

ASSIGNMENT 1

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1 PROBLEM

Find the equation of the plane which contains the line intersection of the planes

$$(1 \ 2 \ 3)\mathbf{x} = 4 \quad (1.0.1)$$

$$(2 \ 1 \ -1)\mathbf{x} = -5 \quad (1.0.2)$$

and which is perpendicular to the plane

$$(5 \ 3 \ -6)\mathbf{x} = -8 \quad (1.0.3)$$

2 SOLUTION

we converted these line vectors into the augmented form :

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 1 & -1 & -5 \end{array} \right) \quad (2.0.1)$$

Now we apply the row elementary operation to convert left part of matrix to the identity matrix,

$$\xleftrightarrow{R_2=R_2-2R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & -3 & -7 & -13 \end{array} \right) \quad (2.0.2)$$

$$\xleftrightarrow{R_2=\frac{-R_3}{3}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 1 & \frac{7}{3} & \frac{13}{3} \end{array} \right) \quad (2.0.3)$$

$$\xleftrightarrow{R_1=R_1-2R_2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{-5}{3} & \frac{-14}{3} \\ 0 & 1 & \frac{7}{3} & \frac{13}{3} \end{array} \right) \quad (2.0.4)$$

From the above equation we can get the direction vector of the straight line which is the intersection of the two planes.

The equation of the line

$$\mathbf{m} = a + b\lambda \quad (2.0.5)$$

Where

$$a = \begin{pmatrix} \frac{-14}{3} \\ \frac{13}{3} \\ 0 \end{pmatrix} \quad (2.0.6)$$

$$b = \begin{pmatrix} \frac{5\lambda}{3} \\ \frac{-7\lambda}{3} \\ \lambda \end{pmatrix} \quad (2.0.7)$$

Then from (2.0.5)

$$\mathbf{m} = \begin{pmatrix} \frac{-14}{3} + \frac{5\lambda}{3} \\ \frac{13}{3} - \frac{7\lambda}{3} \\ \lambda \end{pmatrix} \quad (2.0.8)$$

Perform $\mathbf{m}^T \mathbf{n}_1 = 0$ where $\mathbf{n}_1 = \begin{pmatrix} 5 \\ 3 \\ -6 \end{pmatrix}$

$$\mathbf{m}^T \mathbf{n}_1 = 0 \quad (2.0.9)$$

$$\left(\frac{-14}{3} + \frac{5\lambda}{3} \quad \frac{13}{3} - \frac{7\lambda}{3} \quad \lambda \right) \begin{pmatrix} 5 \\ 3 \\ -6 \end{pmatrix} = 0 \quad (2.0.10)$$

$$\left(\frac{-14}{3} + \frac{5\lambda}{3} \right) 5 + \left(\frac{13}{3} - \frac{7\lambda}{3} \right) 3 + \lambda(-6) = 0 \quad (2.0.11)$$

$$\frac{25\lambda}{3} - 13\lambda = \frac{70}{3} - 13 \quad (2.0.12)$$

$$\frac{(25 - 39)\lambda}{3} = \frac{31}{3} \quad (2.0.13)$$

$$\lambda = \frac{-31}{14} = -2.214 \quad (2.0.14)$$

Substitute the λ in the \mathbf{m} then

$$\mathbf{m} = \begin{pmatrix} -8.357 \\ 9.5 \\ -2.214 \end{pmatrix} \quad (2.0.15)$$

Unit vector $\mathbf{n} = \frac{\mathbf{m}}{\|\mathbf{m}\|}$

$$\|\mathbf{m}\| = \sqrt{(-8.357)^2 + (9.5)^2 + (-2.214)^2} \quad (2.0.16)$$

$$\|\mathbf{m}\| = 12.844 \quad (2.0.17)$$

Then unit vector

$$\mathbf{n} = \begin{pmatrix} -0.650 \\ 0.739 \\ -0.172 \end{pmatrix} \quad (2.0.18)$$

3 ANSWER

The equation of the plane $\mathbf{n}^T \mathbf{x} = 1$

$$\begin{pmatrix} -0.650 \\ 0.739 \\ -0.172 \end{pmatrix}^T \mathbf{x} = 1 \quad (3.0.1)$$

$$\begin{pmatrix} -0.65 & 0.739 & -0.172 \end{pmatrix} \mathbf{x} = 1 \quad (3.0.2)$$