EE5609 Assignment 3

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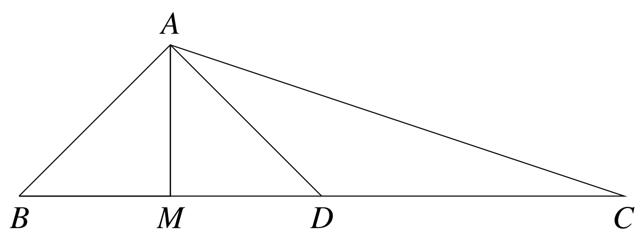
1 Problem

AD is the median of a ΔABC and $AM \perp BC$. prove that
$AB^2 = AD^2 - BC.DM + (\frac{BC}{2})^2$

2 Solution

In a right angle triangle, square of the hypotenuse is equal to the sum of the squares of the other two sides. Also, a medain devides the sides into two equal halves.



from the above figure

$$\|B - D\| = \|D - C\| \tag{2.0.1}$$

$$AM \perp BC \tag{2.0.2}$$

then as per the pythagoras theorem

$$\|A - B\|^2 = \|B - M\|^2 + \|A - M\|^2 \tag{2.0.3}$$

$$= \|B - M\|^2 + \|A - D\|^2 - \|M - D\|^2 \tag{2.0.4}$$

$$= (\|B - D\| - \|M - D\|)^2 + \|A - D\|^2 - \|M - D\|^2 \tag{2.0.5}$$

$=\|B - D\|^2 + \|M - D\|^2 - 2\|B - D\|\|M - D\| + \|A - D\|^2 - \|M - D\|^2$

$$= \|B - D\|^2 - 2\|B - D\|\|M - D\| + \|A - D\|^2 \tag{2.0.6}$$

$$= \|A - D\|^2 + \left\|\frac{B - C}{2}\right\|^2 - 2\|B - D\|\|M - D\| \tag{2.0.7}$$

$$= \|A - D\|^2 + \left\|\frac{B - C}{2}\right\|^2 - 2\left\|\frac{B - C}{2}\right\|\|M - D\| \tag{2.0.8}$$

$$= \|A - D\|^2 + \left\|\frac{B - C}{2}\right\|^2 - \|B - C\|\|M - D\| \tag{2.0.9}$$

Hence proved.