ASSIGNMENT 2

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1 Problem

If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to
(A) A
(B) $I - A$
(C) I
(D) $3A$

2 solution

Given $A^2 = A \implies A^2 - A = 0$
Let λ is the eigen value then theevery eigen value satisfies its very own characteristic equation

$\implies \lambda^2 = \lambda$ then $\lambda^2 - \lambda = 0$
(2.0.1)

Then $(I + A)^3 - 7A$ can be written as $(I + \lambda)^3 - 7\lambda$
$(I + \lambda)^3 - 7\lambda = I^3 + \lambda^3 + 3I^2\lambda + 3\lambda^2 I - 7\lambda$
we know that $I^3 = I^2 = I$
$= I + \lambda^3 + 3I\lambda + 3\lambda^2 I - 7\lambda$
$= I + \lambda^3 + 3\lambda + 3\lambda^2 - 7\lambda$
$= I + (\lambda^2\lambda) + 3\lambda + 3\lambda - 7\lambda$
$= I + (\lambda\lambda) - \lambda$
$= I + \lambda^2 - \lambda$
From the equation(2.0.1) $\lambda^2 - \lambda = 0$
$= I + 0$
$= I$

3 answer

Option C is the valid answer.