有效论证（Valid Argument）

- 定义：当且仅当不可能出现所有前提为真而结论为假的情况。
- 证明方法：当所有前提为真的时候，结论是否为真，结论为真则是有效论证，否则就不是有效论证。
- 注意：有效论证的结论不一定是真的！（即当条件为假时结论也可以是假的）例：p, ¬p → q, ¬q

表1 推理规则

推理规则	永真式	名称
p $\underline{p \to q}$ $\therefore q$	$(p \land (p \to q)) \to q$	假言推理
$\neg q$ $\underline{p \to q}$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	取拒式
$p \to q$ $\underline{q \to r}$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	假言三段论
$p \lor q$ $\underline{\neg p}$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	析取三段论
$\underline{p}$ $\therefore p \lor q$	$p \to (p \lor q)$	附加律
$\underline{p \land q}$ $\therefore p$	$(p \land q) \to p$	化简律
p $\underline{q}$ $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	合取律
$p \lor q$ $\underline{\neg p \lor r}$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	消解律

表 2　量化命题的推理规则

推理规则	名称
$\dfrac{\forall x P(x)}{\therefore P(c)}$	全称实例
$\dfrac{P(c)\text{，任意 } c}{\therefore \forall x P(x)}$	全称引入
$\dfrac{\exists x P(x)}{\therefore P(c)\text{，对某个元素 } c}$	存在实例
$\dfrac{P(c)\text{，对某个元素 } c}{\therefore \exists x P(x)}$	存在引入

问题一

Decide if these statements are consistent:

"If Sergei takes the job offer then he will get a signing bonus."

"If Sergei takes the job offer, then he will receive a higher salary."

"If Sergei gets a signing bonus, then he will not receive a higher salary."

"Sergei takes the job offer."

You must use symbols and propositional calculus to demonstrate your solution.

设P=“Sergei takes the job offer”

Q=“Sergei get a signing bonus”

R=“Sergei receive a higher salary”

则原题四句话用上述符号表示为:

1. $P \rightarrow Q$
2. $P \rightarrow R$
3. $Q \rightarrow \neg R$
4. P

两种思路：

1. 列一个真值表，看看是否存在某一种情况下所有句子取值均为真。若存在则一致，若不存在则不一致。
2. 将所有句子均当成前提，看看能否推出矛盾式（永假式）。若能则不一致，若不能则一致。

1. $P \rightarrow Q$
2. $P \rightarrow R$
3. $Q \rightarrow \neg R$
4. P

5. $P \rightarrow \neg R$ 1,3 假言三段论
6. $P \rightarrow (R \wedge \neg R)$ 2,5 逻辑等价性
7. $\neg P$ 6
8. $P \vee \neg P$ 4,7 合取律

由8可知其为矛盾式，故上述4句话不一致

问题二

Show that the argument form with premises $p_1, p_2, \cdots, p_n$ and conclusion $q \rightarrow r$ is valid if the argument form with premises $p_1, p_2, \cdots, p_n, q$, and conclusion r is valid.

问题三

Show that the argument form with premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\neg s$ and conclusion $q \rightarrow r$ is valid. You should write a formal proof in the form of page 73 in the text to demonstrate your solution.

思路：
运用问题二的结论将本题的结论的前件加入前提条件来进行证明。
即由$(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, $\neg s$, q来推出r。

1. $(p \wedge t) \rightarrow (r \vee s)$
2. $q \rightarrow (u \wedge t)$
3. $u \rightarrow p$
4. $\neg s$
5. q

6. $(u \wedge t)$	2,5	假言推理
7. u	7	化简律
8. t	7	化简律
9. p	3,7	假言推理
10. $(p \wedge t)$	8,9	合取律
11. $(r \vee s)$	1,10	假言推理
12. r	4,11	析取三段论

问题四

1. Formalise the following theorem:

If n is an even number, then n2 is also an even number, where n is even if and only if there exists some integer k such that n = 2k.

2. Write a math proof using natural language.

3. Write a formal proof similar to that on page 77 in the text.

1.$\forall x(\exists y(x = 2y) \rightarrow \exists z(x^2 = 2z))$

2.假设n是一个偶数，那么由定义存在一个k∈Z，使得n=2k

两边同时平方$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$

因为k是一个整数，所以$2k^2$也是一个整数

因此$n^2 = 2p$，$p = 2k^2 \in Z$

因此，由偶数的定义得n^2也是一个偶数。

3.$\forall$x$(\exists y(x = 2y) \rightarrow \exists z(x^2 = 2z))$

设P(x,y)：x是y的2倍， Q(x,z)： x^2是z的2倍

则上式可以转化为： $\forall$x($\exists y$P(x,y) → $\exists z$Q(x,z))

由全称实例得 ： $\exists y$P(a,y) → $\exists z$Q(a,z))

由存在实例得 ： P(a,b)

由P,Q定义得 ： P(a,b) → Q(a,$2b^2$)

由存在引入得 ： $\exists z$Q(a,z)

证毕

问题五

Suppose that on an island there are three types of people, knights, knaves, and normals. Knights always tell the truth, knaves always lie, and normals sometimes lie and sometimes tell the truth. Detectives questioned three inhabitants of the island-Amy, Brenda, and Claire-as part of the investigation of a crime. The detectives knew that one of the three committed the crime, but not which one. They also knew that the criminal was a knight, and that the other two were not. Additionally, the detectives recorded these statements: Amy: ”I am innocent.” Brenda: ”What Amy says is true." Claire: "Brenda is not a normal." After analyzing their information, the detectives positively identified the guilty party. Who was it?

You would use propositional logic calculus or predicate logic calculus to demonstrate your solution, that is, you introduction propositional variables or predicates to formalize what you know and reason about them.

假设D = {A,B,C} 是论述域，A：Amy B：Brenda C：Claire

由条件可得：

1. $\forall x\,(Murder(x) \leftrightarrow Knight(x))$

2. $\exists x\,(Murder(x)) \wedge \exists x\,(Knight(x))$

3. $Knight(A) \rightarrow \neg Murder(A)$

4. $Knight(B) \rightarrow \neg Murder(A)$

5. $Knave(B) \rightarrow Murder(A)$

6. $Knight(C) \rightarrow Knave(B) \vee Knight(B)$

7. $Knight(A)$	假设
8. $\neg Murder(A)$	(3),(7) 假言推理
9. $Murder(A)$	(1),(7) 假言推理
10. $Murder(A) \wedge \neg Murder(A)$	(8)(9) 合取律
11. $\neg Knight(A)$	(7)(10) 矛盾规则

1. $\forall x\,(Murder(x) \leftrightarrow Knight(x))$

2. $\exists x\,(Murder(x)) \wedge \exists x\,(Knight(x))$

3. $Knight(A) \rightarrow \neg Murder(A)$

4. $Knight(B) \rightarrow \neg Murder(A)$

5. $Knave(B) \rightarrow Murder(A)$

6. $Knight(C) \rightarrow Knave(B)$

12. $Knight(C)$	假设
13. $Knave(B)$	(6)(12) 假言推理
14. $Murder(A)$	(5)(13) 假言推理
15. $Murder(A) \wedge \neg Murder(A)$	(8)(14) 合取律
16. $\neg Knight(C)$	(12)(15) 矛盾规则

1. $\forall x\ (Murder(x) \leftrightarrow Knight(x))$

2. $\exists x\ (Murder(x)) \wedge \exists x\ (Knight(x))$

3. $Knight(A) \rightarrow \neg Murder(A)$

4. $Knight(B) \rightarrow \neg Murder(A)$

5. $Knave(B) \rightarrow Murder(A)$

6. $Knight(C) \rightarrow Knave(B) \vee Knight(B)$

11. $\neg Knight(A)$

16. $\neg Knight(C)$

17. $Knight(B) \vee Knight(C)$ (2) (11) 析取三段论

18. $Knight(B)$ (2) (16) 析取三段论

19. $Murder(B)$ (1)(18)

所以Brenda是murder