ASSIGNMENT 2

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1 PROBLEM

Find the equation of the plane which contains the line intersection of the planes

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \mathbf{x} = 4 \quad (1.0.1)$$

$$\begin{pmatrix} 2 & 1 & -1 \end{pmatrix} \mathbf{x} = -5 \quad (1.0.2)$$

and which is perpendicular to the plane

$$\begin{pmatrix} 5 & 3 & -6 \end{pmatrix} \mathbf{x} = -8 \quad (1.0.3)$$

2 SOLUTION

Let $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

then $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \mathbf{x} = 4 \implies x + 2y + 3Z = 4$
$\begin{pmatrix} 2 & 1 & -1 \end{pmatrix} \mathbf{x} = -5 \implies 2x + y - z = -5$
$\begin{pmatrix} 5 & 3 & -6 \end{pmatrix} \mathbf{x} = -8 \implies 5x + 3y - 6z = -8$

the equation of the plane through intersection of the plane $x + 2y + 3z = 4$ and $2x + y - z = -5$ is

$$(x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$$

$$\implies (2\lambda + 1)x + (\lambda + 2)y + (3 - \lambda)z + (5\lambda - 4) = 0$$
(2.0.1)

the direction ratios a_1, b_1, c_1 of this plane are $(2\lambda + 1), (\lambda + 2), (3-\lambda) respectively$

the plane in the equation (2.0.1) is perpendicular to $5x + 3y - 6z = -8$

its direction ratios a_2, b_2, c_2 are $5, 3, -6$ respectively

since the planes are perpendicular,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$\implies (2\lambda + 1)(5) + (\lambda + 2)(3) + (3 - \lambda)(-6) = 0$$

$$(10\lambda + 5) + (3\lambda + 6) + (6\lambda - 18) = 0$$

$$19\lambda + 11 - 18 = 0$$

$$19\lambda = 7$$

$$\lambda = \tfrac{7}{19}$$

substitute the λ value in the equation (2.0.1) then we obtain

$$(2(\tfrac{7}{19})+1)x+((\tfrac{7}{19})+2)y+(3-(\tfrac{7}{19}))z+(5(\tfrac{7}{19})-4) = 0$$

$$1.736x + 2.368y + 2.631z - 2.157 = 0$$

$$\implies 1.736x + 2.368y + 2.631z = 2.157$$

this is the required equation of the plane.

3 ANSWER

the required plane is

$$\begin{pmatrix} 1.736 & 2.368 & 2.631 \end{pmatrix} \mathbf{x} = 2.157$$