

Construct a neural network for solving the Exclusive-OR problem, showing the values of all the weights and biases of the network. You may assume that a threshold function is used for all the neurons.

The truth table of $(x_1 \text{ XOR } x_2)$:

x_1	x_2	$x_1 \text{ XOR } x_2$
0	0	-1
0	1	1
1	0	1
1	1	-1

Hint: The solution of the XOR problem requires two hidden neurons.

h_1 : $\text{sign}(x_1 + x_2 - 0.5)$

h_2 : $\text{sign}(x_1 + x_2 - 1.5)$

Answer:

The equations of the two separation boundaries are given by:

$$x_1 + x_2 - 0.5 = 0$$

$$x_1 + x_2 - 1.5 = 0$$

As a result, the weights of the first hidden neuron are $w_{11} = 1$, $w_{21} = 1$, $w_{01} = -0.5$, and the weights of the second neuron are $w_{12} = 1$, $w_{22} = 1$, $w_{02} = -1.5$.

Using these two neurons, the inputs are mapped to the hidden output space as follows:

x_1	x_2	h_1	h_2
0	0	-1	-1
0	1	1	-1
1	0	1	-1
1	1	1	1

The set of transformed inputs can be separated by the boundary (1) $h_1 - h_2 - 1 = 0$ or (2) $-h_1 + h_2 + 1 = 0$. Since the expression $h_1 - h_2 - 1$ results in the correct set of outputs, the weights of the output neuron are $u_{11} = 1$, $u_{21} = -1$, $u_{01} = -1$.