Chapter 3.
Cascading properties of Networks.

* Modeling diffusion through a network.
- Here we are going to study the mathematical model for the diffusion through a network.
- "Networked co-ordination game" is the mathematical model

* Networked co-ordination game.
  - It is having v and w players
  - A and B are strategies.
  - a and b are values.
  - In this game the node is trying to adapt behaviour A or behaviour B.

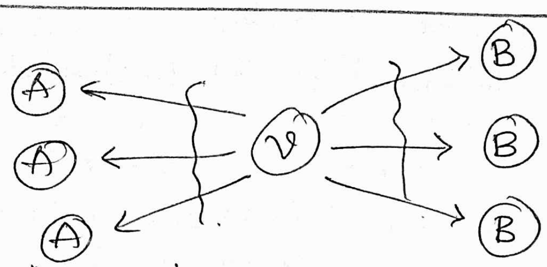
$$
\begin{tikzcd}[row sep=small]
 & A \\
V \arrow[ur] \arrow[dr] & \\
 & B
\end{tikzcd}
$$

  - If v and w adopt A, payoff $a > 0$.
  - If v and w adopt B, payoff $b > 0$
  - If opposite behaviour, payoff $= 0$.
- In terms of payoff matrix it is representated as.

	A(v)	B(w).
A	a,a	0,0
B	0,0	b,b

- To maximize v adaption towards A by using decision rule that is modelling

$V \rightarrow A$



Here d is no. of neighbours.

- This side v is trying to adopt A behaviour.
- Tendency towards A is defined by p fraction of d neighbours they are trying to adopt behaviour A
- pda neighbours use behaviour A

This side v is trying to adopt B behaviour

- Tendency towards B is defined by (1-p) fraction of d neighbours they are trying to adopt behaviour. B.
- (1-p)db neighbours use behaviour B.

* if v chooses A, it gets a payoff of pda
* if v chooses B, it gets a payoff of (1-p)db.

∴ Thus, A is better choice if for v.

$$pda \geq (1-p)db$$

$$p\cancel{d}a \geq \cancel{d}b - p\cancel{d}b$$

$$pa \geq b - pb$$

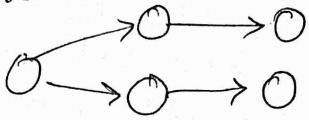
$$p(a+b) \geq b$$

$$p \geq \frac{b}{a+b}$$

$\frac{b}{a+b}$ is assigned by q value where q is threshold value for adaption v.

- if q is small A holds good.
- if q is large B hold good.

* Information or Influence diffusion in network
- Information spreading from one person to another is called cascading.



- Always the individual choices depend on what the other people do.
- It is analysed at 2 different levels of resolution
  1. Amorphous population with aggregate effect (Global decision)
  2. Fine structure level which have been influenced by network neighbous (local decision)
- Most of the examples are of local interaction
- Information diffusion in network has been studied as 2 types of effects.
  1. Informational effects. or indirect-benefit effect
  2. Direct-benefit effects.

* Diffusion of innovation.

Example case study are.

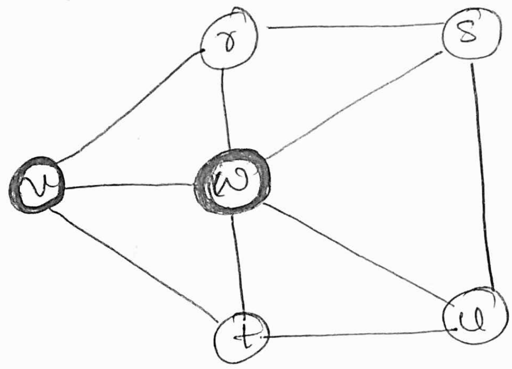
1. Adoption of hybrid seed corn among framers.
   - It was invented by researcher and spread among by third person (salesman) so it is indirect-benefit effect.
2. Adoption of tetracycline medicine by physicians.
   - Here physician is the third person hence it is example of indirect-benefit effect

3. Research communication technologies such as telephone, fax, e-mail.
   - These technologies are experienced by people then spreaded ∴ Hence it is the example for direct benefit effect.

- Success of innovation depends on the
  - ↳ complexity : difficult to understand & implement
  - ↳ observability : Not aware of what others are doing
  - ↳ Triability : gradually and incrementally
  - ↳ compatibility :

- Cascading properties is done in homophily (people with same mind set)
- Adaption is difficult in tightly-knit society

* Example for maximizing the payoff using decision rule.



$p \geq \frac{b}{a+b}$.

assume $a=3$ $b=2$.

- initial adopter are v and w representing behaviour A
- And remaining nodes are representing behaviour B.

step 1 : r and t are occupied by A.

$\frac{2}{3} \geq \frac{2}{3+2}$.

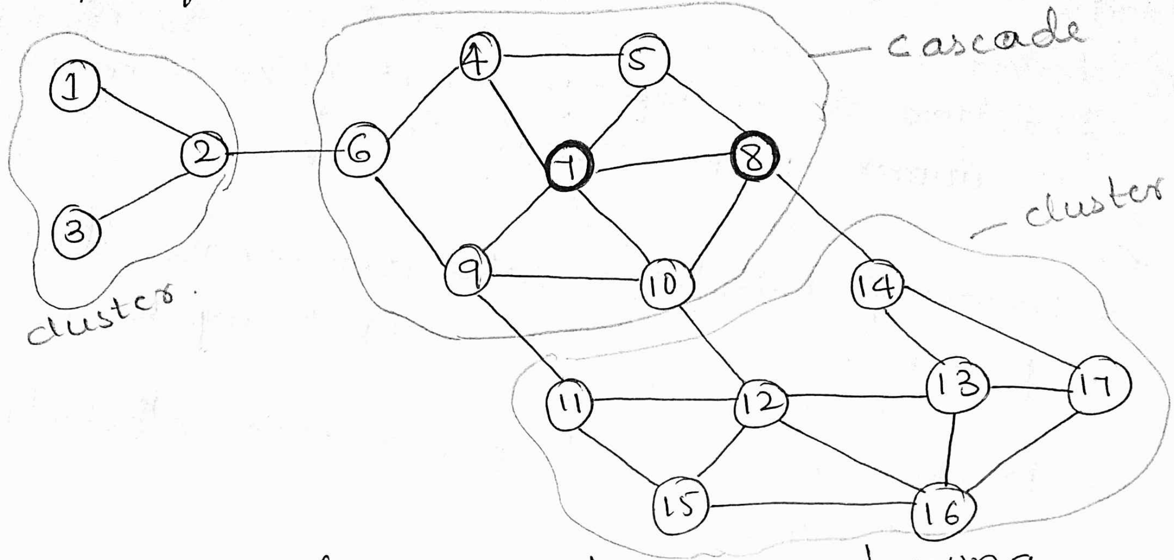
$\frac{2}{3} \geq \frac{2}{5}$.

Step 2 : s and u are occupied by A.

$\frac{2}{3} \geq \frac{2}{5}$

∴ Hence here all the nodes are occupied by A it is example of complete cascading.

Example for cascade and cluster.



- Initially only 7 and 8 are having behaviour A
- Remaining notes are having behaviour B.
- Hence we can conclude that clusters are obstacles to cascades.

* Power laws.
- popularity as a network phenomenon
- Immediate social circles
- Wider visibility
- Global name recognition.
- popularity is measured by centrality, prestige
- Popularity is represented by normal distribution
- The normal distribution is restricted to the power law.

Books are popular } commands an audience (power)
movies are popular

Barack obama } web page. - popularity is measured based on no. of inlink
Bill Gates

f(k) fraction of webpage in link

- large the value of K greater the popularity
- To dervie the power law consider web History
  - fraction of webpage that have K in links is approx. propositional to $\frac{1}{K^2}$

Ex:
- Incomming telephone calls are measured as $\frac{1}{K^2}$
- fraction of books bought by K people $1/K^3$
- fraction of research paper receive K citation are $1/K^3$

- If the value of K is larger then it leads to extreme imbalance.
- So in the law value is fixed that is $\frac{1}{K^c}$

where c is constant.

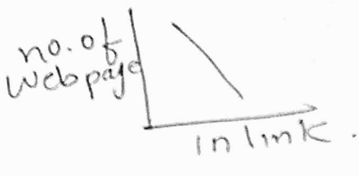
$$f(K) \propto \frac{1}{K^c}$$

$$= \frac{a}{K^c} = aK^{-c}$$

logarithm on both sides

$$\log f(K) = \log a - c\log a K$$

By using power law we can represent it in $y = mx + c$.



loga is slope.
c is y-intercept.

* Rich-Get-Richer Models.

- preferential attachment model.

Ex: city population, DNA mutation, urn process

Algorithm.

1. Pages are created in order and named 1, 2, 3, --- N.
2. When page j is created, it produces a link to an earlier web page according to the following probabilistic rule:
   (a) With probability p, page j chooses a page i uniformly at random from among all earlier pages and creates a link to this page i.
   (b) With probability (1-p), page j instead chooses a page i uniformly at random from among all earlier pages, and creates a link to page that i points to
   (c) creation of a single link from page j, can repeat this process to create multiple, independently generated links from page j.