Time Series

Assignment

Exercise 1:

Consider the process $ARMA(1,1)$: $(1-\phi B)Xt = (1-\theta B)at$

with ϕ being two reals and θ being two reals; |θ| < 1, |ϕ| < 1, at is white noise (0.σ2)

1. Show that there is an absolutely convergent series $\sum_{k=1}^{\infty} b_k$ *such that the optimal prediction of process Xt at date t + 1 is* $\hat{X}_{t+1} = \sum_{k=1}^{\infty} b_k \hat{X}_{t+1-k}$

We know that $\frac{(1-\phi B)}{(1-\theta B)} X_t = a_t$

$\Rightarrow \sum_{k=0}^{\infty} \theta^k X_{t-k} - \phi \sum_{k=0}^{\infty} \theta^k X_{t-k-1} = a_t$

$\Rightarrow X_t + \sum_{k=1}^{\infty} \theta^k X_{t-k} - \sum_{p=1}^{\infty} \phi.\theta^{p-1} X_{t-p} = a_t$

$\Rightarrow X_{t+1} = a_{t+1} - \sum_{k=1}^{\infty} (\theta - \phi)\theta^{k-1} X_{t-k+1}$

$\Rightarrow \hat{X}_{t+1} = \sum_{k=1}^{\infty} -(\theta - \phi)\theta^{k-1} X_{t-k+1}$

and since $t - k + 1 \geq 0 \Rightarrow k \leq t + 1$

then: $\hat{X}_{t+1} = \sum_{k=1}^{\infty} b_k X_{t-k+1}$

white $b_k = -(\theta - \phi)\theta^{k-1}$ et $\sum_{k=1}^{\infty} b_k$ and convergent because $|\theta| < 1$

2. Calculate the best forecast for X_{2012}, X_{2013}

$\hat{X}_{2012} = -(\theta - \phi)(X_{2012} + \theta X_{2010} + \theta^2 X_{2009} + \theta^3 X_{2008} + +\theta^4 X_{2007})$

$= -(0.413047 - 0.495639)(2.4 + 0.413047 \times 2.2 + (0.413047)^2 \times 3.2 + (0.413047)^3 \times 1.9 + (0.413047)^4 \times 1.5)$

$\hat{X}_{2012} = 0.336$

Same for $\hat{X}_{2013}$:

$\hat{X}_{2013} = 0.167$

Exercise 2:

Calculate the best prediction, the variance of the prediction error, and the confidence interval (95%) for each of $X_{2012}, X_{2013}, X_{2014}$

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We know that : $X_t = \mu + \phi X_{t-1} + a_t$

$\Rightarrow E(X_t) = \mu + \phi E(X_t)$

$\Rightarrow (1 - \phi)E(X_t) = \mu$

$\Rightarrow E(X_t) = \frac{\mu}{(1-\phi)} = cte$

So: $\mu = cte(1 - \phi)$

$= 7.125 \times (1 - 0.298)$

$\mu = 5$

And since $\hat{X}_{n+1} = \mu + +\phi X_n$

$\Rightarrow \hat{X}_{2012} = 5 + +0.298 \times X_{2011}$

$\hat{X}_{2012} = 8.1$

$\Rightarrow \hat{X}_{2013} = 5 + +0.298 \times X_{2012}$

$\hat{X}_{2012} = 7,42$

$\Rightarrow \hat{X}_{2014} = 5 + +0.298 \times X_{2013}$

$\hat{X}_{2012} = 7.22$

Since $a_t \sim N(0,1)$ Then $z_{1-\frac{\alpha}{2}} = 1.96$

So: $IC_{2012} = [\hat{X}_{2012} \pm z_{1-\frac{\alpha}{2}}] = [6.14; 10.06]$

$IC_{2013} = [\hat{X}_{2013} \pm z_{1-\frac{\alpha}{2}}\sqrt{1 + 0.298^2}] = [5.36; 9.46]$

$IC_{2014} = [\hat{X}_{2014} \pm z_{1-\frac{\alpha}{2}}\sqrt{1 + 0.298^2 + 0.298^4}] = [5.16; 9.26]$