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C16–EC–102/C16–CHPC–102/C16–PET–102

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16093-EC-233

BOARD DIPLOMA EXAMINATION, (C–16)

OCT/NOV—2017

DECE—FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 hours] [*Total Marks* : 80

PART—A 3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

(3) Answers should be brief and straight to the point and shall not exceed *five* simple sentences.

1. Resolve $\frac{x-4}{(x-2)(x-3)}$ into partial fractions.

2. If $A = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$, then find AA^T.

3. If $A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$, then find $3A - 2B$.

4. Prove that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$.

5. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, show that $A + B = 45°$.

6. Find the mod-amplitude form of the complex number $4 + 3i$.

7. Find the equation of the straight line passing through the points $(1, -3)(-2, -1)$.

8. Find the angle between the straight lines

$$x + 2y + 9 = 0 \text{ and } 3x + y - 7 = 0$$

9. Evaluate $\underset{x \to 0}{\text{Lt}} \dfrac{\sin 3x}{\sin 5x}$.

10. Find $\dfrac{dy}{dx}$, if $y = e^x \sec x$.

PART—B 10×5=50

Instructions : (1) Answer *any* **five** questions.

(2) Each question carries **ten** marks.

(3) Answers should be comprehensive and the criterion for valuation is the content but not the length of the answer.

11. *(a)* Show that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

(b) Solve the equations

$$x + y + z = 6,\ 2x - y + z = 3 \text{ and } x + 2y - z = 2$$

by Cramer's method.

12. *(a)* If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, then show that

$$\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{a}{b}$$

(b) Show that $\tan^{-1}\left(\dfrac{2}{7}\right) + \tan^{-1}\left(\dfrac{1}{4}\right) = \tan^{-1}\left(\dfrac{15}{25}\right)$.

13. *(a)* Solve $4\sin^2\theta - 8\cos\theta + 1 = 0$.

(b) Solve the ΔABC, if $a = 2$, $c = \sqrt{3} + 1$ and $B = 60°$.

14. *(a)* Find the equation of the circle passing through the points (0, 0) (2, 0) and (0, 3).

(b) Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the ellipse

$$16x^2 + 9y^2 = 144$$

15. *(a)* If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.

(b) If $x = a\sec^3\theta$ and $y = a\tan^3\theta$, then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

16. *(a)* If $y = \sin(\log x)$, then prove that $x^2y_2 + xy_1 + y = 0$.

(b) If $u = (x^2 + y^2 + z^2)$, then show that

$$\frac{x\partial u}{\partial x} + \frac{y\partial u}{\partial y} + \frac{z\partial u}{\partial z} = 2u$$

17. *(a)* Show that the curves $y^2 = 4ax$ and $xy = c^2$ cut each other orthogonally if $c^4 = 32a^4$.

(b) A spherical balloon is being inflated so that the radius is increasing at the rate of 3 cm/sec. Find the rate at which the volume is increasing when $r = 10$ cm.

18. *(a)* Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

(b) If time T of a complete oscillation of a simple pendulum of length l is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error 2% in the value of l.

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