ASSIGNMENT 1

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1 Problem

Find the equation of the plane which contains the line intersection of the planes

$$\begin{pmatrix}1 & 2 & 3\end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

$$\begin{pmatrix}2 & 1 & -1\end{pmatrix}\mathbf{x} = -5 \tag{1.0.2}$$

and which is perpendicular to the plane

$$\begin{pmatrix}5 & 3 & -6\end{pmatrix}\mathbf{x} = -8 \tag{1.0.3}$$

2 Solution

we converted these line vectors into the augmented form :

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 1 & -1 & -5 \end{array}\right) \tag{2.0.1}$$

Now we apply the row elemintary operation to convert left part of matrix to the identity matrix,

$$\xleftrightarrow{R_2=R_2-2R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & -3 & -7 & -13 \end{array}\right) \tag{2.0.2}$$

$$\xleftrightarrow{R_2=\frac{-R_3}{3}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 1 & \frac{7}{3} & \frac{13}{3} \end{array}\right) \tag{2.0.3}$$

$$\xleftrightarrow{R_1=R_1-2R_2} \left(\begin{array}{ccc|c} 1 & 0 & \frac{-5}{3} & \frac{-14}{3} \\ 0 & 1 & \frac{7}{3} & \frac{13}{3} \end{array}\right) \tag{2.0.4}$$

From the above equation we can get the direction vector of the straight line which is the intersection of the two planes.

The equation of the line

$$\mathbf{m} = \mathbf{a} + \mathbf{b}\lambda \tag{2.0.5}$$

Where

$$\mathbf{a} = \begin{pmatrix} \frac{-14}{3} \\ \frac{13}{3} \\ 0 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{b} = \begin{pmatrix} \frac{5}{3} \\ \frac{-7}{3} \\ 1 \end{pmatrix} \tag{2.0.7}$$

Then from (2.0.5)

$$\mathbf{m} = \begin{pmatrix} \frac{-14}{3} + \frac{5\lambda}{3} \\ \frac{13}{3} - \frac{7\lambda}{3} \\ \lambda \end{pmatrix} \tag{2.0.8}$$

Perform $\mathbf{m}^T\mathbf{n_1} = 0$ where $\mathbf{n_1} = \begin{pmatrix} 5 \\ 3 \\ -6 \end{pmatrix}$

Let the equation of the desired plane be $\mathbf{n}^T\mathbf{x} = 1$ then you know that $\begin{pmatrix}5 & 3 & -6\end{pmatrix}\mathbf{m} = 0$ since the desired plane is perpendicular to (1.0.3) and $\mathbf{m} = \mathbf{a} + \lambda\mathbf{b}$ in (2.0.5) satisfies $\mathbf{n}^T\mathbf{x} = 1$

$$\mathbf{m}^T\mathbf{n_1} = 0 \tag{2.0.9}$$

$$\begin{pmatrix} \frac{-14}{3} + \frac{5\lambda}{3} & \frac{13}{3} - \frac{7\lambda}{3} & \lambda \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ -6 \end{pmatrix} = 0 \tag{2.0.10}$$

$$\left(\frac{-14}{3} + \frac{5\lambda}{3}\right)5 + \left(\frac{13}{3} - \frac{7\lambda}{3}\right)3 + \lambda(-6) = 0 \tag{2.0.11}$$

$$\frac{25\lambda}{3} - 13\lambda = \frac{70}{3} - 13 \tag{2.0.12}$$

$$\frac{(25-39)\lambda}{3} = \frac{31}{3} \tag{2.0.13}$$

$$\lambda = \frac{-31}{14} = -2.214 \tag{2.0.14}$$

Substitute the λ in the $\mathbf{m}$ then

$$\mathbf{m} = \begin{pmatrix} \frac{-117}{14} \\ \frac{19}{2} \\ \frac{-31}{14} \end{pmatrix} \tag{2.0.15}$$

Unit vector $\mathbf{n} = \frac{\mathbf{m}}{\|\mathbf{m}\|}$

$$\|\mathbf{m}\| = \sqrt{(\frac{-117}{14})^2 + (\frac{19}{2})^2 + (\frac{-31}{14})^2} \tag{2.0.16}$$

$$\|\mathbf{m}\| = 12.844 \tag{2.0.17}$$

Then unit vector

$$\mathbf{n} = \begin{pmatrix} -0.650 \\ 0.739 \\ -0.172 \end{pmatrix} \tag{2.0.18}$$

3 Answer

The equation of the plane $\mathbf{n}^T\mathbf{x} = 1$

$$\begin{pmatrix} \frac{-13}{20} \\ \frac{125}{169} \\ \frac{-17}{100} \end{pmatrix}^T \mathbf{x} = 1 \tag{3.0.1}$$

$$\begin{pmatrix} \frac{-13}{20} & \frac{125}{169} & \frac{-17}{100} \end{pmatrix} \mathbf{x} = 1 \tag{3.0.2}$$