Assignment

Exercise 1: Consider the ARMA(1,1) process: $(1 - \phi B)X_t = (1 - \theta B)a_t$

with ϕ and θ are two real numbers; $|\theta| < 1$, $|\phi| < 1$, a_t is white noise $(0, \sigma^2)$

1. Show that there exists an absolutely convergent series $\sum_{k=1}^{\infty} b_k$, such that the optimal prediction of the process X_t at time $t + 1$, is $\hat{X}_{t+1} = \sum_{k=1}^{t+1} b_k \hat{X}_{t+1-k}$

You are given the following five last annual observations of a series X_t :

t	2007	2008	2009	2010	2011
X_t	1.5	1.9	3.2	2.2	2.4

After analysis, the following model is considered adequate for this series:

variable	Coefficient	Std Error	t-Statistic	Prob
$AR(1)$	0.495639	0.011520	43.02579	0.00000
$MA(1)$	0.413047	0.012079	34.19510	0.00000

2. Calculate the best forecast for X_{2012}, X_{2013}?

Exercise 2: You are given the following observations of a series X_t.

t	2008	2009	2010	2011
x_T	11.2	13.5	11.2	10.4

After analysis, the following model is deemed appropriate for this series:

Variable	Coefficient	STD ERROR	T-Statistic	Prob
C	7.124608	0.014099	505.3234	0.00000
$AR\ (1)$	0.297718	0.009525	31,25600	0.00000

with A_T i. i.d. $\leftarrow\cdots N(0,\ 1)$

Compute the best prediction, the variance of the prediction error, and the confidence interval (95%) for each of X_{2012}, X_{2013}, X_{2014}.