ASSIGNMENT 2

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1 Problem

If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to
(A) A
(B) $I - A$
(C) I
(D) $3A$

2 solution

Given $A^2 = A \implies A^2 - A = 0$
Let λ is the eigen value then theevery eigen value satisfies its very own characteristic equation

$\implies \lambda^2 = \lambda$ (2.0.1)

$\lambda^2 - \lambda = 0$ (2.0.2)

Then $(I + A)^3 - 7A$ can be written as $(I + \lambda)^3 - 7\lambda$

$$(1+\lambda)^3 - 7\lambda = 1^3 + \lambda^3 + 3(1^2)\lambda + 3\lambda^2(1) - 7\lambda \tag{2.0.3}$$

we know that

$$I^3 = I^2 = I = 1 \tag{2.0.4}$$

$$= 1+\lambda^3 + 3(1)\lambda + 3\lambda^2(1) - 7\lambda \tag{2.0.5}$$

$$= 1+\lambda^3 + 3\lambda + 3\lambda^2 - 7\lambda \tag{2.0.6}$$

$$=1+(\lambda^2\lambda) + 3\lambda + 3\lambda - 7\lambda \tag{2.0.7}$$

$$= 1+(\lambda\lambda) - \lambda \tag{2.0.8}$$

$$= 1+\lambda^2 - \lambda \tag{2.0.9}$$

From the equation (2.0.2) $\lambda^2 - \lambda = 0$

$$= 1+0 \tag{2.0.10}$$

$$= 1 \tag{2.0.11}$$

3 answer

Option C is the valid answer.