ASSIGNMENT 1

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1 PROBLEM

Find the equation of the plane which contains the line intersection of the planes

$$\begin{pmatrix}1 & 2 & 3\end{pmatrix}\mathbf{x} = 4 \tag{1.0.1}$$

$$\begin{pmatrix}2 & 1 & -1\end{pmatrix}\mathbf{x} = -5 \tag{1.0.2}$$

and which is perpendicular to the plane

$$\begin{pmatrix}5 & 3 & -6\end{pmatrix}\mathbf{x} = -8 \tag{1.0.3}$$

2 SOLUTION

we converted these line vectors into the augmented form :

$$\begin{pmatrix}1 & 2 & 3 & |4\\ 2 & 1 & -1 & |-5\end{pmatrix}$$

Now we apply the row elimintary operation to convert left part of matrix to the identity matrix,

$$\xleftrightarrow{R_2=R_2-2R_1}\begin{pmatrix}1 & 2 & 3 & |4\\ 0 & -3 & -7 & |-13\end{pmatrix}$$
$$\xleftrightarrow{R_2=\frac{-R_3}{3}}\begin{pmatrix}1 & 2 & 3 & |4\\ 0 & 1 & \frac{7}{3} & |\frac{13}{3}\end{pmatrix}$$
$$\xleftrightarrow{R_1=R_1-2R_2}\begin{pmatrix}1 & 0 & \frac{-5}{3}|\frac{-14}{3} & \\ 0 & 1 & \frac{7}{3} & |\frac{13}{3}\end{pmatrix}$$

From the above equation we can get the direction vector of the straight line which is the intersection of the two planes.
Left part is coverted into identity matrix then the intersection vector is as below.

$$\mathbf{m} = \begin{pmatrix}\frac{-14}{3}+\frac{5\lambda}{3}\\ \frac{13}{3}-\frac{7\lambda}{3}\\ \lambda\end{pmatrix}$$

Perform $\mathbf{m}^T\mathbf{n_1} = 0$ where $\mathbf{n_1} = \begin{pmatrix}5\\3\\-6\end{pmatrix}$

$$\mathbf{m}^T\mathbf{n_1} = 0$$

$$\begin{pmatrix}\frac{-14}{3}+\frac{5\lambda}{3} & \frac{13}{3}-\frac{7\lambda}{3} & \lambda\end{pmatrix}\begin{pmatrix}5\\3\\-6\end{pmatrix} = 0$$

$$(\frac{-14}{3}+\frac{5\lambda}{3})5+(\frac{13}{3}-\frac{7\lambda}{3})3+\lambda(-6) = 0$$

$$\frac{25\lambda}{3}-13\lambda = \frac{70}{3}-13$$

$$\frac{(25-39)\lambda}{3} = \frac{31}{3}$$

$$\lambda = \frac{-31}{14} = -2.214$$

Substitute the λ in the $\mathbf{m}$ then

$$\mathbf{m} = \begin{pmatrix}-8.357\\ 9.5\\ -2.214\end{pmatrix}$$

Unit vector $\mathbf{n} = \frac{\mathbf{m}}{||\mathbf{m}||}$

The equation of the plane $\mathbf{n}^T\mathbf{x} = 1$

$$||\mathbf{m}|| = \sqrt{(-8.357)^2+(9.5)^2+(-2.214)^2}$$

$$||\mathbf{m}|| = 12.844$$

Then unit vector $\mathbf{n} = \begin{pmatrix}-0.650\\ 0.739\\ -0.172\end{pmatrix}$

The equation of the plane

$$\begin{pmatrix} -0.650 \\ 0.739 \\ -0.172 \end{pmatrix}^T \mathbf{x} = 1$$