_{Bj})\right). \end{aligned} \tag{4}$$

We take $Var(X_{Aj})$ and $Var(X_{Bj})$ as the theoretical variances of the binomially distributed, observed voter numbers and estimate $Var(X_{Aj} - X_{Bj})$ based on the observed differences between the party shares. Having done so, one can estimate the covariance $Cov(X_{Aj}, X_{Bj})$ and accordingly also the correlation. As the binomial distribution is directly proportional to the sample size, the effective sample size n_{eff} can be defined as the ratio between the estimated variance for the pooled sample and the theoretical variance of a sample of size one:

$$n_{\text{eff}} = \frac{Var(\text{pooled})}{Var(\text{sample of size 1})},$$

with, in the case of two surveys,

$$Var(\text{pooled}) = Var(X_A + X_B) = Var(X_A) + Var(X_B) + 2Cov(X_A, X_B)$$

and Var(sample of size 1) the theoretical variance of the pooled share.
Looking at the party-specific correlations between 20 surveys conducted by the two most regular German polling agencies, Emnid and Forsa, we on average end up with a medium high correlation, using mean party shares and sample sizes per institute for the theoretical variances. Other institute comparisons were not performed as too few published surveys were conducted cover comparable time frames. For simplicity, we do not recalculate the correlation for each simulation, but rather set the correlation used in our methodology to 0.5. For convenience, the calculation of n_{eff} is based on the party with most votes, as the specific party choice only marginally affects the results.

Pooling in practice
- time window of 14 days
- extended (downweighted) time window in the case of only few published polls
- evtl no a Rechenbeispui fias Pooling gem, d.h. 'beim Poolen vo 2 Umfragen mit $n_1 = x$ und $n_2 = y$ kriagt ma a $n_{eff} = z$ raus

4 Discussion

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5 Conclusion

We presented an approach to estimate probabilities for specific election outcomes based on publicly available opinion polls. Pooling allows for the inclusion of information from multiple surveys. Visualizing the results on a publicly available website for chosen elections, our long-term goal is to make proper uncertainty assessment in general opinion poll-based reporting the rule, rather than an exception.

References

Bender, A. and Bauer, A. (2018). *shiny: Web Application Framework for R.* URL `http://doi.org/10.5281/zenodo.1172595`. adibender/coalitions (Version v0.5.7). Zenodo.
Chang, W., Cheng, J., Allaire, J., Xie, Y., and McPherson, J. (2017). *shiny: Web Application Framework for R.* URL `https://CRAN.R-project.org/package=shiny`. R package version 1.0.5.
FAZ.net (2017). Umfrage zur Bundestagswahl: Schwarz-Gelb verliert die Mehrheit. `http://archive.is/SuXVt`. Accessed 15 February 2018.
Forsa (2013). Wenn am nächsten Sonntag Bundestagswahl wäre... `http://archive.is/f9vse`. Accessed 15 February 2018.

Gelitz, C. (2017). Können die aktuellen Umfragen noch falschliegen? http://archive.is/JydHd. Accessed 15 February 2018.

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., and Rubin, D. B. (2013). *Bayesian data analysis*, volume 3. CRC press Boca Raton, FL.

Graefe, A. (2017). The pollyvotes long-term forecast for the 2017 german federal election. *PS: Political Science & Politics*, **50**(3), 693–696.

Hanley, J. A., Negassa, A., and Forrester, J. E. (2003). Statistical analysis of correlated data using generalized estimating equations: an orientation. *American journal of epidemiology*, **157**(4), 364–375.

Norpoth, H. and Gschwend, T. (2010). The chancellor model: Forecasting german elections. *International Journal of Forecasting*, **26**(1), 42–53.

R Core Team (2017). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

Wilke, C. O. (2017). *ggridges: Ridgeline Plots in 'ggplot2'*. URL https://CRAN.R-project.org/package=ggridges. R package version 0.4.1.

ZEIT ONLINE (2013). Serie: Wahlistik. http://archive.is/1SU1I. Accessed 15 February 2018.