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C16-COMMON–102

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BOARD DIPLOMA EXAMINATION, (C-16)

MAY/JUNE—2023

FIRST YEAR (COMMON) EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 Hours] [*Total Marks* : 80

PART—A 3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

1. Resolve $\frac{1}{(x+1)(x+3)}$ into partial fractions.

2. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 4 \\ 5 & 3 & 2 \end{bmatrix}$, then find $2A - B$.

3. If the matrix $\begin{bmatrix} 3 & 2x & x+1 \\ 2 & & 4 \end{bmatrix}$ is a singular matrix, then find the value of x.

4. Prove that $\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ = \frac{1}{2}$

5. Prove that $\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$

6. Find the modulus of the complex number $3 - 2i$.

7. Find the equation of a line whose inclination with the X-axis is 45° and which passes through the point (0, 5).

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8.* Find the distance between the parallel lines $4x - 3y + 5 = 0$ and $4x - 3y + 7 = 0$.

9. Evaluate $\lim_{x \to 0} \frac{\sin 2x}{x}$

10. Find the derivative of $(2x + 3)(3x - 5)$ w.r.t. x.

PART—B

10×5=50

Instructions : (1) Answer *any* **five** questions.
(2) Each question carries **ten** marks.

11. *(a)* If $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 0 \end{bmatrix}$, then show that $(AB)^T = B^T A^T$.

(b) Solve the system of linear equations $x + 2y - z = 3$, $3x + y + z = 4$ and $x - y + 2z = 6$ by Cramer's rule.

12. *(a)* Prove that $\frac{\sin 7A + \sin 17A}{\cos 7A + \cos 17A} = \tan 12A$

(b) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then show that $xy + yz + zx = 1$.

13. *(a)* Solve $\cos\theta + \sqrt{3}\sin\theta = 1$

(b) In any ΔABC, prove that $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2} = s$.

14. *(a)* Find the equation of the circle with (1, 2) and (4, 5) as the end points of a diameter of the circle.

(b) Find the eccentricity, coordinates of the vertices and foci, and length of the latus-rectum of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

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15.* *(a)* Differentiate $e^{\sin x}$ w.r.t. cos x.

(b) If $x^3 + y^3 = 3axy$, then find $\frac{dy}{dx}$.

16. *(a)* Find $\frac{d^2y}{dx^2}$, if $x = at^2$ and $y = 2at$.

(b) If $u(x,y) = x^2 + xy + y^2$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

17. *(a)* Find the lengths of the tangent, normal, sub-tangent and sub-normal to the curve $y = x^3$ at the point (1, 1).

(b) A spherical soap bubble is expanding so that its radius is increasing at the rate of 0.02 cm/sec. At what rate is the surface area increasing when its radius is 5 cm?

18. *(a)* Find the maximum and minimum values of the function

$$f(x) = x^3 - 6x^2 + 9x + 1$$

(b) A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate, if the radius of the plate before heating is 10 cm.

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