

(Q). Find θ and P if $(\sqrt{3} - 1)x = -2$ is equivalent to $(\cos\theta - \sin\theta)x = P$?

Given, $(\sqrt{3} - 1)x = -2$ is equivalent to $(\cos\theta - \sin\theta)x = P$

The given equation of line is,

$$\vec{n}^T \cdot \vec{x} = c \quad (1)$$

$$\text{where } \vec{n} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} \text{ and } c = -2 \quad (2)$$

Now obtain a new equation,

$$\frac{\vec{n}}{\|\vec{n}\|} \cdot \vec{x} = \frac{c}{\|\vec{n}\|} \quad \text{where } \|\vec{n}\| \text{ is the norm of the } \vec{n} \quad (3)$$

$$\implies \vec{u} \cdot \vec{x} = P \quad \text{where } \vec{u} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \text{ and } P = \frac{c}{\|\vec{n}\|} \quad (4)$$

Substituting the values of \vec{n} and c, we get

$$\|\vec{n}\| = 2 \quad (5)$$

$$\begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} \cdot \vec{x} = -1 \quad (6)$$

$$(7)$$

From (4) and (5). we derive,

$$\cos\theta = \sqrt{3}/2 ; \quad \sin\theta = 1/2 \quad (8)$$

$$P = -1 \quad (9)$$

$$\therefore \theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ \quad \text{and} \quad P = -1 \quad (10)$$