ASSIGNMENT 2

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1 PROBLEM

If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(A) A

(B) $I - A$

(C) I

(D) $3A$

2 SOLUTION

Given

$$A^2 = A \implies A^2 - A = 0 \quad (2.0.1)$$

Let λ is the eigen value then the every eigen value satisfies its very own characteristic equation

$$\implies \lambda^2 = \lambda \implies \lambda^2 - \lambda = 0 \quad (2.0.2)$$

Then $(I + A)^3 - 7A$ can be written as $(I + \lambda)^3 - 7\lambda$

$$(I + \lambda)^3 - 7\lambda = I^3 + \lambda^3 + 3I^2\lambda + 3\lambda^2 I - 7\lambda \quad (2.0.3)$$

we know that

$$I^3 = I^2 = 1 \quad (2.0.4)$$

$$= 1 + \lambda^3 + 3(1)\lambda + 3\lambda^2(1) - 7\lambda \quad (2.0.5)$$

$$= 1 + \lambda^3 + 3\lambda + 3\lambda^2 - 7\lambda \quad (2.0.6)$$

$$= 1 + (\lambda^2\lambda) + 3\lambda + 3\lambda - 7\lambda \quad (2.0.7)$$

$$= 1 + (\lambda\lambda) - \lambda \quad (2.0.8)$$

$$= 1 + \lambda^2 - \lambda \quad (2.0.9)$$

From the equation (2.0.2) $\lambda^2 - \lambda = 0$

$$= 1 + 0 \quad (2.0.10)$$

$$= 1 \quad (2.0.11)$$

3 ANSWER

Option C is the valid answer.