

# ASSIGNMENT 2

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## 1 PROBLEM

If  $A$  is a square matrix such that  $A^2 = A$ , then  $(I + A)^3 - 7A$  is equal to

- (A)  $A$
- (B)  $I - A$
- (C)  $I$
- (D)  $3A$

## 2 SOLUTION

Given  $A^2 = A \implies A^2 - A = 0$

Let  $\lambda$  is the eigen value then the every eigen value satisfies its very own characteristic equation

$$\implies \lambda^2 = \lambda \text{ then } \lambda^2 - \lambda = 0 \quad (2.0.1)$$

Then  $(I + A)^3 - 7A$  can be written as  $(I + \lambda)^3 - 7\lambda$

$$(I + \lambda)^3 - 7\lambda = I^3 + \lambda^3 + 3I^2\lambda + 3\lambda^2I - 7\lambda \quad (2.0.2)$$

we know that

$$I^3 = I^2 = I \quad (2.0.3)$$

$$= I + \lambda^3 + 3I\lambda + 3\lambda^2I - 7\lambda \quad (2.0.4)$$

$$= I + \lambda^3 + 3\lambda + 3\lambda^2 - 7\lambda \quad (2.0.5)$$

$$= I + (\lambda^2\lambda) + 3\lambda + 3\lambda - 7\lambda \quad (2.0.6)$$

$$= I + (\lambda\lambda) - \lambda \quad (2.0.7)$$

$$= I + \lambda^2 - \lambda \quad (2.0.8)$$

From the equation  $\lambda^2 - \lambda = 0$

$$= I + 0 \quad (2.0.9)$$

$$= I \quad (2.0.10)$$

## 3 ANSWER

Option C is the valid answer.