6002

C16-COMMON–102

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BOARD DIPLOMA EXAMINATION, (C-16)

DECEMBER—2022

FIRST YEAR (COMMON) EXAMINATION

ENGINEERING MATHEMATICS – I

Time : 3 hours] [*Total Marks* : 80

PART—A

3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

1. Resolve $\frac{1}{(x-5)(x+7)}$ into partial fractions.

2. If $A=\begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, find $A+B$.

3. If $A=\begin{bmatrix} 7 & 8 \\ 9 & 1 \end{bmatrix}$, find A^{2}.

4. If $A+B=45^{\circ}$, show that $(1+\tan A)(1+\tan B)=2$.

5. Prove that $\frac{1-\cos 2\theta}{\sin 2\theta}=$.

6. Find the multiplicative inverse of $2+3i$.

7. Find the equation of the straight line passing through the points (–5, 2) and (3,–2).

8. Find the perpendicular distance from the point (2, –1) to the line $3x + 4y + 5 = 0$.

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9. Evaluate $\lim_{x \to 0} \frac{\sin 5x}{\sin 3x}$.

10. Find the derivative of xe^x.

PART—B

10×5=50

Instructions : (1) Answer *any* **five** questions.

(2) Each question carries **ten** marks.

(3) Answers should be comprehensive and the criteria for valuation are the content but not the length of the answer.

11. *(a)* $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

(b) Solve the equations $x + y - z = 0$, $2x + y - z = 1$ and $3x + 2y + 2z = 5$ using Cramer's rule.

12. *(a)* Prove that $\frac{\cos 3A - \cos A}{\sin A - \sin 3A} = \tan 2A$.

(b) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$ show that $xy + yz + zx = 1$.

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13. *(a)* Solve the equation $\sin\theta + \cos\theta = \sqrt{2}$.

(b) In a $\triangle ABC$, if A=60º, then prove that $\frac{c}{a+b} + \frac{b}{a+b} = 1$.

14. *(a)* Find the centre and radius of the circle $x^2 + y^2 + 3x - 4y = 0$

(b) Find the equation of the parabola whose focus is the point (3, –4) and directrix is the line $x - y + 5 = 0$

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15. *(a)* Differentiate $\sin^{-1}\left(3x - 4x^3\right)$ with respect to x.

(b) Find $\frac{dy}{dx}$, if $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots}}}$

16. *(a)* If $y = a\cos\left(\log x\right) + b\sin\left(\log x\right)$, then show that $x^2y_2 + xy_1 + y = 0$.

(b) If $z = \log\left(e^x + e^y\right)$, then show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$

17. *(a)* Find the equations of tangent and normal to the curve $y = x^2 + 2x - 1$ at (1, 2).

(b) Each side of a square increases at the rate of 1·5 cm/sec. Find the rate at which the area of the square increases, when the side is 12 cm. Also find the rate at which its perimeter increases.

18. *(a)* The sum of two numbers is 24. Find the numbers when the sum of their square is a minimum.

(b) The time T of a complete oscillation of a simple pendulum of length l is given by $T = 2\pi\sqrt{l/g}$, where g is a constant. Show that the approximate percentage error in the calculated value of T corresponding to an error of 4% in the value of l is 2%

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