# ASSIGNMENT 1

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## 1 PROBLEM

Find the equation of the plane which contains the line intersection of the planes

$$\begin{pmatrix}1 & 2 & 3\end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

$$\begin{pmatrix}2 & 1 & -1\end{pmatrix}\mathbf{x} = -5 \tag{1.0.2}$$

and which is perpendicular to the plane

$$\begin{pmatrix}5 & 3 & -6\end{pmatrix}\mathbf{x} = -8 \tag{1.0.3}$$

## 2 SOLUTION

we converted these line vectors into the augmented form :

$$\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & -1 & -5\end{pmatrix} \tag{2.0.1}$$

Now we apply the row elemintary operation to convert left part of matrix to the identity matrix,

$$\xleftrightarrow{R_2=R_2-2R_1}\begin{pmatrix}1 & 2 & 3 & 4\\ 0 & -3 & -7 & -13\end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow{R_2=\frac{-R_3}{3}}\begin{pmatrix}1 & 2 & 3 & 4\\ 0 & 1 & \frac{7}{3} & \frac{13}{3}\end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_1=R_1-2R_2}\begin{pmatrix}1 & 0 & \frac{-5}{3} & \frac{-14}{3}\\ 0 & 1 & \frac{7}{3} & \frac{13}{3}\end{pmatrix} \tag{2.0.4}$$

From the above equation we can get the direction vector of the straight line which is the intersection of the two planes.

The equation of the line

$$a + \lambda b \tag{2.0.5}$$

The left part of the eq(2.0.3) is identity matrix then the direction vector $\mathbf{m}$ of the line can be written as follows

$$\mathbf{m} = \begin{pmatrix}\frac{-14}{3} + \frac{5\lambda}{3}\\ \frac{13}{3} - \frac{7\lambda}{3}\\ \lambda\end{pmatrix} \tag{2.0.6}$$

Perform $\mathbf{m}^T\mathbf{n_1} = 0$ where $\mathbf{n_1} = \begin{pmatrix}5\\3\\-6\end{pmatrix}$

$$\mathbf{m}^T\mathbf{n_1} = 0 \tag{2.0.7}$$

$$\begin{pmatrix}\frac{-14}{3} + \frac{5\lambda}{3} & \frac{13}{3} - \frac{7\lambda}{3} & \lambda\end{pmatrix}\begin{pmatrix}5\\3\\-6\end{pmatrix} = 0 \tag{2.0.8}$$

$$(\frac{-14}{3} + \frac{5\lambda}{3})5 + (\frac{13}{3} - \frac{7\lambda}{3})3 + \lambda(-6) = 0 \tag{2.0.9}$$

$$\frac{25\lambda}{3} - 13\lambda = \frac{70}{3} - 13 \tag{2.0.10}$$

$$\frac{(25-39)\lambda}{3} = \frac{31}{3} \tag{2.0.11}$$

$$\lambda = \frac{-31}{14} = -2.214 \tag{2.0.12}$$

Substitute the $\lambda$ in the $\mathbf{m}$ then

$$\mathbf{m} = \begin{pmatrix}-8.357\\9.5\\-2.214\end{pmatrix} \tag{2.0.13}$$

Unit vector $\mathbf{n} = \frac{\mathbf{m}}{\|\mathbf{m}\|}$

$$\|\mathbf{m}\| = \sqrt{(-8.357)^2 + (9.5)^2 + (-2.214)^2} \tag{2.0.14}$$

$$\|\mathbf{m}\| = 12.844 \tag{2.0.15}$$

Then unit vector

$$\mathbf{n} = \begin{pmatrix} -0.650 \\ 0.739 \\ -0.172 \end{pmatrix} \tag{2.0.16}$$

## 3 Answer

The equation of the plane $\mathbf{n}^T\mathbf{x} = 1$

$$\begin{pmatrix} -0.650 \\ 0.739 \\ -0.172 \end{pmatrix}^T \mathbf{x} = 1 \tag{3.0.1}$$

$$\begin{pmatrix} -0.65 & 0.739 & -0.172 \end{pmatrix} \mathbf{x} = 1 \tag{3.0.2}$$