Matrix Theory (EE5609)
Assignment-4

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1 PROBLEM

Find the equation of the tangent and normal to the circle

$$\mathbf{x}^T\mathbf{x} + \begin{pmatrix} -6 & 4 \end{pmatrix}\mathbf{x} - 12 = 0 \tag{1.0.1}$$

at the point $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$

2 SOLUTION

The general equation of a second degree can be expressed as:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

comparing 1.0.1 and 2.0.1

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, f = -12 \tag{2.0.2}$$

The vector equation of the line can be expressed as

$$\mathbf{x} = \mathbf{q} + \mu\mathbf{m} \tag{2.0.3}$$

Given point is

$$\mathbf{q} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \tag{2.0.4}$$

The direction vector of the line joining point $\mathbf{q_1}$ and the centre $\mathbf{c}$ can be expressed as :

$$\mathbf{n} = \mathbf{q} - \mathbf{c} \tag{2.0.5}$$

$$\implies \mathbf{n} = \mathbf{q} + \mathbf{u} \tag{2.0.6}$$

where,

$$\mathbf{c} = -\mathbf{u} \tag{2.0.7}$$

The vector $\mathbf{n}$ is the normal to the tangent drawn at $\mathbf{q}$. From (2.0.2) and (2.1.1) we get,

$$\mathbf{n}^T = \begin{pmatrix} 3 & 4 \end{pmatrix} \tag{2.0.8}$$

We know,

$$\mathbf{m}^T\mathbf{n} = 0 \tag{2.0.9}$$

$$\implies \mathbf{m}^T\begin{pmatrix} 3 \\ 4 \end{pmatrix} = 0 \tag{2.0.10}$$

$$\mathbf{m}^T = \begin{pmatrix} -1 & \frac{3}{4} \end{pmatrix} \tag{2.0.11}$$

If $\mathbf{q}$ be a point on the line and $\mathbf{n}$ is the normal vector then the equation of the line can be expressed from (2.0.3) is :

$$\mathbf{n}^T(\mathbf{x} - \mathbf{q}) = 0 \tag{2.0.12}$$

$$\implies \mathbf{n}^T\mathbf{x} = \mathbf{c} \tag{2.0.13}$$

where

$$\mathbf{c} = \mathbf{n}^T\mathbf{q} \tag{2.0.14}$$

$$\implies \mathbf{c} = \begin{pmatrix} 3 & 4 \end{pmatrix}\begin{pmatrix} 6 \\ 2 \end{pmatrix} \tag{2.0.15}$$

$$\mathbf{c} = 26 \tag{2.0.16}$$

Then equation of tangent as from (2.0.13)

$$\begin{pmatrix} 3 & 4 \end{pmatrix}\mathbf{x} = 26 \tag{2.0.17}$$

we have an direction vector of the tangent $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

normal vector $\mathbf{n} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

line passing through $\mathbf{q} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ with the normal vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ is,

$$\begin{pmatrix} -4 & 3 \end{pmatrix}\left(\mathbf{x} - \begin{pmatrix} 6 \\ 2 \end{pmatrix}\right) = 0 \tag{2.0.18}$$

$$\begin{pmatrix}-4 & 3\end{pmatrix}\mathbf{x} - \begin{pmatrix}-4 & 3\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix} = 0 \tag{2.0.19}$$

normal equation

$$\implies \begin{pmatrix}-4 & 3\end{pmatrix}\mathbf{x} = -18 \tag{2.0.20}$$