KOALA: Estimating coalition probabilities in multi-party electoral systems

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Abstract: Common election poll reporting is often misleading as sample uncertainty is either not covered at all or only insufficiently. For a more comprehensive coverage, we propose shifting the focus towards reporting survey-based probabilities for specific election outcomes. We present such an approach for multi-party electoral systems, focusing on probabilities of coalition majorities. A Monte Carlo based Bayesian Multinomial-Dirichlet model is used for estimation. The method utilizes published opinion polls and is accompanied by a pooling approach to summarize multiple current surveys, accounting for dependencies between polling agencies. Sample uncertainty-based probabilities are estimated, assuming the election was held today. An implementation in `R` is freely available.

Keywords: Election analysis; Opinion polls; Election reporting; Multinomial-Dirichlet; Pooling.

1 Introduction and data

Election polls try to represent the public opinion based on a finite sample. Current reporting on such surveys is most often limited to the observed shares, while sample uncertainty is usually ignored. Often e.g., a coalition – i.e. a union of multiple parties, formed to reach a governing majority – is stated to "lose" its majority just because the joint poll share drops under 50% (cf. "Umfrage zur Bundestagswahl", 2017). In our opinion, the focus in survey reporting in multi-party electoral systems should be shifted towards *how probable* an events is. We present our KOALA (Coalition Analysis) approach to estimate such probabilities to bring more value to opinion poll-based reporting. Prior to the German federal elections 2013 and 2017, results based on (an earlier iteration of) our approach already entered general media reporting (cf. "Serie: Wahlistik", 2013, or Gelitz, 2017).

We use data from established polling agencies, quantifying the electoral behavior *if an election was held today.* Our approach is to be differentiated

from prediction-aimed methods (cf. Graefe, 2017 or Norpoth & Gschwend, 2010) as potential shifts until election day are not taken into consideration. A Bayesian Multinomial-Dirichlet model with Monte Carlo simulations is used for estimation. Also, a pooling approach is presented to summarize multiple current opinion polls to reduce sample uncertainty.
All methods were implemented in `R` and are available in the open-source package `coalitions` on GitHub (Bender & Bauer, 2018). An interactive `shiny`-based (Chang et al., 2017) website `koala.stat.uni-muenchen.de` visualizes the results and is used for communication to the general public. The process of fetching new polls, updating the website and sending out Twitter messages based on the newest results is automated.

2 Calculation of probabilities

In the last opinion poll conducted before the German federal election 2013 (Forsa, 2013), special interest was on whether CDU/CSU-FDP (also "Union-FDP") would obtain enough votes to form the governing coalition:

TABLE 1. Observed voter shares in the Forsa opinion poll published September 20th, 2013 with $n = 1995$ respondents

Union	SPD	Greens	FDP	The Left	Pirates	AfD	Others
40%	26%	10%	5%	9%	2%	4%	4%

The German election system mandates a 5% votes share for parties to enter the parliament. Votes for parties below this threshold are redistributed (proportionally) to parties above it. Here, Union-FDP with its 45% raw voter share would get exactly 50% of parliament seats after redistribution. Thus, ingoring uncertainty one would conclude that a majority is slightly missed. However, it is clear that this only holds with a certain probabilitiy and particularly depends on whether FDP and/or AfD pass the 5% hurdle.
To estimate coalition probabilities, we choose a Multinomial-Dirichlet model with uninformative prior for the true party shares θ_j (Gelman et al., 2013):

$$(\theta_1, \ldots, \theta_k)^{\mathrm{T}} \sim Dirichlet(\alpha_1, \ldots, \alpha_k), \quad \text{with} \quad \alpha_1 = \ldots = \alpha_k = \frac{1}{2}$$

Given one (pooled) survey, the posterior also is a Dirichlet distribution with $\alpha_j = x_j + \frac{1}{2}$ for each party j and its observed vote counts x_j.
Using Monte Carlo simulations of election outcomes, one can obtain specific event probabilities by taking their relative frequency of occurence. As vote shares are usually rounded before publication, we adjust the available data by adding random noise to x_j before calculating the Bayesian posterior.
To visualize the development of such probabilities together with the underlying uncertainty for a specific coalition we recommend using ridgeline

plots (Wilke, 2017) for the simulated seat distributions (Fig. 1). Looking at the probabilities based on the last opinion poll before the German election 2013, the posterior distribution is bimodal, based on the distinction whether FDP and/or AfD pass the 5% hurdle. The resulting probability for a Union-FDP majority is 27.2%, based on $10,000$ simulations.

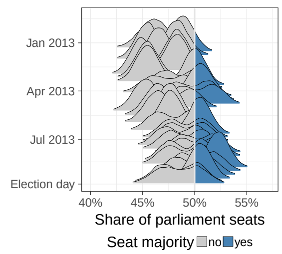

FIGURE 1. Development of simulated parliament seat share densities for the coalition Union-FDP before the German federal election in September 2013 based on Forsa opinion polls. The parts of the densities encoding for seat majorities are colored blue.

3 Pooling approach

Pooling is used to summarize multiple polls to reduce sample uncertainty. To reliably estimate the current public opinion, we use polls published within the past 14 days, only using the most recent survey per polling agency. As vote counts X_{ij} of party j in poll i are multinomially distributed, so are the summed number of votes $\sum_i X_{ij}$ when pooling multiple independent polls. Further analyses, however, show that polls from different polling agencies are correlated. Therefore, we adjust the distribution by using an *effective sample size* (Hanley et al. ,2003). Party-specific correlations were estimated based on 20 surveys of polling agencies Emnid and Forsa, using

$$Cov(X_{Aj}, X_{Bj}) = \frac{1}{2} \cdot \left(Var(X_{Aj}) + Var(X_{Bj}) - Var(X_{Aj} - X_{Bj})\right),$$

with $Var(X_{Aj})$, $Var(X_{Bj})$ the theoretical variances of binomial distributions and $Var(X_{Aj} - X_{Bj})$ estimated from the party share differences. For simplicity, we set the correlation to a fixed value of 0.5. The effective sample size n_{eff} is then defined as the ratio between the estimated variance for the pooled sample and the theoretical variance for a sample of size one:

$$n_{\text{eff}} = \frac{Var(\text{pooled})}{Var(\text{sample of size one})}.$$

For convenience, this calculation is based on the party with most votes, as the specific party choice only marginally affects the results.

4 Conclusion

We presented an approach to estimate probabilities for specific election outcomes based on publicly available opinion polls. Pooling allows for the inclusion of information from multiple surveys. Visualizing the results on a publicly available website for chosen elections, our long-term goal is to make proper uncertainty assessment in general opinion poll-based reporting the rule, rather than an exception.

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