Data Mining

CS57300
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- Regression
- Posteriors
- Working with Data

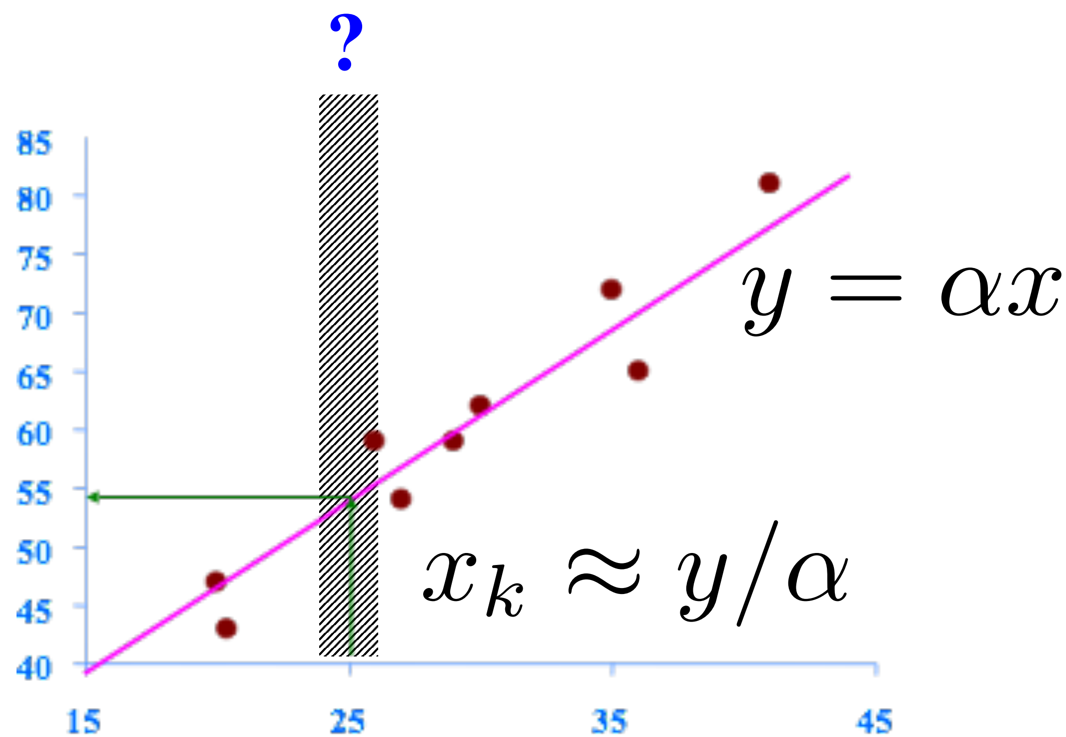
Linear Regression: Review

Linear Regression (use A)

- Interpolation
  (something is missing)

- $(x_1, \ldots, x_t)$

- $(y_1, \ldots, y_t)$



Auto-regression: Predicting Next Value After t Steps Linear Regression (use B)

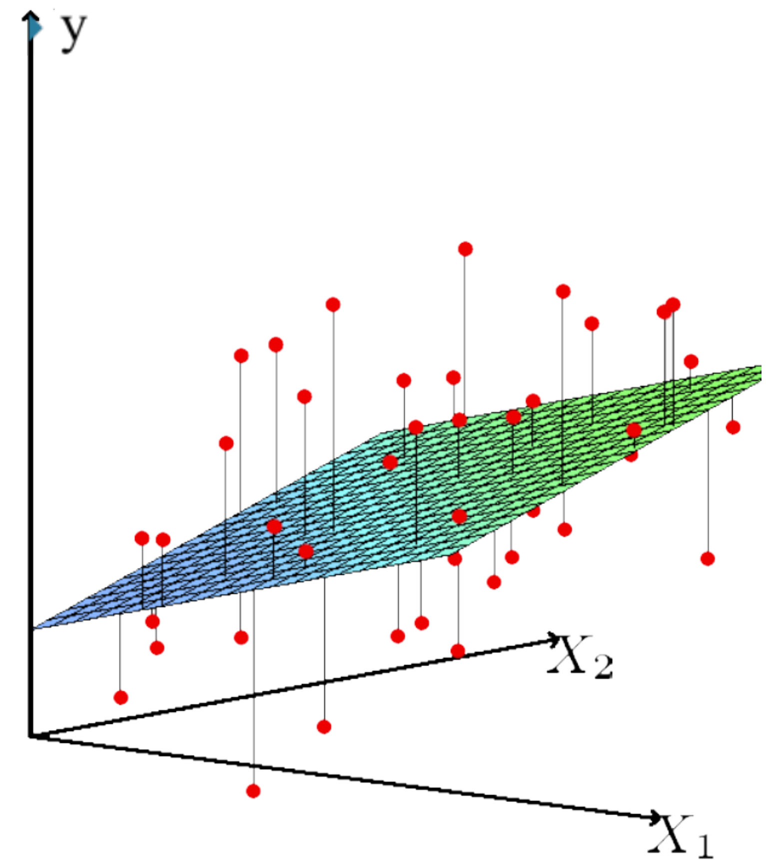
Google Finance

$$x_{t+1} = \sum_{i=t-w}^{t} a_i x_i + \epsilon_{\text{noise}}$$

Similar problem to linear regression:
express unknowns as a linear function of knowns

Predictions from High-Dimensional Historical Data

$$\mathbf{y}_{[t \times 1]} = \mathbf{X}_{[t \times w]} \mathbf{a}_{[w \times 1]}$$



- Over-constrained problem
  - $\mathbf{a}$ is the vector of the regression coefficients
  - $\mathbf{X}$ has the *t* values of the *w* independent variables. These independent variables can mix user characteristics with a window of past observations
  - $\mathbf{y}$ has the *t* values of the dependent variable

Ack: C. Faloutosos

Looking Into Multiplication

may want to add social media variables

$$\mathbf{y}_{[t\times 1]} = \mathbf{X}_{[t\times w]}\mathbf{a}_{[w\times 1]}$$

Predicting corn prices over time…

time

$$\begin{bmatrix} X_{11}, X_{12}, \ldots, X_{1w} \\ X_{21}, X_{22}, \ldots, X_{2w} \\ \vdots \\ \vdots \\ \vdots \\ X_{t1}, X_{t2}, \ldots, X_{tw} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_w \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_t \end{bmatrix}$$

How to Estimate a?

- $\mathbf{a} = (\mathbf{X}^{\mathrm{T}} \cdot \mathbf{X})^{-1} \cdot (\mathbf{X}^{\mathrm{T}} \cdot \mathbf{y})$

$\mathbf{X}^{+} = (\mathbf{X}^{\mathrm{T}} \cdot \mathbf{X})^{-1} \cdot \mathbf{X}^{\mathrm{T}}$ is the Moore–Penrose pseudoinverse

Or: $\mathbf{a} = \mathbf{X}^{+}\, \mathrm{y}$

$\mathbf{a}$ is the vector that minimizes the
Root Mean Squared Error (RMSE) of $(\mathbf{y} - \mathbf{X} \cdot \mathbf{a}^{\mathrm{T}})$

Details: Least Squares Optimization

- Least squares cost function:

$$C = \frac{1}{2}\sum_{i=1}^{t}(\mathbf{y}_i - \mathbf{x}_i^T\mathbf{a})^2 = \frac{1}{2}(y - \mathbf{X}a)^T(y - \mathbf{X}a)$$

$$\underbrace{\begin{bmatrix} X_{11}, X_{12}, \ldots, X_{1w} \\ X_{21}, X_{22}, \ldots, X_{2w} \\ \vdots \\ \vdots \\ \vdots \\ X_{t1}, X_{t2}, \ldots, X_{tw} \end{bmatrix}}_{\mathbf{X}} \times \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_w \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_t \end{bmatrix}}_{\mathbf{y}}$$

- Find **a** that minimizes cost C

$$\frac{\partial C}{\partial \mathbf{a}} = \frac{1}{2}\frac{\partial}{\partial \mathbf{a}}(\mathbf{y} - \mathbf{X}\mathbf{a})^T(\mathbf{y} - \mathbf{X}\mathbf{a})$$
$$= -(\mathbf{y} - \mathbf{X}\mathbf{a})^T\mathbf{X}$$

- Optimal value at:

$$\frac{\partial C}{\partial \mathbf{a}} = 0 \implies \mathbf{X}^T\mathbf{y} = \mathbf{X}^T\mathbf{X}\mathbf{a} \implies \mathbf{a} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

How to Estimate a?

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Or: $\mathbf{a} = \mathbf{X}^+ \mathrm{y}$

a is the vector that minimizes the
Root Mean Squared Error (RMSE) of $(\mathrm{y} - \mathrm{X} \cdot \mathrm{a}^T)$

Problems:

Matrix X grows over time & needs matrix inversion

- $O(t \cdot w^2)$ computation
- $O(t \cdot w)$ storage

Recursive Least Squares

At time t we know $\mathbf{X}_t = (x_1, \ldots, x_t)$, $\quad \mathbf{y}_t = (y_1, \ldots, y_t)$

Least squares is solving

$$\underset{\mathbf{a}^\star}{\operatorname{argmax}} \|\mathbf{a}^T \mathbf{X}_t - \mathbf{y}_t\|^2$$

which gives

$$\mathbf{a}^\star = \mathbf{X}^+ \mathbf{y}$$

where $\mathbf{X}^+ = (\mathbf{X}^{\mathrm{T}} \cdot \mathbf{X})^{-1} \cdot \mathbf{X}^{\mathrm{T}}$

Let

$$\mathbf{\Phi}_t = \mathbf{X}_t^T \mathbf{X}_t \qquad \theta_t = \mathbf{X}_t^T y_t$$

Then $\mathbf{\Phi}_{t+1}^{-1}$ is

$$\mathbf{\Phi}_{t+1}^{-1} = (\mathbf{\Phi}_t + \mathbf{x}_{t+1}\mathbf{x}_{t+1}^T)^{-1} = \mathbf{\Phi}_t^{-1} - \frac{\mathbf{\Phi}_t^{-1}\mathbf{x}_{t+1}\mathbf{x}_{t+1}^T\mathbf{\Phi}_t^{-1}}{1 + \mathbf{x}_{t+1}^T\mathbf{\Phi}_t^{-1}\mathbf{x}_{t+1}}$$

Recursive Least Squares Algorithm

$$\mathbf{\Phi}_{t+1}^{-1} = \mathbf{\Phi}_t^{-1} - \frac{\mathbf{\Phi}_t^{-1}\mathbf{x}_{t+1}^T\mathbf{x}_{t+1}^T\mathbf{\Phi}_t^{-1}}{1 + \mathbf{x}_{t+1}^T\mathbf{\Phi}_t^{-1}\mathbf{x}(t+1)}$$

$$\theta_{t+1} = \theta_t + \mathbf{x}_{t+1}^T\mathbf{y}_{t+1}$$

$$\mathbf{a}_{t+1} = \mathbf{\Phi}_{t+1}^{-1}\theta_{t+1}$$

Exponentially Weighted Recursive Least Squares Algorithm

for $\lambda > 1$

$$\mathbf{\Phi}_{t+1}^{-1} = \frac{1}{\lambda}\mathbf{\Phi}_t^{-1} - \frac{1}{\lambda^2}\frac{\mathbf{\Phi}_t^{-1}\mathbf{x}_{t+1}^T\mathbf{x}_{t+1}^T\mathbf{\Phi}_t^{-1}}{1 + \mathbf{x}_{t+1}^T\mathbf{\Phi}_t^{-1}\mathbf{x}(t+1)}$$

$$\theta_{t+1} = \lambda\theta_t + \mathbf{x}_{t+1}^T\mathbf{y}_{t+1}$$

$$\mathbf{a}_{t+1} = \mathbf{\Phi}_{t+1}^{-1}\theta_{t+1}$$

Comparison

Original Least Squares

- Needs large matrix (growing in size) $O(t \times w)$
- Costly matrix operation $O(t \times w^2)$

Recursive LS

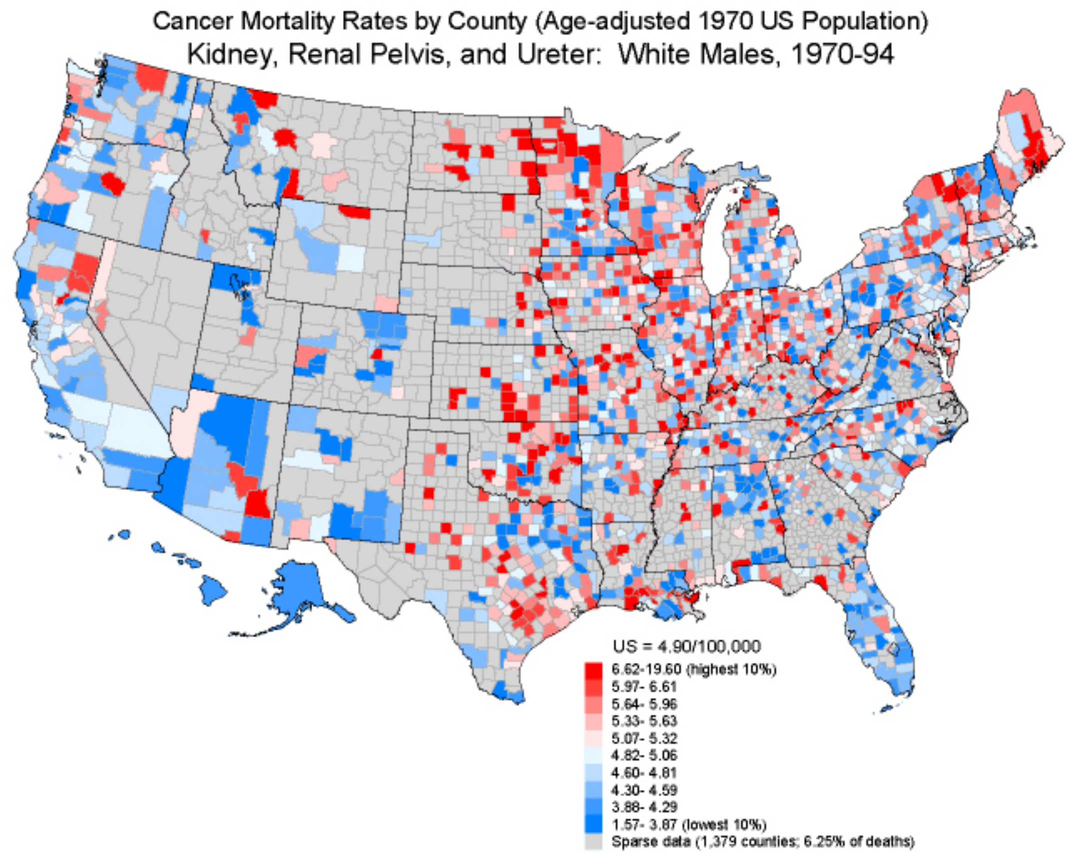
- Need much smaller, fixed size matrix $O(w \times w)$
- Fast, incremental computation $O(1 \text{ x } w^2)$
- no matrix inversion

Ack: Christos Faloutsos

Posteriors

Finding (Real) Patterns in Data

- **Data shows:** rural counties have the highest average mortality rates. But rural counties also have small populations



- Why rural counties have the highest rates of cancer?
  - **A:** Small sample variance
  - Solution?
  - P[cancer rate | data]

Posteriors (e.g. Representing Fractions)

- Consider creating a model that predicts if a soccer striker will score a goal in a game
- Data includes *no. shots on goals* and *no. goals* during the player's career
- Problems with absolute values:
  - number of goals
    (older players have larger values than young players)
  - no. shots on goals
    (does not reflect rate of shot → goal conversion)
- Feature: % shots on goal resulting in goal
  - Alice (Novice): 1 out of 1 → 100%
  - Bob (Senior): 300 out of 1000 → 30%
- Solution? (same problem as in the previous slide (cancer rate). The solution is also Bayesian.

Reaching (Real) Conclusions with Data

- Whatever conclusion using ”cold data”, there are always assumptions
- Example: Simpson’s Paradox (a.k.a. Yule–Simpson effect)
  - At Berkeley, women applicants overall have lower acceptance rate than men
  - But if you look at every department women are accepted at the same rate as men
  - How? What was our wrong assumption?
    - O_i is a random variable that denotes candidate i gets an offer from Berkeley.
    - $O_{i,j}$ is a random variable that denotes candidate i gets an offer from department j at Berkeley.
    - $A_{i,j}$ is a random variable that denotes candidate i has applied to department j.

$$P[O_i] = \sum_j P[O_{i,j}] = \sum_j P[O_{i,j}|A_{i,j}]P[A_{i,j}]$$

Incorrect assumption:
$P[A_{i,j}]$ is the same for men and women

Working with Data

Data Issues

- How to represent the data plays a huge role in the question we can ask and the answers we get
  - It influences similarity metrics
  - It influences the data models we can use
- Data biases (last class)
- Missing data also plays a huge role
- And the problem of outliers in the data
  - Outlier chicken-and-egg problem:
    - Outliers may skew decisions
    - But defining what constitutes an outlier requires deciding a model that describes the “normal” data
    - Deciding such a model requires fitting the data, which may fit the outliers

Missing values

- Reasons for missing values
  - Information is not collected (e.g., people decline to give their age)
  - Attributes may not be applicable to all cases (e.g., annual income is not applicable to children)
- Ways to handle missing values
  - Eliminate entities with missing values
  - Estimate attributes with missing values
  - Ignore the missing values during analysis
  - Replace with all possible values (weighted by their probabilities)
  - Impute missing values

Tan, Steinbach, Kumar. Introduction to Data Mining, 2004.

Duplicate Data

- Data set may include data entities that are duplicates, or almost duplicates of one another
  - Major issue when merging data from heterogeneous sources
  - Example: same person with multiple email addresses
- Sometimes “duplication” happens (different users, same features).
  - Issues with some models.
- Data cleaning
  - Finding and dealing with duplicate entities
  - Finding and correcting measurement error
  - Dealing with missing values